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Design of Fault-Tolerant Controllers for Guaranteed $\mathcal{H}_2$-Performance over Digital Networks with Time-Varying Communication Delays ⋆

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Abstract: This paper addresses the problem of fault tolerant control over communication networks which induce time-varying delays in bounded intervals. The problem is formulated in a discrete-time setting by modeling the controlled plant which may be subject to failures as an uncertain discrete-time process connected to a discrete-time controller via a digital communication network. A procedure is presented for the design of a bank of fault-tolerant controllers capable to stabilize and guarantee an $\mathcal{H}_2$-performance bound for all faulty plant modes in the presence of network-induced time-varying delays.

Keywords: Time-varying delays, LMI, network-based control, guaranteed performance, fault-tolerant control

1. INTRODUCTION

The problem of designing fault-tolerant control (FTC) systems has drawn considerable attention these several last years, see e.g. Blanke et Al. (2003) and references therein. The most important feature of FTC systems is that of a supervisory control which relies on a real-time fault detection and isolation (FDI) algorithm and a controller reconfiguration mechanism. Such a structure allows a flexibility for selecting different controllers according to different component failures, and therefore better performance can be expected for the closed-loop system . However, this holds true when the FDI process does not make an incorrect or *delayed* decision, see e.g. Mariton (1989); Maki et Al. (2004); Mahmoud et Al. (2003). The issue of fault tolerant control in the presence of delays in the feedback loop seems to have been given little attention in the literature. Such delays are very common in networked-based control systems. Indeed, in such networks, it is well known that the information transfer from sensors to controllers and from controllers to actuators is not instantaneous but suffers communication delays. These communication delays can be highly variable due to their strong dependence on variable network conditions such as congestion and channel quality. Clearly, such network-induced delays impact adversely on the stability and performance of the control system (Zhang et Al. (2001); Tipsuwan Chow (2003); Proceedings of the IEEE (2007)). Due to their long-running real-time feature, networked control systems (NCS) should function in a correct manner even in the presence of failures. This makes the issue of fault tolerant control in NCS an important one and entails designing strategies to cope with some of the fundamental problems introduced by the network such as bandwidth limitations, quantization and sampling effects, message scheduling and communication delays. In actual control systems implementation, actuator outages or partial degradation are likely to occur during system operation and this imposes strong requirements on the dependability of the overall control system. Motivated by the above facts, we address the problem of fault-tolerant control for a plant, subject to model *uncertainties* and *actuator faults*, which is controlled over a communication network that induces time-varying delays in bounded intervals.

The paper is organized as follows. In section 2, a networked-based control model for an uncertain plant subject to actuator failures is proposed in a discrete-time setting and the $\mathcal{H}_2$-guaranteed cost control problem is formulated. In section 3, a design procedure for the NCS-based fault-tolerant controller is given. Section 4 presents a numerical example to illustrate the benefit of the proposed FTC design procedure.

2. PROBLEM FORMULATION

The dynamics of the plant we consider in this paper are assumed to be described by the following state space equation

$$\begin{aligned} x(k+1) &= (A + D\Delta(k)E)x(k) + B\mathcal{L}u(k) + x_0\delta(k) \\ z(k) &= C_1x(k) + D_{12}u(k) \\ y(k) &= x(k) \end{aligned} \tag{1}$$

where $x(k) \in \mathcal{R}^n$ is the state of the uncertain plant, $u(k) \in \mathcal{R}^m$ is the control input, $z(k)$ is the regulated (or *performance*) signal and $y(k)$ the measured output. The vector x_0 might be viewed as the initial condition on the

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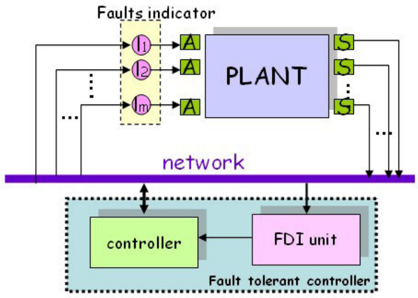

Fig. 1. NCS-based fault tolerant control

state $x(k)$. The matrices A, B, D, E are all real constant matrices and $\Delta(k)$ is an uncertain time-varying matrix belonging to the set

$$\mathbf{\Delta} = \{\Delta : \Delta^T \Delta \leq I\}$$

where I denotes the identity matrix with appropriate dimension. The "performance" matrices are $C_1 = \begin{bmatrix} Q_1^{1/2} & 0 \end{bmatrix}^T$, $D_{12} = \begin{bmatrix} 0 & Q_2^{1/2} \end{bmatrix}^T$ where matrices $Q_1^{1/2}$, $Q_2^{1/2}$ are positive square roots of the positive definite symmetric matrices Q_1, Q_2 of dimension n. The practical operational state of the m actuators is described by the diagonal matrix $\mathcal{L}$ (fault indicator matrix) given by

$$\mathcal{L} = diag\{l_1, \dots, l_m\} \tag{2}$$

where each actuator is ideally modeled as a simple gain l_j, $j = 1, 2, ..., m$ such that $l_j \in \{0, 1\}$. This means that any actuator has only two states, namely in *healthy state* corresponding to $l_j = 1$, and *out of operation* corresponding to $l_j = 0$. Having a finite number of actuators, the set of possible related failure modes is also finite and, by abuse of notation, we denote this set by

