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An Adaptive Controller for Bilateral Teleoperators: Variable Time-Delays Case

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Abstract: It is well known that in most bilateral teleoperation systems, variable time-delays arise in the communications. Motivated by this fact, the present work reports the extension to the (asymmetric) variable time-delays case of the adaptive controller for bilateral teleoperators with uncertain parameters and constant time-delays, reported in Nuño et al (2010) [Nuño, E., Ortega, R. and Basañez, L. (2010). An adaptive controller for nonlinear teleoperators. Automatica, 46(1), 155–159]. Compared to the previous work, the stability analysis does not rely on the *cascade interconnection* structure of the local and remote nonlinear dynamics and the linear interconnection map. Instead, the paper employs a different Lyapunov candidate function that incorporates a strictly positive term, the local and remote position error. With the only assumption that a bound of the time-delays is known, the paper also presents a sufficient condition that ensures the asymptotic convergence of position errors and velocities to zero. Some simulations, in free space and interacting with a stiff wall, illustrate the performance of the proposed control scheme in the presence of variable time–delays.

Keywords: Bilateral teleoperation, adaptive control, time-delays.

1. INTRODUCTION

The control of bilateral teleoperators is a highly active field, that is challenging due to the complexity of their nonlinear dynamics, to the time–delays in the communications as well as to the wide range of practical real-life applications. A major breakthrough to the treatment of this problem has been the use of scattering signals (wave variables) to transform the pure time–delays of the communications into a passive transmission line. Assuming that the human operator and the environment are passive (from force to velocity) and using a damping injection term on the local and the remote manipulators to transform the passive mechanical manipulators into output strictly passive systems, asymptotic convergence to zero of velocities can be ensured (Anderson and Spong, 1989; Niemeyer and Slotine, 1991). For a recent historical survey along this research line the reader may refer to (Hokayem and Spong, 2006) and, for a tutorial on teleoperators control, to (Nuño et al., 2011a).

Position tracking is rarely ensured by scattering–based schemes mostly because, if a position error term is added in the controller, the communications lose their passive behavior, due to the extra energy generated by such error term (Chopra et al., 2006). PD–like schemes that overcome this obstacle, without employing the scattering transformation, have been reported in (Lee and Spong, 2006; Nuño et al., 2008, 2009). Chopra and Spong (2006) proposed to formulate the position tracking problem in terms of synchronization, which also avoids the scattering transformation. An adaptive version of this scheme is proposed in (Chopra et al., 2008b) where the aim is to synchronize the local and remote positions and velocities using a synchronizing signal that is a linear combination of such positions and velocities.

Nuño et al. (2010) reports an adaptive controller, for uncertain bilateral teleoperators with constant time-delays, that is capable of ensuring asymptotic convergence to zero of both, local and remote, position errors and velocities. The main, simple but essential, difference between the controller in (Nuño et al., 2010) and the one in (Chopra et al., 2008b) is the use of a linear combination of the velocity and the position error —instead of the position— in the, so-called, synchronizing signal. This idea has been latter adopted in (Nuño et al., 2011c) to the problem of synchronization and consensus of networks of nonidentical Euler–Lagrange systems (an exception to the constant time-delays is the recent work (Hashemzadeh et al., 2012), where an adaptive controller together with a high-gain sliding term is proposed). Recently, based on the small gain theorem and assuming that the physical parameters are known, (Polushin et al., 2013) proposes a controller for the asymptotical stabilization of a cooperative teleoperation system with variable time-delays.

Motivated by the wide number of applications of bilateral teleoperators and networked robotic systems that exhibit variable time–delays in their communications (Chopra et al., 2008a; Ryu et al., 2010; Secchi et al., 2008; Polushin et al., 2006; Kang et al., 2013), the present paper reports an extension to the asymmetric variable time–delays case of the controller of Nuño et al. (2010). It should be underscored that the proof of the convergence in (Nuño et al., 2010) relies on the analysis of a *cascade interconnection* between the nonlinear dynamics of the local and remote manipulators and a linear map, that accounts for their interconnection and contains some time–delayed terms, with the synchronizing signal as an input. The linear map stability is studied in the frequency domain using the Laplace transform (see (Nuño et al., 2011b)). Clearly, such technique cannot be employed in the case of variable time–delays. Instead, this proposal employs a different Lyapunov function candidate that incorporates a strictly positive term with regards to the local and remote position error. By doing so, the paper derives a sufficient condition for the asymptotic convergence of position errors and velocities to zero. Up to the authors knowledge, this is the first work that deals with variable time-delays using an adaptive controller to estimate the robot physical parameters in a bilateral teleoperation scenario. Finally, some simulations in free space and in contact with a stiff environment are presented to confirm the performance of the proposed approach.

To streamline the presentation, throughout the paper the following *notation* is introduced. Lower case letters denote scalar functions, *e.g.* t, bold lower case letters denote vectors, *e.g.* $\mathbf{x}$, and bold upper case letters denote matrices, *e.g.* $\mathbf{A}$. Moreover, $\mathbf{I}, \mathbf{\emptyset}$ will be the identity and all-zero matrices, respectively, of appropriate dimensions. Additionally, we define $\mathbb{R} := (-\infty, \infty)$, $\mathbb{R}_{>0} := (0, \infty)$, $\mathbb{R}_{\geq 0} := [0, \infty)$. $\lambda_m\{\mathbf{A}\}$ and $\lambda_M\{\mathbf{A}\}$ represent the minimum and maximum eigenvalues of matrix $\mathbf{A}$, respectively, while $||\mathbf{A}||$ denotes the matrix–induced 2-norm. $|\mathbf{x}|$ stands for the standard Euclidean norm of vector $\mathbf{x}$. For any function $\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, the $\mathcal{L}_\infty$-norm is defined as $\|\mathbf{f}\|_\infty := \underset{t\geq 0}{sup}|\mathbf{f}(t)|$, and the $\mathcal{L}_2$-norm as $\|\mathbf{f}\|_2 := (\int_0^\infty |\mathbf{f}(t)|^2 dt)^{\frac{1}{2}}$. The $\mathcal{L}_\infty$ and $\mathcal{L}_2$ spaces are defined as the sets $\{\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n : \|\mathbf{f}\|_\infty < \infty\}$ and $\{\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n : \|\mathbf{f}\|_2 < \infty\}$, respectively.

