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A Gradient Method for the Static Output Feedback Mixed H_2/H_∞ Control ⋆

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Abstract: This paper is concerned with the mixed H_2/H_∞ control problem via static output feedback control. The main purpose of this paper is to give an iterative method for finding a sub-optimal static output-feedback controller for the mixed H_2/H_∞ control problem. The contribution of this paper is to derive a gradient of the H_2 cost function. Using this gradient, we propose a gradient method for H_2 and mixed H_2/H_∞ control problems. Numerical examples show the efficiency of our methods.

Keywords: Optimization based controller synthesis, Robust controller synthesis

1. INTRODUCTION

One of major requirements for designing control systems is to achieve optimal performance and robust stabilization against uncertainty simultaneously. Since H_2 and H_∞ norms are measures for these requirements such control systems can be designed through the so-called mixed H_2/H_∞ control problem which is an important example of multi-objective control problem. On the other hand some practical limitations, e.g., we can only measure part of state variables, make us use an output-feedback controller. Hence, the output-feedback mixed H_2/H_∞ control problem is very important control problem from a point of view of practical applications. However, it is difficult to obtain the globally optimal solution, because this control problem is described as bilinear matrix inequality (BMI) problem.

Recently, many sub-optimization methods for multi-objective control problems have been proposed Chilali et al. [1996]–Shimomura [2005]. One well-known technique is to fix some variables so as to reduce BMI problems to LMI problems. Another well-known technique is to use common Lyapunov variables at the expense of conservatismChilali et al. [1996]–Scherer et al. [1997]. Moreover, some techniques using non-common Lyapunov variables are proposed Ebihara and Hagiwara [2004]–Shimomura [2005]. However there is no efficient method for obtaining the globally optimal solution of multi-objective contorl problems and there are few methods which guarantee the properties of obtained controller.

In this paper, we tackle the mixed H_2/H_∞ controller design with static output feedback. The purpose of this paper is to give a sub-optimization method for this control problem. The main contribution of this paper is to derive a gradient of the H_2 cost function. Using the gradient, we propose an iterative method for the mixed H_2/H_∞ control problem, which guarantees that the obtained controller is a locally optimal controller or on the boundary of the H_∞ norm constarint. Numerical examples show the efficiency of our methods.

The following notations are used in this paper: A (p, q)-th element of a matrix M is shown as M_{pq}. He$\{M\}$ and $\begin{bmatrix} A & * \\ B^T & C \end{bmatrix}$ denote $M + M^T$ and the symmetric matrix $\begin{bmatrix} A & B \\ B^T & C \end{bmatrix}$, respectively.

2. PROBLEM FORMULATION

In this paper, consider the following LTI system:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + B_1w_1(t) + B_2w_2(t), \\ z_1(t) = C_1x(t) + D_1u(t), \\ z_2(t) = C_2x(t) + D_2u(t), \\ y(t) = Cx(t), \end{cases} \tag{1}$$

where x is the plant state, $w_i (i = 1, 2)$ are any exogenous inputs, u is the control input, $z_i (i = 1, 2)$ are the performance outputs, and y is the measured output. Throughout this paper, the following assumptions are made:

(1) (A, B, C) is stabilizable and detectable.
(2) $D_2^T D_2 = I$.

Let us consider the static output-feedback controller:

$$u(t) = Ky(t). \tag{2}$$

Via the output feedback control low the closed-loop system is described as

$$\begin{cases} \dot{x}_{cl}(t) = A_{cl}x_{cl}(t)B_1w_1(t) + B_2w_2(t), \\ z_1(t) = C_{cl1}x_{cl}(t), \\ z_2(t) = C_{cl2}x_{cl}(t), \end{cases} \tag{3}$$

where

$$A_{cl} := A + BKC,\ C_{cl_i} := C_i + D_iKC,\ (i = 1, 2). \tag{4}$$

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For this system we define the mixed H_2/H_∞ control problem as follows.

The mixed H_2/H_∞ control problem 1: *Given an achievable H_∞ norm bound γ, find a controller that satisfies*

$$\min_K \|T_{z_2w_2}(K)\|_2 \text{ s.t. } \|T_{z_1w_1}(K)\|_\infty < \gamma, \tag{5}$$

where $\|\cdot\|_2$ and $\|\cdot\|_\infty$ denote the H_2 and H_∞ norms, respectively, and $T_{z_iw_i}$ $(i = 1, 2)$ denote the closed-loop transfer functions from w_i to z_i.