$$\mathcal{L} = \{\mathcal{L}^1, \mathcal{L}^2, \dots, \mathcal{L}^N\} \tag{3}$$

with $N = 2^m - 1$. Each failure mode $\mathcal{L}^i, (i = 1, 2, ..., N)$ is therefore an element of the set $\mathcal{L}$. We also view $\mathcal{L}^i$ as a matrix, i.e., as a particular pattern of matrix $\mathcal{L}$ in (2) depending on the values of $l_j (j = 1, 2, ...m)$. Throughout, when $\mathcal{L}$ is invoked as a matrix, it will mean that matrix $\mathcal{L}$ varies over the set of matrices in (3). Note that the faulty mode $\mathcal{L}^i$ is estimated by an FDI unit. In order to ensure that system (1) should remain controllable, we assume that the set $\mathcal{L}$ excludes the element $diag\{0, 0, \dots, 0\}$, i.e., at least one actuator should be healthy.

The networked fault-tolerant control system architecture is shown in figure 1 and consists of the single uncertain plant with its sensors and actuators, controlled by a digital controller through the communication network. The digital communication network induces time-delays from the sensors to the FTC controller and from the FTC controller to the actuators. The total delays in the closed-loop path are modelled as time-varying quantities $\tau(k)$ at time k and they are assumed to lie between the following integer bounds τ_m and τ_M, i.e.,

$$\tau_m \leq \tau(k) \leq \tau_M \tag{4}$$

From the control viewpoint, the actual effect of these communication time-delays can be modeled as a delayed control law

$$u(k) = Kx(k - \tau(k)) \tag{5}$$

where K is a matrix gain of appropriate dimension. The problem addressed is that of finding a matrix gain K such that when the plant is controlled through the digital network, the resulting closed-loop system (1)-(5) is stable and the $\mathcal{H}_2$-norm of the transfer function from $w = \delta(k)$ to z is less than or equal to some $\sqrt{\gamma^*} > 0$ for all possible $\Delta \in \mathbf{\Delta}$ and all failure modes $\mathcal{L}^i \in \mathcal{L}$, i.e.,

$$J = \|T_{wz}\|_2^2 \triangleq \sum_{k=0}^{\infty} z(k)^T z(k) \leq \gamma^* \ \forall \Delta \in \mathbf{\Delta}, \forall \mathcal{L}^i \in \mathcal{L}$$

For such a matrix gain, the control law (5) is referred to as an $\mathcal{H}_2$-guaranteed cost control.

We will proceed through two main steps to design an $\mathcal{H}_2$-guaranteed cost fault-tolerant control in the NCS framework. These steps are :

- (i) construct a fault-tolerant controller (i.e., a robust controller), with structure as given by (5), which achieves the smallest possible value for γ^* under all admissible plant uncertainties and *all* actuator failure modes in the set (2)
- (ii) redesign that part of the above robust controller which is associated to only *one* fault-free actuator in order to improve the robust performance without loss of the stability property of the design in step (i). Step (ii) will be repeated for all m actuators and results in a bank of m controllers.

It follows from inequality $m \leq N = 2^m - 1$, that the cardinality of the bank of controllers (which is equal to the number of actuators) is less than the cardinality of the set $\mathcal{L}$ of faulty modes. For each faulty mode $\mathcal{L}^i$, the controller to be switched-on should be the best as ranked with respect to some closed-loop performance index. In this paper, we will not address the switching and reconfiguration mechanism which will be reported elsewhere, we instead focus on the design of the bank of m controllers.

3. FAULT-TOLERANT CONTROLLER SYNTHESIS

3.1 Robust Performance

The control law (5) applied to plant (1) results in the following system:

$$x(k+1) = A_1 x(k) + B\mathcal{L}Kx(k - \tau(k)) + x_0 \delta(k) \tag{6}$$

where $A_1 = A + D\Delta(k)E$. The $\mathcal{H}_2$-norm of the system (6) is therefore

$$J = \sum_{k=0}^{\infty} z(k)^T z(k) = \textstyle\sum_{k=0}^{\infty} \tilde{x}^T(k) Q \tilde{x}(k) \tag{7}$$

where $\tilde{x}^T(k) = [x^T(k), \ x^T(k - \tau(k))]$, and $Q = diag\{Q_1, \ K^T Q_2 K\}$. Under the assumptions made in section 2, we can state the following sufficient condition for the existence of an $\mathcal{H}_2$-guaranteed cost controller for the uncertain plant (1):

Theorem 1. If there exists a gain matrix K, a scalar $\epsilon > 0$, symmetric positive-definite matrices $P_1 \in \mathcal{R}^{n \times n}$, $R \in \mathcal{R}^{n \times n}$, $S \in \mathcal{R}^{n \times n}$, and matrices $P_2 \in \mathcal{R}^{n \times n}$, $P_3 \in \mathcal{R}^{n \times n}$, $W \in \mathcal{R}^{2n \times 2n}$, $M \in \mathcal{R}^{2n \times n}$ such that the following matrix inequalities are satisfied:

$$\begin{bmatrix} \Gamma & P^T\begin{bmatrix} 0 \\ B\mathcal{L}K \end{bmatrix} - M & \begin{bmatrix} E^T \\ 0 \end{bmatrix} \\ * & -R+K^TQ_2K & 0 \\ * & * & -\epsilon I \end{bmatrix} < 0 \quad (8)$$

