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Neuro-Adaptive Motion Controller with Velocity Observer for Operational Space Formulation

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Abstract:
An operational space controller that employs a three-layer neural network (NN) adaptive controller with the velocity observer is presented in this paper. This incorporates the versatility of NN based adaptive controller with the performance and effective formulation of the operational space framework, which accommodates unified force/motion control as well as highly redundant mechanisms postures. In this paper, it is shown that the trajectory tracking errors, the estimation tracking errors, and the NN weight errors are bounded even when the actual and accurate velocity feedbacks are not available, such as often the case in a physical robot. Consequently, the controller with velocity observer is shown to be stable. Realtime experiments on PUMA 560 robot was carried out to compare the effectiveness of the proposed NN adaptive control strategy, the inverse-dynamics control, and the Proportional-plus-Derivative (PD) control with gravity compensation.

1. INTRODUCTION

It is well-established that the operational space formulation [Khatib, 1987] provides a natural extension to unified force-motion (compliant) control [Jamisola et al., 2005] as well as to the elegant posture management of highly redundant and branching mechanisms [Russakow et al., 1995]. Control approach for this framework can be described as inverse-dynamics, which requires the *a priori* knowledge of robot dynamics. However, it is also known that the system identification process for a robot is difficult to perform accurately [Armstrong et al., 1986].

The identification of a robot Lagrangian dynamics is a two step process: the derivation of symbolic expressions for the dynamic parameters through the equations of motion and the measurement of the numerical values of the dynamics parameters of the physical robot. Some experimental techniques have been proposed in the past [Jamisola et al., 1999], to measure a robot dynamics through the responses obtained from sending certain commands to the joint actuators, however, it is difficult to obtain an accurate result from the measurements. Additionally, joint friction of a real robot is also difficult to obtain, as friction parameters depend on the current ambient condition, so friction identification should ideally be performed immediately prior to the operation of the robot.

Linear-In-Parameter (LIP) adaptive control [Craig et al., 1986, Slotine and Li, 1987, Middleton and Goodwin, 1988, Ortega and Spong, 1988] was explored in the early effort to formulate an easy-to-use high performance robot control strategy. However, LIP adaptive strategy still requires the derivation of the regression matrix out of the symbolic expressions of the dynamics equations, prior to the parameters estimation. Furthermore the dynamics equations in Cartesian space becomes more complicated than in joint space, making it difficult for implementation.

A three-layer Neural Network (NN) joint space adaptive motion control was proposed in [Lewis et al., 1996] with satisfactory performance in simulation. Several studies then followed [Hu et al., 2000] [Kwan et al., 1994] where the task space force/motion NN adaptive control was formulated based upon model-based framework of [McClamroch and Wang, 1988]. However, this framework still required the contact surface geometry. A position only NN adaptive control was demonstrated in [Ge et al., 1997] which lacked the orientation control of the end-effector. Furthermore, only the second layer NN weights were tuned. It has been established that dual-layer weight NN can approximate any function, only if both layers are tuned.

As most physical robots do not have a joint velocity feedback and have to rely on joint displacement sensors to obtain velocity measurements, the formulation becomes more complicated. The adverse consequence of utilizing backward difference of joint displacement feedback in the absence of a clean joint velocity signal is shown in [Soewandito et al., 2008], where an adaptive NN formulation was implemented on a PUMA 560.

In this paper, an NN adaptive control in Operational Space without velocity feedback is proposed, based on the joint space model-based motion controller with velocity observer by [Berghuis and Nijmeijer, 1993] and the three-layer NN controller by [Lewis et al., 1996]. The proposed controller is then constructed within the framework of op-

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erational space motion formulation [Khatib, 1987]. Stability proof as well as comparison to the inverse-dynamics and the PD with gravity control strategies are presented in this paper. Real-time experimentation shows the effectiveness of the proposed strategy. The implementation results on Puma 560 are provided and discussed.

