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MODEL ALGORITHM TRACKING CONTROL OF WHEELED MOBILE ROBOTS

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Abstract: This paper proposed a model algorithm control (MAC) method for trajectory tracking control of the differentially steered wheeled mobile robots (WMRs) subject to nonholonomic constraint. The dynamic model of the wheeled mobile robot is presented and used as the model to be controlled. The performance of the proposed control algorithm is verified via computer simulations in which the WMR is controlled to track two different reference paths. It is shown that the control strategy is feasible.

Keywords: Nonlinear systems, Discrete event systems modeling and control, Tracking, Nonlinear system control, Mobile robots.

1. INTRODUCTION

The differentially steered wheeled mobile robots (WMRs) possess the advantages of high mobility, high traction with pneumatic tires, and a simple wheel configuration. Due to these advantages, the differentially steered WMRs have been utilized for automating highway maintenance and constructions. For these applications, tracking ability is essential since most tasks involve tracking a predefined path and/or a detected path in a real time manner. Extensive research has been conducted in the area of mobile robotics in last decade and many different types of mobile robots for industrial applications were developed for the operations that must follow a reference path.

In recent years, the research on designing wheeled mobile robots (WMRs) controllers subject to nonholonomic contrains is both extensive and diverse. There are tow fundamental status in controlling a mobile robot: posture stabilization and trajectory tracking. The aim of posture stabilization is to stabilize the robot to a reference point, while the aim of trajectory tracking is to have the robot follow a reference trajectory. The trajectory tracking problem, indeed, is particularly relevant in practical applications, since WMR modules are usually required to follow a previously planned collision-free path. Therefore, the problem of controlling these robots needs to be studied in order to have good and robust path or trajectory tracking algorithms for different types of automated tasks. Tracking control of nonholonomic mobile robots aims at controlling robots to tracking a given time varying trajectory (reference trajectory). It is a fundamental motion control problem and has been intensively investigated in the robotic domain. Arthur and Hugh (2000) addressed the problem of fully decentralized data fusion and control for a modular wheeled mobile robot. Corradini, *et al.* (2002) addresses the trajectory tracking problem for a wheeled mobile base, considering the presence of disturbances that violate the nonholonomic constraint, and using an approximated discrete-time model for the vehicle. Chen, *et al.* (2006) developed a visual servo tracking controller for a monocular camera system mounted on an underactuated wheeled mobile robot subject to nonholonomic motion constraints. Paulo and Urbano (2005) presented the implementation of a new control strategy, Kalman-based active observer controller for the path following of wheeled mobile robots subject to nonholonomic constrains. Yang, *et al.* (2005) proposed a robust tracking scheme for nonholonomic wheeled mobile robots with parameter uncertainty, external disturbance and input constraints. Tan and Gu (2005) proposed a control design method for autonomous mobile platforms basing on way point guidance approach combining with model reference trajectory control method. Zhang, *et al.* (2003) discussed dynamic modeling and robust control of a differentially steered mobile robot subject to wheel slip and external loads. Shim, *et al.* (1995) proposed a variable structure controller for a nonholonomic wheeled mobile robot for tracking desired trajectories.

This paper proposed a Model Algorithm Control (MAC) method for tracking control of the wheeled mobile robots. MAC is a one-step-ahead predictive controller, in which the control law is obtained by minimizing the output error at time $k+r$. It basically involves an impulse response model for system representation and prediction, a reference trajectory, an optimality criterion and a consideration of the state and control constraints. The main idea of the MAC strategy is to predict the deviation of the future system outputs from the reference path based on the model, define an optimality criterion that reflects the deviations, and obtain an optimality control strategy to minimize the criterion over a certain horizon in the future. A closed-loop MAC that incorporates process uncertainties by adjusting the discrepancy between the process output and its predicted value is particularly robust against process model errors and disturbances. Two reference paths are chosen to do the simulation. The two paths used in this simulation refer to a time history of position and velocity for the mobile robot. That means that the robot track the desired path using the desired velocity based on the proposed control algorithm.

2. MATHEMATICAL MODEL OF THE WHEELED MOBILE ROBOT

Consider a WMR with differentially driven wheels as shown in Fig. 1.

