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Roll angle estimation for smart munitions under GPS jamming environment

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Abstract: A smart munition revolves by 2~3Hz when it flights. Before guidance starts, the roll estimation of a smart munition is an important factor to improve the navigation performance. The roll estimation algorithm is already known using IMU and GPS. But GPS can't be available as a measurement under GPS jamming environment. This paper explains how to estimate the roll angle of a smart munition under jamming environment by using IMU, earth magnetism measurements. The system and measurement models of the extended Kalman filter are designed for GPS jamming environment. Under GPS jamming environment, the proposed method shows better performance than the previous method by Monte Carlo simulation.

1. INTRODUCTION

A smart munition is a low cost military weapon with a guidance system. When it is fired from an artillery or tank, it flights by the inertial force before guidance starts. Then a guidance usually starts after its height is a maximum value. A smart munition revolves by 2~3Hz in flight. So it is difficult to know the roll angle of it. The roll angle estimation is very important factor to reduce the navigation error of a smart munition.

To estimate the roll angle of a smart munition, IMU(Inertial Measurement Unit) and GPS(Global Positioning System) can be used(Lucia, 1995). The performance analysis of a MEMS(Micro Electro Mechanical System) device under high gravity environment is already studied(T.Gordon, 2003). And the performance analysis of a MEMS device under high spin condition is already studied(Bradford, 1998). A low cost MEMS gyroscope can't measure the angular rate of a smart munition. So another method is needed to know the roll angle. A roll angle is related to a pitch rate and yaw rate(Lucia, 1995). If the pitch rate and yaw rate of a smart munition are given, we can calculate the roll angle from it. The pitch rate and yaw rate of a smart munition can be obtained from the GPS velocity of the smart munition. In chapter 2, the extended Kalman filter(EKF) with the GPS measurement is introduced, which is already known by previous paper(HeeYoung, 2007).

Under GPS jamming environment, a pitch rate and yaw rate can't be obtained. In this case, we can assume that the pitch rate and yaw rate are constant which is calculated by fall free trajectory of a smart munition. But this assumption makes the roll error increased. To reduce roll error, other measurement is needed in the EKF. The earth magnetism is a good measurement to reduce the roll error.

In Chapter 3, this paper explains the roll estimation method using the earth magnetism as a measurement. In general, the flight distance of a smart munition is short within 100Km. Within this area, the earth magnetism is hardly changed. So we can assume that the earth magnetism is constant. To represent the earth magnetism with respect to the navigation frame, an earth magnetism frame is defined. After the transformation between two coordinates is explained, the system and measurement models are designed.

In chapter 4, we explain three simulation results, which are the previous method without GPS jamming, the previous method with GPS jamming and the proposed method with GPS jamming.

2. The EKF design using the GPS measurement

2.1 The roll angle from a pitch rate and yaw rate

The attitude of a gyroscope is aligned with the body frame. The X axis output of a gyroscope can't measure its roll rate because the roll rate is usually larger than 2Hz. If the pitch rate and yaw rate are given from GPS velocity, The roll angle is calculated by the following equations(Lucia, 1995).

$$\begin{bmatrix} \omega_y^{\ b} \\ \omega_z^{\ b} \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ \dot{\psi} \end{bmatrix} \quad (1)$$

$\omega_y^{\ b}$: *Y axis output of a gyroscope aligned with body frame*
$\omega_z^{\ b}$: *Z axis output of a gyroscope aligned with body frame*
ϕ : *roll angle of a smart munition*
$\dot{\theta}$: *pitch rate of a smart munition*
$\dot{\psi}$: *yaw rate of a smart munition*

From the equation 1, the following equation holds.

$$\cos\phi = \frac{\dot{\theta}\omega_y{}^b + \dot{\psi}\omega_z{}^b}{\dot{\theta}^2 + \dot{\psi}^2} \tag{2}$$

$$\sin\phi = \frac{\dot{\psi}\omega_y{}^b - \dot{\theta}\omega_z{}^b}{\dot{\theta}^2 + \dot{\psi}^2} \tag{3}$$

Figure 1 shows the roll angle related to the pitch rate and yaw rate of a smart munition when the roll angle is 90 degrees. In this case, The Y axis of a gyroscope measures the yaw rate of the smart munition and the Z axis measures its negative pitch rate. This result is coincident with the equation 1. The equation 1 can be explained by the concept of a coordinate rotation for any angles as well as 90 degrees.