2. EFFECTOR DYNAMICS AND PROPERTIES

The effector dynamics of a serial manipulator in this paper is expressed as:

$$\mathbf{M_x}(\mathbf{q})\ddot{\mathbf{x}} + \mathbf{B_x}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{x}} + \mathbf{g_x}(\mathbf{q}) + \mathbf{f_x}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{F} \quad (1)$$

where $\mathbf{x} \in \Re^n$ denotes the operational space coordinates, $\mathbf{q} \in \Re^n$ the joint space coordinates, $\mathbf{M_x}(\mathbf{q}) \in \Re^{n\times n}$ the kinetic energy matrix, $\mathbf{B_x}(\mathbf{q},\dot{\mathbf{q}}) \in \Re^{n\times n}$ the Coriolis and centrifugal matrix, $\mathbf{g_x}(\mathbf{q}) \in \Re^n$ the gravity forces vector, $\mathbf{f_x}(\mathbf{q},\dot{\mathbf{q}}) \in \Re^n$ the friction forces vector, and $\mathbf{F} \in \Re^n$ the operational space forces as control input. The subscript '$_\mathbf{x}$'indicates that the matrices and vectors are expressed in operational space. The relationships between the components of the joint space dynamic and the operational space dynamic of non-redundant manipulator in non-singular configuration can be expressed as

$$\mathbf{M_x}(\mathbf{q}) = \mathbf{J}^{-\mathrm{T}}(\mathbf{q})\mathbf{M}(\mathbf{q})\mathbf{J}^{-1}(\mathbf{q}) \quad (2)$$

$$\mathbf{B_x}(\mathbf{q},\dot{\mathbf{q}}) = [\mathbf{J}^{-\mathrm{T}}(\mathbf{q})\mathbf{B}(\mathbf{q},\dot{\mathbf{q}}) - \mathbf{M_x}(\mathbf{q})\dot{\mathbf{J}}(\mathbf{q},\dot{\mathbf{q}})]\mathbf{J}^{-1}(\mathbf{q}) \quad (3)$$

$$\mathbf{g_x}(\mathbf{q}) = \mathbf{J}^{-\mathrm{T}}(\mathbf{q})\mathbf{g}(\mathbf{q}) \quad (4)$$

$$\mathbf{f_x}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{J}^{-\mathrm{T}}(\mathbf{q})\boldsymbol{\tau}_f(\dot{\mathbf{q}}) \quad (5)$$

where $\mathbf{J}(\mathbf{q})$ is the basic Jacobian and $\mathbf{M}(\mathbf{q})$, $\mathbf{B}(\mathbf{q},\dot{\mathbf{q}})$, $\mathbf{g}(\mathbf{q})$ are as stated above, but expressed in joint space. The friction term $\boldsymbol{\tau}_f(\dot{\mathbf{q}})$ can be obtained by Xia et al. [2004]:

$$\boldsymbol{\tau}_f(\dot{\mathbf{q}}) = \boldsymbol{\tau}_{vis}\dot{\mathbf{q}} + \left[\boldsymbol{\tau}_{cou} + \boldsymbol{\tau}_{sti}\mathrm{exp}^{(-\boldsymbol{\tau}_{dec}\dot{\mathbf{q}}^2)}\right]\mathrm{sgn}(\dot{\mathbf{q}}) \quad (6)$$

where $\mathrm{sgn}(\dot{q}) = +1, -1, 0$ if $\dot{q} =$ positive, negative and zero respectively and $\boldsymbol{\tau}_{vis}$, $\boldsymbol{\tau}_{cou}$, $\boldsymbol{\tau}_{sti}$, and $\boldsymbol{\tau}_{dec}$ are the viscous friction, coulomb friction, stiction, and Stribeck effect, respectively.

In order to develop the NN adaptive controller in the operational space framework, the following properties of the effector dynamics (1) are utilized:

Property 1. The operational space kinetic energy matrix $\mathbf{M_x}(\mathbf{q})$ is symmetric and positive definite due to (2) and the joint space kinetic energy $\mathbf{M}(\mathbf{q})$ is symmetric and positive definite by definition. Hence according to Rayleigh-Ritz theorem:

$$M_m\|\mathbf{z}\|^2 \le \mathbf{z}^{\mathrm{T}}\mathbf{M_x}(\mathbf{q})\mathbf{z} \le M_M\|\mathbf{z}\|^2 \quad (7)$$

where M_m and M_M denote the minimum and maximum eigenvalues of $\mathbf{M_x}(\mathbf{q})$ respectively. Moreover any positive definite matrix $\mathbf{A}(\mathbf{y})$ satisfies:

$$A_m \le \|\mathbf{A}(\mathbf{y})\| \le A_M \quad (8)$$

where A_m and A_M denote the minimum and maximum eigenvalues of $\mathbf{A}(\mathbf{y})$ respectively. Unless otherwise specified, all norms are defined as 2-norm (Frobenius norm).