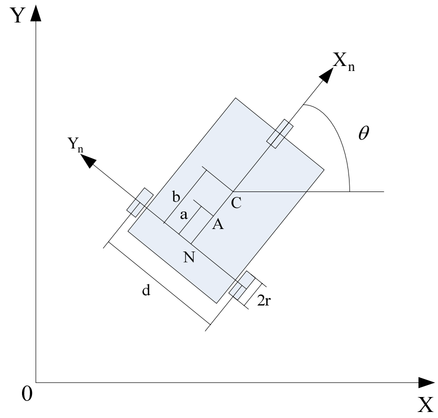

Fig. 1. Model of a nonholonomic wheeled mobile robot.

2.1 Kinematic Model

Fig. 1 presents a geometrical model of the wheeled mobile robot defining the necessary variables to obtain the kinematic model. This WMR has two driving wheels (radius r) and one support wheel. And the two driving wheels are independently actuated by two DC motors. $N(x_n, y_n)$ defines the intersection of the axis of symmetry with driving wheel axis, and is assumed to be the origin of coordinate frame $X_n - Y_n$. C is the center of mass of the robot with coordinate (x_c, y_c) . The point A represents the point being tracked by the controller. a is the distance between point N and point A , b is the distance between point N and point C , and d is the wheel track.

For this kind of WMR there are three constraints. The first one is that the robot must move in the direction of the axis of symmetry.

$$\dot{x}_n \sin\theta - \dot{y}_n \cos\theta = 0 \quad (1)$$

The other two constrains are the rolling constraints, which are that the driving wheels do not slip.

$$\dot{x}_n \cos\theta + \dot{y}_n \sin\theta + \frac{d}{2}\dot{\theta} - r\dot{\varphi}_r = 0 \quad (2)$$

$$\dot{x}_n \cos\theta + \dot{y}_n \sin\theta - \frac{d}{2}\dot{\theta} - r\dot{\varphi}_l = 0 \quad (3)$$

where θ is the orientation angle of the mobile robot, and ϕ_r, ϕ_l are the angles of the right and left driving wheels.

Let $q = (x_n, y_n, \theta, \phi_r, \phi_l)$, the three constraints can be written in the form:

$$A(q)\dot{q} = 0 \tag{4}$$

Where

$$A(q) = \begin{bmatrix} \sin\theta & -\cos\theta & 0 & 0 & 0 \\ \cos\theta & \sin\theta & \frac{d}{2} & -r & 0 \\ \cos\theta & \sin\theta & -\frac{d}{2} & 0 & -r \end{bmatrix} \tag{5}$$

Let $S(q)$ spans the null space of $A(q)$ and a full-rank matrix formed by a set of smooth and linearly independent vector fields, such that:

$$A(q)S(q) = 0 \tag{6}$$

Considering the mobile robot kinematics, we have:

$$S(q) = \frac{r}{2}\begin{bmatrix} \cos\theta & \cos\theta \\ \sin\theta & \sin\theta \\ \frac{2}{d} & -\frac{2}{d} \\ \frac{2}{r} & 0 \\ 0 & \frac{2}{r} \end{bmatrix} \tag{7}$$

2.2 Dynamic Model

The dynamic model can be described as follows:

$$M(q)\ddot{q} + V(q,\dot{q})\dot{q} = E(q)\tau - A^T(q)\lambda \tag{8}$$

where $M(q) \in R^{n\times n}$ is the symmetric and positive definite inertia matrix, $V(q,\dot{q}) \in R^{n\times n}$ is the centripetal and Coriolis force Matrix, $A(q) \in R^{m\times n}$ is the matrix associated with the constraints, $\lambda \in R^m$ is the Lagrangian multiplier vector, $E(q) \in R^{n\times r}$ is the input transformation matrix and $\tau \in R^r$ is the torque input vector.

