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Model-plant mismatch expression for classically controlled systems

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Abstract: This paper presents a closed-form expression for the model-plant mismatch that may be present in a feedback control system. The main limitation on the expression is that the controller and plant models should be representable by means of transfer functions, i.e. they should be linear and time invariant. This includes a variety of controllers, among which the ubiquitous Proportional, Integral and Derivative (PID) controller. The expression can then be used to identify the true plant transfer function. The MPM expression is shown to work for single-input single-output as well as a multiple-input multiple-output systems.

Keywords: model-based control, model-plant mismatch, PID control, transfer function

1. INTRODUCTION

The situation where only poor process models are available for control is a common one. This is especially true in, but not limited to, the mineral-processing industry where the availability of only poor process models is typical (Hodouin, 2011). For this class of processes Hodouin (2011) states that the peripheral control tools are as important as the controller itself. Peripheral control tools constitute all the elements in the control loop, other than the controller itself, that function to improve controller performance. These include fault detection and isolation, data reconciliation, observers, soft sensors, optimisers and model parameter tuners.

Many controller design methods make use of the plant model that is available. This usually helps to improve controller performance, but the dynamics of industrial processes can change significantly over time (as is shown for the example of a milling circuit in Olivier and Craig (2013)). As soon as the plant dynamics change, model-plant mismatch (MPM) is present and the controller designed based on the original model will produce sub-optimal control moves. Examples of the source of a change in plant dynamics are maintenance on equipment or a change in operating conditions/parameters. In order to restore the controller performance the process needs to be re-identified and the controller redesigned, which is a costly and time-consuming exercise (Conner and Seborg, 2005). Apart from the formerly mentioned problems, process re-identification also involves intrusive plant tests that disturb the normal operation of the plant (Badwe et al., 2009).

An alternative to full process re-identification, is to firstly identify the elements in the process transfer function matrix that contain significant mismatch and to only re-identify these. Algorithms for MPM detection have been proposed by Badwe et al. (2009) and Kano et al. (2010). These algorithms identify the transfer function matrix elements that contain mismatch as well as the significance of the mismatch. This is useful information that can be used to help assess the need for process re-identification. These algorithms do however not supply any additional information about the true plant. This still leads to the need for process re-identification (although not as expensive as full process re-identification) and ad-hoc controller re-tuning.

Model identification techniques making use of closed-loop data have been introduced some time ago (see for example Gustavsson et al. (1977) and Söderström and Stoica (1989)). A good overview of closed-loop identification is given by Van den Hof (1998). In this article different closed-loop identification techniques are discussed and their characteristic properties are compared. The methods described by Van den Hof (1998) are mostly based on statistical approaches and do not make explicit use of the transfer functions representing the system, unlike the method presented in this article. A more recent approach to on-line closed loop identification is given in Pingkang et al. (2006).

This paper presents, as its main objective, a closed-form expression for the model-plant mismatch, which can be used to update the model to be the same as the actual plant. Although this method is related to closed-loop identification, it does make use of the explicit expression for the mismatch to identify the true plant. This implies that the model structure is known *a priori* and can simply be updated through the mismatch expression.

The newly identified model may then be used to update the controller, such that it can perform in an optimal

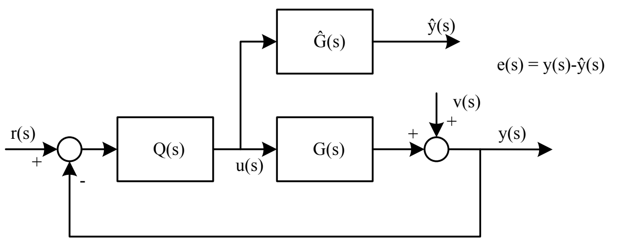

Fig. 1. Block diagram of a control loop with model outputs being generated.

manner. The expression is however only valid for systems that contain a controller and plant model that can be expressed by means of transfer functions. This does include an array of controllers, but probably most importantly it includes PID controllers.

PID control is still very predominant in the mineral-processing industry (as it is in other industries as well). An industrial survey on grinding mill circuits by Wei and Craig (2009) found that more than 60% of the respondents make use of PID control, which implies a large scope for implementation of the presented expression.