3. PRELIMINARIES

For $\|T_{z_2w_2}(K)\|_2$ and $\|T_{z_1w_1}(K)\|_\infty$ the following lemmas hold Boyd et al. [1994].

Lemma 1. (H_2 norm optimization) For $\|T_{z_2w_2}(K)\|_2$, the following statements hold:

(1) K stabilizes the closed-loop sysytem (3) and minimizes $\|T_{z_2w_2}(K)\|_2$.
(2) $K = K_2^*$, where K_2^* is the solution of the problem

$$\inf \text{ trace } Q \text{ s.t.}$$
$$\begin{bmatrix} -Q & * \\ VB_2 & -V \end{bmatrix} < 0 \tag{6}$$
$$\text{He}\{VA_{cl}\} + C_{cl2}^T C_{cl2} < 0, \tag{7}$$
$$Q > 0, V > 0 \tag{8}$$

where

$$A_{cl} := A + BKC,\ C_{cl2} := C_2 + D_2KC. \tag{9}$$

(3) $K = K_2^*$, where K_2^* is the solution of

$$\inf J(K) := \|T_{z_2w_2}(K)\|_2^2 = \text{trace}\left(B_2^T G B_2\right), \tag{10}$$
$$\text{s.t. He}\{GA_{cl}\} + C_{cl2}^T C_{cl2} = 0. \tag{11}$$

Lemma 2. (H_∞ norm constraint) For $\|T_{z_1w_1}(K)\|_\infty$ the following statements hold:

(1) K stabilizes the closed-loop sysytem (3) and achieves $\|T_{z1w1}(K)\|_\infty < \gamma$.
(2) There exists X which satisfies

$$\begin{bmatrix} \text{He}\{A_{cl}X\} + B_1B_1^T & * \\ C_{cl1}X & -\gamma^2 I \end{bmatrix} < 0,\ X > 0. \tag{12}$$

Using Lemmas 1-(2) and 2-(2), the mixed H_2/H_∞ control problem 1 can be often described as follows:

Mixed H_2/H_∞ control problem 2 : *Given an achievable H_∞ norm bound γ, find a controller that satisfies*

$$\inf_K \text{trace } Q \text{ s.t. } (6), (7),\ (8),\ \text{and } (12). \tag{13}$$

Since there are bilinear terms in (7) and (12), the mixed H_2/H_∞ control problem 2 is a bilinear matrix inequality (BMI) problem. In general, it is difficult to obtain the globally optimal solution of BMI problem. Hence, many researchers have proposed interesting sub-optimization methods for such BMI problems Chilali et al. [1996]–Shimomura [2005]. Classically, the next iterative method which uses the property that BMI's become LMI's with some variables fixed is used for obtaining a sub-optimal solution:

Classical Iterative Method

Step 1 Find K_0 which achieves $\|T_{z_1w_1}(K_0)\|_\infty < \gamma$ and let $i := 0$.
Step 2 Letting $K := K_i$, find V and X which are the solutions of (13) and let them be V_i and X_i, respectively.
Step 3 Letting $V := V_i$ and $X = X_i$, find K which satisfies (13) and let it be K_{i+1}.
Step 4 If $\|T_{z_1w_1}(K_{i+1})\|_\infty < \gamma$ and $\|T_{z_2w_2}(K_i)\|_2 > \|T_{z_2w_2}(K_{i+1})\|_2$ then let $i := i + 1$ and go to Step 2. Otherwise let $K_a = K_i$ and exit.

However this method has a critical drawback such that K_i can not improve the H_2 norm of $T_{z_2w_2}$ in some cases, i.e., $\|T_{z_2w_2}(K_i)\|_2$ is not so lower than $\|T_{z_2w_2}(K_0)\|_2$. On the other hand, various different methods for mixed H_2/H_∞ control problems have been proposed Kami and Nobuyama [2002] – Shimomura [2005]. However there are few papers which guarantee the properties of the obtained controller. The purpose of this paper is to propose an iterative method for the mixed H_2/H_∞ control problem with output feedback control, which guarantees that the obtained controller is a locally optimal controller or on the boundary of the H_∞ norm constraint.