$$\begin{bmatrix} W & M \\ * & S \end{bmatrix} \geq 0 \quad (9)$$

with

$$\Gamma = P^T \begin{bmatrix} 0 & I \\ A-I & -I \end{bmatrix} + \begin{bmatrix} 0 & I \\ A-I & -I \end{bmatrix}^T P + \epsilon P^T \begin{bmatrix} 0 & 0 \\ 0 & DD^T \end{bmatrix} P + \begin{bmatrix} \mu R + Q_1 & 0 \\ 0 & P_1 + \tau_M S \end{bmatrix} + \tau_M W + [\, M \;\; 0\,] + [\, M \;\; 0\,]^T$$

$$\mu = 1 + (\tau_M - \tau_m), \quad P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}$$

Then, system (6) is asymptotically stable and the performance (7) satisfies the inequality:

$$J \leq x_0^T P_1 x_0 + \sum_{l=-\tau_M}^{-1} x^T(l) R x(l) + \sum_{\theta=-\tau_M+1}^{0} \sum_{l=-1+\theta}^{-1} y^T(l) S y(l) + \sum_{\theta=-\tau_M+2}^{-\tau_m+1} \sum_{l=\theta-1}^{-1} x^T(l) R x(l) \quad (10)$$

where $y(l) = x(l+1) - x(l)$.

Proof. See the appendix.

Remark 1. The $*$ sign represents blocks that are readily inferred by symmetry

Remark 2. The upper bound in equation (10) depends on the initial condition x_0 of system (6). To remove this dependence, we assume that that the initial state of system (6) might be arbitrary but belongs to the set $\mathcal{S} = \{x(l) \in \mathcal{R}^n : x(l) = Uv,\ v^T v \leq 1,\ l = -\tau_M, -\tau_M+1, \ldots, -\tau_m\}$, where U is a given matrix. Then inequality (10) leads to:

$$J \leq \lambda_{\max}(U^T P_1 U) + \rho_1 \lambda_{\max}(U^T R U) + \rho_2 \lambda_{\max}(U^T S U) \quad (11)$$

where $\lambda_{\max}(\cdot)$ denotes the maximum eigenvalue of matrix $(\cdot)$, $\rho_1 = \mu(\tau_M + \tau_m)/2$ and $\rho_2 = 2\tau_M(\tau_M + 1)$.

3.2 Step (i): Controller Design

Now, we derive the guaranteed cost controller in terms of the feasible solutions to a set of linear matrix inequalities.

We use matrix inversion formula to get:

$$P^{-1} = \begin{bmatrix} P_1^{-1} & 0 \\ -P_3^{-1} P_2 P_1^{-1} & P_3^{-1} \end{bmatrix}$$

and we set $X = P_1^{-1}$, $Y = P_3^{-1}$ and $Z = -P_3^{-1} P_2 P_1^{-1}$. We further restrict M to the following form in order to obtain a linear matrix inequality (LMI) (see e.g. Chen et Al. (2003)):

$$M = \delta P^T \begin{bmatrix} 0 \\ B\mathcal{L}K \end{bmatrix}$$

where δ is a scalar parameter. Pre- and post-multiply equation (8) by $diag\{(P^{-1})^T,\ P_1^{-1},\ I\}$ and $diag\{P^{-1},\ P_1^{-1},\ I\}$ respectively; also pre- and post-multiply equation (9) by $diag\{(P^{-1})^T, P_1^{-1}\}$ and $diag\{P^{-1}, P_1^{-1}\}$ and denote:

$$L = P_1^{-1} R P_1^{-1},\ F = K P_1^{-1},\ \bar{S} = S^{-1},$$

$$(P^{-1})^T W P^{-1} = \begin{bmatrix} \bar{W}_1 & \bar{W}_2 \\ * & \bar{W}_3 \end{bmatrix}.$$

Applying the Schur complement and expanding the block matrices, we obtain the following result under the assumptions made in section 2.

Theorem 2. Suppose that for a prescribed scalar δ, there exists a scalar $\epsilon > 0$, matrices $X > 0$, Y, Z, F, $L > 0$, $\bar{S} > 0$, $\bar{W}_1$, $\bar{W}_2$, $\bar{W}_3$, such that the following matrix inequalities are satisfied:

$$\begin{bmatrix} \Psi_1 & \Psi_2 & 0 & \Psi_{41} \\ * & \Psi_3 & (1-\delta)B\mathcal{L}F & \Psi_{42} \\ * & * & -L & \Psi_{43} \\ * & * & * & \Psi_5 \end{bmatrix} < 0 \quad (12)$$

$$\begin{bmatrix} \bar{W}_1 & \bar{W}_2 & 0 \\ * & \bar{W}_3 & \delta B\mathcal{L}F \\ * & * & X\bar{S}^{-1}X \end{bmatrix} \geq 0 \quad (13)$$

where

$$\Psi_1 = Z + Z^T + \mu L + \tau_M \bar{W}_1$$
$$\Psi_2 = Y + X(A-I)^T - Z^T + \tau_M \bar{W}_2 + \delta(B\mathcal{L}F)^T$$
$$\Psi_3 = -Y - Y^T + \tau_M \bar{W}_3 + \epsilon DD^T$$
$$\begin{bmatrix} \Psi_{41} \\ \Psi_{42} \\ \Psi_{43} \end{bmatrix} = \begin{bmatrix} XE^T & \tau_M Z^T & 0 & X & Z^T \\ 0 & \tau_M Y^T & 0 & 0 & Y^T \\ 0 & 0 & F^T & 0 & 0 \end{bmatrix}$$
$$\Psi_5 = diag\{-\epsilon I, -\tau_M \bar{S}, -Q_2^{-1}, -Q_1^{-1}, -X\}$$