Another limitation on the expression is that it requires the input signals to be sufficiently exciting in order to make the implementation sensible. This limitation is however also present for the MPM detection algorithms presented by Badwe et al. (2009) and Kano et al. (2010).

Identifying the mismatch in the manner proposed in this paper is equivalent to identifying the uncertainty in the model (Skogestad and Postlethwaite, 2005). Another example of how this may be done is presented by Böling et al. (2004).

The paper firstly gives the derivation of the MPM expression and shows how the true plant transfer function may be obtained from it. Thereafter the expression is used in a single-input single-output (SISO) application example to show its usefulness. Next some provisions for multiple-input multiple-output (MIMO) applications are presented before a MIMO application example is shown.

2. MODEL-PLANT MISMATCH EXPRESSION

Consider the control loop shown in Fig. 1 in which all signals and transfer functions are represented in the Laplace domain. G is the plant that generates the true output $y(s)$, $\hat{G}$ is the model of the plant that generates the model output $\hat{y}(s)$, Q is the controller, $v(s)$ is any disturbance that may be present and $r(s)$ is the reference signal (set-point).

The derivation of the MPM expression which follows is done for a general, internally stable (Skogestad and Postlethwaite, 2005), MIMO system in which all signals may be vectors and all transfer functions may be matrices. The reference to the Laplace operator (s) will be dropped for ease of representation. Let the residual ($e(s)$) be the difference between the actual output and the model output as

$$e = y - \hat{y}, \tag{1}$$
$$e = Gu + v - \hat{G}u, \tag{2}$$
$$e = \Delta u + v, \tag{3}$$

where $\Delta = G - \hat{G}$ is the mismatch. This definition for the mismatch is equivalent to the definition for additive uncertainty presented by (Skogestad and Postlethwaite, 2005, p.293). For notational simplicity during the derivation however we will use Δ to represent uncertainty in general, as opposed to weighted uncertainty. The control signal ($u(s)$) is given by

$$u = Q\left(r - y\right), \tag{4}$$
$$u = Q\left(r - [Gu + v]\right), \tag{5}$$
$$u = Qr - QGu - Qv, \tag{6}$$
$$\left(I + QG\right)u = Qr - Qv, \tag{7}$$
$$u = \left(I + QG\right)^{-1} Q\left(r - v\right), \tag{8}$$
$$u = Q\left(I + GQ\right)^{-1}\left(r - v\right), \tag{9}$$

where the push-through rule for matrix manipulation (Skogestad and Postlethwaite, 2005, p.68) was used to go from (8) to (9). Substitution of (9) into (3) then gives

$$e = \Delta Q\left(I + GQ\right)^{-1}\left(r - v\right) + v, \tag{10}$$
$$e = \Delta Q\left(I + \{\Delta + \hat{G}\}Q\right)^{-1}\left(r - v\right) + v, \tag{11}$$
$$e = \Delta Q\left(I + \Delta Q + \hat{G}Q\right)^{-1}\left(r - v\right) + v. \tag{12}$$

The expression $G = \Delta + \hat{G}$ is used to go from (10) to (11). After this substitution all the terms in (11) are known, save for the disturbance if it is unmeasured. Further matrix algebra leads to

$$\left(e - v\right)\left(r - v\right)^{-1} = \Delta Q\left(I + \Delta Q + \hat{G}Q\right)^{-1}, \tag{13}$$
$$\left(e - v\right)\left(r - v\right)^{-1}\left(I + \Delta Q + \hat{G}Q\right) = \Delta Q, \tag{14}$$
$$\begin{aligned}\left(e - v\right)\left(r - v\right)^{-1}\left(I + \hat{G}Q\right) &= \Delta Q \\ &- \left(e - v\right)\left(r - v\right)^{-1}\Delta Q,\end{aligned} \tag{15}$$
$$\begin{aligned}\left(e - v\right)\left(r - v\right)^{-1}\left(I + \hat{G}Q\right) &= \\ \left[I - \left(e - v\right)\left(r - v\right)^{-1}\right]\Delta Q.\end{aligned} \tag{16}$$

