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Modern Ability of Optimization-Simulation Approach

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Abstract: Optimization-simulation technology is developed simultaneously with progress in numeric methods, computer science and different fields of modern mathematics. This paper gives the list of problems, which are formulated and successfully solved, and two primers of numeric procedures for decision of multiparameter multicriteria optimization problem. Iterative procedure for structural grows of large-scale system is considered on primer of fuel and energy system. For LP_τ-search with averaging methodology a set of application is discussed. A part of existent proofs of convergence of numeric procedures is described. *Copyright © 2008 IFAC*

1. INTRODUCTION

The optimization-simulation is very popular technology. It bases on high speed of modern computers, advanced numerical algorithms, manifold macro language for creation of simulation models. This approach permits to find only "rational" decisions of optimization problems. This technology consists in multiple repetition numerical experiments with simulation model.

Optimizations problem for large-scale system with vector of input parameters α and index of quality $F(\alpha)$ may be given as follows:

To define such parameters values and systems structure, which provide

$$extr\ F(\alpha)$$

under restrictions

$$f_i(\alpha) \le 0,\ i \in I\ ; \qquad (1)$$

$$f_j(\alpha) \le 0,\ j \in J\ . \qquad (2)$$

Set of restrictions (1) is given in form of mathematical expressions. Set of restrictions (2) is given in algorithmic form, i.e. it involves both procedures for testing parameters constructions and fragments of mathematical expressions. Algorithmic restrictions are completely examined only after imitation experiments with simulation model. The functional of optimisation problem $F(\alpha)$ may be given in algorithmic form too. There are seven large directions for using optimization-simulation approach (Tsvirkun *et. al.*, 1985a, Antonova *et. al*, 2005). Problem definitions for them are formulated under follows assumptions.

The first. Different structures of large-scale system are chosen.

The second. All restrictions are examined.

The third. A set of acceptable variants of system for restrictions (1) is denoted as I_0 and a set of acceptable variants of system for restrictions (2) is denoted as J_0.

2. PROBLEMS FOR OPTIMIZATION LARGE-SCALE SYSTEMS

2.1 First optimization problem

Efficiency function is given in analytical form. Acceptable variants are chosen from I_0. In this case both efficiency function and restrictions have analytical form. It means that problem has high level of formalization. So all methods of mathematical programming are acceptable. However there is very small quantity of such problems in real practice. In addition stochastic character of system parameters and internal and external noises is ignored.

A few operations are fulfilled for optimal variant structure synthesis. They involve creation of simulation model, realization of procedure of optimisation, analyse of results and repetition for some stage if it is need.

2.2 Second optimization problem

Efficiency function is given in analytical form. Restrictions are chosen from J_0 and have algorithmic form. In this case creation of simulation model is obligatory.

Very large quantity of imitation experiments with simulation model must be fulfilled for optimal variant structure synthesis. There are many numeric methods for reduction of laboriousness of optimization procedures. They involve design of experiments, directional simulation modelling,

LP_τ-search with averaging methodology (Antonova, 2001, 2007) and others.

2.3 Third optimization problem

Efficiency function is given in analytical form. Restrictions are chosen from $I_0 \cap J_0$ and have mixed character, i.e. one part of restrictions has analytical form but another part has algorithmic form.

Special procedures are needed for imitation experiments results processing. Profitable procedure consists in checking analytical restrictions on first stage and checking algorithmic restrictions after choosing of appropriate variant on second stage. Overall quantity of imitation experiments is reduced. If obtained variant of large-scale system structure is satisfactory, procedure is over. Otherwise some iteration is needed for choosing suitable variant.

2.4 Forth optimization problem

Efficiency function has algorithmic form. Restrictions are chosen from I_0 and given in analytical form.

Simulation modelling allows calculate efficiency function values. Rational variant of parameters values or systems structure is determined after large quantity of imitation experiments. All simulation modelling methods, design of experiments, directional simulation modelling, optimization-simulation methods realise for solving of this problem.

2.5 Fifth optimization problem

Efficiency function and restrictions have algorithmic form. Restrictions are chosen from J_0. Such problems usually have not even partial analytic description.

