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Semiactive Control System based on MR Damper for Suppressing Vibration of Stay Cable under Wind Load

Hyung-Jo Jung*, Dong-Doo Jang*, Seung-Woo Lee*, Heon-Jae Lee*, In-Ho Kim**

*Department of Civil and Environmental Engineering, KAIST, Daejeon, Korea,
(e-mail: hjung@kaist.ac.kr, ddjjang@kaist.ac.kr, swlee@kaist.ac.kr, wildhawk@kaist.ac.kr)
**Department of Civil and Environmental Engineering, Sejong University, Seoul, Korea,
(e-mail: kih119@sju.ac.kr)

Abstract: This paper investigates the performance of the semiactive control system based on magnetorheological fluid (MR) dampers for suppressing excessive vibration of stay cable installed in cable-stayed bridges under wind load. The cable model is extracted from a 156.3 m long stay cable with high tension. The external wind load is generated from the widely used wind load spectrum such as the Kaimal spectrum. Several semiactive control algorithms such as the Lyapunov stability theory-based control, the maximum energy dissipation and the clipped-optimal control are considered to find the appropriate control strategy for the cable-damper system employing MR dampers. Numerical simulations are carried out to demonstrate the effectiveness of the semiactive control systems based on MR dampers and their control performances are compared with those of passively operated control systems.

1. INTRODUCTION

The modern concept of vibration control of structures was proposed by John Milne more than 100 years ago (Housner et al. 1997). He demonstrated that a structure could be isolated from an earthquake. Another milestone of structural control was reached as J.T.P. Yao presented his conceptual study in 1972 (Yao 1972). Since then, considerable attention has been given increasingly to structural control of civil infrastructures such as buildings and bridges for mitigating excessive vibration of structures under the action of earthquakes, wind and man-made loads. Structural control can be largely divided into passive, active, hybrid, and semiactive control. Among them, semiactive control systems which combine the best features of both passive and active control systems (i.e., reliability and adaptability, respectively) could be considered as one of the most promising schemes for vibration mitigation of large-scale civil infrastructures (Housner et al. 1997).

Stay cables and their connections are the most crucial elements in a cable-stayed bridge. Long stay cables are vulnerable to wind-induced vibrations, because they have low inherent damping characteristics. To mitigate vibrations of stay cables, several methods such as adopting mechanical dampers, adding crossing ties or spacers, or providing cable surface treatments have been developed, and some of them have been implemented to real cable structures, though each has its limitations. In recent years, several studies on a semiactive control system based on a magnetorheological fluid (MR) damper to suppress cable vibration have been carried out. Johnson et al. (2000) developed a control-oriented model using a static deflection shape in a series expansion for the cable. Ni et al. (2002) proposed neural network-based controllers incorporated with an MR damper for reducing the excessive vibration of stay cables. Johnson et al. (2003) extended their previous work by adding sag and inclination to the cable model, and showed that the response of the cable was significantly reduced by semiactive dampers for a wide range of cable sag and damper locations. Also, Christenson (2001) experimentally verified the effectiveness of MR damper-based semiactive control technology using a small-scaled cable model. Recently, MR dampers were installed on stay cables of the Dongting Lake Briege and the Binzhou Yellow River Bridge in China to reduce cable vibration, which are the full-scale implementations of MR dampers for bridge structures (Spencer and Nagarajaiah 2003). In most of the previous works on MR damper-based semiactive control systems for stay cables, however, the applicability and performance of various semiactive control algorithms were not taken into account.

In this paper, the effectiveness of a semiactive control system using an MR fluid damper in mitigating cable responses is investigated. The equations of motion of the cable-damper system are derived using a standard Galerkin approach with a control-oriented model. The external wind load is generated from the widely used wind load spectrum such as the Kaimal spectrum. Several semiactive control strategies, such as the control based on Lyapunov stability theory, the maximum energy dissipation, and the clipped optimal control algorithms, are considered. Numerical simulations consider a 156.3 m long stay cable, which is installed in a cable-stayed bridge under construction in Korea. It is excited by the external load distributed along the cable and controlled by MR dampers. The control performances of each control algorithm have been compared with those of the passive-type control systems employing an MR damper.

