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Exponential Tracking Control of Nonlinear Systems with Actuator Nonlinearity ⋆

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Abstract: In this paper, the problem of adaptive tracking control is addressed for a class of nonlinear systems with parametric uncertainty, unknown actuator nonlinearity and disturbance. Two type of actuator nonlinearities, that is, backlash-like hysteresis and symmetric dead-zone, are considered simultaneously. Without constructing the inverse function of actuator nonlinearity, unified control framework is established. An adaptive control scheme, capable of guaranteeing the exponential tracking with zero tracking error, is proposed. Two simulation examples are provided to clarify and verify the proposed approach.

Keywords: Adaptive control, actuator nonlinearity, hysteresis, dead zone, tracking control.

1. INTRODUCTION

Generally speaking, there are two design methods to eliminate the effects of actuator nonlinearities. One is to construct the inverse function of actuator nonlinearity (see, e.g. Tao and Kokotovic (1994, 1995); Zhou et al. (2006, 2007, 2012)), and the other is to directly design robust adaptive controller without the inverse problem (see, e.g. Chen et al. (2008, 2010); Su et al. (2000); Zhou et al. (2004); Su et al. (2003); Wen and Zhou (2007); Su et al. (2005); Wang and Su (2006); Wang et al. (2004); Ibrir et al. (2007); Hua et al. (2008); Hua and Ding (2011)). In the above two research directions, considerable efforts have been made to mitigate the influences of actuator nonlinearity. For example, in order to compensating for actuator nonlinearity, pioneering work was done in Tao and Kokotovic (1994, 1995, 1996), where adaptive inverse methods were developed. In Zhou et al. (2006, 2007, 2012), smooth inverse functions of dead-zone, backlash nonlinearity and Bouc-Wen hysteresis were respectively introduced and the adaptive backstepping output-feedback control schemes were proposed. In order to avoid the direct inversion of the hysteresis model, in Chen et al. (2008, 2010), the concept of implicit inversion was introduced. For discrete and continuous linear systems with Prandtl-Ishlinskii hysteresis, the approximate and perfect implicit inversions were respectively incorporated into adaptive control designs. In Su et al. (2000), Zhou et al. (2004), Su et al. (2003) and Wen and Zhou (2007), the backlash nonlinearity was approximated by a continuous differential equation model, which was called as backlash-like hysteresis. Instead of constructing a hysteresis inverse, robust adaptive controllers were directly designed to minimize the effects of the hysteresis nonlinearity. Motivated by Su et al. (2000) and Zhou et al. (2004), robust adaptive control techniques were applied to deal with Prandtl-Ishlinskii type hysteresis (Su et al., 2005; Wang and Su , 2006). In Wang et al. (2004), Ibrir et al. (2007), Hua et al. (2008) and Hua and Ding (2011), symmetric and non-symmetric dead-zones were investigated, respectively. New descriptions on the dead-zone models, that is, the combination of linear input function with constant or time-varying coefficients and a bounded time-varying function, were introduced, and new control strategies were proposed.

Despite the great process in the control of dynamical systems with actuator nonlinearity, some challenging problems still remain. One of the main drawbacks in the current literature is that most of the proposed robust adaptive tracking controllers do not produce asymptotic tracking. Instead, the so-called bounded-error trajectory tracking is achieved (see, e.g. Tao and Kokotovic (1994, 1995); Zhou et al. (2006, 2007); Chen et al. (2008); Su et al. (2000); Wen and Zhou (2007); Su et al. (2005); Wang and Su (2006); Wang et al. (2004); Ibrir et al. (2007); Hua et al. (2008); Hua and Ding (2011)) . It is noted that asymptotic tracking was obtained in Zhou et al. (2012) and Zhou et al. (2004) (see Scheme 1 in Zhou et al. (2004)). However, it is worth pointing out that additional cost had to be paid. In Zhou et al. (2012), by constructing a new inverse function of the hysteresis, the tracking error was proved to converge to zero asymptotically. However, the parameters in the hysteresis model were required to be known. In