Then, the control law

$$u(k) = FX^{-1}x(k - \tau(k)) \quad (14)$$

is a $\mathcal{H}_2$-guaranteed cost networked control law for system (1) and the corresponding performance satisfies:

$$J \leq \lambda_{\max}(U^T X^{-1} U) + \rho_1 \lambda_{\max}(U^T X^{-1} L X^{-1} U) + \rho_2 \lambda_{\max}(U^T \bar{S}^{-1} U) \quad (15)$$

where $\rho_1 = \mu(\tau_M + \tau_m)/2$ and $\rho_2 = 2\tau_M(\tau_M + 1)$.

Remark 3. From (15), we establish the following inequalities:

$$\begin{bmatrix} -\alpha I & U^T \\ * & -X \end{bmatrix} < 0, \quad \begin{bmatrix} -\beta I & U^T \\ * & -XL^{-1}X \end{bmatrix} < 0, \quad \begin{bmatrix} -\gamma I & U^T \\ * & -\bar{S} \end{bmatrix} < 0 \quad (16)$$

where α, β, and γ are scalars to be determined. It is worth noting that condition (16) is not a LMI because of the term $-XL^{-1}X$. This is also the case for condition (13) which is not a LMI because of the term $X\bar{S}^{-1}X$. Note that for any matrix $X > 0$, we have:

$$X\bar{S}^{-1}X \geq 2X - \bar{S}, \quad XL^{-1}X \geq 2X - L$$

Given a prescribed scalar δ, the design problem of the optimal guaranteed cost controller can be formulated therefore as the following optimization problem:

$$\textbf{OP1:} \quad \min_{\epsilon, X, Y, Z, F, L, \bar{S}, \bar{W}_1, \bar{W}_2, \bar{W}_3} (\alpha + \rho_1 \beta + \rho_2 \gamma)$$

$$\text{s.t.} \begin{cases} \text{(i)} & \text{Equation}(12) \\ \text{(ii)} & \begin{bmatrix} \bar{W}_1 & \bar{W}_2 & 0 \\ * & \bar{W}_3 & \delta B\mathcal{L}F \\ * & * & 2X - \bar{S} \end{bmatrix} \geq 0 \\ \text{(iii)} & \begin{bmatrix} -\alpha I & U^T \\ * & -X \end{bmatrix} < 0, \quad \begin{bmatrix} -\beta I & U^T \\ * & -2X + L \end{bmatrix} < 0, \\ & \begin{bmatrix} -\gamma I & U^T \\ * & -\bar{S} \end{bmatrix} < 0 \end{cases} \quad (17)$$

Clearly, the above optimization problem (17) is a convex optimization problem which can be effectively solved by existing LMI software (Gahinet et Al. (1995)). Thus, the minimization of $\alpha + \rho_1\beta + \rho_2\gamma$ implies the minimization of the cost in (7). By applying a simple one-dimensional search over δ for a certain τ_M, a global optimum cost can be found.

3.3 Robust performance with at least a fault-free actuator

Based on the controller designed in Theorem 2, let us assume that actuator i is fault-free, then we can redesign the i-th row of controller gain matrix K to improve the robust performance for the system against actuator failures. We can rewrite the overall control system as

$$x(k+1) = A_1 x(k) + (B_{\bar{i}}\mathcal{L}_{\bar{i}}K_{\bar{i}} + b_i k_i)x(k-\tau(k)) \qquad (18)$$

where $A_1 = A + D\Delta(k)E$, matrix $K_{\bar{i}}$ is obtained by deleting the i-th row from K, $B_{\bar{i}}$ is obtained by deleting the i-th column from B and $\mathcal{L}_{\bar{i}}$ is obtained by deleting i-th row and i-th column from $\mathcal{L}$. The cost function related to system (18) reads as:

$$J = \sum_{k=0}^{\infty} \tilde{x}^T(k)Q\tilde{x}(k) \qquad (19)$$

with $\tilde{x}^T(k) = [x^T(k),\ x^T(k-\tau(k))]$, $Q = diag\{Q_1,\ k_i^T Q_{2i}k_i + K_{\bar{i}}^T Q_{2\bar{i}}K_{\bar{i}}\}$, where $Q_{2\bar{i}}$ is obtained by deleting the i-th row and i-th column from Q_2. With regard to system (18) where $K_{\bar{i}}$ is assumed to be known, we have the following result