Simulation model is very complicated in this case. Design of experiments is obligatory. For fast search of rational variant of parameters values or systems structure it is needed combine optimization-simulation methods with design of experiments. LP_τ-search with averaging methodology and directional simulation modelling, allow accelerate solving of this problem.

2.6 Sixth optimization problem

Efficiency function has algorithmic form. Restrictions are chosen from $I_0 \cap J_0$, i.e. part of restrictions is given in analytical form.

Simulation modelling allows calculate efficiency function values. Methods of searching of rational variant of parameters values or systems structure are similar them, pointed above.

2.7 Seventh optimization problem

Noticeable problems quantity is formulated as multiparameter multicriteria problems. So there are many variants of problem statements. In general case optimizations problem may be given as follows:

To define such parameters values and systems structure, which provide join extremum for the set of criteria

$$F_l(\alpha),\ l=\overline{1,L_{crit}}$$

under restrictions

$$f_{li}(\alpha) \le 0,\ i \in I\ ; \qquad (3)$$

$$f_{lj}(\alpha) \le 0,\ j \in J\ . \qquad (4)$$

Separate single variant of decision in such task usually is inaccessible. Really the Pareto set of variants is defined and then final decision is determined according to integral criterion or additional condition. Complicated special procedures are needed for multicreteria problems solving.

In (Antonova, 2001, 2007) an application of grid's methods for investigation of complicated systems, represented in form of simulation model, is considered. Values of indices of quality for multiparameter multicriteria optimization task are calculated in the form of multidimensional integrals

$$K_j = \int_0^\infty \int_G \int_\Omega f_j(\alpha,\omega) w_{\mathrm{G}}(\alpha,\omega)\, d\omega\, d\alpha\, dt,\ j=\overline{1,J}\ ,$$

where J is the total number of quality indices characterizing the considered object, $K_j, j=\overline{1,J}$ – the set of indices of quality at system's output, G – the region of efficiency, which is estimated in the process of searching a decision of optimization problem, Ω – the domain of variation of values of stochastic parameters, $f_j(\alpha(t),\omega)$ – the function describing the j-th quality index, $\alpha(t)$ – input parameters vector with dimensionality n_1, ω – external and internal stochastic noise vector with dimensionality n_2, t – time,

$$\mathrm{w}_{\mathrm{G}}(\alpha,\omega) = w(\alpha,\omega) / \int_G \int_\Omega w(\alpha,\omega)\, d\omega\, d\alpha$$

– the distribution density, normalized relatively the region G, $w(\alpha,\omega)$ – the distribution density satisfying the condition:

$$\int_{-\infty}^{\infty} \int_\Omega w(\alpha,\omega)\, d\omega\, d\alpha = 1\, .$$

In the process of solving the problem the estimate of the region of the Euclidean space or the set of estimates from the space of estimates with given metric (Antonova, 2007) are determined, for which the quality indices constitute the Pareto set. In the region of efficiency joint extremum of indices of quality is present in the sense of definition of multicriteria optimization task decision. Choosing conditions must be fulfilled

$$K_j(G) \geq K_{jz}, j = \overline{1,q}, \quad (5)$$

$$K_j(G) \leq K_{jz}, j = \overline{q+1,J}. \quad (6)$$

The procedure for approximate solution search combines the methods of statistical modeling, the Monte Carlo method and LP_τ-search with averaging methodology (Antonova, 2001, 2007). In the domain of integration G (the region of efficiency) the averaged values of indices of quality will be better then in other regions.

Values of indices of quality are estimated as a sum of imitation experiments results

$$K_j = \int_0^\infty \int_G \int_\Omega f_j(\alpha,\omega) w_G(\alpha,\omega) d\omega d\alpha dt \cong$$

$$\cong \frac{1}{L_G} \sum_{l=1}^{L_G} \frac{1}{N} \sum_{i=1}^{N} f_j^l(\xi_i), \ j = \overline{1,J}, \quad (7)$$

where L_G is the size of the subsequence of the points of the LP_τ-sequence that fall into the region of efficiency, N is the number of experiments with the simulation model, ξ_i is the vector of coordinates of i-th point for imitation experiment's conducting, $f_j^l(\xi_i)$ is the function, describing the j-th quality index and calculating in l-th point of the LP_τ-sequence.