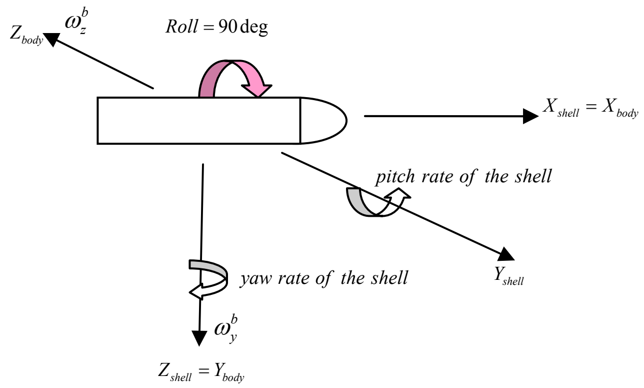

Fig. 1. The roll angle related to the pitch rate and yaw rate

The pitch rate and yaw rate of a smart munition can be obtained by GPS data. The pitch angle and yaw angle of the smart munition are calculated by the equations 4, 5 and their rates are calculated by the equations 6, 7(Lucia, 1995).

$$\tan\theta = -\frac{V_D}{\sqrt[2]{V_N{}^2 + V_E{}^2}} \tag{4}$$

$$\tan\psi = \frac{V_E}{V_N} \tag{5}$$

In the equations (4), (5), V_N, V_E, V_D are the GPS velocity of a smart munition, which is represented in the navigation frame.

$$\dot{\theta} \cong \frac{\Delta\theta}{\Delta t} \tag{6}$$

$$\dot{\psi} \cong \frac{\Delta\psi}{\Delta t} \tag{7}$$

2.2 The EKF design using the GPS measurement

The EKF using GPS measurement is known by previous paper(HeeYoung, 2007). The state variables of this filter are the roll rate(p), initial roll(ϕ_0), Y axis bias of the gyroscope(ω_y^{bias}) and Z axis bias of the gyroscope(ω_z^{bias}).

The system equation is 4×4 identity matrix. It is assumed that all state variables are constant regardless of time(Lucia, 1995). The system equation is as follows.

$$\dot{x}(k+1) = F(k)x(k) + w(k) \tag{8}$$

$$\begin{aligned} &\textit{where}\ \ w(k): \textit{system noise} \\ &x(k) = \begin{bmatrix} p & \phi_0 & \omega_y^{bias} & \omega_z^{bias} \end{bmatrix}^T \\ &F(k) = I_{4\times4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

The equations (2), (3) are used to drive the measurement equations. We can also use the gyroscope bias as a measurement. They are obtained by the following equations.

$$\sum_1^N \frac{\omega_y^b}{N} = \omega_y^{bias} \tag{9}$$

$$\sum_1^N \frac{\omega_z^b}{N} = \omega_z^{bias} \tag{10}$$

By applying perturbation method to the equations (2), (3), (9), (10), the measurement equations are as follows.

$$\delta z(k+1) = H_1(k)\delta x(k) + v(k) \tag{11}$$

$$\begin{aligned} &\textit{where}\quad v = \textit{measurement noise} \\ &\delta z(k) = \left[\delta\left(\sin\hat{\phi}(t)\right) \ \ \delta\left(\cos\hat{\phi}(t)\right) \ \ \delta\left(\sum \frac{\omega_y{}^b}{k}\right) \ \ \delta\left(\sum \frac{\omega_z{}^b}{k}\right) \right]^T \\ &H_1(k) = \begin{bmatrix} t\cos(pt+\phi_0) & \cos(pt+\phi_0) & \frac{\dot{\psi}}{\dot{\theta}^2+\dot{\psi}^2} & -\frac{\dot{\theta}}{\dot{\theta}^2+\dot{\psi}^2} \\ -t\sin(pt+\phi_0) & -\sin(pt+\phi_0) & \frac{\dot{\theta}}{\dot{\theta}^2+\dot{\psi}^2} & \frac{\dot{\psi}}{\dot{\theta}^2+\dot{\psi}^2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned}$$

The extended Kalman filter is given as follows(E.W. Kamen, 1999).

