

## Tension Control with ARHC Scheme for Hot Strip Finishing Mills

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**Abstract:** This paper presents a design procedure of the ARHC(Adaptive Receding Horizon Control) of the looper-tension controller in hot strip finishing mill. The controller is applied to satisfy the constraints of the control input and attenuate the disturbance of the actuator. The system matrices of the looper model are periodically updated during online simulation. Moreover the closed loop stability of the controller is analyzed.

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### 1. INTRODUCTION

The width of the hot strip process is important to determine the productivity of the steel company, because the width shrinkage of the strip is trimmed after the process. The width of the finishing mill mainly depends on the tension between two neighboring stands which is controlled by the looper system. Therefore the tension deviation can be minimized by the looper-tension control system. Normally, it has to satisfy the constraints on the control input and state, attenuate the variations of the system parameters and the speed disturbance of the actuator.

The researchers have developed the looper-tension control system using the conventional PI, inverse linear quadratic,  $H_\infty$  control, model predictive control, and so on (Park [2007]). However, they have the weak robustness with respect to the input constraints, the variation of model parameters, and uncertainties, and so on.

The purpose of this paper is the construction of the adaptive RHC(ARHC) control system based on the state space model for the looper system. The system matrices of the looper model are updated by the online identification. The proposed RHC is based on the finite terminal weighting matrix instead of the terminal equality constraint. Moreover, the closed loop stability of the control system is analyzed.

The paper is organized as follows: Section 2 gives a brief description of the RHC control scheme and its adaptation. In Section 3, the closed loop stability of the RHC is introduced. Conclusions are presented in Section 4.

### 2. ADAPTIVE RECEDING HORIZON CONTROL

#### 2.1 Design of RHC System

Nonlinear dynamics of the looper system is linearized by the approximation technique using Taylor's series expansion, where the considered operating points are the strip tension(8.6[N/mm<sup>2</sup>]) and the looper angle(18[degree]) (Hesketh et al. [1998], Imanari et al. [1998]). To design the RHC controller which satisfies some constraints subject to

control input/state variables, the time-invariant discrete system is represented:

$$x_{k+1} = A_k x_k + B_k u_k, \quad (1)$$

$$y_k = C_k x_k + D_k u_k, \quad (2)$$

where  $x_k \in \mathbb{R}^n$  is a state, and  $u_k \in \mathbb{R}^m$  and  $y_k \in \mathbb{R}^m$  are a control input and a measured output( $n=5, m=2$ ), respectively, and defined as  $x_k = [\delta\sigma \ \delta\theta \ \delta\dot{\theta} \ \delta v_{Re} \ \delta i_{lm}]^T$ ,  $u_k = [\delta v_{Re}^{ref} \ \delta i_{lm}^{ref}]^T$ ,  $y_k = [\delta\sigma \ \delta\theta]^T$ .

The performance criterion is represented as follows:

$$J(x_k, k) = \sum_{i=0}^{N-1} (x_{k+i}^T Q x_{k+i} + u_{k+i}^T R u_{k+i}) + x_{k+N}^T \Psi x_{k+N} \quad (3)$$

where  $Q \geq 0$ ,  $R > 0$  and  $\Psi > 0$  are the state weighting matrix, the input weighting matrix and the terminal weighting matrix, respectively.

To design the RHC controller, we define state equations (1) and (2) of the looper-tension system and set the horizon  $N$ , input/state constraints and weighting matrix( $Q, R$ ), respectively (Lee [1998], Kwon [2005]). The augmented variables are defined as follows:

$$X_k = \begin{bmatrix} x_{k|k} \\ x_{k+1|k} \\ \vdots \\ x_{k+N|k} \end{bmatrix}, U_k = \begin{bmatrix} u_{k|k} \\ \vdots \\ u_{k+N-1|k} \end{bmatrix} \quad (4)$$

Then we calculate the performance index (3) as follows:

$$J(x_k, k) = X_k^T \hat{Q} X_k + U_k^T \hat{R} U_k, \quad (5)$$

where

$$\hat{Q} = \begin{bmatrix} Q & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & 0 \\ \vdots & \ddots & Q & 0 \\ 0 & \cdots & 0 & \Psi \end{bmatrix}, \hat{R} = \begin{bmatrix} R & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & R \end{bmatrix}.$$

The RHC problem minimizing (3) with constraints can be formulated into a semidefinite programming(SDP) with variable  $t \in \Re$  as follows (Boyd et al. [1994]):

$$U_k^* = \text{Minimize } t \quad (6)$$

subject to

$$\begin{cases} \text{input and state constraints,} \\ \begin{bmatrix} I & W_k^{1/2}U_k \\ (W_k^{1/2}U_k)^T & \omega_k^T U_k + \omega_{0k} + t \end{bmatrix} \geq 0, \\ \begin{bmatrix} I & (A_k^N x_k + \bar{B}_k U_k)^T \\ A_k^N x_k + \bar{B}_k U_k & \Psi^{-1} \end{bmatrix} \geq 0, \end{cases} \quad (7)$$

where

$$\begin{aligned} W_k &= \hat{W}_k^T \hat{Q} \hat{W}_k + \hat{R}, \quad \omega_k^T = 2\hat{\omega}_{0k}^T \hat{Q} \hat{W}_k, \\ \omega_{0k} &= \hat{\omega}_{0k}^T \hat{Q} \hat{\omega}_{0k}, \\ \hat{W}_k &= [I - \hat{A}_k]^{-1} \hat{B}_k, \quad \text{and} \quad \hat{\omega}_{0k} = [I - \hat{A}_k]^{-1} X_{k0}, \\ \bar{B}_k &= [A_k^{N-1} B_k \quad A_k^{N-2} B_k \quad \cdots \quad B_k]. \end{aligned}$$