2. BACKGROUND

This section presents the dynamical model of the nonlinear bilateral teleoperator and the previous adaptive controller.

2.1 Nonlinear Dynamical Model

The local and remote robot manipulators are modeled as a pair of n–Degree Of Freedom (DOF), fully actuated, Euler–Lagrange systems. Their corresponding nonlinear dynamics are given by

$$\begin{aligned}\mathbf{M}_l(\mathbf{q}_l)\ddot{\mathbf{q}}_l + \mathbf{C}_l(\mathbf{q}_l, \dot{\mathbf{q}}_l)\dot{\mathbf{q}}_l + \mathbf{g}_l(\mathbf{q}_l) + \mathbf{d}_l &= \boldsymbol{\tau}_h - \boldsymbol{\tau}_l \\ \mathbf{M}_r(\mathbf{q}_r)\ddot{\mathbf{q}}_r + \mathbf{C}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r)\dot{\mathbf{q}}_r + \mathbf{g}_r(\mathbf{q}_r) + \mathbf{d}_r &= \boldsymbol{\tau}_r - \boldsymbol{\tau}_e,\end{aligned} \quad (1)$$

where $\ddot{\mathbf{q}}_i, \dot{\mathbf{q}}_i, \mathbf{q}_i \in \mathbb{R}^n$ are the acceleration, velocity and joint position, respectively. The mapping $\mathbf{M}_i : \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is the inertia matrix; $\mathbf{C}_i : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times n}$ is the Coriolis and centrifugal effects matrix, defined via the Christoffel symbols of the first kind; $\mathbf{g}_i : \mathbb{R}^n \to \mathbb{R}^n$ is the vector of gravitational forces; $\boldsymbol{\tau}_i \in \mathbb{R}^n$ is the control signal; $\boldsymbol{\tau}_h \in \mathbb{R}^n$, $\boldsymbol{\tau}_e \in \mathbb{R}^n$ are the joint torques corresponding to the forces exerted by the human operator and the environment interaction, respectively, and $\mathbf{d}_i$ is an external disturbance which is assumed unknown but constant. The subscript $i = \{l, r\}$ refers to the local and remote manipulators, respectively.

With regards to the dynamics (1), the following standard assumption is adopted:

A1. The generalized inertia matrix is positive definite and bounded, i.e., $\forall \mathbf{q}$, $m_m^i\mathbf{I} \leq \mathbf{M}_i(\mathbf{q}) \leq m_M^i\mathbf{I}$, where $m_m^i := \lambda_{\rm m}\{\mathbf{M}_i(\mathbf{q})\}$ and $m_M^i := \lambda_{\rm M}\{\mathbf{M}_i(\mathbf{q})\}$.

Further, it is well known that dynamics (1) enjoy the following properties (Kelly et al., 2005; Spong et al., 2005; Nuño et al., 2009):

P1. For all $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{x}^\top[\dot{\mathbf{M}}_i(\mathbf{q}_i) - 2\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)]\mathbf{x} = 0$.

P2. For all $\mathbf{q}, \mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, $\exists k_c^i \in \mathbb{R}_{>0}$ such that $|\mathbf{C}_i(\mathbf{q}, \mathbf{x})\mathbf{y}| \leq k_c^i|\mathbf{x}||\mathbf{y}|$. Hence $|\mathbf{C}_i(\mathbf{q}, \mathbf{x})\mathbf{x}| \leq k_c^i|\mathbf{x}|^2$.

P3. The dynamics are linearly parameterizable. Thus, for all $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$,
$\mathbf{M}_i(\mathbf{q}_i)\mathbf{y}+\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\mathbf{x}+\mathbf{g}_i(\mathbf{q}_i)+\mathbf{d}_i = \mathbf{Y}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i, \mathbf{x}, \mathbf{y})\boldsymbol{\theta}_i$
where $\mathbf{Y}_i : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times p}$ is a map of known functions and $\boldsymbol{\theta}_i \in \mathbb{R}^p$ is the constant vector containing the manipulator physical parameters (link masses, moments of inertia, etc.).