Time propagation

$$x^{-}(k) = F(k-1)x(k-1)$$

$$P^{-}(k) = F(k-1)P(k-1)F^{T}(k-1) + Q(k-1)$$

Measurement update (12)

$$K(k) = P^{-}(k)H^{T}(k)\left(H(k)P^{-}(k)H^{T}(k) + R(k)\right)^{-1}$$

$$P(k) = \left(I - K(k)H(k)\right)^{-1} P^{-}(k)$$

$$x(k) = x^{-}(k) + K(k)\left(z(k) - h(k, x^{-}(k))\right)$$

3. The EKF design using the earth magnetism measurement

3.1 The earth magnetism expressed in the navigation frame

The earth magnetism consists two parts. One is declination, the other is inclination. The magnetic declination is defined as the angle between magnetic north and geographic north. The magnetic inclination is defined as the angle between magnetic vector and the tangential plane of the horizon at the position where is measured.

In figure 2 and figure 3, the axes of the navigation frame(X_n, Y_n, Z_n) are defined as geographic north, east, down respectively. In figure 2, the attitude of a magnetic sensor is exactly aligned with the body frame(X_b, Y_b, Z_b). The magnetic sensor output consists of the values measured by earth magnetism in the body frame. We normalize magnetic sensor output by dividing its magnitude. The normalized magnetic sensor output can be expressed with respect to the navigation frame by coordinate transformation. The equation (13) represents these relations. The matrix C_b^n is a coordinate transform matrix from the body frame to the navigation frame.

In figure 3, we define the earth magnetism frame (X_{em}, Y_{em}, Z_{em}). The X axis of the earth magnetism frame is the direction of earth magnetic vector. The Y, Z axis are determined by the coordinate transformation from the navigation frame to the earth magnetism frame. In this coordinate transformation, the yaw is declination, the pitch is inclination and the roll is zero. In other words, the earth magnetism frame can be aligned with the navigation frame by coordinate transformation. The normalized magnetic sensor output expressed in the earth magnetism frame is $\begin{bmatrix}1 & 0 & 0\end{bmatrix}^T$, which is theoretical value. The equation (14) represents these relations. In this equation, the matrix C_n^m is the coordinate transform matrix from the navigation frame to the earth magnetism frame.

In Seoul(the capital city of Korea) the magnetic declination is 6.5 degrees(west) and the magnetic inclination is 53 degrees(down). This means that the yaw(ψ) is -6.5 degrees and the pitch(θ) is -53 degrees in coordinate transformation. We think these values as constant because the trajectory of a smart munition is within 100Km.