Property 2. As shown in Lewis et al. [1993], $\dot{\mathbf{M}}_\mathbf{x}(\mathbf{q}) - 2\mathbf{B_x}(\mathbf{q},\dot{\mathbf{q}})$ is skew-symmetric, hence

$$\mathbf{z}^{\mathrm{T}}\left(\dot{\mathbf{M}}_\mathbf{x}(\mathbf{q}) - 2\mathbf{B_x}(\mathbf{q},\dot{\mathbf{q}})\right)\mathbf{z} = 0, \quad \mathbf{z} \in \Re^n \quad (9)$$

Property 3. In joint space, $\boldsymbol{\tau}_f(\dot{\mathbf{q}})$ by parts satisfies, Lewis et al. [1993]:

$$\|\boldsymbol{\tau}_{vis}\dot{\mathbf{q}}\| \le \tau_{vis,M}\|\dot{\mathbf{q}}\| \quad (10)$$

$$\|\boldsymbol{\tau}_{cou}\mathrm{sgn}(\dot{\mathbf{q}})\| \le \tau_{cou,M} \quad (11)$$

$$\|\boldsymbol{\tau}_{sti}\mathrm{exp}^{(-\boldsymbol{\tau}_{dec}\dot{\mathbf{q}}^2)}\mathrm{sgn}(\dot{\mathbf{q}})\| \le \tau_{sti,M} \quad (12)$$

Property 4. The operational space gravity vector $\mathbf{g_x}(\mathbf{q})$ is upper-bounded, this is direct from (4):

$$\|\mathbf{g_x}(\mathbf{q})\| \le g_M < \infty \quad (13)$$

Property 5. The operational space Coriolis and centrifugal matrix $\mathbf{B_x}(\mathbf{q},\dot{\mathbf{q}})$ can be expressed as a function of $\mathbf{q}$ and $\dot{\mathbf{x}}$ since

$$\mathbf{B_x}(\mathbf{q},\dot{\mathbf{x}}) = [\mathbf{J}^{-\mathrm{T}}(\mathbf{q})\mathbf{B}(\mathbf{q},\dot{\mathbf{x}}) - \mathbf{M_x}(\mathbf{q})\dot{\mathbf{J}}(\mathbf{q},\dot{\mathbf{x}})]\mathbf{J}^{-1}(\mathbf{q}) \quad (14)$$

This can be obtained directly by substituting $\mathbf{B}(\mathbf{q},\dot{\mathbf{q}}) = \mathbf{B}(\mathbf{q},\dot{\mathbf{x}})$ and $\dot{\mathbf{J}}(\mathbf{q},\dot{\mathbf{q}}) = \dot{\mathbf{J}}(\mathbf{q},\dot{\mathbf{x}})$ (since $\dot{\mathbf{q}} = \mathbf{J}^{-1}\dot{\mathbf{x}}$) into (3).

Property 6. The operational space Coriolis and centrifugal matrix $\mathbf{B_x}(\mathbf{q},\dot{\mathbf{x}})$ is upper-bounded

$$\|\mathbf{B_x}(\mathbf{q},\dot{\mathbf{x}})\| \le B_{\mathbf{x},M}\|\dot{\mathbf{x}}\| \quad (15)$$

where $B_{\mathbf{x},M}$ is a positive scalar constant. It is a direct result from property 5 combined with the joint space properties $\|\mathbf{B}(\mathbf{q},\dot{\mathbf{x}})\| \le B_M\|\dot{\mathbf{x}}\|$ and $\|\dot{\mathbf{J}}(\mathbf{q},\dot{\mathbf{x}})\| \le \dot{J}_M\|\dot{\mathbf{x}}\|$.