4. GRADIENT METHOD FOR THE H_2 CONTROL PROBLEM

In this section, we derive the gradient of the H_2 cost function $J(K)$ with respect to the controller variable K. Using the gradient, we propose a gradient method for the H_2 control problem.

4.1 Gradient of the H_2 cost function

The next theorem gives a gradient othe H_2 cost function $J(K)$:

Theorem 3. Let K be a stabilizing controller. Then the partial differentiation of $J(K)$ with respect to K_{pq} is given as follows:

$$\frac{\partial J(K)}{\partial K_{pq}} = 2\text{trace}(MY), \tag{14}$$
$$M := (GB + C_2^T D_2 + C^T K^T)E_{pq}C, \tag{15}$$

where $E_{pq} := \dfrac{\partial K}{\partial K_{pq}}$, i.e., E_{pq} is the matrix such that (p, q)-th element is equal to 1 and the others are equal to 0, and G and Y are solutions of the Lyapunov equations (11) and

$$\text{He}\{A_{cl}Y\} + B_2B_2^T = 0, \tag{16}$$

respectively.

Proof: From (10), $\dfrac{\partial J(K)}{\partial K_{pq}}$ is given as follows:

$$\frac{\partial J(K)}{\partial K_{pq}} = \text{trace}\left(B_2^T \frac{\partial G}{\partial K_{pq}} B_2\right). \tag{17}$$

For obtaining $\dfrac{\partial G}{\partial K_{pq}}$ differentiating (11) with respect to (p, q)-th element of K to get the following Lyapunov equation:

$$\mathrm{He}\left\{\frac{\partial G}{\partial K_{pq}}A_{cl}+M\right\}=0. \tag{18}$$

Since K is the stabilizing controller, $\dfrac{\partial G}{\partial K_{pq}}$ is given by

$$\frac{\partial G}{\partial K_{pq}}=\int_0^\infty e^{A_{cl}^Tt}\mathrm{He}\{M\}e^{A_{cl}t}dt. \tag{19}$$

Substituting (19) in (17) to get

$$\begin{aligned}\frac{\partial J(K)}{\partial K_{pqj}}&=\mathrm{trace}\left(B_2^T\int_0^\infty e^{A_{cl}^Tt}\mathrm{He}\{M\}e^{A_{cl}t}dtB_2\right)\\&=\mathrm{trace}\left(\mathrm{He}\{M\}\int_0^\infty e^{A_{cl}t}B_2B_2^Te^{A_{cl}^Tt}dt\right)\\&=2\mathrm{trace}\left(MY\right),\end{aligned} \tag{20}$$

where

$$Y:=\int_0^\infty e^{A_{cl}t}B_2B_2^Te^{A_{cl}^Tt}dt, \tag{21}$$

and since A_{cl} is stable Y is the unique solution of (16).

4.2 A Gradient method for the H_2 control problem

Using (14), the (p,q)-th element of a descent direction ΔK is defined as

$$\Delta K_{pq}:=-2\mathrm{trace}(MY). \tag{22}$$

Then, a gradient method for the H_2 control problem is propsed as follows:

Algorithm 1: Gradient Method for the H_2 control problem

Step 1 Find K_0 which stabilizes the closed-loop system (3) and let $i:=0$. For example, an exterior-point approach Kami and Nobuyama [2004] can be used for finding K_0.

Step 2 Get G_i and Y_i which are the solutions of

$$\mathrm{He}\{G_iA_{cli}\}+C_{cl2i}^TC_{cl2i}=0, \tag{23}$$

and

$$\mathrm{He}\{A_{cli}Y_i\}+B_2B_2^T=0, \tag{24}$$

respectively, where

$$A_{cli}=A+BK_iC, C_{cl2i}=C_2+D_2K_iC. \tag{25}$$

Step 3 Caliculate the partial derivative of $J(K)$ with respect to K_{pq} via (14) and define the descent direction ΔK via (22). If ΔK is a zero matrix then let $K^*:=K_i$ and exit. Otherwise go to the next step.

Step 4 Let $K_{i+1}:=K_i+d_i\Delta K$, where $d_i>0$ is a step size which is the solution of

$$\min_d\|T_{z_2w_2}(K_i+d\Delta K)\|_2. \tag{26}$$

Let $i:=i+1$ and go to Step 2.

Lemma 4. Algorithm 1 has the next property:

(1) $J(K_i)\ge J(K_{i+1})$ holds i.e., $\|T_{z_2w_2}(K_i)\|_2$ is monotonically decreasing.
(2) K^* is a loccaly optimal solution of the H_2 control problem.