2.2 Previous Approach for Constant Time–Delays

Let $\mathbf{e}_i \in \mathbb{R}^n$ denote the position errors, defined, for a constant time-delay T, by

$$\mathbf{e}_l = \mathbf{q}_l - \mathbf{q}_r(t - T); \quad \mathbf{e}_r = \mathbf{q}_r - \mathbf{q}_l(t - T). \quad (2)$$

The control objective of (Nuño et al., 2010) is to drive the position errors and the velocities to zero independently of the constant time–delay T and without using the scattering transformation. For, the following adaptive controllers are proposed

$$\begin{aligned}\boldsymbol{\tau}_l &= \lambda\hat{\mathbf{M}}_l\dot{\mathbf{e}}_l + \lambda\hat{\mathbf{C}}_l\mathbf{e}_l - \hat{\mathbf{g}}_l - \hat{\mathbf{d}}_l + \mathbf{K}_l\boldsymbol{\epsilon}_l + \mathbf{B}\dot{\mathbf{e}}_l \quad (3)\\ \boldsymbol{\tau}_r &= -\lambda\hat{\mathbf{M}}_r\dot{\mathbf{e}}_r - \lambda\hat{\mathbf{C}}_r\mathbf{e}_r + \hat{\mathbf{g}}_r + \hat{\mathbf{d}}_r - \mathbf{K}_r\boldsymbol{\epsilon}_r - \mathbf{B}\dot{\mathbf{e}}_r,\end{aligned}$$

where, to shorten the equation, the explicit dependence on $\mathbf{q}_i$ and $\dot{\mathbf{q}}_i$ has been withdrawn from $\hat{\mathbf{M}}_i, \hat{\mathbf{C}}_i$ and $\hat{\mathbf{g}}_i$ which are the estimated inertia matrix, Coriolis matrix and gravity vector, respectively, and $\hat{\mathbf{d}}_i$ is the estimated external disturbance. The controller gains $\mathbf{K}_i$ and $\mathbf{B}$ are diagonal and positive definite $n \times n$ matrices. Additionally, the synchronizing signal $\boldsymbol{\epsilon}_i$ is defined as

$$\boldsymbol{\epsilon}_i = \dot{\mathbf{q}}_i + \lambda\mathbf{e}_i, \quad (4)$$

for any $\lambda \in \mathbb{R}_{>0}$. Note that using Property P3, controllers (3) can be written as $\boldsymbol{\tau}_l = \mathbf{Y}_l(\mathbf{q}_l, \dot{\mathbf{q}}_l, \mathbf{e}_l, \dot{\mathbf{e}}_l)\hat{\boldsymbol{\theta}}_l + \mathbf{K}_l\boldsymbol{\epsilon}_l + \mathbf{B}\dot{\mathbf{e}}_l$ and $\boldsymbol{\tau}_r = -\mathbf{Y}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r, \mathbf{e}_r, \dot{\mathbf{e}}_r)\hat{\boldsymbol{\theta}}_r - \mathbf{K}_r\boldsymbol{\epsilon}_r - \mathbf{B}\dot{\mathbf{e}}_r$, and thus $\mathbf{Y}_i\hat{\boldsymbol{\theta}}_i = \lambda\hat{\mathbf{M}}_i\dot{\mathbf{e}}_i + \lambda\hat{\mathbf{C}}_i\mathbf{e}_i - \hat{\mathbf{g}}_i - \hat{\mathbf{d}}_i$.

The dynamics of the estimations of the uncertain parameters $\hat{\boldsymbol{\theta}}_i(t)$ is given by

$$\dot{\hat{\boldsymbol{\theta}}}_i = \boldsymbol{\Gamma}_i\mathbf{Y}_i^\top\boldsymbol{\epsilon}_i, \quad (5)$$

where $\boldsymbol{\Gamma}_i = \boldsymbol{\Gamma}_i^\top \in \mathbb{R}^{p\times p}$ are positive definite matrices.

The following proposition states the convergence claims of the previous result (Nuño et al., 2010).

Proposition 1. (Nuño et al., 2010) Consider the bilateral teleoperator (1) in free motion ($\boldsymbol{\tau}_h = \boldsymbol{\tau}_e = \mathbf{0}$) controlled by (3) and using the parameter update law (5) together with (4). Then, for any constant time-delay T, all signals in the system are bounded. Moreover, position errors and velocities asymptotically converge to zero, *i.e.*, $\lim_{t\to\infty} |\mathbf{e}_i(t)| = \lim_{t\to\infty} |\dot{\mathbf{q}}_i(t)| = 0$. □

The proof of the previous proposition exploited the cascade interconnection structure between the closed–loop teleoperator, (1) and (3), and the linear interconnection map (4). First, using the Lyapunov-Krasovskiĭ candidate function $V = \frac{1}{2} \sum_{i\in\{l,r\}} V_i$ with

$$V_i = \boldsymbol{\epsilon}_i^\top \mathbf{M}_i \boldsymbol{\epsilon}_i + \tilde{\boldsymbol{\theta}}_i^\top \boldsymbol{\Gamma}_i^{-1} \tilde{\boldsymbol{\theta}}_i + \lambda \mathbf{e}_i^\top \mathbf{B} \mathbf{e}_i + \int_{t-T}^{t} \dot{\mathbf{q}}_i^\top \mathbf{B} \dot{\mathbf{q}}_i d\sigma,$$

it has been shown that $\lim_{t\to\infty} |\boldsymbol{\epsilon}_i(t)| = \lim_{t\to\infty} |\dot{\mathbf{e}}_i(t)| = 0$. Then using the matrix representation of (4), together with a proper change of variables and its transformation to the frequency domain —using the Laplace transform [1]— it is proved that $\lim_{t\to\infty} |\mathbf{e}_i(t)| = \lim_{t\to\infty} |\dot{\mathbf{q}}_i(t)| = 0$.

3. THE ADAPTIVE CONTROLLER FOR THE VARIABLE TIME-DELAYS CASE

In this section the paper reports its main contribution, that is, the extension of controller (3) to the variable time–delays. With regards to the time–delays, the following standard assumption is used:

A2. The variable time-delays have known upper bounds ${}^*T_i$, i.e., $0 \leq T_i(t) \leq {}^*T_i < \infty$. Additionally, the time–derivatives $\dot{T}_i(t)$ are bounded.