2. DYNAMIC MODEL OF CABLE

The dynamic motion of the cable with small sag may be modeled by the motion of a taut string (Irvine 1981). Cables installed in cable-stayed bridges typically have small sag (1 % sag-to-length ratio or less) with high tension-to-weight ratios. Therefore, the transverse motion of the cable with a semiactive damper attached transverse to the cable is depicted as shown in Fig 1.

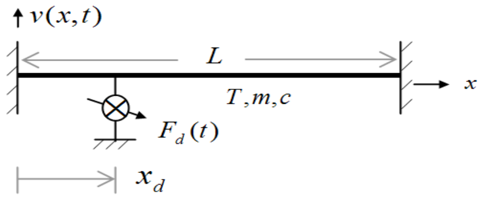

Fig. 1. Cable with attached semi-active damper

In the figure, $v(x,t)$ represents the transverse deflection of the cable, L is the length of cable, x_d is the location of the damper, $F_d(t)$ is the damper force, T is the cable tension, m is the cable mass per unit length, and c is the cable viscous damping per unit length.

The dynamic equation of motion of the taut cable in the linear range is expressed as

$$m\ddot{v}(x,t)+c\dot{v}(x,t)-Tv''(x,t)=f(x,t)+F_d(t)\delta(x-x_d) \tag{1}$$

where $f(x,t)$ is the external load and c is the viscous damping coefficient per unit length. The transverse deflection of the cable may be approximated as

$$v(x,t)=\sum_{j=1}^{n}\phi_j(x)\,\eta_j(t) \tag{2}$$

where n is the number of modes considered, $\eta_j(t)$ is the generalized displacement and $\phi_j(x)$ is a shape function.

Johnson et al. (2000) showed that introducing shape function based on the deflection due to a static force at the damper location (i.e., Eq. (3)) can dramatically reduce the number of terms required for the comparable accuracy. The static deflection-based shape function is given by

$$\phi_1(x)=\begin{cases} x/x_d & 0\le x\le x_d \\ (L-x)\,/(L-x_d) & x_d\le x\le L \end{cases} \tag{3}$$

The other shape functions remain sinusoidal as follows:

$$\phi_{j+1}(x)=\sin\pi\, j\frac{x}{L} \qquad j=1,2\cdots n-1 \tag{4}$$

Substituting Eq. (2) into Eq. (1) gives

$$M\ddot{\eta}(t)+C\dot{\eta}(t)+K\eta(t)=\mathrm{f}(t)+\varphi F_d(t) \tag{5}$$

resulting in the mass matrix M,

$$M=[m_{ij}],\;\; m_{ij}=m\int_0^L\phi_i(x)\phi_j(x)dx \tag{6}$$

the stiffness matrix K,

$$K=[k_{ij}],\;\; k_{ij}=-T\int_0^L\phi_i'(x)\phi_j'(x)dx \tag{7}$$

the load vector f,

$$\mathrm{f}=[f_i],\;\; f_i=\int_0^L f(x,t)\phi_i(x)dx \tag{8}$$

and the damper force vector φ,

$$\varphi=\phi(x_d)=[\phi_1(x_d)\;\;\phi_2(x_d)\;\;\cdots\;\;\phi_n(x_d)]^T \tag{9}$$

Eq. (5) can be written in state-space form as

$$\dot{z}=Az+BF_d(t)+Gf \tag{10}$$

$$Z=C_Zz+D_ZF_d(t)+H_Zf \tag{11}$$

$$y=C_yz+D_yF_d(t)+H_yf+v \tag{12}$$

where $z=[\eta\;\;\dot{\eta}]^T$ is the state vector, $Z=[\eta\;\;\dot{\eta}\;\;\ddot{\eta}]^T$ is a vector of quantities to be regulated, η is the generalized displacement, y is a vector of noisy sensor measurements, v is a vector of stochastic sensor noise processes, and the matrices are

$$A=\begin{bmatrix}0 & I\\ -M^{-1}K & -M^{-1}C\end{bmatrix},\; B=\begin{bmatrix}0\\ M^{-1}\varphi\end{bmatrix},\; G=\begin{bmatrix}0\\ M^{-1}\end{bmatrix},$$

$$C_Z=\begin{bmatrix}I & 0\\ 0 & I\\ -M^{-1}K & -M^{-1}C\end{bmatrix},\; D_Z=\begin{bmatrix}0\\ 0\\ M^{-1}\varphi\end{bmatrix},\; H_Z=\begin{bmatrix}0\\ 0\\ M^{-1}\end{bmatrix},$$

$$C_y=\begin{bmatrix}\varphi^T & 0\\ -\varphi^TM^{-1}K & -\varphi^TM^{-1}C\end{bmatrix},\; D_y=\begin{bmatrix}0\\ \varphi^TM^{-1}\varphi\end{bmatrix},\; H_y=\begin{bmatrix}0\\ \varphi^TM^{-1}\end{bmatrix}.$$