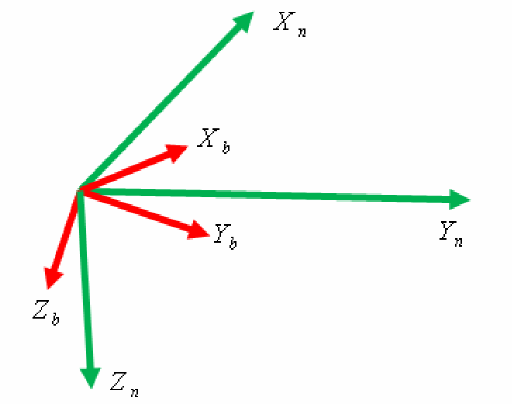

Fig. 2. Navigation frame and body frame

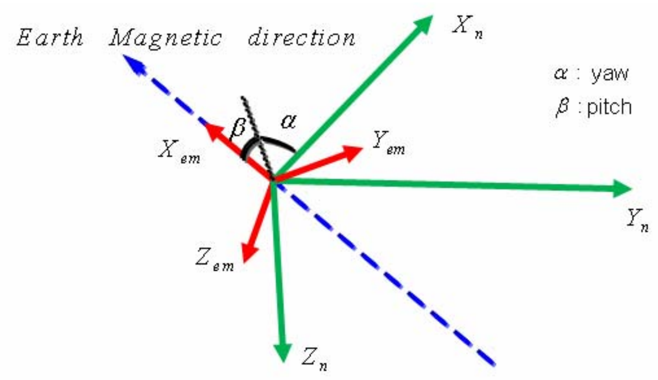

Fig. 3. Navigation frame and earth magnetism frame

$$\begin{bmatrix} X_n \\ Y_n \\ Z_n \end{bmatrix} = C_1^T C_2^T C_3^T \begin{bmatrix} X_b \\ Y_b \\ Z_b \end{bmatrix} = C_b^n \begin{bmatrix} X_b \\ Y_b \\ Z_b \end{bmatrix} \tag{13}$$

where

$$C_1^T = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}, C_2^T = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}, C_3^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

$$\begin{bmatrix}1\\0\\0\end{bmatrix}=\begin{bmatrix}X_{em}\\Y_{em}\\Z_{em}\end{bmatrix}$$
$$=\begin{bmatrix}\cos\beta & 0 & -\sin\beta\\ 0 & 1 & 0\\ \sin\beta & 0 & \cos\beta\end{bmatrix}\begin{bmatrix}\cos(\alpha) & \sin(\alpha) & 0\\ -\sin(\alpha) & \cos(\alpha) & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}X_n\\Y_n\\Z_n\end{bmatrix} \quad (14)$$
$$=C_n^m\begin{bmatrix}X_n\\Y_n\\Z_n\end{bmatrix}$$

In figure 2 and figure 3, these two normalized magnetic outputs expressed by the navigation frame must be same value. In the equation (13), $[X_n, Y_n, Z_n]^T$ is substituted into the equation (14), the following equation holds. We will use this equation (15) as a measurement equation in the EKF.

$$\begin{bmatrix}X_m\\Y_m\\Z_m\end{bmatrix}=\left(C_b^n\right)^T\left(C_n^m\right)^T\begin{bmatrix}1\\0\\0\end{bmatrix} \quad (15)$$

3.2 The EKF design using an earth magnetism measurement

If GPS signal can't be available because of GPS jamming, we can use the magnetic sensor output as a measurement to reduce the navigation error. The state variables are as follows.

$$x(k)=\begin{bmatrix}\omega_y^{bias} & \omega_z^{bias} & p & \phi_0 & \theta & \psi\end{bmatrix}^T \quad (16)$$

The system equation is 6×6 identity matrix as the equation (8). It is assumed that all state variables are constant regardless of time.

The measurement equation is as follows.

$$\delta z(k+1)=H(k)\delta x(k)+\nu(k) \quad (17)$$

$$H(k)=\begin{bmatrix} & H_1(k) & & O_{4\times 2} & \\ O_{3\times 2} & H_2(k) & H_3(k) & H_4(k) & H_5(k)\end{bmatrix}$$

In the equation (17), H_1 is given by the equation (11). From H_2 to H_5, all components of these matrices are obtained by partial derivative in the equation (15) as follows.

$$\begin{aligned}H_2(k)&=\frac{\partial([X_m,Y_m,Z_m]^T)}{\partial p}\\ &=\frac{\partial C_3}{\partial p}\times C_2\times C_1\times C_n^{mT}\times[1\ \ 0\ \ 0]^T\end{aligned} \quad (18)$$

$$\begin{aligned}H_3(k)&=\frac{\partial([X_m,Y_m,Z_m]^T)}{\partial \phi_0}\\ &=\frac{\partial C_3}{\partial \phi_0}\times C_2\times C_1\times C_n^{mT}\times[1\ \ 0\ \ 0]^T\end{aligned} \quad (19)$$