Rewriting the equation with Δ isolated on the left-hand side gives the closed-form mismatch expression as:

$$\begin{aligned}\Delta = \left[I - \left(e - v\right)\left(r - v\right)^{-1}\right]^{-1} \cdot \\ \left(e - v\right)\left(r - v\right)^{-1}\left(I + \hat{G}Q\right)Q^{-1}.\end{aligned} \tag{17}$$

This expression may be used to derive the mismatch if the disturbances are known. If the disturbances are however unmeasured, data from a period of operation free from significant disturbances can be used (if this is possible), and with $v = 0$, (17) becomes

$$\Delta = \left[I - er^{-1}\right]^{-1} er^{-1}\left(I + \hat{G}Q\right)Q^{-1}. \tag{18}$$

If however unmeasured disturbances cannot be ignored, disturbance estimation techniques (see for example Lee and Ricker (1994)) may be used to account for their values.

Usually signals (such as $r(s)$) will not be square for MIMO systems and will consequently not have an inverse in the true sense. This issue will be discussed further in Section 4 before we present the MIMO application example. Sufficient excitation is required in either the disturbance or the reference signal in order for the application of (17) to be sensible.

The expression $G = \Delta + \hat{G}$ may now again be used to obtain the transfer function of the actual plant as

$$G = \left[I - (e - v)\,(r - v)^{-1}\right]^{-1} \cdot \\ (e - v)\,(r - v)^{-1}\left(I + \hat{G}Q\right)Q^{-1} + \hat{G}. \tag{19}$$

If (18) was used as the mismatch expression, the plant transfer function is given by

$$G = \left[I - er^{-1}\right]^{-1} er^{-1}\left(I + \hat{G}Q\right)Q^{-1} + \hat{G}. \tag{20}$$

3. SISO APPLICATION EXAMPLE

The derived MPM expression is firstly applied to a simple SISO system to illustrate its use. Consider the first order plus time delay model

$$G = \frac{K}{\tau s + 1}e^{-\theta s}, \tag{21}$$

which is arbitrarily specified as the model to be

$$\hat{G} = \frac{2}{4s + 1}e^{-s}, \tag{22}$$

for illustration of this example. The controller is derived based on the simple internal model control (SIMC) tuning rules given by Skogestad (2003) which gives a PI-controller with a gain and integral time constant of

$$K_c = \frac{1}{K}\frac{\tau}{\tau_c + \theta};\ \tau_I = \tau, \tag{23}$$

where τ_c is the desired closed-loop bandwidth. Skogestad (2003) recommends selecting $\tau_c = \theta$ to achieve a good trade-off between output performance and robustness. This results in the PI controller with $K_c = 1$ and $\tau_I = 4$. The gain is however reduced to $K_c = 0.7$ such that robust performance is achieved for 10% uncertainty in all 3 plant parameters. See Skogestad and Postlethwaite (2005) for a complete discussion on how such a robust performance analysis may be performed. * The plant is then perturbed with a 15% increase in the gain, time constant, and time delay to be

$$G = \frac{2.3}{4.6s + 1}e^{-1.15s}. \tag{24}$$

These perturbations are less severe than what may be present on an industrial process (Olivier et al., 2012) but severe enough to violate the robust performance requirement. In this article perturbations that cause the controller to violate the robust performance requirement are regarded as severe. In this situation where robust performance is no longer achieved, the uncertainty and consequently the true plant model should be calculated such that the controller can be updated. The nominal and perturbed step-responses of the system are shown in Fig. 2.

* In this analysis we used $w_I = 0.1\frac{16s+1}{0.7s+1}$ and $w_P = 0.2\frac{10s+1}{10s}$ according to the discussion in Skogestad and Postlethwaite (2005). The performance weight specifies integral action and a closed-loop bandwidth of 0.02.