In which, $\varphi=\phi(x_d)=[\phi_1(x_d)\;\;\phi_2(x_d)\;\;\cdots\;\;\phi_n(x_d)]^T$

A Kalman-Bucy filter is used as the observer for state estimation, because measurements are assumed to be available only at the damper location in the numerical example. The displacement and velocity at the damper location are measured for inputs of the Kalman-Bucy filter, and then the state estimation vector $\hat{z}=[\hat{\eta}\;\;\dot{\hat{\eta}}]^T$ can be obtained from

$$\dot{\hat{z}}=(A-L_{kf}C_y)\hat{z}+\begin{bmatrix}L_{kf} & B-L_{kf}D_y & G-L_{kf}H_y\end{bmatrix}\begin{Bmatrix}y\\ F_d(t)\\ f\end{Bmatrix} \tag{13}$$

$$\begin{bmatrix} \hat{y} \\ \hat{z} \end{bmatrix} = \begin{bmatrix} C_y \\ I \end{bmatrix} \hat{z} + \begin{bmatrix} 0 & D_y & H_y \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} y \\ F_d(t) \\ f \end{Bmatrix} \tag{14}$$

where L_{kf} is the estimator gain obtained by solving an algebraic Riccati equation (Burl 1999).

3. SEMIACTIVE CONTROL ALGORITHMS FOR CABLE VIBRATION

Various semiactive control algorithms have been proposed for control of MR dampers. In this study, three control algorithms are considered. Each algorithm is partially modified for application to the cable-damper system. Detailed information of each algorithm can be found in Jansen and Dyke (2000).

3.1 Control Algorithm Based on Lyapunov Stability Theory

Lyapunov's direct approach was applied to design a semi-active controller. In this approach, the goal of the control law is to choose control inputs that will result in making the rate of change of the Lyapunov function as negative as possible. In an MR damper-based control system, therefore, the control law can be used as

$$V = V_{\max} H((-z)^T P_L B F_d) \tag{15}$$

where $V_{\max}$ is the maximum voltage input to an MR damper, P_L is the real, symmetric, positive definite matrix satisfying the Lyapunov equation (i.e., $AP_L + P_L A^T = -Q_p$) and $H(\cdot)$ is the heaviside step function.

3.2 Maximum Energy Dissipation Algorithm

The maximum energy dissipation algorithm was presented as a variation of the decentralized bang-bang approach. In the maximum energy dissipation algorithm, the Lyapunov function was chosen to represent the relative total vibratory energy in the system (Jansen and Dyke 2000). The control law for an MR damper-based controller attached to stay cable is obtained as

$$V = V_{\max} H(-\dot{\eta}\varphi F_d) \tag{16}$$

This control algorithm commands the maximum control voltage when the cable system dissipates energy.

3.3 Clipped-Optimal Control Algorithm

The clipped-optimal control algorithm proposed by Dyke et al. (1996) is the one that has been shown to be effective for use with an MR damper. This algorithm consists of two parts of controller. The primary controller is the LQR control design which gives the optimal control force, F_{dci} , that minimizes the cost function. In this study, LQR controller which is designed by Johnson et al. (2000) and proved to perform well for stay cables is adapted. This controller uses force proportional to an estimate of the state of the system using feedback gain that minimizes the cost function

$$J = \lim_{T\to\infty} E\left[\frac{1}{T} \int_0^T (z^T Q z + R F_{d_{ci}}{}^2) dt \right] \tag{17}$$

The secondary controller, which accounts for the characteristics of MR dampers that can only exert dissipative forces, is given by

$$V = V_{\max} H(\{F_{dci} - F_d\} F_d) \tag{18}$$

The control law means that when the force produced by the damper is smaller than the desired optimal force and the two forces have the same direction, the controller will command the maximum voltage to control device.