Theorem 3. If there exists a gain matrix k_i, a scalar $\epsilon > 0$, symmetric positive-definite matrices $P_1 \in \mathcal{R}^{n\times n}$, $R \in \mathcal{R}^{n\times n}$, $S \in \mathcal{R}^{n\times n}$, and matrices $P_2 \in \mathcal{R}^{n\times n}$, $P_3 \in \mathcal{R}^{n\times n}$, $W \in \mathcal{R}^{2n\times 2n}$, $M \in \mathcal{R}^{2n\times n}$ such that the following matrix inequalities are satisfied:

$$\begin{bmatrix} \Gamma & P^T\begin{bmatrix} 0 \\ B_{\bar{i}}\mathcal{L}_{\bar{i}}K_{\bar{i}} + b_i k_i \end{bmatrix} - M & \begin{bmatrix} E^T \\ 0 \end{bmatrix} \\ * & -R + k_i^T Q_{2i}k_i + K_{\bar{i}}^T Q_{2\bar{i}}K_{\bar{i}} & 0 \\ * & * & -\epsilon I \end{bmatrix} < 0, \qquad (20)$$

$$\begin{bmatrix} W & M \\ * & S \end{bmatrix} \geq 0. \qquad (21)$$

Then, system (18) is asymptotically stable and the performance (19) satisfies inequality (10).

Proof. Similar to the proof of Theorem 1.

3.4 Step (ii): Controller Redesign

Proceeding as in Step (i), we restrict M to the following case in order to obtain a LMI:

$$M = \delta P^T \begin{bmatrix} 0 \\ b_i k_i \end{bmatrix}$$

where δ is a scalar parameter. Pre- and post-multiply equation (20) with $diag\{(P^{-1})^T,\ P_1^{-1},\ I\}$ and $diag\{P^{-1},\ P_1^{-1},\ I\}$ respectively; also pre- and post-multiply equation (21) with $diag\{(P^{-1})^T, P_1^{-1}\}$ and $diag\{P^{-1}, P_1^{-1}\}$ respectively and denote:

$$L = P_1^{-1}RP_1^{-1},\ F^* = k_i P_1^{-1},\ \bar{S} = S^{-1},$$
$$(P^{-1})^T W P^{-1} = \begin{bmatrix} \bar{W}_1 & \bar{W}_2 \\ * & \bar{W}_3 \end{bmatrix}.$$

The Schur complement trick leads to the following controller redesign result.

Theorem 4. Suppose that for a prescribed scalar δ, there exists a scalar $\epsilon > 0$, matrices $X > 0$, Y, Z, F^*, $L > 0$, $\bar{S} > 0$, $\bar{W}_1$, $\bar{W}_2$, $\bar{W}_3$, such that the following matrix inequalities are satisfied:

$$\begin{bmatrix} \tilde{\Psi}_1 & \tilde{\Psi}_2 & 0 & \tilde{\Psi}_{41} \\ * & \tilde{\Psi}_3 & B_{\bar{i}}\mathcal{L}_{\bar{i}}K_{\bar{i}}X + (1-\delta)b_i F^* & \tilde{\Psi}_{42} \\ * & * & -L & \tilde{\Psi}_{43} \\ * & * & * & \tilde{\Psi}_5 \end{bmatrix} < 0 \qquad (22)$$

$$\begin{bmatrix} \bar{W}_1 & \bar{W}_2 & 0 \\ * & \bar{W}_3 & \delta b_i F^* \\ * & * & X\bar{S}^{-1}X \end{bmatrix} \geq 0 \qquad (23)$$

where

$$\begin{aligned} \tilde{\Psi}_1 &= Z + Z^T + \mu L + \tau_M \bar{W}_1 \\ \tilde{\Psi}_2 &= Y + X(A-I)^T - Z^T + \tau_M \bar{W}_2 + \delta(b_i F^*)^T \\ \tilde{\Psi}_3 &= -Y - Y^T + \tau_M \bar{W}_3 + \epsilon DD^T \\ \begin{bmatrix} \tilde{\Psi}_{41} \\ \tilde{\Psi}_{42} \\ \tilde{\Psi}_{43} \end{bmatrix} &= \begin{bmatrix} XE^T & \tau_M Z^T & 0 & 0 & X & Z^T \\ 0 & \tau_M Y^T & 0 & 0 & 0 & Y^T \\ 0 & 0 & (F^*)^T & XK_{\bar{i}}^T & 0 & 0 \end{bmatrix} \\ \tilde{\Psi}_5 &= diag\{-\epsilon I, -\tau_M\bar{S}, -Q_{2i}^{-1}, -Q_{2\bar{i}}^{-1}, -Q_1^{-1}, -X\} \end{aligned}$$

Then, the ith control law

$$u_i(k) = F^* X^{-1} x(k-\tau(k)) \qquad (24)$$

is a $\mathcal{H}_2$-guaranteed cost networked control law of system (18) and the corresponding performance satisfies:

$$J \leq \lambda_{\max}(U^T X^{-1}U) + \rho_1\lambda_{\max}(U^T X^{-1}LX^{-1}U) + \rho_2\lambda_{\max}(U^T\bar{S}^{-1}U) \qquad (25)$$

where $\rho_1 = \mu(\tau_M + \tau_m)/2$ and $\rho_2 = 2\tau_M(\tau_M + 1)$.