Proof : Obvious from the construction of Algorithm 1.

Remark 1 It is difficult to get the globally optimal solution of the problem (26), because the search area for d_i is not bounded. Therefore, when Algorithm 1 is implemented we limit the search area of d_i to $0\le d_i\le\bar{d}$, where $\bar{d}>0$ is a prescribed upper bound of d_i, i.e., the next problem is solved by grid search instead of (26):

$$\min_{0\le d_i\le\bar{d}}\|T_{z_2w_2}(K_i+d\Delta K)\|_2. \tag{27}$$

Remark 2 In the case that $C=I$, i.e., K is static state feedback controller, K^* is the globally optimal solution of the H_2 control problem, because a stationary point of $J(K)$ is unique.

5. A GRADIENT METHOD FOR THE MIXED H_2/H_∞ CONTROL PROBLEM

In this section, we extend Algorithm 1 to an iterative method for the mixed H_2/H_∞ control problem. The key idea of the extention is to choose K_{i+1} on the descent direction so as to achieve the H_∞ norm constraint.

An iterative method for the mixed H_2/H_∞ control problem is propsed as follows:

Algorithm 2: Gradient Method for the mixed H_2/H_∞ control problem

Step 1 Find K_0 which achieves $\|T_{z_1w_1}(K)\|_\infty<\gamma$ and let $i:=0$. For example, an exterior-point approach Kami and Nobuyama [2004] can be used for finding K_0.

Step 2 Get G_i and Y_i which are the solutions of (23) and (24), respectively.

Step 3 Caliculate the partial derivative of $J(K)$ with respect to K_{pq} by (14) and define the descent direction ΔK via (22). If ΔK is a zero matrix then let $K^*:=K_i$ and exit. Otherwise go to the next step.

Step 4 Let $K_{i+1}:=K_i+d_i\Delta K$, where $d_i>0$ is a step size which is the solution of

$$\begin{aligned}&\min_d\|T_{z_2w_2}(K_i+d\Delta K)\|_2 \text{ s.t.}\\&\|T_{z_1w_1}(K_i+d\Delta K)\|_\infty<\gamma.\end{aligned} \tag{28}$$

Step 5 For sufficiently small ε_1 and ε_2, if $|d_i|<\varepsilon_1$ and $\|T_{z_1w_1}(K_{i+1})\|_\infty>\gamma-\varepsilon_2$, then let $K^*:=K_{i+1}$ and exit. Otherwise let $i:=i+1$ and go to Step 2.

Lemma 5. Algorithm 2 has the next property:

(1) $J(K_i)\ge J(K_{i+1})$ holds i.e., $\|T_{z_2w_2}(K_i)\|_2$ is monotonically decreasing.
(2) $K_i(i=0,1,2,\cdots)$ achieve the H_∞ norm constraint, i.e., $\|T_{z_1w_1}(K_i)\|_\infty<\gamma$.
(3) If Algorithm 2 stops at Step 2, then K^* is a loccaly optimal solution of the mixed H_2/H_∞ control problem.
(4) If Algorithm 2 stops at Step 5, then K^* is on the boundary of the H_∞ norm constaint of the mixed H_2/H_∞ control problem.

Proof : Obvious from the construction of Algorithm 2.

Remark 3 From the same reason as described in Remark 1, when Algorithm 2 is implemented we limit the search area of d_i to $0 \le d_i \le \bar{d}$, i.e., the next problem is solved by grid search instead of (28):

$$\begin{aligned} \min_{0\le d\le \bar{d}} & \|T_{z_2w_2}(K_i + d\Delta K)\|_2 \text{ s.t.} \\ & \|T_{z_1w_1}(K_i + d\Delta K)\|_\infty < \gamma. \end{aligned} \tag{29}$$

Remark 4 In the case that $C = I$, i.e., K is static state-feedback controller, K^* satisfies a necessary condition for K to be the globally optimal solution of the mixed H_2/ H_∞ control problem Kami and Nobuyama [2003].

6. NUMERICAL EXAMPLES

To demonstrate the efficency of Algorithms 1 and 2, we consider two examples: one is output feedback case and the other is state feedback case. For both examples we consider the unconstrained H_2 control problem and the mixed H_2/H_∞ control problem.