4. NUMERICAL SIMULATION

4.1 Numerical Model of Cable Damping Model

To numerically evaluate the control performance of several semiactive control algorithms for an MR fluid damper-based semiactive control systems of cable vibration, a numerical model was extracted from a 156.3 m long high-tension stay cable. The geometric and material properties of the cable are demonstrated in Table 1.

Table 1. Cable Characteristics

parameter	Value
Length (*l*)	156.3 m
Mass per unit length (*m*)	60.1 kg/m
Tension (*T*)	3,460 kN
Diameter	125 mm

Two MR dampers are introduced to provide controllable damping forces and the parameters of the dampers identified by Terasawa et al. (2004) are used after adjusting them by magnification factors. The damping force $F_d(t)$ in the dynamic model of the damper can be expressed by

$$F_d = \sigma_a w + \sigma_0 w V + \sigma_1 \dot{w} + \sigma_2 \dot{v}_d + \sigma_b \dot{v}_d V \tag{19}$$

$$\dot{w} = \dot{v}_d - a_0 |\dot{v}_d| w \tag{20}$$

where $w(t)$ is the internal state variable (m), $v_d(t)$ is the displacement of cable at damper location (m), $V(t)$ is the input voltage to the MR fluid damper, σ_0 is the stiffness of $w(t)$ influenced by $V(t)$ (N/(mV)), σ_1 is the damping coefficient of $w(t)$ (Ns/m), σ_2 is the viscous damping coefficient (Ns/m), σ_a is the stiffness of $w(t)$ (N/m), σ_b is the viscous damping coefficient influenced by $V(t)$ (Ns/(mV)), and a_0 is the constant value (1/m). Numerical values for the parameters used in this study are addressed in Table 2. In the table, MF represents the magnification factor of the parameters.

Table 2. Parameters for damper model

parameter	value	parameter	value
σ_0 $(N/(mV))$	28,815MF*	σ_a (N/m)	30,542MF
σ_1 (Ns/m)	0.131MF	σ_b $(Ns/(mV))$	16.3MF
σ_2 (Ns/m)	29.6MF	a_0 (V/N)	3,198

The maximum capacity of each damper is assumed to be about 2,800 N and maximum voltage input to the damper, $V_{\max}$, is 10 volts. The two dampers are positioned at 4.690 m (3% of the cable length) from the bottom support and the twin damper setup with the angle between two dampers of 60 degree can provide in-plane forces transverse to the cable.

4.2 Evaluation Criteria

The three semiactive control algorithms considered in this study are evaluated using a set of the evaluation criteria. The first and second evaluation criteria are measurements of the displacement at mid-span and quarter-span, respectively. The third and forth evaluation criteria are root mean square (RMS) of cable deflection and velocity. Each evaluation criterion is normalized by the uncontrolled value. All the evaluation criteria are defined by

$$J_1 = \frac{\max(v_{midspan}(t))}{\max(v_{midspan}(t))|_{uncontrolled}} \tag{21}$$

$$J_2 = \frac{\max(v_{quarterspan}(t))}{\max(v_{quarterspan}(t))|_{uncontrolled}} \tag{22}$$

$$J_3 = \frac{\sigma^2_{displacement}(t)}{\sigma^2_{displacement}(t)|_{uncontrolled}} \tag{23}$$

where

$$\sigma^2_{displacement}(t) = E\left[\int_0^L v^2(x,t)dx\right] = trace\left\{M^{1/2}E[\eta(t)\eta^T(t)]M^{1/2}\right\} \tag{24}$$

$$J_4 = \frac{\sigma^2_{velocity}(t)}{\sigma^2_{velocity}(t)|_{uncontrolled}} \tag{25}$$

where

$$\sigma^2_{velocity}(t) = E\left[\int_0^L \dot{v}^2(x,t)dx\right] = trace\left\{M^{1/2}E[\dot{\eta}(t)\dot{\eta}^T(t)]M^{1/2}\right\} \tag{26}$$

4.3 External Load

In the numerical example, a specifically generated time history of dynamic load is used as the external wind load. The wind load can be expressed as Eq. (27) which combines static and dynamic components.

$$F(t) = \frac{1}{2}\cdot\rho\cdot C\cdot A\cdot\bar{V}^2 + \rho\cdot C\cdot A\cdot\bar{V}\cdot v'(t) + \frac{1}{2}\cdot\rho\cdot C\cdot A\cdot v'(t)^2 \tag{27}$$

where ρ, C, A, $\bar{V}$ and v' are the air density, the wind force coefficient, the area under the wind load, the mean wind velocity and the wind velocity fluctuation, respectively.