Given a prescribed scalar δ, the redesign problem of the optimal guaranteed cost controller can be formulated as the following convex optimization problem:

$$\textbf{OP2:} \quad \min_{\epsilon,X,Y,Z,F^*,L,\bar{S},\bar{W}_1,\bar{W}_2,\bar{W}_3} (\alpha + \rho_1\beta + \rho_2\gamma)$$

$$\text{s.t.} \begin{cases} \text{(i)} \quad \text{Equation}(22) \\ \text{(ii)} \quad \begin{bmatrix} \bar{W}_1 & \bar{W}_2 & 0 \\ * & \bar{W}_3 & \delta b_i F^* \\ * & * & 2X - \bar{S} \end{bmatrix} \geq 0 \\ \text{(iii)} \quad \begin{bmatrix} -\alpha I & U^T \\ * & -X \end{bmatrix} < 0,\ \begin{bmatrix} -\beta I & U^T \\ * & -2X + L \end{bmatrix} < 0, \\ \qquad \begin{bmatrix} -\gamma I & U^T \\ * & -\bar{S} \end{bmatrix} < 0 \end{cases} \qquad (26)$$

4. EXAMPLE

The dynamics are described by the following matrices:

$$A = \begin{bmatrix} 0.9 & 0 \\ 0.2 & 0.5 \end{bmatrix},\ B = \begin{bmatrix} 0.2 & 0.1 \\ 0 & -0.1 \end{bmatrix},$$
$$D = \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix},\ E = \begin{bmatrix} 0.1 & 0 \\ 0.1 & -0.1 \end{bmatrix},$$

and the simulation parameters are given as:

$$Q_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ Q_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},\ U = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

When $\tau_m = 1, \tau_M = 2$ and $\delta = 1$, by OP1 (17), the cost is obtained as $J_1 = 61.6653$ and the fault-tolerant controller can be designed for Step (i):

$$\begin{bmatrix} k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} -0.0812\times 10^{-5} & -0.1333\times 10^{-5} \\ -0.1865\times 10^{-5} & -0.3060\times 10^{-5} \end{bmatrix}.$$

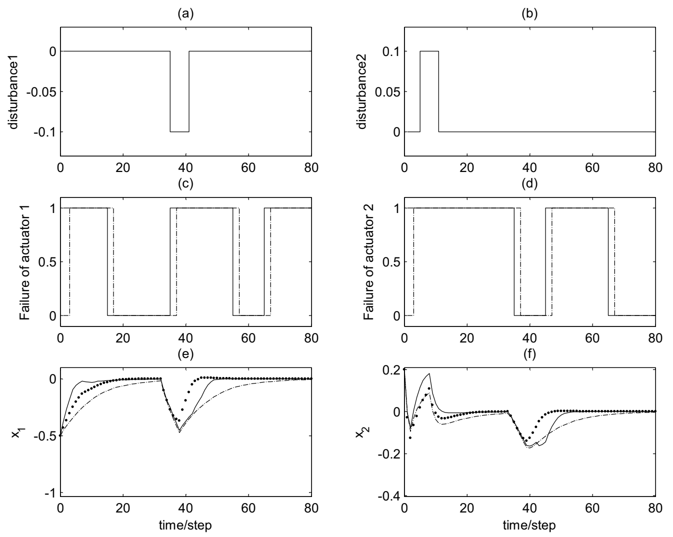

Fig. 2. Disturbance, Actuator Failures and State Response

In Step (ii), by OP2 (26), the cost and the feedback gains are redesigned as

$$J_2 = 39.0026,\ k_1^* = [\,-0.8776\ \ -0.2857\,],$$
$$J_3 = 49.9616,\ k_2^* = [\,-0.6494\ \ -0.4161\,].$$

As a result, the two controllers are determined as follows:

$$\sharp 1 : \begin{bmatrix} k_1^* \\ k_2 \end{bmatrix} = \begin{bmatrix} -0.8776 & -0.2857 \\ -0.1865 \times 10^{-5} & -0.3060 \times 10^{-5} \end{bmatrix},$$

$$\sharp 2 : \begin{bmatrix} k_1 \\ k_2^* \end{bmatrix} = \begin{bmatrix} -0.0812 \times 10^{-5} & -0.1333 \times 10^{-5} \\ -0.6494 & -0.4161 \end{bmatrix}.$$

In the simulation, the step disturbance 1 as shown in Fig. 2-(a) enters into the system at time instant 35 and disappears at time instant 40. The step disturbance 2 as shown in Fig. 2-(b) enters into the system at time instant 5 and disappears at time instant 10 . In figure 2-(c), the solid line represents the failure of actuator 1 which occurs at time instant 15 and disappears at time instant 35, occurs again at time instant 55 and disappears at time instant 65. In figure 2-(d), the solid line represents the failure of actuator 2 which occurs at time instant 35 and disappears at time instant 45, occurs again at time instant 65 and disappears at time instant 80. The delay of fault detection is assumed to be 3 time steps, which is represented by dash-dotted lines as shown in figure 2 (c) and (d). Under the above simulation setting, the state responses are shown in figures 2 (e) and (f):

- the dotted line represents the state response for controller-switching sequence N°1: $\sharp 2$ is the initial controller, and $\sharp 1$ is switched-on at time instant 38, then $\sharp 2$ at time instant 48, $\sharp 1$ at time instant 68;
- the solid line represents the state response for controller-switching sequence N°2: $\sharp 1$ is the initial controller, and $\sharp 2$ is switched-on at time instant 38, then $\sharp 1$ at time instant 48, $\sharp 2$ at time instant 68;
- the dashed line represents the state response under the fault tolerant control of step (i);

The trace of matrices $(x^*)(x^*)^T/(simulation\ time)$ is used as a performance measure for comparison, where x^* represents the state trajectory in the different schemes. After computation, we obtain for the above three scenarios the traces $Tr_1 = 0.0279, Tr_2 = 0.0338, Tr_3 = 0.0499$, which means that $Tr_1 < Tr_2 < Tr_3$. We can draw the conclusion that the proposed method for sequence N°1 is the best control scheme, and in all possible switching scenarios with controllers in the designed bank, the proposed FTC is able to guarantee at least the closed-loop stability of the overall system.