6.1 Example 1: output feedback case

Let's consider the following state space matrices:

$$A = \begin{bmatrix} 0 & 1 \\ -0.5 & -0.2 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ B_1 = \begin{bmatrix} 0 & 0 \\ -1 & -1 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ C_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix},\ D_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\ D_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ C = [\,1\ \ 0\,], \gamma = 1.$$

For this example, the globally optimal H_2 value of the unconstrained H_2 control problem is 1.7579, and we set $\bar{d} = 0.5$.

Figures 1 shows behaviours of $\|T_{z_2w_2}(K_i)\|_2$ as a function of iteration number i on Classical iterative method and Algorithm 1. This figure shows that $\|T_{z_2w_2}(K_i)\|_2$ is monotonically decreasing as i increases and converges to the globally optimal H_2 value while Classical iterative method cannot improve $\|T_{z_2w_2}(K_i)\|_2$.

Figures 2 and Figure 3 show behaviours of $\|T_{z_2w_2}(K_i)\|_2$ and $\|T_{z_1w_1}(K_i)\|_\infty$ as a function of iteration number i on Algorithm 2, respectively. Figure 2 also show a behaviour of $\|T_{z_2w_2}(K_i)\|_2$. Figures 2 shows that $\|T_{z_2w_2}(K_i)\|_2$ is monotonically decreasing as i increases while Classical iterative method cannot improve $\|T_{z_2w_2}(K_i)\|_2$. Figure 3 shows that $\|T_{z_1w_1}(K_i)\|_\infty$ reaches the H_∞ norm bound as i increases, which implies that the obtained controller is on the boundary of the H_∞ norm constraint.

6.2 Example 2: state feedback case

Let's consider the system shown by figure 4, where $m_1 = m_2 = 1$ and the spring constant k is an uncetain parameter which satisfies $1 \le k \le 1.5$. Moreover, a coefficient matrix of a control input includes 10% uncertainty. Then the state-space matrices of this system and the uncertainty structure are given as follows:

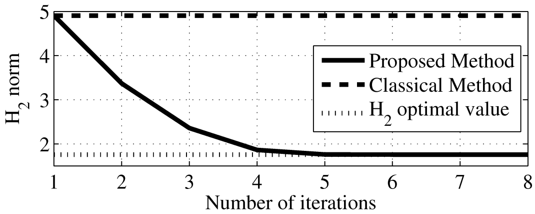

Fig. 1. $\|T_{z_2w_2}(K_i)\|_2$ on Algorithm 1 for Example 1

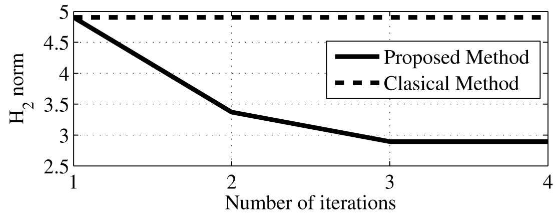

Fig. 2. $\|T_{z_2w_2}(K_i)\|_2$ on Algorithm 2 for Example 1

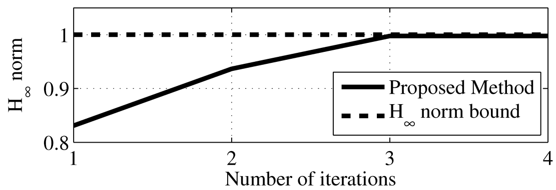

Fig. 3. $\|T_{z_1w_1}(K_i)\|_\infty$ on Algorithm 2 for Example 1

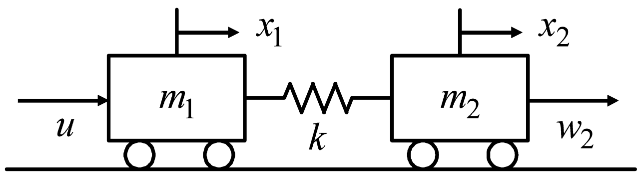

Fig. 4. Two-mass and one-spring system.