The wind velocity fluctuation, $v'(t)$, can be generated by using spectral representation method (or Shinozuka-Deodatis method). The wind velocity fluctuation is defined as the sum of three vectors of along-wind direction, cross-wind direction and vertical direction in general, however, only the component of along-wind direction is considered in this investigation.

The power spectral density of the along-wind velocity fluctuation can be written as follows (Kaimal, 1972):

$$S_v(\omega) = \frac{1}{2}\frac{1}{2\pi}200U_*^2\frac{z}{\bar{V}(z)}\frac{1}{\left(1+50\dfrac{|\omega|z}{2\pi\bar{V}(z)}\right)^{5/3}} \tag{28}$$

where ω is the angular frequency, z is the height of the cable, $\bar{V}(z)$ is the mean wind velocity at the height of z and U_* is the shear velocity. Fig. 2 shows the Kaimal Spectrum considered in this study.

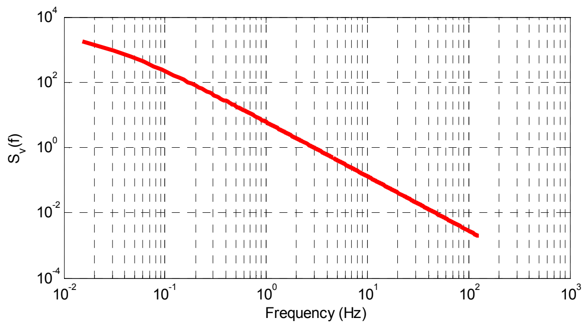

Fig. 2. Kaimal spectrum

Then, the time history of the wind velocity fluctuation can be obtained through the spectral representation method as follows (Deodatis, 1996):

$$v'(t) = \sqrt{2\Delta\omega}\sum_{l=1}^{N}\sqrt{S(\omega_l)}\cos(\omega_l t + \phi_l) \tag{29}$$

where ϕ is phase angle randomly defined at the range of $0 \sim 2\pi$.

Fig. 3 shows the dynamic components of the generated wind load, which is used in the numerical simulation.

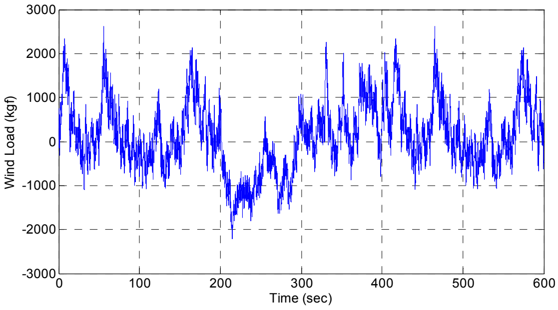

Fig. 3. Dynamic components of the generated wind load

4.4 Numerical Results

A series of numerical simulations were conducted. In all the numerical simulations, 20 shape functions are used. To compare the performance of the semiactive control systems employing several semiactive control algorithms with that of passively operated control systems, two cases are considered in which the MR fluid dampers are used in a passive mode by maintaining constant voltages to the devices: passive-off (V = 0 volts) and passive-on (V = 10 volts). For all the semiactive control algorithms considered except the maximum energy dissipation algorithm (MED), the optimal parameters for each controller should be obtained to make the well-performed controller. In the case of the control algorithm based on the Lyapunov stability theory (LYAP), the several tries are carried out by varying the values in Q_p because of no standard method for selecting Q_p. The resulting Q_p is the unity matrix with order of 40. In the case of the clipped optimal control algorithm (CO), after varying weight R from 10^0 to 10^{-11} with the fixed weight $Q = diag(0.5M, 0.5M)$, the optimal control weighting matrix is obtained as $R = 10^{-6}$.

Fig. 4 shows the normalized responses of control algorithms with respect to the uncontrolled system. As shown in the figure, all the control algorithms significantly reduce the responses compared with the uncontrolled system by about 20 % to 60%. In two passively operated control systems, the performance of the passive-on case is much better than that of the passive-off case. It also can be seen from the figure that the control performances of two semiactive control algorithms such as the maximum energy dissipation algorithm and the clipped-optimal control algorithms are slightly better than that of the passive-on. On the other hand, the Lyapunov stability theory-based control algorithm has the comparable performance with the passive-on case. The performance of the clipped-optimal control algorithm is a little better than the maximum energy dissipation algorithm.