It should be noted that, contrary to (Nuño et al., 2010), the time–delays can be also asymmetric.

Since delays are now time–varying, the position errors in (2) change to [2]:

$$\mathbf{e}_l = \mathbf{q}_l - \mathbf{q}_r(t - T_r(t)); \quad \mathbf{e}_r = \mathbf{q}_r - \mathbf{q}_l(t - T_l(t)). \quad (6)$$

In this case,

$$\begin{aligned} \dot{\mathbf{e}}_l &= \dot{\mathbf{q}}_l - (1 - \dot{T}_r(t))\dot{\mathbf{q}}_r(t - T_r(t)) \\ \dot{\mathbf{e}}_r &= \dot{\mathbf{q}}_r - (1 - \dot{T}_l(t))\dot{\mathbf{q}}_l(t - T_l(t)). \end{aligned} \quad (7)$$

The local and remote proposed controllers are

$$\begin{aligned} \boldsymbol{\tau}_l &= \lambda \hat{\mathbf{M}}_l(\mathbf{q}_l)\dot{\mathbf{e}}_l + \lambda \hat{\mathbf{C}}_l(\mathbf{q}_l, \dot{\mathbf{q}}_l)\mathbf{e}_l - \hat{\mathbf{g}}_l(\mathbf{q}_l) - \hat{\mathbf{d}}_l + \mathbf{K}_l \boldsymbol{\epsilon}_l \\ \boldsymbol{\tau}_r &= -\lambda \hat{\mathbf{M}}_r(\mathbf{q}_r)\dot{\mathbf{e}}_r - \lambda \hat{\mathbf{C}}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r)\mathbf{e}_r + \hat{\mathbf{g}}_r(\mathbf{q}_r) + \hat{\mathbf{d}}_r - \mathbf{K}_r \boldsymbol{\epsilon}_r. \end{aligned} \quad (8)$$

Compared to (3), these controllers do not contain the term $\mathbf{B}\dot{\mathbf{e}}_i$ and the position and velocity errors are calculated using (6) and (7), respectively.

Invoking Property P3 and using (4), the teleoperator (1) in closed–loop with (8) is given by

$$\begin{aligned} \mathbf{M}_l(\mathbf{q}_l)\dot{\boldsymbol{\epsilon}}_l + \mathbf{C}_l(\mathbf{q}_l, \dot{\mathbf{q}}_l)\boldsymbol{\epsilon}_l + \mathbf{K}_l \boldsymbol{\epsilon}_l &= \mathbf{Y}_l \tilde{\boldsymbol{\theta}}_l + \boldsymbol{\tau}_h \\ \mathbf{M}_r(\mathbf{q}_r)\dot{\boldsymbol{\epsilon}}_r + \mathbf{C}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r)\boldsymbol{\epsilon}_r + \mathbf{K}_r \boldsymbol{\epsilon}_r &= \mathbf{Y}_r \tilde{\boldsymbol{\theta}}_r - \boldsymbol{\tau}_e \end{aligned} \quad (9)$$

where $\tilde{\boldsymbol{\theta}}_i = \boldsymbol{\theta}_i - \hat{\boldsymbol{\theta}}_i$, $\mathbf{Y}_i \hat{\boldsymbol{\theta}}_i = \lambda \hat{\mathbf{M}}_i \dot{\mathbf{e}}_i + \lambda \hat{\mathbf{C}}_i \mathbf{e}_i - \hat{\mathbf{g}}_i - \hat{\mathbf{d}}_i$ and $\mathbf{K}_i = \mathbf{K}_i^\top > 0 \in \mathbb{R}^{n\times n}$.

We are now ready to state the main result of this paper.

Proposition 2. Consider the nonlinear teleoperator (1) in free motion ($\boldsymbol{\tau}_h = \boldsymbol{\tau}_e = \mathbf{0}$) and in closed-loop with the controller (8) and using the parameter estimation law (5). If the controller gain λ is set as

$$1 \geq \lambda^2({}^*T_l + {}^*T_r)^2, \quad (10)$$

then, for any variable time–delay fulfilling Assumption A2, all signals in the system are bounded. Moreover, the local and remote position errors and velocities asymptotically converge to zero, *i.e.*, $\lim_{t\to\infty} |\mathbf{e}_i(t)| = \lim_{t\to\infty} |\dot{\mathbf{q}}_i(t)| = 0$. □

Proof. As usual in adaptive control, let us start by defining $W_i = \frac{1}{2}\boldsymbol{\epsilon}_i^\top \mathbf{M}_i(\mathbf{q}_i)\boldsymbol{\epsilon}_i + \frac{1}{2}\tilde{\boldsymbol{\theta}}_i^\top \boldsymbol{\Gamma}_i^{-1} \tilde{\boldsymbol{\theta}}_i$.

From Assumption A1, W_i is positive definite and radially unbounded with regards to $\boldsymbol{\epsilon}_i$ and $\tilde{\boldsymbol{\theta}}_i$. It can be easily verified, using Property P1, that $\dot{W}_i$ along the closed-loop trajectories (9) is $\dot{W}_i = -\boldsymbol{\epsilon}_i^\top \mathbf{K}_i \boldsymbol{\epsilon}_i$. Since $W_i \geq 0$, $\dot{W}_i \leq 0$ we conclude that $\boldsymbol{\epsilon}_i \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ and $\tilde{\boldsymbol{\theta}}_i \in \mathcal{L}_\infty$.