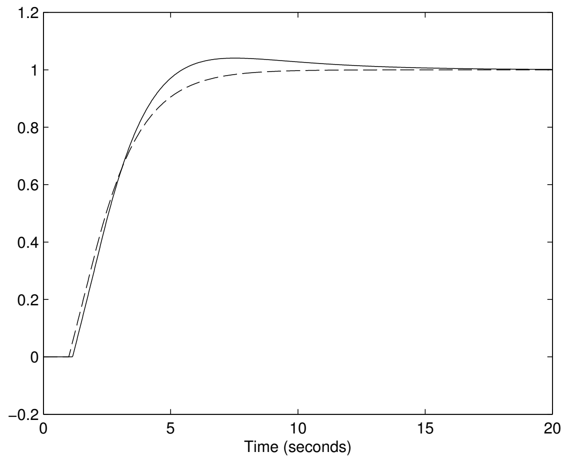

Fig. 2. Nominal (dashed line) and perturbed (solid line) step responses for the first order SISO system.

For this simple system the step function set-point is given in the Laplace domain by $r = \frac{1}{s}$. $\hat{G}$ and Q are known and e is simply the difference between the actual output and the model output (see equation 1). Equation (18) is then applied to produce the mismatch as

$$\Delta = \frac{4.6\,s - \mathrm{e}^{0.15s} - 4.6\,s\,\mathrm{e}^{0.15s} + 1.15}{\mathrm{e}^{1.15s}\,(9.2\,s^2 + 4.3\,s + 0.5)}, \tag{25}$$

from which the true plant may be calculated as

$$G = \Delta + \hat{G}, \tag{26}$$

$$= \frac{2.3}{4.6s + 1}e^{-1.15s}, \tag{27}$$

which is exactly the same as the true plant given in (24).

4. MIMO APPLICATION PROVISIONS

It was stated in Section 2 that signals such as $r(s)$ are usually not square, which is a problem for MIMO applications. This is because a non-square matrix does not have an inverse in the traditional sense. Say for example we have an $n \times 1$ output ($y(s)$) that is generated by applying an $n \times 1$ input signal ($u(s)$) to an $n \times n$ plant ($G(s)$) as

$$y = Gu, \tag{28}$$

$$\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} g_{11} & \cdots & g_{1n} \\ \vdots & \ddots & \vdots \\ g_{n1} & \cdots & g_{nn} \end{bmatrix} \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}, \tag{29}$$

from which $y(s)$ is calculated to be

$$\begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} g_{11}u_1 + \cdots + g_{1n}u_n \\ \vdots \\ g_{n1}u_1 + \cdots + g_{nn}u_n \end{bmatrix}. \tag{30}$$

What the method is then basically doing is to try and determine the transfer function by inverting the input signal as

$$G = yu^{-1}. \tag{31}$$

In the SISO case this is not a problem as both $y(s)$ and $u(s)$ are scalars. In the MIMO case however the expression cannot be applied as such because the non-square signal

$u(s)$ does not have an inverse. If however the input signal is rewritten as the diagonal matrix

$$U = \begin{bmatrix} u_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & u_n \end{bmatrix}, \tag{32}$$

the output becomes

$$Y = \begin{bmatrix} g_{11}u_1 & \cdots & g_{1n}u_n \\ \vdots & \ddots & \vdots \\ g_{n1}u_1 & \cdots & g_{nn}u_n \end{bmatrix}. \tag{33}$$

Now U is square and does have a matrix inverse. Applying equation (31) now gives

$$G = YU^{-1}, \tag{34}$$

$$G = \begin{bmatrix} g_{11}u_1 & \cdots & g_{1n}u_n \\ \vdots & \ddots & \vdots \\ g_{n1}u_1 & \cdots & g_{nn}u_n \end{bmatrix} \begin{bmatrix} u_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & u_2 \end{bmatrix}^{-1}, \tag{35}$$

$$= \begin{bmatrix} g_{11} & \cdots & g_{1n} \\ \vdots & \ddots & \vdots \\ g_{n1} & \cdots & g_{nn} \end{bmatrix}, \tag{36}$$

which is equal to the original transfer function.

The input signal can easily be written in the form of a square matrix as in (32). The output is however not usually available as a square matrix. It is however apparent that the first entry of (33) is equal to the first output in (30) if $u_2 \cdots u_n = 0$. This means that a portion of the output signal generated without excitation in $u_2 \cdots u_n$ can be used to calculate the first entry of (33). The same argument holds for the calculation of the other entries of (33).