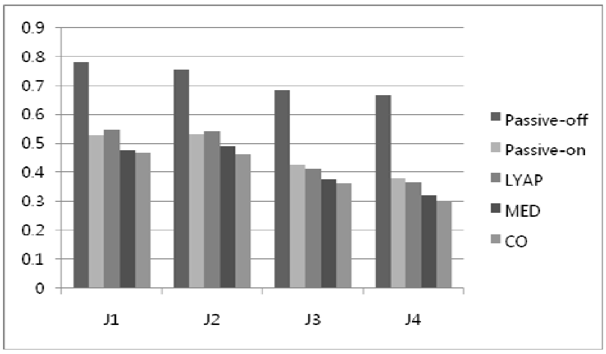

Fig. 4. Normalized responses with respect to the uncontrolled system

Fig. 5 shows the maximum and RMS control forces of MR dampers in each control algorithm. Among the semiactive control algorithms considered in this study, the maximum damper force of the maximum energy dissipation algorithm is the smallest except the passive-off case. On the other hand, the clipped-optimal control algorithm has the smallest RMS damper force except the passive-off case, which means that this algorithm might consume the smallest energy compared with the others.

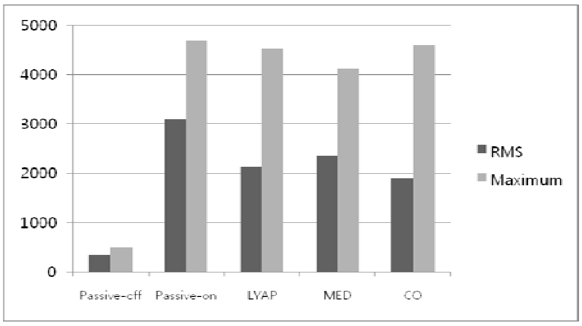

Fig. 5. Control forces of MR damper in each control algorithm (unit: N)

To preliminarily investigate the adaptability of each control method with respect to the variation of the magnitude of the excitation, the additional simulation is carried out by using the input excitation with one-fourth of the magnitude of the original excitation. As shown in Fig. 6, it can be seen that the performance of the passive-on case gets a little worse compared to the original excitation case. On the other hand, the performance of the clipped optimal control algorithm is superior to those of the other control cases.

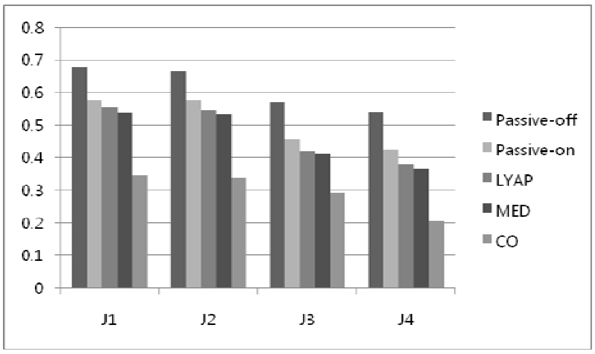

Fig. 6. Normalized responses in the case of the input excitation with one-fourth of the magnitude of the original excitation

5. CONCLUSIONS

In this study, the effectiveness of an MR damper-based semiactive control systems for vibration suppression of stay cables has been numerically investigated. The three commonly used semiactive control algorithms, such as the control based on Lyapunov stability theory, the maximum energy dissipation, and clipped optimal control algorithms, are applied to semiactive control systems for a stay cable, and the performance of each control algorithm is numerically evaluated. It is verified from the numerical simulation results that all the control algorithms including passively operated cases has a good control performance to mitigate cable vibration. In addition, it is demonstrated from the numerical simulation results that the clipped optimal control algorithm shows the best control performance compared with the other two semiactive control algorithms as well as the passive cases. Therefore, it might be considered as one of the most appropriate control algorithms for the MR damper-based semiactive control systems to suppress vibration of a cable in a cable-stayed bridge. However, since numerical simulations in this study are not sufficient to validate it, more comprehensive numerical as well as experimental investigation should be followed.

6. ACKNOWLEDGEMENTS

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