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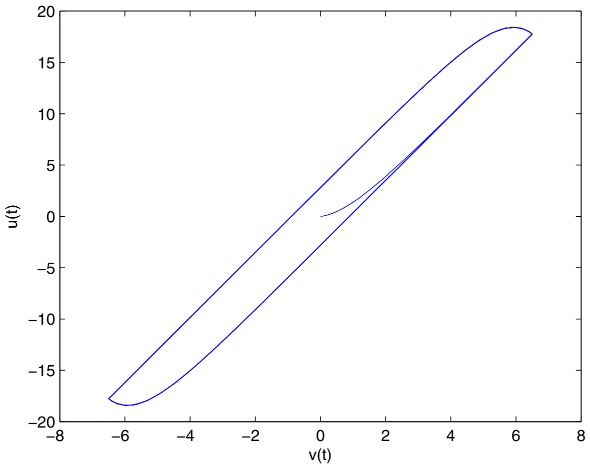

Fig. 1. Backlash-like hysteresis

Zhou et al. (2004), two adaptive control schemes were proposed by employing the backstepping approach. In the first scheme, asymptotic tracking was achieved. However, a discontinuous sign function was introduced and the chattering phenomenon may occur. In the second scheme, a smooth control scheme was presented by defining a novel differentiable function. However, perfect tracking could not be ensured. It is well known that exacting output tracking has considerable theoretical and practical significance. Therefore, it is highly desirable to design new adaptive compensation control scheme with zero tracking error and without the aforementioned cost.

2. PROBLEM FORMULATION

2.1 System model

Consider a class of uncertain nonlinear systems with actuator nonlinearity

$$x^{(n)}(t) + \sum_{i=1}^{r} a_i Y_i(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)) = bu(t) + d(t),$$
$$u(t) = N(v(t)), \tag{1}$$

where plant parameters a_i are unknown constants, Y_i are known smooth functions, control gain b is unknown constant, $d(t)$ denotes bounded external disturbance, $N(\cdot)$ represents an actuator uncertainty, $v(t)$ is the applied control, and $u(t)$ is not available for measurement. In this paper, two type of actuator nonlinearity characteristics are considered.

Backlash-like hysteresis (Su et al., 2000):

$$\dot{u} = \alpha|\dot{v}|(cv - u) + B_1\dot{v}, \tag{2}$$

where α, c and B_1 are constants, and $c > B_1$. Fig. 1 illustrates the backlash-like hysteresis dynamics described by (2), where $\alpha = 1, c = 3.1635, B_1 = 0.345, v(t) = 6.5\sin(2.3t), u(0) = 0$.

Symmetric dead-zone (Tao and Kokotovic , 1994):

$$u = \begin{cases} m(v - b_r), & v \geq b_r, \\ 0, & b_l < v < b_r, \\ m(v - b_l), & v \leq b_l, \end{cases} \tag{3}$$

where m is the slope of the dead-zone characteristic, b_r and b_l represent the break points.

Before proceeding further, we introduce two useful lemmas for the actuator nonlinearity models.

Lemma 1 (Su et al., 2000): The solution of (2) is

$$u(t) = cv(t) + d_1(v), \tag{4}$$
$$d_1(v) = [u(0) - cv(0)]e^{-\alpha(v - v(0))\mathrm{sgn}(\dot{v})}$$
$$+e^{-\alpha v(\mathrm{sgn}(\dot{v}))} \int_{v(0)}^{v} (B_1 - c)e^{\alpha\zeta(\mathrm{sgn}\dot{v})} d\zeta, \tag{5}$$

and $d_1(v)$ is bounded.

Lemma 2 Wang et al. (2004): The dead-zone model (3) can be represented as

$$u(t) = mv(t) + d_2(v), \tag{6}$$
$$d_2(v) = \begin{cases} -mb_r, & v \geq b_r, \\ -mv, & b_l < v < b_r, \\ -mb_l, & v \leq b_l, \end{cases} \tag{7}$$

and $d_2(v)$ is bounded.

The control objective is to design a control law $v(t)$ in (1) such that all the closed-loop signals are bounded, while the system state vector $X = [x, \dot{x}, \ldots, x^{(n-1)}]^T$ exponentially tracks a specified desired trajectory $X_d = [x_d, \dot{x}_d, \ldots, x_d^{(n-1)}]^T$ with zero tracking error, where $x_d(t)$ is a given reference signal.

To this end, we make the following assumptions.

Assumption 1: The reference signal $x_d(t)$ and its first n derivatives are known and bounded.

Assumption 2: The parameters of actuator nonlinearity such as α, c, B_1 in the Backlash-like hysteresis and m, b_r, b_l in the dead-zone model are unknown. Moreover, the uncertain parameters b, c, m are such that $bc > 0, bm > 0$.