Property 7. For any vectors $\mathbf{y},\mathbf{z} \in \Re^n$, the operational space Coriolis and centrifugal matrix $\mathbf{B_x}(\mathbf{q},\dot{\mathbf{x}})$ satisfies the following relationship

$$\mathbf{B_x}(\mathbf{q},\mathbf{y})\mathbf{z} = \mathbf{B_x}(\mathbf{q},\mathbf{z})\mathbf{y} \quad (16)$$

which is obtained by using the following properties: the joint space coriolis and Centrifugal matrix property $\mathbf{B}(\mathbf{q},\mathbf{y})\mathbf{z} = \mathbf{B}(\mathbf{q},\mathbf{z})\mathbf{y}$ [Nicosia and Tomei, 1990] and $\dot{\mathbf{J}}(\mathbf{q},\mathbf{y})\mathbf{z} = \dot{\mathbf{J}}(\mathbf{q},\mathbf{z})\mathbf{y}$ and the fact that $\mathbf{J}^{-1}(\mathbf{q})\dot{\hat{\mathbf{x}}} = \dot{\hat{\mathbf{q}}}$.

3. NN ADAPTIVE MOTION CONTROLLER-OBSERVER FORMULATION

3.1 NN Adaptive Motion Controller-Observer

In this section, the operational space NN-based adaptive motion controller is proposed as:

$$\mathbf{F} = \mathbf{K_v}(\mathbf{r}_1 + \mathbf{r}_2) + \hat{\mathbf{M}}_\mathbf{x}(\mathbf{q})\ddot{\mathbf{x}}_r + \hat{\mathbf{B}}_\mathbf{x}(\mathbf{q},\dot{\mathbf{x}}_0)\dot{\mathbf{x}}_r + \hat{\mathbf{g}}_\mathbf{x}(\mathbf{q}) + \hat{\mathbf{f}}_\mathbf{x}(\mathbf{q},\dot{\hat{\mathbf{q}}}) \quad (17)$$

The estimate of a dynamic parameter m is represented as $\hat{m}$ and the error dynamics are represented by $\tilde{m} = m - \hat{m}$.

The following terms are defined

$$\ddot{\mathbf{x}}_r = \ddot{\mathbf{x}}_d + \boldsymbol{\Lambda}_1(\dot{\mathbf{x}} - \dot{\hat{\mathbf{x}}}) + \boldsymbol{\Lambda}_i\mathbf{e} \quad (18)$$

$$\dot{\mathbf{x}}_r = \dot{\mathbf{x}}_d + \boldsymbol{\Lambda}_1(\mathbf{x}_d - \hat{\mathbf{x}}) + \boldsymbol{\Lambda}_i\int_0^{\tau=t}\mathbf{e}\,\mathrm{d}\tau \quad (19)$$

$$\mathbf{r}_1 = \dot{\mathbf{x}}_r - \dot{\mathbf{x}} = \dot{\mathbf{e}} + \boldsymbol{\Lambda}_1\mathbf{e} + \boldsymbol{\Lambda}_1\tilde{\mathbf{x}} + \boldsymbol{\Lambda}_i\int_0^{\tau=t}\mathbf{e}\,\mathrm{d}\tau \quad (20)$$

$$\dot{\mathbf{x}}_0 = \dot{\hat{\mathbf{x}}} - \boldsymbol{\Lambda}_2\tilde{\mathbf{x}} \quad (21)$$

$$\mathbf{r}_2 = \dot{\mathbf{x}} - \dot{\mathbf{x}}_0 = \dot{\tilde{\mathbf{x}}} + \boldsymbol{\Lambda}_2\tilde{\mathbf{x}} \quad (22)$$

$$\mathbf{r}_1 + \mathbf{r}_2 = \dot{\mathbf{x}}_r - \dot{\mathbf{x}}_0 \quad (23)$$

where $\boldsymbol{\Lambda}_1, \boldsymbol{\Lambda}_2, \boldsymbol{\Lambda}_i \in \Re^{n\times n}$ are positive diagonal matrices, $\mathbf{e} = \mathbf{x}_d - \mathbf{x}$ and $\dot{\mathbf{e}} = \dot{\mathbf{x}}_d - \dot{\mathbf{x}}$, are the trajectory tracking errors, and $\mathbf{x}_d, \dot{\mathbf{x}}_d, \ddot{\mathbf{x}}_d$ are the desired trajectories. The estimation motion tracking errors are $\tilde{\mathbf{x}} = \mathbf{x} - \hat{\mathbf{x}}$ and $\dot{\tilde{\mathbf{x}}} = \dot{\mathbf{x}} - \dot{\hat{\mathbf{x}}}$, while $\hat{\mathbf{x}}$ is the estimated operational space coordinates. Combining the robot dynamics (1) with property 5, and then with proposed controller (17), a *general closed-loop dynamics* is obtained as:

$$\mathbf{M_x}(\mathbf{q})\dot{\mathbf{r}}_1 = -\mathbf{K_v}(\mathbf{r}_1 + \mathbf{r}_2) - \mathbf{B_x}(\mathbf{q},\dot{\mathbf{x}}_0)\dot{\mathbf{x}}_r + \mathbf{B_x}(\mathbf{q},\dot{\mathbf{x}})\dot{\mathbf{x}} + \mathbf{f_x}(\mathbf{q},\dot{\mathbf{q}}) - \mathbf{f_x}(\mathbf{q},\dot{\hat{\mathbf{q}}}) + \boldsymbol{\eta} \quad (24)$$

where the uncertainties of the system $\boldsymbol{\eta}$

$$\boldsymbol{\eta} = \tilde{\mathbf{M}}_{\mathbf{x}}(\mathbf{q})\ddot{\mathbf{x}}_r + \tilde{\mathbf{B}}_{\mathbf{x}}(\mathbf{q}, \dot{\mathbf{x}}_0)\dot{\mathbf{x}}_r + \tilde{\mathbf{g}}_{\mathbf{x}}(\mathbf{q}) + \tilde{\mathbf{f}}_{\mathbf{x}}(\mathbf{q}, \dot{\hat{\mathbf{q}}}). \quad (25)$$

From property 3

$$\begin{aligned} \mathbf{f}_{\mathbf{x}}(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{f}_{\mathbf{x}}(\mathbf{q}, \dot{\hat{\mathbf{q}}}) &= \mathbf{J}^{-\mathrm{T}}[\boldsymbol{\tau}_{vis}\dot{\tilde{\mathbf{q}}} + \boldsymbol{\tau}_{cou}(\mathrm{sgn}(\dot{\mathbf{q}}) - \mathrm{sgn}(\dot{\hat{\mathbf{q}}})) \\ &+ \boldsymbol{\tau}_{sti}(\exp^{(-\boldsymbol{\tau}_{dec}\dot{\mathbf{q}}^2)}\mathrm{sgn}(\dot{\mathbf{q}}) - \exp^{(-\boldsymbol{\tau}_{dec}\dot{\hat{\mathbf{q}}}^2)}\mathrm{sgn}(\dot{\hat{\mathbf{q}}}))]. \end{aligned} \quad (26)$$

Using property 7, $\mathbf{B}_{\mathbf{x}}(\mathbf{q}, \dot{\mathbf{x}}_0)\dot{\mathbf{x}}_r - \mathbf{B}_{\mathbf{x}}(\mathbf{q}, \dot{\mathbf{x}})\dot{\mathbf{x}}$ in (24) equals to $\mathbf{B}_{\mathbf{x}}(\mathbf{q}, \dot{\mathbf{x}})\mathbf{r}_1 - \mathbf{B}_{\mathbf{x}}(\mathbf{q}, \dot{\mathbf{x}}_r)\mathbf{r}_2$. This yields the *controller closed-loop dynamics*:

$$\begin{aligned} \mathbf{M}_{\mathbf{x}}(\mathbf{q})\dot{\mathbf{r}}_1 &= -\mathbf{K}_{\mathbf{v}}(\mathbf{r}_1 + \mathbf{r}_2) - \mathbf{B}_{\mathbf{x}}(\mathbf{q}, \dot{\mathbf{x}})\mathbf{r}_1 \\ &+ \mathbf{B}_{\mathbf{x}}(\mathbf{q}, \dot{\mathbf{x}}_r)\mathbf{r}_2 + \mathbf{f}_{\mathbf{x}}(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{f}_{\mathbf{x}}(\mathbf{q}, \dot{\hat{\mathbf{q}}}) + \boldsymbol{\eta} \end{aligned} \quad (27)$$

The proposed observer is introduced as:

$$\dot{\hat{\mathbf{x}}} = \dot{\mathbf{x}} - \dot{\tilde{\mathbf{x}}} = \mathbf{z} + \mathbf{L}_D\tilde{\mathbf{x}} \quad (28)$$

$$\dot{\mathbf{z}} = \ddot{\mathbf{x}}_r + \mathbf{L}_P\tilde{\mathbf{x}} \quad (29)$$

where $\mathbf{L}_D, \mathbf{L}_P \in \Re^{n\times n}$ are positive diagonal matrices. Combining (29) and the first derivative of (28), and taking into account (20), we obtain:

$$\ddot{\tilde{\mathbf{x}}} + \mathbf{L}_D\dot{\tilde{\mathbf{x}}} + \mathbf{L}_P\tilde{\mathbf{x}} = -(\ddot{\mathbf{x}}_d - \ddot{\mathbf{x}} + \boldsymbol{\Lambda}_1(\dot{\mathbf{x}}_d - \dot{\hat{\mathbf{x}}})) = -\dot{\mathbf{r}}_1 \quad (30)$$