5. CONCLUSION

In this paper, the $\mathcal{H}_2$-guaranteed performance against actuators failure in network-based control system with time-varying but bounded delays has been addressed. Plants with norm-bounded parameter uncertainty have been considered, where the uncertainty may appear in the state matrix. Modeling network-induced delays as communication delays between sensors and actuators, linear memoryless state feedback fault-tolerant controllers have been developed through LMI-based methods. A simulation example has been presented to show the potentials of the proposed method for fault-tolerant control in networked control systems.

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REFERENCES

M. Blanke, M. Kinnaert, J. Lunze and M. Staroswiecki, *Diagnosis and Fault-Tolerant Control*, Springer-Verlag, Berlin, 2003

M. Mariton, "Detection delays, false alarm rates and the reconfiguration of control systems," *International Journal of Control*, vol. 49, pp. 981-992, 1989

M. Mahmoud, J. Jiang, and Y. Zhang, "Stabilization of active fault tolerant control systems with imperfect fault detection and diagnosis," *Stochastic Analysis and Applications*, vol. 21, no. 3, pp. 673–701, 2003.

M. Maki, J. Jiang, and K. Hagino, "A stability guaranteed active fault-tolerant control against actuator failures," *International Journal of Robust and Nonlinear Control*, vol. 14, pp. 1061–1077, 2004.

W. Zhang, M. S. Branicky, and S.M. Phillips, "Stability of networked control systems", *IEEE Control Systems Magazine*, vol. 21, no. 1, pp. 8499, 2001.

Special Issue on Technology of Networked Control Systems, *Proceedings of the IEEE*, vol. 95, no. 1, 2007.

Y. Tipsuwan, M-Y. Chow, "Control methodologies in networked control systems", *Control Engineerong Practice*, vol. 11, pp. 1099-1111, 2003.

W. H. Chen, Z. H. Guan, and X. Lu, "Delay-dependent guaranteed cost control for uncertain discrete-time systems with delay," *IEE Proc.-Control Theory Appl.*, vol. 150, no. 4, pp. 412–416, July 2003.

P. Gahinet, A. Nemirovski, A. J. Laub, and M. Chilali, *LMI Control Toolbox-for Use with Matlab.* The Math Works Inc., 1995.

Y. S. Moon, P. Park, W. H. Kwon, and Y. S. Lee, "Delay-dependent robust stabilization of uncertain state-delayed systems," *Int. J. Control*, vol. 74, no. 14, pp. 1447–1455, 2001.

Y. Wang, L. Xie, and C. E. D. Souza, "Robust control of class uncertain nonlinear systems," *Systems and Control Letters*, vol. 19, no. 3, pp. 139–149, 1992.

Appendix A. PROOF OF THEOREM 1

The following matrix inequalities are essential for the proof of theorem 1:

Lemma 1. (Moon et Al. (2001)) Assume that $a(\cdot) \in \mathcal{R}^{n_a}, b(\cdot) \in \mathcal{R}^{n_b}$, and $N(\cdot) \in \mathcal{R}^{n_a \times n_b}$. Then, for any matrices $X \in \mathcal{R}^{n_a \times n_a}$, $Y \in \mathcal{R}^{n_a \times n_b}$, and $Z \in \mathcal{R}^{n_b \times n_b}$, the following holds:

$$-2a^T Nb \leq \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} X & Y-N \\ Y^T - N^T & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

where $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \geq 0.$

Lemma 2. (Wang et Al. (1992)) Let Y be a symmetric matrix and H, E be given matrices with appropriate dimensions, then

$$Y + HFE + E^T F^T H^T < 0$$

holds for all F satisfying $F^T F \leq I$, if and only if there exists a scalar $\epsilon > 0$ such that:

$$Y + \epsilon HH^T + \epsilon^{-1} E^T E < 0$$

Proof. Note that $x(k-\tau(k)) = x(k) - \sum_{l=k-\tau(k)}^{k-1} y(l)$, where $y(l) = x(l+1) - x(l)$. Then from system (6), we have:

$$0 = (A_1 + B\mathcal{L}K - I)x(k) - y(k) - B\mathcal{L}K \sum_{l=k-\tau(k)}^{k-1} y(l) \tag{A.1}$$

Consider the following Lyapunov-Krasovskii candidate-functional:

$$V(k) = V_1(k) + V_2(k) + V_3(k) \tag{A.2}$$

where

$$V_1(k) = x^T(k) P_1 x(k)$$

$$V_2(k) = \sum_{l=k-\tau(k)}^{k-1} x^T(l) R x(l)$$

$$V_3(k) = \sum_{\theta=-\tau_M}^{-1} \sum_{l=k+\theta}^{k-1} y^T(l) S y(l) + \sum_{\theta=-\tau_M+2}^{-\tau_m+1} \sum_{l=k+\theta-1}^{k-1} x^T(l) R x(l)$$