Now, consider $W = W_l + \frac{k_l}{k_r} W_r + \lambda k_l |\mathbf{q}_l - \mathbf{q}_r|^2$, where $k_i := \lambda_m\{\mathbf{K}_i\}$. Clearly, W is positive definite and radially unbounded with regards to $\boldsymbol{\epsilon}_i, \tilde{\boldsymbol{\theta}}_i$ and $|\mathbf{q}_l - \mathbf{q}_r|$. Its time-derivative is $\dot{W} = \dot{W}_l + \frac{k_l}{k_r}\dot{W}_r + 2\lambda k_l(\mathbf{q}_l - \mathbf{q}_r)^\top(\dot{\mathbf{q}}_l - \dot{\mathbf{q}}_r)$.

Using $\dot{W}_i \leq -k_i|\boldsymbol{\epsilon}_i|^2$ yields

$$\frac{1}{k_l}\dot{W} \leq -|\boldsymbol{\epsilon}_l|^2 - |\boldsymbol{\epsilon}_r|^2 + 2\lambda(\mathbf{q}_l - \mathbf{q}_r)^\top(\dot{\mathbf{q}}_l - \dot{\mathbf{q}}_r).$$

At this point, it is useful to note that

$$\mathbf{q}_i - \mathbf{q}_i(t - T_i(t)) = \int_{t-T_i(t)}^{t} \dot{\mathbf{q}}_i(\sigma)d\sigma \quad (11)$$

Now, using (4) and (11) on $\dot{W}_i$ returns

$$\begin{aligned} \frac{1}{k_l}\dot{W} \leq& -\lambda^2(|\mathbf{e}_l|^2 + |\mathbf{e}_r|^2) - |\dot{\mathbf{q}}_l|^2 - |\dot{\mathbf{q}}_r|^2 - \\ & - 2\lambda\dot{\mathbf{q}}_l^\top \int_{t-T_r(t)}^{t} \dot{\mathbf{q}}_r(\sigma)d\sigma - 2\lambda\dot{\mathbf{q}}_r^\top \int_{t-T_l(t)}^{t} \dot{\mathbf{q}}_l(\sigma)d\sigma \end{aligned}$$

Integrating $\dot{W}_i$, from 0 to t, yields

$$\begin{aligned} \frac{1}{k_l}W(t) - \frac{1}{k_l}W(0) \leq& -\lambda^2(\|\mathbf{e}_l\|_2^2 + \|\mathbf{e}_r\|_2^2) - \|\dot{\mathbf{q}}_l\|_2^2 - \\ & - \|\dot{\mathbf{q}}_r\|_2^2 - 2\lambda \int_0^t \dot{\mathbf{q}}_l^\top(\theta) \int_{\theta-T_r(\theta)}^{\theta} \dot{\mathbf{q}}_r(\sigma)d\sigma d\theta - \\ & - 2\lambda \int_0^t \dot{\mathbf{q}}_r^\top(\theta) \int_{\theta-T_l(\theta)}^{\theta} \dot{\mathbf{q}}_l(\sigma)d\sigma d\theta. \end{aligned}$$

Invoking Lemma 1 of Nuño et al. (2009) to the double integral terms, with α_l and α_r, respectively, yields

$$\frac{1}{k_l}W(t) - \frac{1}{k_l}W(0) \leq -\lambda^2(\|\mathbf{e}_l\|_2^2 + \|\mathbf{e}_r\|_2^2) - \psi_l\|\dot{\mathbf{q}}_l\|_2^2 - \psi_r\|\dot{\mathbf{q}}_r\|_2^2$$

where $\psi_l := 1 - \lambda\alpha_l - \frac{\lambda^* T_l^2}{\alpha_r}$, $\psi_r := 1 - \lambda\alpha_r - \frac{\lambda^* T_r^2}{\alpha_l}$. Solving simultaneously for $\psi_i > 0$ and for $\alpha_i > 0$, it is

[1] Note that, if a signal $\mathbf{x}(t) \in \mathbb{R}^n$ accepts the Laplace transform $\mathfrak{L}$ and $\mathfrak{L}\{\mathbf{x}(t)\} = X(s)$, then $\mathfrak{L}\{\mathbf{x}(t - T)\} = e^{-sT}X(s)$, where s is the Laplace variable.

[2] With some abuse of notation, the rest of the paper uses $\mathbf{e}_i$ for the position errors with variable delays.

straightforward to show that there exist possible solutions if λ is set fulfilling $1 \geq \lambda^2({}^*T_l + {}^*T_r)^2$. If this inequality holds, then there exists $\psi_i > 0$ and thus

$$\frac{1}{k_l}W(t)+\lambda^2(\|\mathbf{e}_l\|_2^2+\|\mathbf{e}_r\|_2^2)+\psi_l\|\dot{\mathbf{q}}_l\|_2^2+\psi_r\|\dot{\mathbf{q}}_r\|_2^2 \leq \frac{1}{k_l}W(0)$$

Clearly $\mathbf{e}_i, \dot{\mathbf{q}}_i \in \mathcal{L}_2$ and $W \in \mathcal{L}_\infty$. This last and the fact that W is radially unbounded with respect to $|\mathbf{q}_l - \mathbf{q}_r|$, shows that $|\mathbf{q}_l - \mathbf{q}_r| \in \mathcal{L}_\infty$.