$\mathbf{L}_D$ and $\mathbf{L}_P$ can be written as

$$\mathbf{L}_D = \mathbf{l}_D + \boldsymbol{\Lambda}_2 \quad (31)$$

$$\mathbf{L}_P = \mathbf{l}_D\boldsymbol{\Lambda}_2 \quad (32)$$

where $\mathbf{l}_D, \boldsymbol{\Lambda}_2 \in \Re^{n\times n}$ are positive diagonal matrices. Substituting $\mathbf{r}_2 = \dot{\tilde{\mathbf{x}}} + \boldsymbol{\Lambda}_2\tilde{\mathbf{x}}$ into (30) and multiply with $\mathbf{M}_{\mathbf{x}}(\mathbf{q})$, it yields

$$\mathbf{M}_{\mathbf{x}}(\mathbf{q})\dot{\mathbf{r}}_2 + \mathbf{M}_{\mathbf{x}}(\mathbf{q})\mathbf{l}_D\mathbf{r}_2 = -\mathbf{M}_{\mathbf{x}}(\mathbf{q})\dot{\mathbf{r}}_1 \quad (33)$$

Using property 7, $\mathbf{B}_{\mathbf{x}}(\mathbf{q}, \dot{\mathbf{x}}_0)\dot{\mathbf{x}}_r - \mathbf{B}_{\mathbf{x}}(\mathbf{q}, \dot{\mathbf{x}})\dot{\mathbf{x}}$ in (24) equals to $\mathbf{B}_{\mathbf{x}}(\mathbf{q}, \dot{\mathbf{x}}_0)\mathbf{r}_1 - \mathbf{B}_{\mathbf{x}}(\mathbf{q}, \dot{\mathbf{x}})\mathbf{r}_2$ and (33), the *observer closed-loop dynamics* is obtained as:

$$\begin{aligned} \mathbf{M}_{\mathbf{x}}(\mathbf{q})\dot{\mathbf{r}}_2 &= -(\mathbf{M}_{\mathbf{x}}(\mathbf{q})\mathbf{l}_D - \mathbf{K}_{\mathbf{v}})\mathbf{r}_2 + \mathbf{K}_{\mathbf{v}}\mathbf{r}_1 \\ &- \mathbf{B}_{\mathbf{x}}(\mathbf{q}, \dot{\mathbf{x}})\mathbf{r}_2 + \mathbf{B}_{\mathbf{x}}(\mathbf{q}, \dot{\mathbf{x}}_0)\mathbf{r}_1 - (\mathbf{f}_{\mathbf{x}}(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{f}_{\mathbf{x}}(\mathbf{q}, \dot{\hat{\mathbf{q}}})) \\ &- \boldsymbol{\eta}. \end{aligned} \quad (34)$$

3.2 Three-Layer Neural Networks

Given N_1, N_2 and N_3 are the number of neurons in layer 1, 2 and 3, respectively, an input vector $\mathbf{z} \in \Re^{N_1}$ can be defined where v_{kl} is the first-to-second layer node weights, $l = 1, \ldots, N_1$, $k = 1, \ldots, N_2$, θ_k is the threshold offset, and w_{ik} is second-to-third layer node weights where $i = 1, \ldots, N_3$. For a three-layer NN, the output vector $y \in \Re^{N_3}$ can be expressed as

$$y_i = \sum_{k=1}^{N_2} w_{ik}\ \sigma_k\left(\sum_{l=1}^{N_1} v_{kl}z_l\ +\ \theta_k\right); i = 1, \ldots, N_3, \quad (35)$$

or an output matrix $y \in \Re^{N_3\times N_4}$, which can be expressed as:

$$y_{ij} = \sum_{k=1}^{N_2} w_{ijk}\ \sigma_k\left(\sum_{l=1}^{N_1} v_{kl}z_l\ +\ \theta_k\right); \quad i = 1, \ldots, N_3, j = 1, \ldots, N_4 \quad (36)$$