3. ADAPTIVE CONTROL DESIGN

Now, substituting (4) or (6) into (1) yields

$$x^{(n)}(t) + \sum_{i=1}^{r} a_i Y_i(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t))$$
$$= \rho v(t) + D(t), \tag{8}$$

where

$$\rho = bc, \quad D(t) = bd_1(v) + d(t), \tag{9}$$

or

$$\rho = bm, \quad D(t) = bd_2(v) + d(t). \tag{10}$$

From Assumption 2, it follows that ρ is unknown but $\rho > 0$. Applying Lemmas 1-2 together with the boundedness of $d(t)$, we know that $D(t)$ is bounded. Let the tracking error

$$e = X - X_d, \tag{11}$$

that is, $e = [e_1, e_2, \ldots, e_n]^T$,

$$e_1 = x - x_d, \quad e_2 = \dot{x} - \dot{x}_d, \quad \cdots, \quad e_n = x^{(n-1)} - x_d^{(n-1)}. \tag{12}$$

From (8) and (12), the dynamics of the tracking error is governed by

$$\dot{e} = Ae + B\left[-\sum_{i=1}^{r} a_i Y_i(X) + \rho v(t) + D(t) - x_d^{(n)}\right], \tag{13}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ & \ddots & & \ddots & \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}. \tag{14}$$

It is observed that (A, B) is controllable. So is $(A+\sigma_1 I, B)$, where $\sigma_1 > 0$ is a design parameter. Thus, there exists a constant matrix k satisfying that $(A + \sigma_1 I) + Bk$ is stable and hence $P = P^T > 0$ exists such that

$$[(A + \sigma_1 I) + Bk]^T P + P[(A + \sigma_1 I) + Bk] = -Q, \tag{15}$$

where $Q = Q^T > 0$ is a given matrix, that is,

$$(A + Bk)^T P + P(A + Bk) = -Q - 2\sigma_1 P < -2\sigma_1 P. \tag{16}$$

Before adaptive control scheme is presented, we define some variables used in what follows:

$$f(X, t) = \sum_{i=1}^{r} \sqrt{|Y_i(X)|^2 + h_i} + \sqrt{\|e(t)\|^2 + h} + 1, \tag{17}$$

$$d_0 = \sup_{t\geq 0}\left[|D(t)| + |x_d^{(n)}|\right], \tag{18}$$

$$\theta = \max\{|a_1|, \ldots, |a_r|, \|k\|, d_0\}, \tag{19}$$

$$\theta^* = \frac{\theta}{\rho}, \tag{20}$$

where $h, h_i, i = 1, 2, \ldots, r$, are positive design constants, $\|\cdot\|$ denotes the Euclidean norm of a vector. By Assumption 1 and the boundedness of $D(t)$, we know that d_0 does exist. Then, the control law and parameter update law are designed as, respectively,

$$v(t) = -\frac{e^T PB\hat{\theta}^2(t) f^2(X, t)}{e^T PB \tanh\left[l^{-1} e^T PB \exp(2\sigma_2 t)\right]\hat{\theta}(t) f + l\exp(-2\sigma_2 t)}, \tag{21}$$

$$\dot{\hat{\theta}} = \gamma \exp(2\sigma_1 t)|e^T PB| f(X, t), \tag{22}$$

where l, σ_2 are positive design constants, $\gamma > 0$ is adaptive gain, σ_2 satisfies $\sigma_2 > \sigma_1$, $\hat{\theta}(t)$ is the estimate of θ^*, $\hat{\theta}(0) \geq 0$.

Theorem 1: Consider the closed-loop system consisting of the system (1) with actuator nonlinearity (2) or (3), control law (21) and adaptive law (22) based on Assumptions 1-2. Then, all the closed-loop signals remain bounded, and the tracking error converges to zero exponentially with the rate of not less than σ_1.