Further, using (6), $\dot{\mathbf{q}}_i \in \mathcal{L}_2$ and $|\mathbf{q}_l - \mathbf{q}_r| \in \mathcal{L}_\infty$ ensure that $\mathbf{e}_i \in \mathcal{L}_\infty$. Immediately, from (4) with $\boldsymbol{\epsilon}_i, \mathbf{e}_i \in \mathcal{L}_\infty$ it is also shown that $\dot{\mathbf{q}}_i \in \mathcal{L}_\infty$ which in turn —together with Assumption A2— implies that $\dot{\mathbf{e}}_i \in \mathcal{L}_\infty$.

From the closed–loop system (9), Assumption A1, Properties P2 and P3, and all the previous bounded signals it is proved that $\dot{\boldsymbol{\epsilon}}_i \in \mathcal{L}_\infty$. This concludes the boundedness part of the proof.

Finally, the fact that $\dot{\boldsymbol{\epsilon}}_i, \dot{\mathbf{e}}_i \in \mathcal{L}_\infty$ and $\boldsymbol{\epsilon}_i, \mathbf{e}_i \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ ensures, from Barbalăt's lemma, that $\lim_{t\to\infty} \boldsymbol{\epsilon}_i(t) = \lim_{t\to\infty} \mathbf{e}_i(t) = \mathbf{0}$. Since $\boldsymbol{\epsilon}_i = \dot{\mathbf{q}}_i + \lambda\mathbf{e}_i$ it is also concluded that $\lim_{t\to\infty} \dot{\mathbf{q}}_i(t) = \mathbf{0}$. This concludes the proof.

Remark 1. The proposed stability analysis for the *variable* time-delays case is based on simple Lyapunov–like functions and thus is more straightforward than the one appearing in (Nuño et al., 2010). In such work, apart from the fact that it considers only *constant* time-delays, the stability analysis is cumbersome and involves the use of Lyapunov-Krasovskiĭ functionals and frequency domain techniques.

Remark 2. Compared to (Nuño et al., 2010), no additional damping terms, see $-\mathbf{B}\dot{\mathbf{e}}_i$, need to be incorporated to prove the convergence of position errors and velocities to zero.

Remark 3. As it has already been observed in the literature, see for example (Nuño et al., 2011a), it can be readily shown that if the human and environmental input forces are bounded then the velocities and position error are also bounded. In addition, and although not presented here due to space limitations, it can be shown that if the external forces $\boldsymbol{\tau}_i$ are assumed to belong to the $\mathcal{L}_2$ space then $\mathbf{e}_i, \dot{\mathbf{q}}_i \in \mathcal{L}_2$, while if $\boldsymbol{\tau}_i \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ then $\lim_{t\to\infty} |\mathbf{q}_l(t) - \mathbf{q}_r(t)| = \lim_{t\to\infty} |\dot{\mathbf{q}}_i(t)| = 0$.

Remark 4. In order to compute $\dot{T}_i(t)$ at both ends, the value of a new function $f_i(t)$ is sent through the communications together with position and velocity data. Thus, when $f_i(t)$ arrives to its destination it has the value $f_i(t - T_i(t))$. Hence, we can estimate $\dot{T}_i(t)$, indirectly, from $\dot{f}_i(t - T_i(t)) = \dot{f}_i(t)[1 - \dot{T}_i(t)]$. Designing $f_i(t)$ s.t. $\dot{f}_i(t) = 1$, yields $\dot{T}_i(t) = 1 - \dot{f}_i(t - T_i(t))$. Hence, $\dot{T}_i(t)$ can be obtained without knowledge of $T_i(t)$.

4. SIMULATIONS

To show the effectiveness of the proposed scheme, some simulations, in which the local and remote manipulators are modeled as a pair of 2 DOF serial links with revolute joints (cf. Fig. 1), are presented. Their corresponding nonlinear dynamics are modeled by (1). In what follows

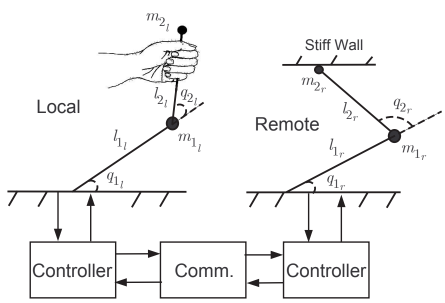

Fig. 1. Simulations scheme.

$\alpha_i := l_{2_i}^2 m_{2_i} + l_{1_i}^2(m_{1_i} + m_{2_i})$, $\beta_i := l_{1_i} l_{2_i} m_{2_i}$ and $\delta_i := l_{2_i}^2 m_{2_i}$. The inertia matrices $\mathbf{M}_i(\mathbf{q}_i)$ are given by

$$\mathbf{M}_i(\mathbf{q}_i) = \begin{bmatrix} \alpha_i + 2\beta_i c_{2_i} & \delta_i + \beta_i c_{2_i} \\ \delta_i + \beta_i c_{2_i} & \delta_i \end{bmatrix}.$$

c_{2_i} is the short notation for $\cos(q_{2_i})$. q_{k_i} is the articular position of link k of manipulator i, with $k \in \{1, 2\}$. The Coriolis and centrifugal effects are modeled by

$$\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) = \begin{bmatrix} -2\beta_i s_{2_i}\dot{q}_{2_i} & -\beta_i s_{2_i}\dot{q}_{2_i} \\ \beta_i s_{2_i}\dot{q}_{1_i} & 0 \end{bmatrix}.$$

s_{2_i} is the short notation for $\sin(q_{2_i})$. $\dot{q}_{1_i}$ and $\dot{q}_{2_i}$ are the respective revolute velocities of the two links. The gravity forces $\mathbf{g}_i(\mathbf{q}_i)$ for each manipulator are represented by

$$\mathbf{g}_i(\mathbf{q}_i) = \begin{bmatrix} \frac{1}{l_{2_i}} g\delta_i c_{12_i} + \frac{1}{l_{1_i}}(\alpha_i - \delta_i)c_{1_i} \\ \frac{1}{l_{2_i}} g\delta_i c_{12_i} \end{bmatrix}.$$

c_{12_i} stands for $\cos(q_{1_i} + q_{2_i})$. l_{k_i} and m_{k_i} are the respective lengths and masses of each link. For simplicity, the external disturbance $\hat{\mathbf{d}}_i$ is set to zero.