The nonholonomic mobile robot (8) is transformed to and divided into the following two equations:

$$\dot{q} = S(q)\eta(t) \tag{9}$$

$$\bar{M}\dot{\eta} + \bar{V}\eta = \bar{B}\tau \tag{10}$$

where

$$\bar{M}(q) = \begin{bmatrix} r^2\frac{(\frac{md^2}{4}+I)}{d^2} + I_w & r^2\frac{(\frac{md^2}{4}-I)}{d^2} \\ r^2\frac{(\frac{md^2}{4}-I)}{d^2} & r^2\frac{(\frac{md^2}{4}+I)}{d^2} + I_w \end{bmatrix}$$

$$\bar{V} = \begin{bmatrix} 0 & \frac{r^2 m_c b\dot{\theta}}{d} \\ -\frac{r^2 m_c b\dot{\theta}}{d} & 0 \end{bmatrix}$$

$$\bar{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{11}$$

and $\tau = \begin{bmatrix}\tau_r & \tau_l\end{bmatrix}^T$ is the torque applied on the right and left wheel, $\eta = \begin{bmatrix}\eta_r & \eta_l\end{bmatrix}^T$ represents the angular velocity of the right and left wheel, $I = m_c b^2 + \frac{m_w d^2}{2} + I_c + 2I_m$, $m = m_c + 2m_w$. Here m_c and m_w are the mass of the mobile robot platform and the mass of one driving wheel with the actuator respectively, I_c, I_w and I_m are the moment of inertia of the platform about the vertical axis through point N, the wheel with the actuator about the wheel axis, and the wheel with the actuator about the wheel diameter respectively.

Assume the linear velocity and the orientation angular velocity of the mobile robot at point N are v and w, therefore we have:

$$\begin{bmatrix}\eta_r \\ \eta_l\end{bmatrix} = \begin{bmatrix} \frac{1}{r} & \frac{d}{2r} \\ \frac{1}{r} & -\frac{d}{2r} \end{bmatrix}\begin{bmatrix} v \\ w \end{bmatrix}$$

$$\dot{q} = \begin{bmatrix}\dot{x}_n \\ \dot{y}_n \\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1\end{bmatrix}\begin{bmatrix} v \\ w\end{bmatrix} \tag{12}$$

Then it is easy to show that the dynamics equation for point A leads to the following:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} v\cos\theta - aw\sin\theta \\ v\sin\theta + aw\cos\theta \\ w \\ \beta_1 w^2 \\ \beta_2 vw \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \alpha_1 & 0 \\ 0 & \alpha_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \qquad (13)$$

where $\beta_1 = \dfrac{m_c b r^2}{\Theta_u}$, $\beta_2 = -\dfrac{2m_c b r^2}{\Theta_w}$, $\alpha_1 = \dfrac{r}{\Theta_u}$, $\alpha_2 = \dfrac{rd}{\Theta_w}$, $\Theta_u = mr^2 + 2I_w$, $\Theta_w = 2r^2 I + I_w d^2$, and $u_{1,2} = (\tau_r \pm \tau_l)$.

3. MODEL ALGORITHMIC CONTROL FOR NONLINEAR SYSTEM

Consider the nonlinear systems described by a discrete-time state-space model in the form:

$$\begin{aligned} x_m(k+1) &= \Phi[x_m(k), u(k)] \\ y_m(k) &= h[x_m(k)] \end{aligned} \qquad (14)$$

where x denotes the vector of state variables, u denotes the manipulated input, y represents an output (to be controlled), all in the form of deviation variables, and the subscript m is added to indicate estimates of x and y obtained in model simulations and differentiate the measured y . It is assumed that $x \in X \subset R^n$, and $u \in U \subset R^m$, where X and U are open-connected sets that contain the origin (that is, the nominal equilibrium point). $\Phi(x,u)$ is an analytic vector function on $X \times U$, and $h(x)$ is an analytic scalar function on X .

We suppose that system (14) has the relative order r , i.e. r is the smallest number of sampling periods after which the manipulated input $u(k)$ affects the output y . That means that:

$$\begin{aligned} &\left[\frac{\partial h(x)}{\partial x}\right]\left[\frac{\partial \Phi(x,u)}{\partial x}\right]^l \left[\frac{\partial \Phi(x,u)}{\partial u}\right] = 0, \; l = 0,1,\ldots,r-2 \\ &\left[\frac{\partial h(x)}{\partial x}\right]\left[\frac{\partial \Phi(x,u)}{\partial x}\right]^{r-1} \left[\frac{\partial \Phi(x,u)}{\partial u}\right] \neq 0 \end{aligned} \qquad (15)$$