Algorithm and program package in C++ environment is created for LP_τ-search with averaging methodology realization. It allows solving a set of problems for optimizing parameters of dynamical stochastic systems.

3. A FEW PROOFS OF CONVERGENCE OF NUMERIC PROCEDURES IN OPTIMIZATION-SIMULATION

There are the most difficult seventh optimization problem are considered. Two primers of numeric procedures for search of multiparameter multicriteria optimization task decision are discussed.

3.1 Iterative method for optimization of large-scale systems

For multilevel large-scale system in (Akinfiev *et. al.*, 1985b) was proposed compound optimization method. Iterative method for search of multiparameter multicriteria optimization problem solving is described on primer of optimization procedures for the fuel and energy complex (FEC). If there are N subsystems in structure of large-scale system, N indices of quality for all subsystems and simulation model for evaluation of them are chosen.

$$R_n(\mathrm{Y}_n), \ \mathrm{Y}_n \in q_n(\mathrm{X}), \quad (8)$$

where X is vector, which specifies optimal fuel and energy resources (FER) output, optimal capacity growth for extraction, processing and shipment of FER, optimal investment by period of functioning,

$\mathrm{Y}_n, \ n = \overline{1,N}$ are the set of vectors, which specifies for subsystem number n for separate periods of functioning the composition of new and renovated production and transportation facilities, speed of this operations, the sequence of introduction of new capacities, placing of orders in the subsystems.

In addition general index of quality is used,

$F(\Omega, \mathrm{X}), \ \mathrm{X} \in G$, (9)

where $\Omega = \left| w_{nl}^t \right|$ specifies aggregated indices for cost per unit of various resources required to produce a FER of type l in subsystem n during the period of functioning t. They involve investment, building activities, labour resources, etc.

An iterative technique has following steps.

1. The initial approximation of the aggregated indices Ω^0 is given. Optimization problem (9) at top level is solved after dynamic programming is used. As result vectors $\mathrm{X}_n, \ n = \overline{1,N}$ are obtained.

2. Optimization problems (8) for all subsystems are formulated and $\mathrm{X}_n, \ n = \overline{1,N}$ are used as restrictions for optimization problem. Their solution yields vectors $\mathrm{Y}_n, \ n = \overline{1,N}$. Linear programming method is applied.

3. Vectors $\mathrm{Y}_n, \ n = \overline{1,N}$ are used to calculate the first approximation of the aggregated indices Ω^1. Special numeric algorithm for "updated" Ω is constructed.

4. Procedure repeats many times because of all variables are interconnected. Algorithmic form of restrictions and indices of quality doesn't permit correct optimal decision.

M.M. Kapranov in (Akinfiev *et. al.*, 1985b) proved a few theorems about properties of this iterative procedure.

Definition 1. Decision X^* and corresponding decision Y^* are called optimal if an appropriate aggregate $\Omega^* = w(\mathrm{Y}^*(\mathrm{X}^*))$, corresponding to it, may be found such that

$$\mathrm{X}^* = \pi(\Omega^*)$$

and X^* provides an extremum of the goal function (9).

X^* is evaluated by the following iterative technique:

$$\mathrm{X}^* = \lim_{k \to \infty} (p \circ w)^k p(\Omega^0).$$

There are two possible conditions for finishing of the iterative process.

First condition. Two consecutive evaluations of X coincide, i.e. $\left|X^{(k)} - X^{(k-1)}\right| \le \varepsilon$.

Second condition. Two consecutive evaluations of the goal function values coincide, i.e. $\left|F(\Omega(X^{(k)}), X^{(k)}) - F(\Omega(X^{(k-1)}), X^{(k-1)})\right| \le \varepsilon_1$ Values of $\varepsilon,\ \varepsilon_1$ must be given previously.

Let in the iterative procedure *p* be a mapping specified for global optimisation problem (9), while *w* be a mapping specified by local optimisation problems (8).