The following parametrization $\mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \mathbf{e}, \dot{\mathbf{e}})$ is proposed for both manipulators

$$\mathbf{Y} = \begin{bmatrix} \lambda\dot{e}_1 & \lambda Y_{12} & \lambda\dot{e}_2 & -gc_{12} & -gc_1 \\ 0 & \lambda(c_2\dot{e}_1 + s_2\dot{q}_1 e_1) & \lambda(\dot{e}_1 + \dot{e}_2) & gc_{12} & 0 \end{bmatrix},$$

$$\boldsymbol{\theta} = \begin{bmatrix} \alpha & \beta & \delta & \frac{1}{l_2}\delta & \frac{1}{l_1}(\alpha - \delta) \end{bmatrix}^\top,$$

where $Y_{12} = 2c_2\dot{e}_1 + c_2\dot{e}_2 - s_2\dot{q}_2 e_2 - 2s_2\dot{q}_2 e_1$.

The physical parameters for the manipulators are: the length of links l_{1_i} and l_{2_i}, for both manipulators, is 0.38m; the masses of the links are $m_{1_l} = 1.5$kg, $m_{2_l} = 0.75$kg, $m_{1_r} = 2.5$kg and $m_{2_r} = 1.5$kg.

The initial conditions are $\dot{\mathbf{q}}_i(0) = \mathbf{0}$ and $\mathbf{q}_l^\top(0) = [-1/8\pi; 1/8\pi]$, $\mathbf{q}_r^\top(0) = [1/6\pi; -1/4\pi]$.

For simplicity, in the simulations we set $T_l(t) = T_r(t)$, and ${}^*T_l = {}^*T_r = 0.7$. The variable time–delay and its derivative can be seen in Fig. 2.

The controller gains are set as: $\mathbf{K}_l = 5\mathbf{I}_2$, $\mathbf{K}_r = 15\mathbf{I}_2$ and $\boldsymbol{\Gamma}_l = \boldsymbol{\Gamma}_r = 0.5\mathbf{I}_5$. Further, λ is set fulfilling (10) as $\lambda = 0.7$.

The human operator is modeled as the spring–damper system $\boldsymbol{\tau} = K_h(\mathbf{q}_d - \mathbf{q}_l) - d\dot{\mathbf{q}}_l$ where $K_h = 25$ and $d = 5$. Fig. 3 depicts the desired human position $\mathbf{q}_h$.

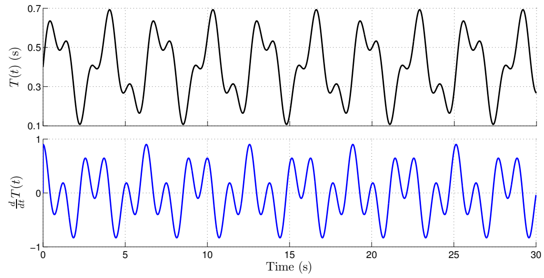

Fig. 2. Variable time-delay employed in the simulations.

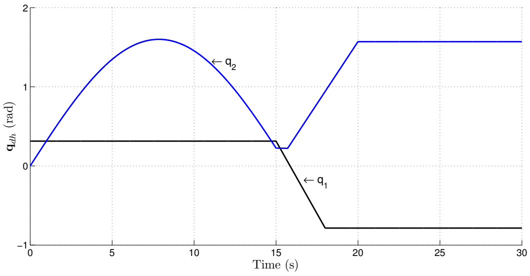

Fig. 3. Desired human position.

4.1 Remote Manipulator in Free Space

In this section, the simulations in which the remote manipulator moves without contact with its environment are presented. In Fig. 4, it can be observed that position tracking between the local and the remote manipulators is established in this case. Moreover, Fig. 5 shows that local and remote velocities asymptotically converge to zero. Finally, Fig. 6 depicts the time evolution of the estimated parameters, which are clearly bounded.

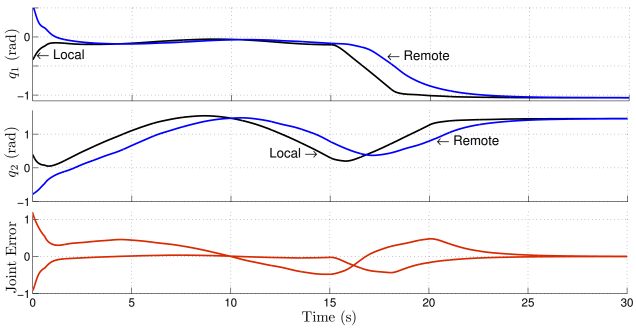

Fig. 4. Joint position an error when the remote manipulator moves freely.