Proof: We first rewrite (13) as

$$\dot{e} = (A + Bk)e + B\left[-\sum_{i=1}^{r} a_i Y_i(X) + \rho v(t) + D(t) - x_d^{(n)} - ke\right] \tag{23}$$

Define a positive Lyapunov function

$$V = e^T Pe + \frac{\rho}{\gamma}\exp(-2\sigma_1 t)\tilde{\theta}^2(t) \tag{24}$$

with the estimation error

$$\tilde{\theta}(t) = \theta^* - \hat{\theta}(t). \tag{25}$$

In view of (23)-(25), the derivative of V is

$$\begin{aligned}\dot{V} = {} & e^T\left[P(A + Bk) + (A + Bk)^T P\right]e \\ & -\frac{2\sigma_1\rho}{\gamma}\exp(-2\sigma_1 t)\tilde{\theta}^2 - \frac{2\rho}{\gamma}\exp(-2\sigma_1 t)\tilde{\theta}\dot{\hat{\theta}} + 2e^T PB \\ & \cdot\left[-\sum_{i=1}^{r} a_i Y_i(X) + \rho v(t) + D(t) - x_d^{(n)} - ke\right].\end{aligned} \tag{26}$$

Substituting (16) and (22) into (26) and noting (24), we have

$$\begin{aligned}\dot{V} \leq {} & -2\sigma_1 e^T Pe - \frac{2\sigma_1\rho}{\gamma}\exp(-2\sigma_1 t)\tilde{\theta}^2 - 2\rho\tilde{\theta}|e^T PB| f(X, t) \\ & +2e^T PB\left[-\sum_{i=1}^{r} a_i Y_i(X) + D(t) - x_d^{(n)} - ke\right] \\ & +2\rho e^T PBv(t) \\ = {} & -2\sigma_1 V - 2\rho\tilde{\theta}|e^T PB| f(X, t) + 2\rho e^T PBv(t) \\ & +2e^T PB\left[-\sum_{i=1}^{r} a_i Y_i(X) + D(t) - x_d^{(n)} - ke\right].\end{aligned} \tag{27}$$

Noting the definitions in (17)-(20), we have

$$\begin{aligned} & e^T PB\left[-\sum_{i=1}^{r} a_i Y_i(X) + D(t) - x_d^{(n)} - ke\right] \\ & \leq \rho|e^T PB|\theta^* f(X, t).\end{aligned} \tag{28}$$

Combining (25), (27) and (28) implies that

$$\begin{aligned}\dot{V} \leq {} & -2\sigma_1 V - 2\rho\tilde{\theta}|e^T PB| f(X, t) + 2\rho e^T PBv(t) \\ & +2\rho|e^T PB|\theta^* f(X, t) \\ = {} & -2\sigma_1 V + 2\rho e^T PBv(t) + 2\rho|e^T PB|\hat{\theta} f(X, t).\end{aligned} \tag{29}$$

Then, substituting (21) into (29) results in

$$\begin{aligned}\dot{V} \leq {} & -2\sigma_1 V + 2\rho|e^T PB|\hat{\theta} f(X, t) \\ & -\frac{2\rho(e^T PB)^2\hat{\theta}^2 f^2(X, t)}{e^T PB\tanh\left[l^{-1}e^T PB\exp(2\sigma_2 t)\right]\hat{\theta} f + l\exp(-2\sigma_2 t)} \\ \leq {} & -2\sigma_1 V + 2\rho l\exp(-2\sigma_2 t) \\ & \cdot\frac{|e^T PB|\hat{\theta} f(X, t)}{|e^T PB|\hat{\theta} f(X, t) + l\exp(-2\sigma_2 t)} \\ \leq {} & -2\sigma_1 V + 2\rho l\exp(-2\sigma_2 t).\end{aligned} \tag{30}$$

Thus, we obtain

$$V(t) \leq \left(V(0) + \frac{\rho l}{\sigma_2 - \sigma_1}\right)\exp(-2\sigma_1 t). \tag{31}$$

Owing to (24), we conclude that

$$e^T Pe \le \left(V(0) + \frac{\rho l}{\sigma_2 - \sigma_1}\right) \exp(-2\sigma_1 t), \quad (32)$$

$$\frac{\rho}{\gamma} \exp(-2\sigma_1 t)\tilde{\theta}^2 \le \left(V(0) + \frac{\rho l}{\sigma_2 - \sigma_1}\right) \exp(-2\sigma_1 t)(33)$$

which further implies that

$$\|e\| \le \sqrt{\frac{V(0) + \frac{\rho l}{\sigma_2 - \sigma_1}}{\lambda_{\min}(P)}} \exp(-\sigma_1 t), \quad (34)$$

$$|\tilde{\theta}| \le \sqrt{\frac{\gamma\left(V(0) + \frac{\rho l}{\sigma_2 - \sigma_1}\right)}{\rho}}. \quad (35)$$

Clearly, it can be seen from (34) that the tracking error converges to zero exponentially, and the convergence rate is not less than σ_1. Moreover, from (25) and (35), it follows that the parameter estimate $\hat{\theta}(t)$ is bounded. By (11), (34) and Assumption 1, it is shown that X is bounded. Examining (17), we obtain the boundedness of $f(X,t)$. Next, we will prove $v(t)$ is bounded. Using (21), we get

$$|v(t)| \le \hat{\theta} f(X,t) + \kappa \hat{\theta}^2 f^2(X,t), \quad (36)$$

where $\kappa = 0.2785$. Noting the boundedness of $\hat{\theta}$ and $f(X,t)$, we can obtain the boundedness of $v(t)$. Therefore, all the closed-loop signals are bounded. This completes the proof.