A similar situation holds true for measured disturbances. If disturbances are however unmeasured, care would need to be taken to use a portion of data that is disturbance free as unmeasured disturbances are not handled by the expression.

5. MIMO APPLICATION EXAMPLE

In order to illustrate the working of the MPM expression in the MIMO case, the algorithm is applied to a 2×2 ball mill grinding circuit for which MPM is introduced. Consider the ball mill grinding circuit of Fig. 3 which is described in Chen et al. (2009).

The manipulated variables are the fresh ore feed rate [u_1 (t/h)] and the dilution water flow rate [u_2 (m^3/h)]. The controlled variables are the product particle size [y_1 ($\% - 200$ mesh)] and the circulating load [y_2 (t/h)]. The nominal values and constraints for the manipulated and controlled variables are given in Table 1. Care should be taken when using the method to not use data where the output or control variable values are saturated against the limits. This is because the saturation function is not linear and therefore not compatible with the MPM expression.

The MIMO transfer function model of the milling circuit is given by

$$\begin{bmatrix} y_1(s) \\ y_2(s) \end{bmatrix} = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix} \begin{bmatrix} u_1(s) \\ u_2(s) \end{bmatrix}, \tag{37}$$

where

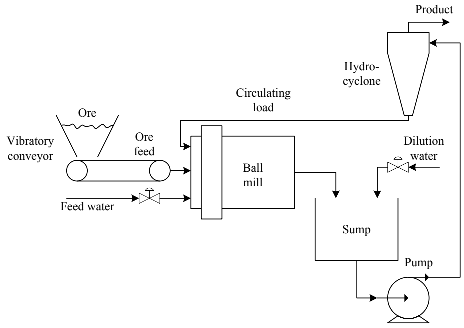

Fig. 3. Ball mill grinding (reproduced from Chen et al. (2009)).

Table 1. Nominal values and constraints for the 2x2 ball mill grinding circuit variables

Var.	Description	Nom	Min	Max	Unit
u_1	Fresh ore feed rate	65	60	70	t/h
u_2	Dilution water flow rate	45	40	50	m^3/h
y_1	Product particle size	70	68	72	%
y_2	Circulating load	150	140	170	t/h

$$g_{11}(s) = \frac{-0.58}{2.5s+1}e^{-0.68s}, \tag{38}$$

$$g_{12}(s) = \frac{4(1-0.9938e^{-0.47s})}{(2s+1)(6s+1)}e^{-0.2s}, \tag{39}$$

$$g_{21}(s) = \frac{2.2}{6s+1}e^{-0.6s}, \tag{40}$$

$$g_{22}(s) = \frac{2.83}{3.5s+1}e^{-0.13s}. \tag{41}$$

Milling circuits are often controlled by decentralized PI(D) controllers (Wei and Craig, 2009; Hodouin, 2011) as was also implemented for this circuit by Chen et al. (2009). The diagonal PI controller is in the form

$$Q(s) = \begin{bmatrix} K_{c_1}\left(1 + \frac{1}{\tau_{I_1}s}\right) & 0 \\ 0 & K_{c_2}\left(1 + \frac{1}{\tau_{I_2}s}\right) \end{bmatrix}, \tag{42}$$

with $K_{c_1} = -2$, $\tau_{I_1} = 3.3$ min, $K_{c_2} = 0.42$ and $\tau_{I_2} = 5.2$ min. A test for robust performance ** on this system shows that with 10% gain uncertainty the performance specification is achieved.