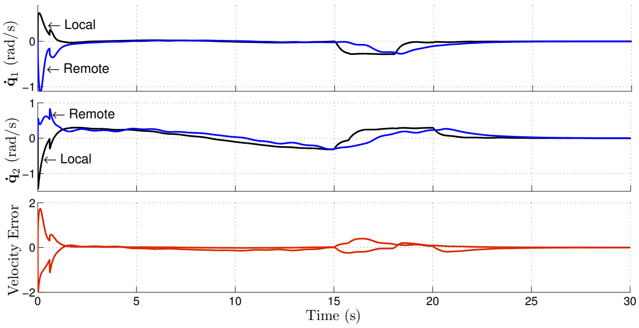

Fig. 5. Joint velocities in free space.

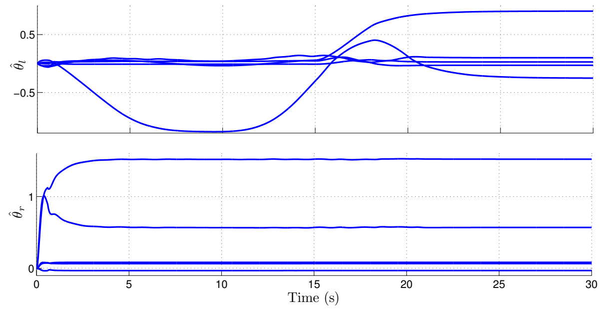

Fig. 6. Dynamic behavior of the estimated parameters.

4.2 Remote Manipulator Interacting with a Stiff Wall

In this set of simulations, a stiff wall is added in the remote environment. The wall is located in the $xz-$plane at $y = 0.3$m. It is modeled as a spring–damper Cartesian system with stiffness equal to 20000Nm and damping equal to 200Nm/s.

For this case, Fig. 7 and Fig. 8 show the position tracking capabilities of the proposed controller in Cartesian space and in joint space, respectively. From these figures it is concluded that, despite variable time–delays and a stiff interaction with the environment, position error converges to zero and hence position tracking is established. Fig. 9 presents the local and remote velocities and Fig. 10 the evolution of the estimated parameters.

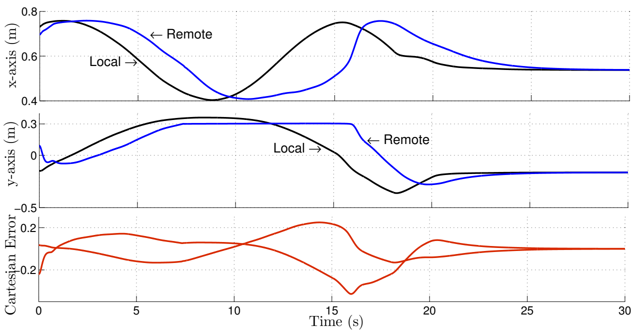

Fig. 7. Cartesian position when interacting with a stiff wall, located at $y_r = 0.3$m.

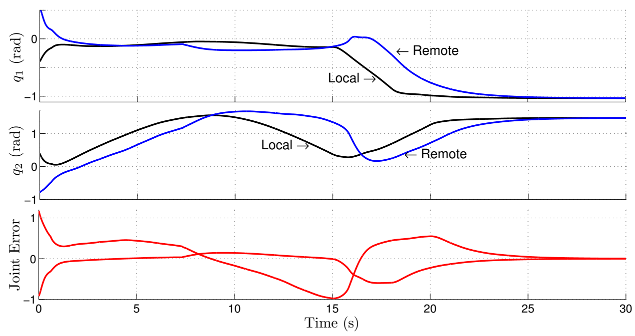

Fig. 8. Joint position and error behavior when interacting with a stiff wall.

5. CONCLUSIONS

This work proposes an adaptive controller for general nonlinear teleoperators with *variable* time–delays. This

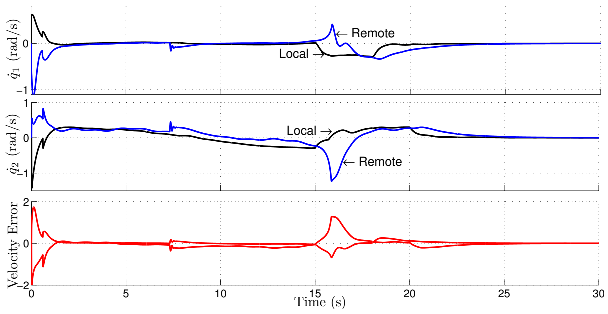

Fig. 9. Joint velocities for the case when interacting with a rigid wall.

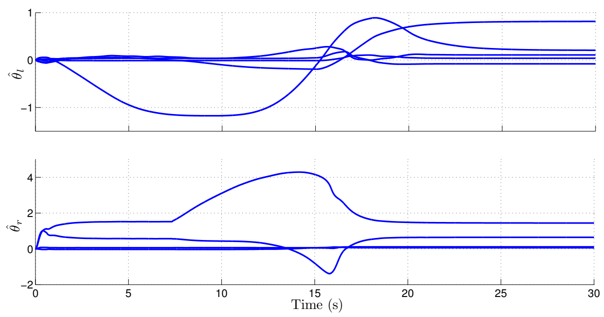

Fig. 10. Time evolution of the estimated parameters $\hat{\boldsymbol{\theta}}_i(t)$

adaptive controller can be seen as an extension of the previous schemes reported in (Nuño et al., 2010). The proposed controller excels that in (Nuño et al., 2010) in two main aspects, the time–delays can be asymmetric and variable and it injects less damping. The reported scheme assures that, in free motion, all signals in the system are bounded and position errors and velocities asymptotically converge to zero. The paper presents some numerical simulations that confirm the theoretical results.

Future work includes the extension of this framework to the more general case of synchronization of networks of Euler–Lagrange systems with variable time-delays in the interconnection.

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