$$\begin{aligned}H_4(k)&=\frac{\partial([X_m,Y_m,Z_m]^T)}{\partial \theta}\\ &=C_3\times\frac{\partial C_2}{\partial \theta}\times C_1\times C_n^{mT}\times[1\ \ 0\ \ 0]^T\end{aligned} \quad (20)$$

$$\begin{aligned}H_5(k)&=\frac{\partial([X_m,Y_m,Z_m]^T)}{\partial \psi}\\ &=C_3\times C_2\times\frac{\partial C_1}{\partial \psi}\times C_n^{mT}\times[1\ \ 0\ \ 0]^T\end{aligned} \quad (21)$$

4. Simulation

In this chapter, the previous method and the proposed method are simulated. Figure 4 shows the simulation trajectory of a smart munition. The start point is (0,0,0), which is marked by circle. Its initial velocity is 1000m/s and total flight time is 108.6 seconds. The initial pitch and yaw are 35 degrees and 20 degrees respectively. The initial roll and its roll rate are random valuables, whose ranges are given by table 1. In this trajectory, we make the gyroscope data, GPS data and magnetic sensor data, which have noise. The repetition of Monte Carlo simulation is 100 and we calculate the mean of the absolute roll error. By this value, we evaluate the performance of three roll estimation methods.

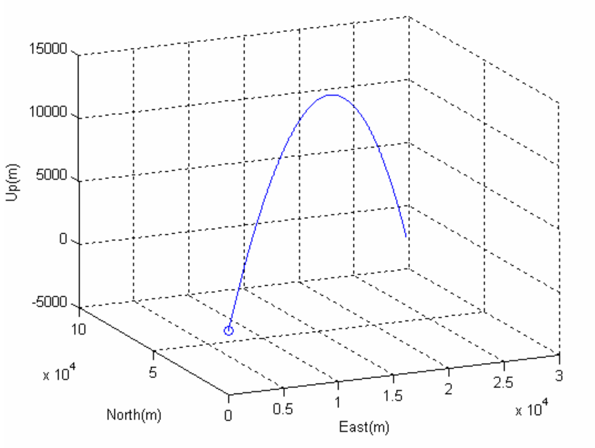

Fig. 4. The trajectory of a smart munition

The detail conditions in this simulation are given in table 1.

Table 1. Simulation condition

Characteristics	Value
Exit velocity	1000 m/s
Initial roll	0~360 deg (random)
Initial pitch	35 deg
Initial yaw	20 deg
Roll rate	1~3 Hz (random)
Gyroscope sampling rate	10 Hz
Gyroscope bias	150 deg/h
Rate noise density	0.05 deg/s/ $\sqrt{Hz}$
Scale factor error	1000 ppm
Non-orthogonal angle	3 deg
Magnetic sensor noise density	2.5×10^{-4}
GPS velocity noise	0.1 m/s
GPS position noise	10 m
GPS sampling rate	1 Hz
Monte Carlo simulation	50 (repetition)

4.1 The simulation with a GPS measurement

In the equations (2), (3), the pitch rate, yaw rate can be calculated by GPS data. The simulation result using the GPS measurement is shown in figure 5. The dot line is the mean of the absolute roll error and upper line is its standard deviation. Within 10 seconds, the trajectory has a big error because the EKF can't know the initial roll and its rate. In real case, it is impossible to know the initial roll and its rate because the smart munition revolves very fast at firing. In figure 5, the EKF estimates the roll error within 1 deg after 30 seconds.