A step of iterative procedure

$$X^{(k+1)} = A(X^{(k)}),$$

where $A(X) = p(w(X), X)$

is a transparent mapping of the iterative process. The convergence of iterative procedure is obviously determined by the properties of mapping, connected with a transparent mapping *A*. The necessary condition of the convergence of such iterative procedure is the presence of some fixed points in the transparent mapping *A*. Because mappings *p* and *w* are specified through solutions of the linear and dynamic programming M.M. Kapranov in (Akinfiev *et. al.*, 1985b) introduced a class of $\boldsymbol{\varepsilon}$ -continuous mappings.

Definition 2. A mapping *A*: $X \to Y$ of metric spaces is called ε -continuous, if for a fixed ε , some ε -dependent δ may be found such, that $dist(X_1, X_2) < \delta$ implies $dist(AX_1, AX_2) < \varepsilon$, where *dist*() define the distance between the points. The mapping *A* is usually ε -continuous for some set of pairs (ε, δ) . The minimal values of ε and $\delta(\varepsilon)$ define the degree of continuity of mapping *A*.

Mapping *A* is called quasicontinuous, if the minimal values of $\boldsymbol{\varepsilon}$ is small enough.

The first theorem, proved by M.M. Kapranov in (Akinfiev *et. al.*, 1985b), concerns the existence of fixed points with quasicontinuous mappings.

Theorem 1. Let *D* be a closed convex domain in R^n and *A*: $D \to D$ be ε -continuous mapping. Then, there exists a point $X_0 \in D$ such, that $dist(X_0, A(X_0)) \le \varepsilon$.

This theorem for above mentioned iterative schemes guarantees the existence of "almost fixed point" within the iterative process among which the optimal solution should be found.

Organization of iterative process may be different. Another two theorems were proved.

Theorem 2. Consider a linear step iteration process. Assume, that there is a positive function $g(X)$ and a strictly concave function $f(X)$ such that $\theta_i(X) = g_i(X)\dfrac{\partial f}{\partial x_i}(X)$. Moreover, assume that θ_i does not turn into zero simultaneously with *D*. Then, 1) with any initial value $X^{(0)}$ the iterative process converges to one of almost fixed points X^{∞} determined by Theorem 1 which is the point of extremum (minimum) of function *f* on $G(X^{\infty})$; 2) the ratio of $f(X)$ to the goal function $F(X)$ differs from 1/*g* less than by $sd_D|D|/\mu$, where d_D is the domain diameter, $|D| = \max\limits_i \max\limits_{X \in D} |x_i|,\ \varepsilon \ge \dfrac{\partial^2 f}{\partial x_i \partial x_j}$ and $\mu = \min\limits_{X \in D} |F(X)|$

Theorem 3. Consider an adaptive step iteration process. Assume that its goal function $F(X) = \sum\limits_{i=1}^{N} \varphi_i(X)$ is strictly concave. The process converges then to the point X^{∞} which provides the extremum of the goal function $F(X)$ in $G(X^{\infty})$.

3.2 Analyse of conditions for LP_τ *-search with averaging using*

LP_τ -search with averaging realize by means numeric procedures. So a proof of convergence of these procedures is needed. According (Antonova, 2007) definition is introduced.

Definition 3. Convergence of LP_τ - search with averaging signify, that under condition $N \to \infty$, it is possible to get as many number points from the domain of integration *G* (estimate of the region of efficiency), as it is necessary for design of Monte-Carlo estimates for indices of quality. Region *G* must be not far to the region of efficiency according to metric

$$r(G_i,\ G_{i+1}) = \int\limits_{G_i \Delta G_{i+1}} d\alpha$$

from (Antonova, 2007).

In (Antonova, 2007) different situations with numeric evaluation of estimates for region of efficiency in procedures LP_τ -search with averaging are considered.

There are some results, connected with Monte-Carlo estimates properties, presented. Estimates must to exist, to converge to integrals (7) and to be continuous function according to rules of LP_τ -search.

Existing of Monte-Carlo estimates for indices of quality in form (7) follows from the uniform low of distribution of the points of the LP_τ -sequence, that fall in multidimensional single hypercube or in its mapping in the region of space of parameters with arbitrary form.

The convergence of Monte-Carlo estimates to values of integrals (7) is checked in assumption, that indices of quality $K_j, j = \overline{1,J}$ for the dynamic stochastic systems are computable and described by the functions of many variables, including random. One of the theorems is following.