Online simulation of the model described by Eq. (14) can be used to predict the future changes in the output y as follows:

$$\begin{aligned} &y_m(k+1) - y_m(k) = h^1[x_m(k)] - h[x_m(k)] \\ &y_m(k+2) - y_m(k) = h^2[x_m(k)] - h[x_m(k)] \\ &\vdots \\ &y_m(k+r-1) - y_m(k) = h^{r-1}[x_m(k)] - h[x_m(k)] \\ &y_m(k+r) - y_m(k) = h^{r-1}[\Phi[x_m(k), u(k)] - h[x_m(k)] \end{aligned} \qquad (16)$$

where r is the relative order of the system, and the following notation will be used:

$$\begin{cases} h^0(x) = h(x) \\ h^l(x) = h^{l-1}\left[\Phi(x,u)\right], \; l = 1,\ldots,r-1 \end{cases} \qquad (17)$$

Here we take into account Eq. (15) that can be represented in the form:

$$\begin{aligned} &\frac{\partial}{\partial u} h^l[\Phi(x,u)] = \left[\frac{\partial h(x)}{\partial x}\right]\left[\frac{\partial \Phi(x,u)}{\partial x}\right]^l \left[\frac{\partial \Phi(x,u)}{\partial u}\right] = 0, \\ &l = 0,1,\ldots,r-2, \\ &\frac{\partial}{\partial u} h^{r-1}[\Phi(x,u)] = \left[\frac{\partial h(x)}{\partial x}\right]\left[\frac{\partial \Phi(x,u)}{\partial x}\right]^{r-1} \left[\frac{\partial \Phi(x,u)}{\partial u}\right] \neq 0 \end{aligned} \qquad (18)$$

Furthermore, the following relations will hold:

$$\begin{cases} y(k+l) = h^l[x(k)], \; l = 0,\ldots,r-1 \\ y(k+r) = h^{r-1}\{\Phi[x(k), u(k)]\} \end{cases} \qquad (19)$$

Therefore r is the smallest number of sampling periods after which the manipulated input $u(k)$ affects the output y .

With a finite relative order r , Eq. (15) implies that the algebraic equation

$$h^{r-1}[\Phi(x,u)] = y \qquad (20)$$

is locally solvable in u . The corresponding implicit function will be denoted by:

$$u = \Psi_0(x,y) \qquad (21)$$

and will be assumed to be well-defined and unique on $X \times h(X)$.

When these predicted changes are added to the measured output signal $y(k)$, one obtains the following closed-loop predictions of the output:

$$\begin{aligned} &\hat{y}(k+1) = y(k) + h^1[x_m(k)] - h[x_m(k)] \\ &\hat{y}(k+2) = y(k) + h^2[x_m(k)] - h[x_m(k)] \\ &\ldots \\ &\hat{y}(k+r-1) = y(k) + h^{r-1}[x_m(k)] - h[x_m(k)] \\ &\hat{y}(k+r) = y(k) + h^{r-1}[\Phi[x_m(k), u(k)]] - h[x_m(k)] \end{aligned} \qquad (22)$$

where the ^ is used to indicate that $\hat{y}$ represents a prediction of the output. It is interesting to observe that the output predictions in Eq. (22) are "closed-loop" predictions in the sense that they make use of the measured output signal. In addition, the manipulated input $u(k)$ affects the output after r sampling periods, and this interprets $r\Delta t$ as the overall delay of the system.

At every time step, the control computer can calculate the output prediction Eq. (22), driven by $u(k)$ and $y(k)$, where $x_m(k)$ is obtained by online simulation of the state equations of Eq.(14): $x_m(k+1) = \Phi[x_m(k), u(k)]$.