Thus we pick the first one up among  $U_k^*$  as

$$u_k^* = [1, 0, \dots, 0] U_k^* \quad (8)$$

Finally the control input( $u_k$ ) is represented as follows :

$$\begin{cases} u_k = u_k^*, & k = 0, 1, \dots, N-1 \\ u_k = Hx_k, & k = N, N+1, \dots \end{cases} \quad (9)$$

where  $H$  is a feedback control gain.

### 2.2 Adaptation of the Looper-Tension System

Since the RHC controller does not theoretically consider modeling errors, its robustness may be weak. To enhance the robustness of the RHC controller, we identify system matrices( $A_k, B_k$ ) in (1) using an on-line subspace identification algorithm, so called 4SID(Subspace-based State Space System IDentification). At first, we form Hankel matrices from the input/output data sets as follows :

$$Y_p = \begin{bmatrix} y(p) & y(p+1) & \cdots & y(p+M-1) \\ y(p+1) & y(p+2) & \cdots & y(p+M) \\ \vdots & \vdots & \ddots & \vdots \\ y(p+j-1) & y(p+j) & \cdots & y(p+j+M-2) \end{bmatrix} \quad (10)$$

$$U_p = \begin{bmatrix} u(p) & u(p+1) & \cdots & u(p+M-1) \\ u(p+1) & u(p+2) & \cdots & u(p+M) \\ \vdots & \vdots & \ddots & \vdots \\ u(p+j-1) & u(p+j) & \cdots & u(p+j+M-2) \end{bmatrix} \quad (11)$$

where  $Y_p \in \Re^{lj \times M}$ ,  $U_p \in \Re^{mj \times M}$  and  $p$  is the acquisition time of the  $(1 \times 1)$  component of the each matrix and *rank*  $U_k = mj(m = 2)$ , which the rank condition ensures the persistently exciting condition of  $u(p)$ . For the looper system,  $u(p) = [\delta v_{Re}^{ref} \quad \delta v_{lm}^{ref}]^T$ ,  $y(p) = [\delta \sigma \quad \delta \theta]^T$ . Since many samples are needed for the on-line adaptation, the parameter  $M$  and  $j$  are set to 200, 50 samples, respectively. Thus the on-line adaptation is executed after 4[sec].

Then  $RQ$  factorization for the input and output matrices is represented as follows :

$$\begin{bmatrix} U_p \\ Y_p \end{bmatrix} = \begin{bmatrix} R_{11} & 0 \\ R_{21} & R_{22} \end{bmatrix}, \quad (12)$$

where  $R_{11} \in \Re^{mj \times mj}$ ,  $R_{21} \in \Re^{lj \times mj}$ ,  $R_{22} \in \Re^{lj \times lj}$ . We perform the singular value decomposition(SVD) for  $R_{22}$  in (12).

$$R_{22} = [U_n \quad U_n^\perp] \begin{bmatrix} \sum_n & 0 \\ 0 & \sum_2 \end{bmatrix} \begin{bmatrix} V_n^T \\ (V_n^\perp)^T \end{bmatrix}, \quad (13)$$

where  $U_n \in \Re^{lj \times n}$ ,  $U_n^\perp \in \Re^{lj \times (lj-n)}$ ,  $\sum_n \in \Re^{n \times n}$ .

Finally, we solve the realization of system matrices  $[A_T, B_T, C_T, D_T]$ , which is similar to  $[A_k, B_k, C_k, D_k]$ .

$$C_T = U_n(1:l, :), \quad U_n^{(1)} A_T = U_n^{(2)}, \quad X_y = X_u \Theta, \quad (14)$$

where  $U_n^{(1)}$  is a submatrix of  $U_n$  composed of the first  $l(j-1)$  rows,  $U_n^{(2)}$  from  $(j+1)$  to  $lj$  rows, respectively and  $\Theta$  is obtained by a least square error method(LS).

### 3. STABILITY ANALYSIS

The closed-loop stability of the RHC is analyzed for time-invariant systems which guarantee the monotonicity of the optimal cost. The cost function can be represented as  $J(x_i, u_{i+}, i, i_f)$  and the optimal cost can be given as  $J^*(x_i, i, i_f)$ , where  $x_i$  is the initial state,  $u_{i+}$  input,  $i$  initial time, and  $i_f$  terminal time, respectively. Assume that the pairs  $(A, B)$  and  $(A, Q^{\frac{1}{2}})$  are stabilizable and observable respectively, and that the receding horizon control associated with the quadratic cost  $J(x_i, i, i+N)$  exists. If  $J^*(x_i, i, i+N+1) \leq J^*(x_i, i, i+N)$ , then asymptotical stability is guaranteed (Kwon [2005]).

### 4. CONCLUSION

This paper investigates the ARHC scheme with online model identification for the looper-tension system. The proposed controller satisfies the constraints of the control input and the closed loop stability criterion. It is also easy to apply the online test because of the simple algorithms.

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