Taking the forward difference for the Lyapunov functional $V_1(k)$, we have:

$$\Delta V_1(k) = 2x^T(k) P_1 y(k) + y^T(k) P_1 y(k) \tag{A.3}$$

From (A.1), we have:

$$2x^T(k)P_1y(k) = 2\eta^T(k) P^T \begin{bmatrix} y(k) \\ (A_1 + B\mathcal{L}K - I)x(k) - y(k) - B\mathcal{L}K \sum_{l=k-\tau(k)}^{k-1} y(l) \end{bmatrix}$$

where $\eta^T(k) = \begin{bmatrix} x^T(k) & y^T(k) \end{bmatrix}$. Choose constant matrices W, M and S satisfying (9), by Lemma 1, we have:

$$\begin{aligned} &-2 \sum_{l=k-\tau(k)}^{k-1} \eta^T(k) P^T \begin{bmatrix} 0 \\ B\mathcal{L}K \end{bmatrix} y(l) \\ &\leq \tau_M \eta^T(k) W \eta(k) + \sum_{l=k-\tau_M}^{k-1} y^T(l) S y(l) \\ &+ 2\eta^T(k) \left\{ M - P^T \begin{bmatrix} 0 \\ B\mathcal{L}K \end{bmatrix} \right\} (x(k) - x(k-\tau(k))) \end{aligned} \tag{A.4}$$

Similarly,

$$\begin{aligned} \Delta V_2(k) &= x^T(k) R x(k) - x^T(k-\tau(k)) R x(k-\tau(k)) \\ &+ \sum_{l=k+1-\tau(k+1)}^{k-1} x^T(l) R x(l) - \sum_{k-\tau(k)+1}^{k-1} x^T(l) R x(l) \end{aligned} \tag{A.5}$$

Note that:

$$\begin{aligned} &\sum_{l=k+1-\tau(k+1)}^{k-1} x^T(l) R x(l) \\ &= \sum_{l=k+1-\tau_m}^{k-1} x^T(l) R x(l) + \sum_{l=k+1-\tau(k+1)}^{k-\tau_m} x^T(l) R x(l) \\ &\leq \sum_{l=k+1-\tau(k)}^{k-1} x^T(l) R x(l) + \sum_{l=k+1-\tau_M}^{k-\tau_m} x^T(l) R x(l) \end{aligned}$$

So, we have:

$$\begin{aligned} \Delta V_2(k) &\leq x^T(k) R x(k) - x^T(k-\tau(k)) R x(k-\tau(k)) \\ &+ \textstyle\sum_{l=k+1-\tau_M}^{k-\tau_m} x^T(l) R x(l) \end{aligned} \tag{A.6}$$

Furthermore, we have:

$$\begin{aligned} \Delta V_3(k) &= \tau_M y^T(k) S y(k) - \textstyle\sum_{l=k-\tau_M}^{k-1} y^T(l) S y(l) \\ &+ (\tau_M - \tau_m) x^T(k) R x(k) - \textstyle\sum_{l=k+1-\tau_M}^{k-\tau_m} x^T(l) R x(l) \end{aligned} \tag{A.7}$$

Combining (7) and (A.3)-(A.7), we have:

$$\begin{aligned} \Delta V(k) &\leq \xi^T(k) [\Theta_0(\tau_m, \tau_M) + \hat{D}\Delta(k)\hat{E} + \hat{E}^T \Delta^T(k) \hat{D}^T] \xi(k) \\ &- \tilde{x}^T(k) Q \tilde{x}(k) \end{aligned}$$

where

$$\begin{aligned} \xi^T(k) &= \begin{bmatrix} \eta^T(k) & x^T(k-\tau(k)) \end{bmatrix}, \\ \hat{D}^T &= \begin{bmatrix} \begin{bmatrix} 0 & D^T \end{bmatrix} P & 0 \end{bmatrix}, \hat{E} = \begin{bmatrix} \begin{bmatrix} E & 0 \end{bmatrix} 0 \end{bmatrix}, \\ \Theta_0(\tau_m, \tau_M) &= \begin{bmatrix} \Gamma_0 & P^T \begin{bmatrix} 0 \\ B\mathcal{L}K \end{bmatrix} - M \\ * & -R + K^T Q_2 K \end{bmatrix}, \\ \Gamma_0 &= \Gamma - \epsilon P^T \begin{bmatrix} 0 & 0 \\ 0 & DD^T \end{bmatrix} P \end{aligned}$$

By Lemma 2, we have:

$$\begin{aligned} \Delta V(k) &\leq \xi^T(k) [\Theta_0(\tau_m, \tau_M) + \epsilon \hat{D}\hat{D}^T + \epsilon^{-1} \hat{E}^T \hat{E}] \xi(k) \\ &- \tilde{x}^T(k) Q \tilde{x}(k) \end{aligned}$$

By Schur complement and from (8), we have:

$$\Delta V(k) \leq -\tilde{x}^T(k) Q \tilde{x}(k)$$

Summing both sides of the above inequality from 0 to ∞ and using system stability yields equation (10), and from the definition of the guaranteed cost, this completes the proof of the theorem.