The plant is perturbed to be

$$g_{11}(s) = \frac{-0.464}{2s+1}e^{-0.68s}, \tag{43}$$

$$g_{12}(s) = \frac{4(1-1.1014e^{-0.47s})}{(2s+1)(6s+1)}e^{-0.2s}, \tag{44}$$

$$g_{21}(s) = \frac{2.2}{6.6s+1}e^{-0.6s}, \tag{45}$$

$$g_{22}(s) = \frac{2.547}{3.5s+1}e^{-0.13s}, \tag{46}$$

**With $W_I = \frac{0.21s+0.1}{0.1s+1}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $W_P = \frac{0.45s+0.05}{s}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ selected according to the discussion in Skogestad and Postlethwaite (2005). The performance weight specifies integral action and a closed-loop bandwidth of 0.05.

which is less severe than the mismatch introduced into the system by Chen et al. (2009) but more severe than allowed by the robust performance analysis weight. Now robust performance is not achieved and the uncertainty (and also the changed plant model) should be calculated. The nominal and perturbed responses for a step in the particle size set-point are shown in Fig. 4 and Fig. 5. The nominal and perturbed responses for a step in the circulating load set-point are shown in Fig. 6 and Fig. 7.

There is a much bigger difference between the nominal and perturbed responses for the particle size than for the circulating load. Once the output signals have been generated the mismatch can be identified. The mismatch is given by

$$\Delta = \begin{pmatrix} \dfrac{0.0232\mathrm{e}^{-0.68s}}{s^2+0.9\,s+0.2} & \dfrac{-0.0066\mathrm{e}^{-0.67\,s}}{s^2+0.667\,s+0.0833} \\ \dfrac{-0.0333\mathrm{e}^{-0.6s}\,s}{s^2+0.3182\,s+0.0253} & \dfrac{-0.0809\mathrm{e}^{-0.13s}}{s+0.2857} \end{pmatrix}, \tag{47}$$

from which the actual transfer function is calculated to be exactly the same as the original transfer function as given in (38) - (41).

6. APPLICATION TO CONTROLLER DESIGN

Once the mismatch has been identified correctly equation (26) may be used to obtain the true transfer function of the plant. Seeing that the tuning of the controller is based on the plant model, it is now possible to update the controller tuning based on the newly obtained plant model. Consider the SISO application example where the perturbed plant transfer function was calculated as shown in (27). It is now possible to redefine the controller based on the tuning relation given by (23). Applying this equation gives the controller with $K_c = 0.909$ and $\tau_I = 4.4$. Again only 70% of this gain is used, resulting in the controller

$$Q = 0.6364 \cdot \left(1 + \frac{1}{4.4s}\right). \tag{48}$$

This controller is less aggressive owing to the smaller gain and larger integral time constant. Considering Fig. 2 it is clear that the perturbed performance can be made similar to the nominal control performance by using a less aggressive controller, in line with the initial tuning objective. The test for robust performance is again performed for the perturbed plant and the returned controller. It is found that this system does achieve robust performance with 10% uncertainty in all 3 plant parameters. This controller retuning method is also applicable to the MIMO case.

One would not apply this controller retuning method unsupervised. It is suggested that controller retuning be done manually if significant mismatch is detected. Here significant mismatch implies that the robust performance specification is violated. This will be less expensive and less time consuming than full re-identification.

7. CONCLUSION

This paper presents a closed-form expression for the MPM that may be present in a feedback control system where the controller is representable by means of a transfer function. The expression may be used to identify an accurate plant transfer function. The expression is directly applicable for SISO systems where the plant is easily identified. In the MIMO case some provisions are needed to ensure correct results. The plant model was correctly identified in both the SISO and MIMO cases. The plant transfer function can then be used to redefine the controller. The expression does need sufficiently exciting signals to make its application sensible.

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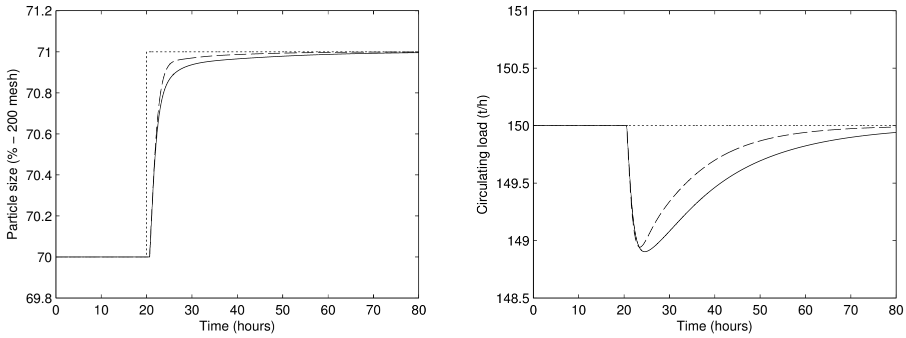

Fig. 4. Response of controlled variables for a step in the particle size showing the set-point (dotted line), the nominal response (dashed line) and the perturbed response (solid line).