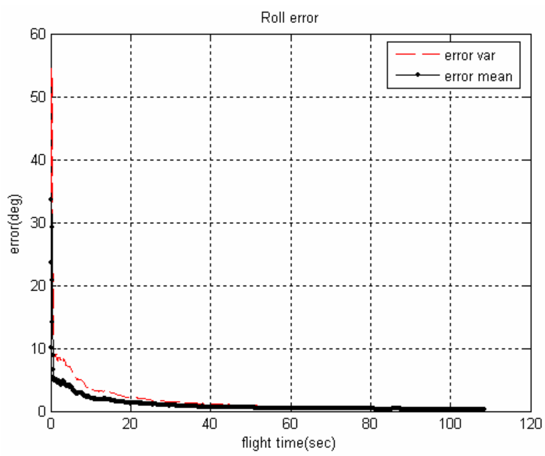

Fig. 5. Roll estimation error using only a GPS measurement

4.2 The simulation with a constrained GPS measurement under GPS jamming environment

If GPS jamming occurs, we can't use GPS signal as a measurement. If the initial pitch angle is 35 degrees and flight time is 108.6 seconds. The angle variation is 70 degrees, so the pitch rate is approximated by -0.64deg/s. We fixed the pitch rate and yaw rate as -0.64deg/s and 0deg/s respectively. In figure 6, before the GPS jamming time(at 38.7 second), roll error trajectory is the same as that in section 4.1. But after jamming time, the roll error increases. So another measurement is needed to reduce this error.

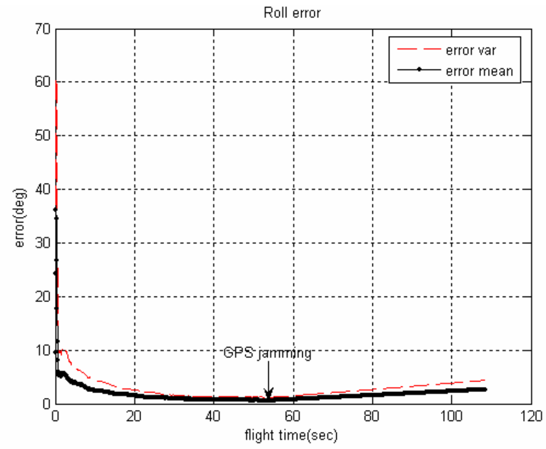

Fig. 6. Roll estimation error using a constrained GPS measurement under jamming environment

4.3 The simulation with an earth magnetism measurement under jamming environment

In this section, we use the earth magnetism as a measurement under GPS jamming environment. We fixed the pitch rate and yaw rate by same reason in section 4.2.

In figure 7, under GPS jamming time, we use the earth magnetism measurement in the EKF and the navigation performance is improved. The simulation result shows very good performance. The roll estimation is stabilized within short time by using earth magnetic sensor. The roll error converges to 0.2 deg within 3 seconds after GPS jamming in figure 7.

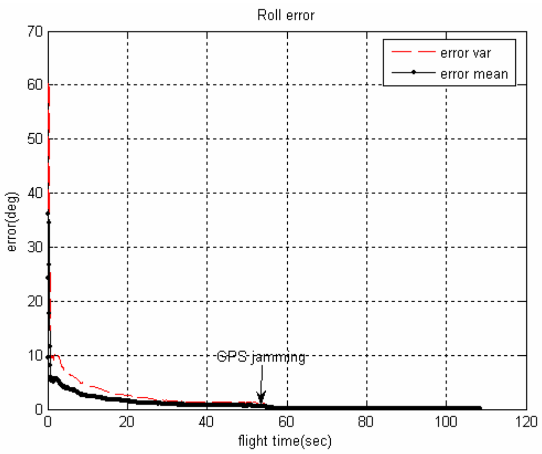

Fig. 7. Roll estimation error using an earth magnetism measurement under jamming environment

5. CONCLUSIONS

The previous roll estimation algorithm of a smart munition is introduced. Based on this knowledge, the roll estimation algorithm using the earth magnetism is suggested. We drive system and measurement equations for the EKF. Without GPS jamming, previous method is 1 deg within 22 seconds. But under GPS jamming environment, the previous method shows that the roll estimation error is increased. To reduce this roll estimation error, the earth magnetism is used as a measurement. Under GPS jamming environment, we didn't use GPS data. We fixed the pitch rate and yaw rate as constant using the initial velocity and flight time of a smart munition. The simulation result of the proposed method shows good performance. The roll estimation error converges to 0.2 deg within 2 seconds under GPS jamming environment.

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