Theorem 4. If probabilities of performing conditions (5) and (6) for all elements of index of quality (7) $P_j > 0,\ j = \overline{1,J}$, i.e. positive, and henceforth region of efficiency exists, therefore the Monte-Carlo estimates of indices of quality

$$K_j = \int_0^\infty \int_G \int_\Omega f_j(\alpha,\omega) w_G(\alpha,\omega) d\omega d\alpha dt, j = \overline{1,J}$$, received with LP_τ-search with averaging methodology using, converge.

Continuity of Monte-Carlo estimates is checked in following statement.

Statement 1. If functions $f_j^l,\ j = \overline{1,J}$ computable, indices of quality satisfy Lipschitz condition and inaccuracy of calculation of Monte-Carlo estimates for them is limited by the value $\varepsilon^0 > 0$, therefore the Monte-Carlo estimates (7), obtained by the LP_τ-search with averaging algorithm, are continuous on the topological space of estimates for region of efficiency *G* with the metric

$$r(G_i,\ G_{i+1}) = \int_{G_i \Delta G_{i+1}} d\alpha$$

4. CONCLUSIONS

A numerous applications of optimisation-simulation approach to improvement of parameters or structure of large-scale system showed, that it is of great value in investigations and optimizations of functioning of important objects.

In above mentioned (Akinfiev *et. al*., 1985b) the primer of optimization the fuel and energy complex is described in detail.

In (Antonova, 1996) statement of seventh optimization problem for investigation of Data Transfer Systems on Short-wave channel with fading and its decision are considered.

An application of LP_τ-search with averaging methodology for defining skip checking block coefficients in Net Satellite Radio-Navigation System is discussed in (Antonova, 1999a).

There is possible extension of 7-th problem to systems with continuous and discrete parameters. For its decision LP_τ-search with averaging repeats several times for fixed values of discrete parameters and variable values of continuous parameters. If there is common region of efficiency after complete investigation final common region of efficiency is determined. An application of this procedure for choosing of the noise-resistant correction codes is described in (Antonova, 1999b, 2002).

For imitation experiments four noise-resistant correcting codes are chosen. Its generating polynomials are listed in (Antonova, 2002). The first code has rate of transmission ½ and free distance equal 5, the second code has rate ½ and free distance equal 6, the third code has rate ½ and free distance equal 8, the fourth code has rate 1/3 and free distance equal 10. A mutual region of efficiency for them is defined as an intersection of separate region of efficiency of various codes. If a mutual region of efficiency exists, and will be found after imitation experiments, discrete problem for choice of noise-resistant correcting code will be decided. LP_τ-search with averaging was fulfilled in LP_τ-sequence points with numbers from 1 to 256 and from 1000 to 1115. The mutual region of efficiency is described:

$$\hat{Gr}(m_x, m_y, \sigma_x^2, \sigma_y^2) < \varepsilon,\ \varepsilon \to 0,$$

where $\hat{Gr}$ is statistics estimate for the index of grouping, which characterize rarity or grouping of errors in channel, $m_x,\ m_y$ are the expected values of the orthogonal components x and y, in to which the module of the complex transfer-coefficient $|\dot{\mu}|$ can be decomposed., $\sigma_x^2,\ \sigma_y^2$ are the variances of the orthogonal components x and y.

Analysis of the results of simulation demonstrates that coding is efficient for those states of the communication channel, where the grouping coefficient is close to zero, i.e. errors are independent. It corroborates with theoretical recommendations. Further analysis shows that the residual error probability decreases appreciable with increase in the free distance of the convolutional code. For Code 3, however, such technical characteristic as the decoder complexity grows, and for Code 4 the rate decreases substantially. The set of experiments with zero residual error probability was obtained for Code 4. For these experiments Codes 2,3,4 create the Pareto set in space of characteristic: the decoder complexity; the free distance; the residual error probability; rate of transmission. Here, Code 2 has minimal complexity, its rate is the same as that of Code 3, but the values of the residual error probability are inferior. Code 3 has the maximal complexity for mean residual error probability and rate greater than that of Code 4. Code 4 has an acceptable complexity, but its rate is minimal in the chosen group of codes. Therefore, technical specialists can use obtained results to solve the problem of expert estimation and choice from discrete set of codes.

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