4. SIMULATION STUDIES

Example 1: Consider the uncertain nonlinear system studied in Zhou et al. (2006), Su et al. (2000), Zhou et al. (2004), Su et al. (2005) and Wang and Su (2006)

$$\dot{x} = a\frac{1 - \exp(-x)}{1 + \exp(-x)} + bu(t), \quad (37)$$

where the parameters $a = 1, b = 1$, are assumed to be unknown constants. The actuator nonlinearity is modeled as backlash-like hysteresis (2) with unknown parameters $\alpha = 1, c = 3.1635, B_1 = 0.345$. The initial condition of the controlled plant (37) is set to be $x(0) = 1.05$. The objective is to design the control v such that x can track the desired trajectory $x_d = 12.5\sin(2.3t)$. According to (11), (14) and (17), we have

$$e = x - x_d,$$
$$A = 0, \quad B = 1,$$
$$f(x,t) = \sqrt{\left[\frac{1 - \exp(-x)}{1 + \exp(-x)}\right]^2 + h_1} + \sqrt{e^2 + h} + 1, (38)$$

where h_1, h are chosen as $h_1 = 0.01, h = 0.01$, respectively.

In the simulation, we choose $\sigma_1 = 0.01, k = -1.01, Q = 10, \gamma = 3, l = 0.1, \sigma_2 = 0.05, u(0) = 0, \hat{\theta}(0) = 0$. Thus, the solution of (15) is $P = 5$. The system responses are shown in Figs. 2-5. From Fig. 3, we can see that the tracking error converges to zero rapidly. At the same time, the boundedness of control signal v is shown in Fig. 5, from which the large difference between hysteresis input v and its output u can also be observed. The boundedness of other signals including plant state x and parameter estimate $\hat{\theta}$ is revealed in Figs. 2 and 4, respectively.