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1.25 & 1.25 & 0 & 0 \\ 1.25 & -1.25 & 0 & 0 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 0 \\ 5 \\ 0 \end{bmatrix}$$
$$B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ -0.25 & 1 \\ 0.25 & 0 \end{bmatrix},\ B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 5 \end{bmatrix}$$
$$C_1 = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},\ C_2 = \begin{bmatrix} 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
$$D_1 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix},\ D_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ C = \mathrm{diag}(1, 1, 1, 1).$$
$$\Delta A := B_{w1}\Delta(t)C_1,$$
$$\Delta(t) := \mathrm{diag}(\delta_1(t), \delta_2(t)),\ |\delta_i(t)| < 1 (i = 1, 2).$$

Then the condition for robust stability against $\Delta(t)$ is given as $\|T_{z_1w_1}(K)\|_\infty < 1$. For this example, the globally optimal H_2 value of the unconstrained H_2 control problem is 1.4036, and we set $\bar{d} = 0.5$.

Figures 5 shows behaviours of $\|T_{z_2w_2}(K_i)\|_2$ as a function of iteration number i on Classical iterative method and Algorithm 1. Figures 2 and Figure 3 show behaviours of $\|T_{z_2w_2}(K_i)\|_2$ and $\|T_{z_1w_1}(K_i)\|_\infty$ as a function of iteration

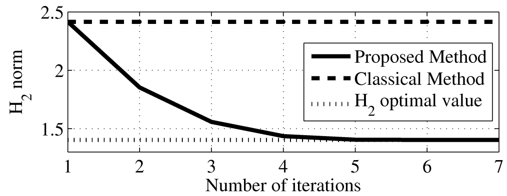

Fig. 5. $\|T_{z_2w_2}(K_i)\|_2$ on Algorithm 1 for Example 2

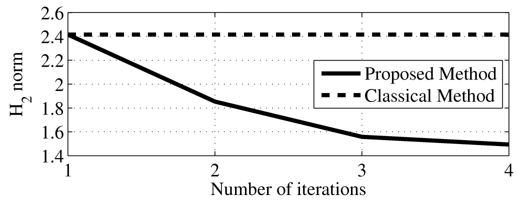

Fig. 6. $\|T_{z_2w_2}(K_i)\|_2$ on Algorithm 2 for Example 2

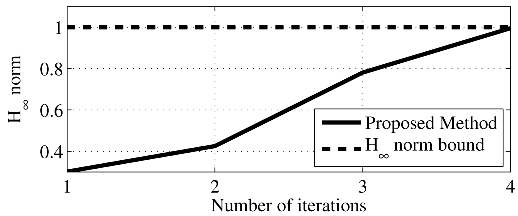

Fig. 7. $\|T_{z_1w_1}(K_i)\|_\infty$ on Algorithm 2 for Example 2

Table 1. Comparison with the obtained H_2 norms

Method	$\|T_{zw}(\Sigma)\|_2$
Common Lyapunov Variables (Initial controller)	2.4153
Classica Method	2.4153
Shimomura [2005]	2.0703
Shimomura and Fujii [2005]	1.4498
Kami and Nobuyama [2003]	1.4948
Proposed method	1.4939

number i on Algorithm 2, respectively. From these figures the same result as described in Example 1 is obtained, i.e.,

- $\|T_{z_2w_2}(K_i)\|_2$ is monotonically decreasing as i increases in Algorithms 1 and 2 while Classical iterative method cannot improve $\|T_{z_2w_2}(K_i)\|_2$.
- Algorithm 2 gives a contrtller on the boundary of the H_∞ norm constraint.

Table 1 shows H_2 norms of the closed-loop system via the controllers obtained by Common Lyapunov Variables, Classical iterative method, Shimomura [2005], Shimomura and Fujii [2005], Kami and Nobuyama [2003], and Algorithm 2. Table 1 shows that the controller obtained by Algorithm 2 achieves lower H_2 norm than Common Lyapunov Variables, Classical iterative method, and Shimomura [2005] and achieves the almost same H_2 norm as those obtained by Shimomura and Fujii [2005] and Kami and Nobuyama [2003]

7. CONCLUSION

In this paper, we consider the H_2 and mixed H_2/H_∞ control problems with static output feedback. Firstly, we derived a partial differentiation of the H_2 cost function. Secondly, we proposed a gradient method for the H_2 control problem. This method guarantees that the obtained controller is a locally optimal solution of the H_2 control problem. Next, we modified the method to the mixed H_2/H_∞ control. This method guarantees that the obtained controller is a a locally optimal solution or on the boundary of the H_∞ norm constaint of the mixed H_2/H_∞ control problem. Finally, we gave numerical examples which showed the efficiency of the proposed methods.

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