The question that arises is what should be the choice of $u(k)$ to obtain a desirable output response after r time steps. If $u(k)$ is chosen so that $\hat{y}(k+r)$ is exactly the set-point value y_{sp}, this would clearly create a non-robust situation since the output can be seriously affected by the disturbances or system errors which can make the system unstable. Instead, one can request $\hat{y}(k+r)$ to be in the right direction and cover a fraction of the "distance" between $\hat{y}(k+r-1)$ and the set-point value. In other words, one can define a desirable value y_d of the output at the $(k+r)$th time step by:

$$y_d(k+r) = (1-\alpha)y_{sp} + \alpha\hat{y}(k+r-1) \tag{23}$$

where α is a tunable filter parameter such that $0 < \alpha < 1$. Clearly, $\alpha \to 0$ corresponds to $y_d(k+r) \to y_{sp}$ and therefore, will try to force the output to go to the set point as soon as possible, whereas $\alpha \to 1$ corresponds to $y_d(k+r) \to \hat{y}(k+r-1)$, leaving the output unaffected. An intermediate choice of α corresponds to a desirable value of the output between y_{sp} and $\hat{y}(k+r-1)$ that tries to bridge the gap to a certain extent. Equation (23) is referred to as the "reference trajectory" in the MAC literature.

One can derive a nonlinear MAC controller by requesting that the output prediction match the reference trajectory in the sense of minimizing the performance index of Eq. (23):

$$\min_{u(k)} [y_d(k+r) - \hat{y}(k+r)]^2 \tag{24}$$

Considering Eqs. (29) and (30), this becomes

$$\begin{aligned}&\min_{u(k)} \{(1-\alpha)e(k) - h^{r-1}\{\Phi[x_m(k), u(k)]\} \\ &+\alpha h^{r-1}[x_m(k)] + (1-\alpha)h[x_m(k)]\}^2\end{aligned} \tag{25}$$

where $e(k) = y_{sp}(k) - y(k)$.

In the absence of input constraints, this minimization problem is trivially solvable. Minimizing $u(k)$ is the solution of the nonlinear algebraic equation:

$$h^{r-1}\{\Phi[x_m(k), u(k)]\} = b(x_m(k),\ e(k)) \tag{26}$$

where $b(x,e) = \alpha h^{r-1}[x] + (1-\alpha)(h[x]+e)$.

Recalling the definition of Ψ_0 (Eq. 28), the solution can be represented as:

$$u(k) = \Psi_0\{x_m(k), b(x_m(k), e(k))\} \tag{27}$$

Therefore, the derived control law is given by Eq. (27), where $x_m(k)$ is obtained by Eq. (14).

4. SIMULATION

We simulated the proposed control law to demonstrate its effectiveness. In a simulation program, dynamic model described in Section 2 are used as the mobile robot model.

The first reference path is a straight line. The tracking performance of the MAC controller for the straight line is shown in Figure 2.

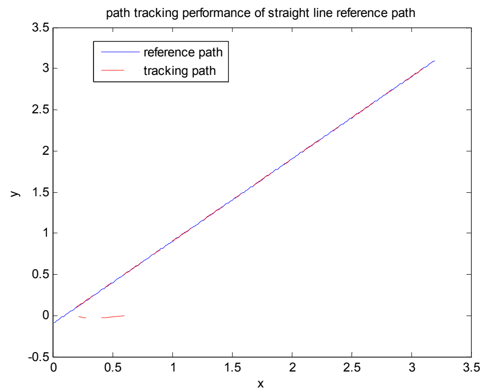

Fig. 2 Path tracking performance of straight line reference path.

Next we choose the reference path as a circular reference path. The tracking performance of the MAC controller for the circular reference path is shown in Figure 3.

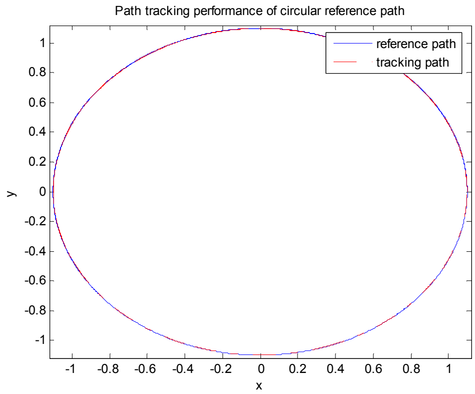

Fig. 3 Path tracking performance of circular reference path

In the above simulations the computation time is about 6% of the robot operation time. The computational load of this proposed algorithm is small which indicates the feasibility of real-time application of this proposed controller.

5. CONCLUSION

In this paper we study the path tracking problem of dynamic WMRs subject to nonholonomic constrains. The MAC control method is proposed for tracking control of the discrete time nonlinear system. Two numerical simulations are done to show the promise of the proposed MAC control method in terms of tracking performance.

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