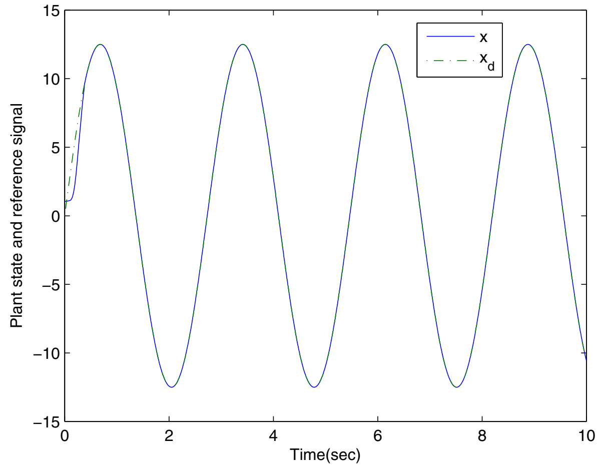

Fig. 2. Example 1: plant state x and reference signal x_d

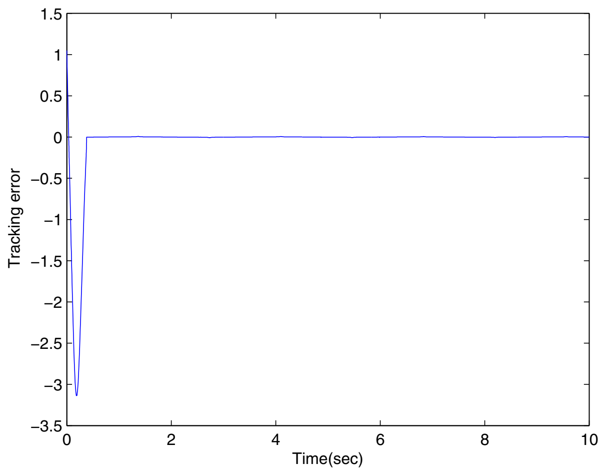

Fig. 3. Example 1: tracking error

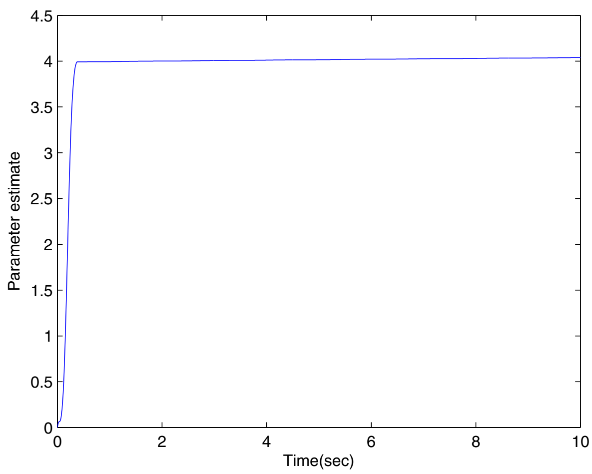

Fig. 4. Example 1: parameter estimate

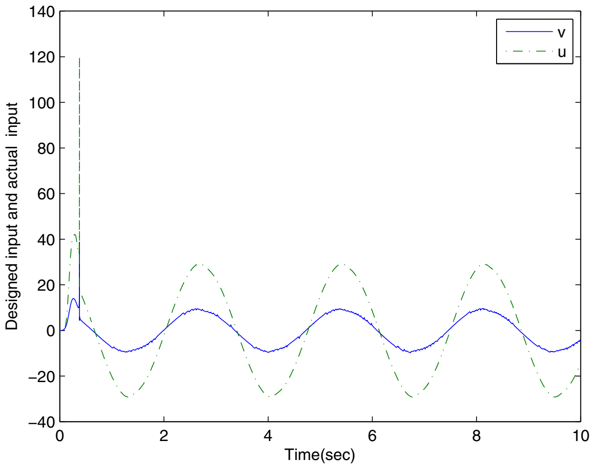

Fig. 5. Example 1: designed input v and actual input u

Example 2: To further illustrate the effectiveness of our design method, we simulate the proposed adaptive compensation controller on a nonlinear system described by Wang et al. (2004)

$$\ddot{x} = a_1 \frac{1-\exp(-x)}{1+\exp(-x)} - a_2(\dot{x}^2 + 2x)\sin(\dot{x}) \\ -0.5a_3 x \sin(3t) + bu. \tag{39}$$

In this example, we consider the dead-zone model (3). The plant parameters and dead-zone parameters are respectively given as $a_1 = a_2 = a_3 = b = 1, m = 1, b_r = 0.5, b_l = -0.6$, which are assumed to be unknown. The initial condition of the controlled plant (39) is set to be $x(0) = -2.5, \dot{x}(0) = 3.5$. The reference signal is $x_d = 2.5\sin(t)$.

From (11), (12), (14) and (17), it follows that

$$e = [e_1, e_2]^T, \quad e_1 = x_1 - x_d, \quad e_2 = x_2 - \dot{x}_d,$$
$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$
$$f(x_1, x_2, t) = \sqrt{\left[\frac{1-\exp(-x_1)}{1+\exp(-x_1)}\right]^2 + h_1} \\ + \sqrt{[(x_2^2 + 2x_1)\sin(x_2)]^2 + h_2} \\ + \sqrt{[0.5x_1\sin(3t)]^2 + h_3} + \sqrt{\|e\|^2 + h} + 1, \tag{40}$$

where $x_1 = x, x_2 = \dot{x}$, h_1, h_2, h_3, h are selected as $h_1 = h_2 = h_3 = h = 1$. Then, we apply the proposed control scheme to this example. The design parameters are chosen as follows: $\sigma_1 = 0.01, k = [-1, -1.1], Q = [6 \quad 3; 3 \quad 6], \gamma = 4, l = 100, \sigma_2 = 0.1, \hat{\theta}(0) = 0$. Solving (15), we have

$$P = \begin{bmatrix} 5.8617 & 3.0586 \\ 3.0586 & 5.5584 \end{bmatrix}. \tag{41}$$

According to Theorem 1, the control law and adaptive law can be derived. The simulation results are shown in Figs. 6-9. From these figures, similar conclusions on the signal boundedness and exponential tracking, as discussed in Example 1, can be drawn.