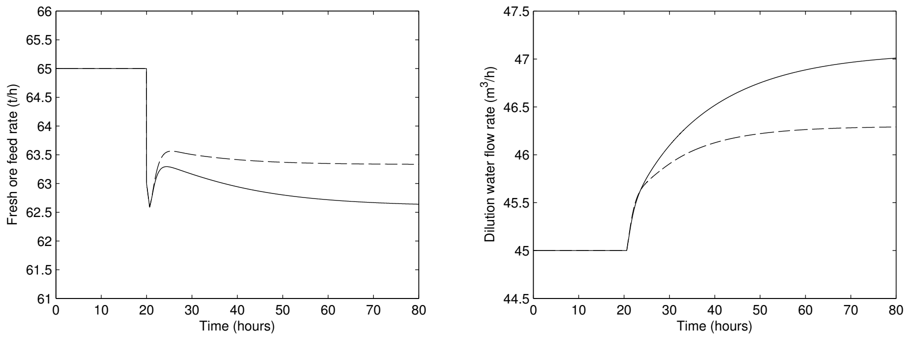

Fig. 5. Response of manipulated variables for a step in the particle size showing the nominal response (dashed line) and the perturbed response (solid line).

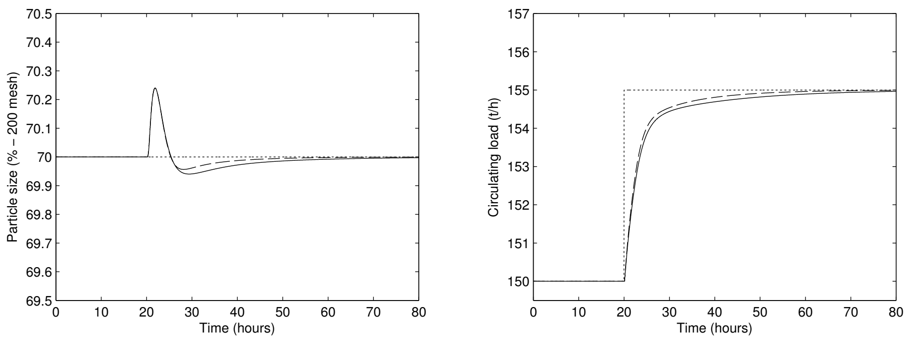

Fig. 6. Response of controlled variables for a step in the circulating load showing the set-point (dotted line), the nominal response (dashed line) and the perturbed response (solid line).

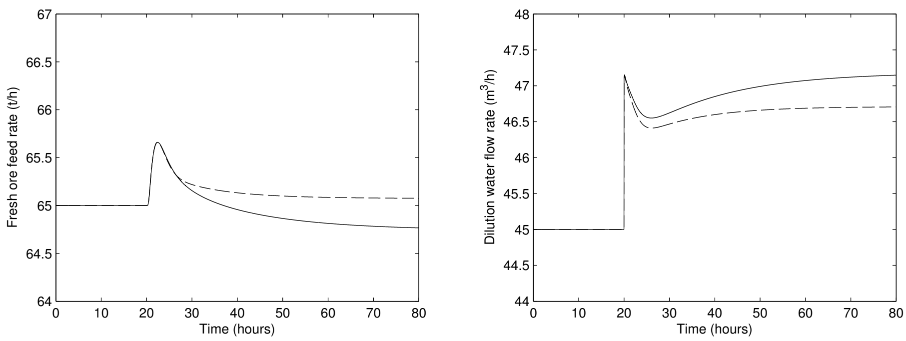

Fig. 7. Response of manipulated variables for a step in the circulating load showing the nominal response (dashed line) and the perturbed response (solid line).