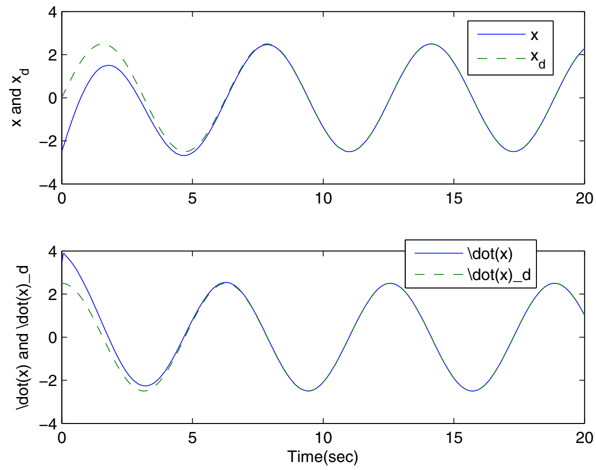

Fig. 6. Example 2: plant states and reference signals (a) x, x_d; (b) $\dot{x}, \dot{x}_d$

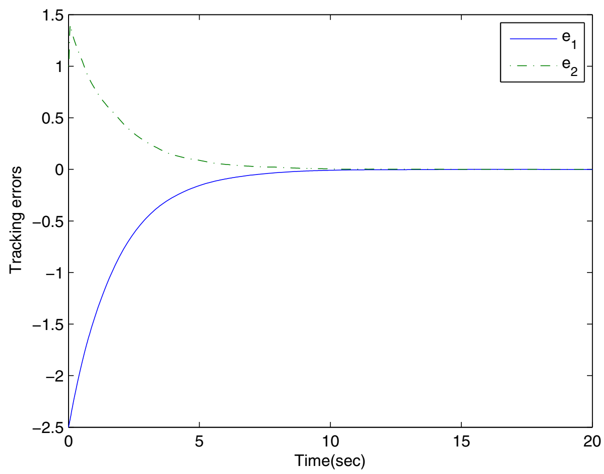

Fig. 7. Example 2: tracking errors

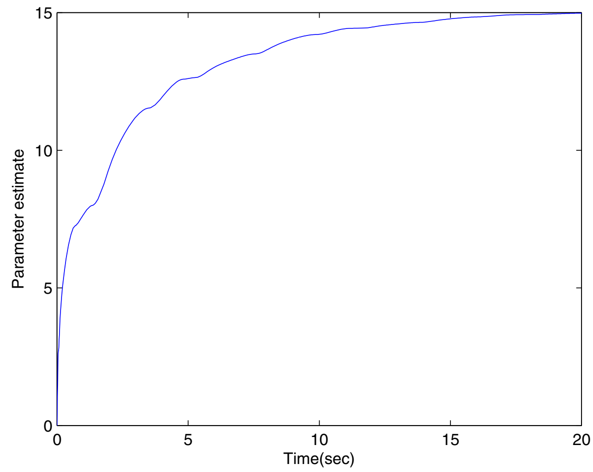

Fig. 8. Example 2: parameter estimate

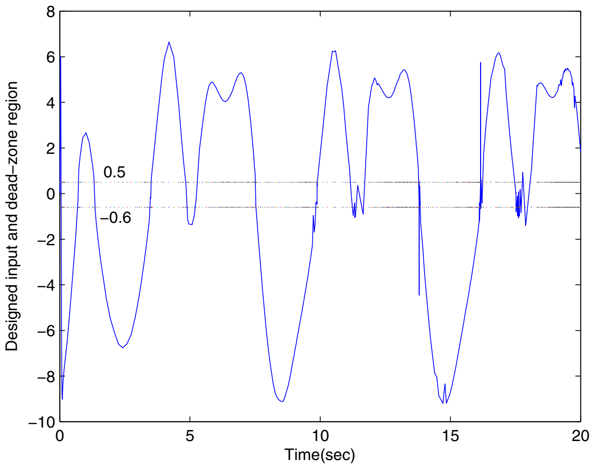

Fig. 9. Example 2: designed input v and dead-zone region

5. CONCLUSION

The problem of adaptive tracking control for a class of uncertain nonlinear systems with two possible actuator nonlinearities has been considered. The plant parameters and the parameters of actuator nonlinearity are assumed to be unknown. We have proposed a class of adaptive controllers for tracking of dynamical signals. We have shown that by employing the presented adaptive tracking controller, the tracking error can be guaranteed to decrease to zero exponentially.

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