where function $\sigma(\cdot)$ is differentiable such as sigmoid and hyperbolic functions. The output (35) and (36) can be compactly expressed in vector form ($\mathbf{y}$) and matrix form ($\mathbf{Y}$):

$$\mathbf{y} = \{\mathbf{W}\}^{\mathrm{T}}\boldsymbol{\sigma}\left(\{\mathbf{V}\}^{\mathrm{T}}\mathbf{z}\right), \quad \mathbf{Y} = \{\mathbf{W}\}^{\mathrm{T}}\boldsymbol{\sigma}\left(\{\mathbf{V}\}^{\mathrm{T}}\mathbf{z}\right) \quad (37)$$

3.3 Uncertainties $\boldsymbol{\eta}$ in NN terms

Since the estimate of a dynamic parameter m is represented as $\hat{m}$ and the error dynamics are represented by $\tilde{m} = m - \hat{m}$ then the uncertainties $\boldsymbol{\eta}$ (25) can be written as

$$\begin{aligned} \boldsymbol{\eta} &= (\mathbf{M}_{\mathbf{x}} - \hat{\mathbf{M}}_{\mathbf{x}})(\mathbf{q})\ddot{\mathbf{x}}_r + (\mathbf{B}_{\mathbf{x}} - \hat{\mathbf{B}}_{\mathbf{x}})(\mathbf{q}, \dot{\mathbf{x}}_0)\dot{\mathbf{x}}_r \\ &+ (\mathbf{g}_{\mathbf{x}} - \hat{\mathbf{g}}_{\mathbf{x}})(\mathbf{q}) + (\mathbf{f}_{\mathbf{x}} - \hat{\mathbf{f}}_{\mathbf{x}})(\mathbf{q}, \dot{\hat{\mathbf{q}}}) \end{aligned} \quad (38)$$

From NN theory, given unlimited number of hidden layer nodes, three layer NNs with ideal weights can approximate any functions. In practice, however, there are only limited number of hidden layer nodes, thus the dynamic terms $\mathbf{M}_{\mathbf{x}}(\mathbf{q})$, $\mathbf{B}_{\mathbf{x}}(\mathbf{q}, \dot{\mathbf{x}}_0)$, $\mathbf{g}_{\mathbf{x}}(\mathbf{q})$, and $\mathbf{f}_{\mathbf{x}}(\mathbf{q}, \dot{\hat{\mathbf{q}}})$ can be approximated by three-layer NNs with "optimum"weights $\{\mathbf{V}\}, \{\mathbf{W}\}$ and approximation error $\boldsymbol{\varepsilon}$:

$$\mathbf{M}_{\mathbf{x}}(\mathbf{q}) = \{\mathbf{W}_{\mathbf{M}}\}^{\mathrm{T}}\boldsymbol{\sigma}_{\mathbf{M}}(\{\mathbf{V}_{\mathbf{M}}\}^{\mathrm{T}}\mathbf{z}_{\mathbf{M}}) + \boldsymbol{\varepsilon}_{\mathbf{M}} \quad (39)$$

$$\mathbf{B}_{\mathbf{x}}(\mathbf{q}, \dot{\mathbf{x}}_0) = \{\mathbf{W}_{\mathbf{B}}\}^{\mathrm{T}}\boldsymbol{\sigma}_{\mathbf{B}}(\{\mathbf{V}_{\mathbf{B}}\}^{\mathrm{T}}\mathbf{z}_{\mathbf{B}}) + \boldsymbol{\varepsilon}_{\mathbf{B}} \quad (40)$$

$$\mathbf{g}_{\mathbf{x}}(\mathbf{q}) = \{\mathbf{W}_{\mathbf{g}}\}^{\mathrm{T}}\boldsymbol{\sigma}_{\mathbf{g}}(\{\mathbf{V}_{\mathbf{g}}\}^{\mathrm{T}}\mathbf{z}_{\mathbf{g}}) + \boldsymbol{\varepsilon}_{\mathbf{g}} \quad (41)$$

$$\mathbf{f}_{\mathbf{x}}(\mathbf{q}, \dot{\hat{\mathbf{q}}}) = \{\mathbf{W}_{\mathbf{f}}\}^{\mathrm{T}}\boldsymbol{\sigma}_{\mathbf{f}}(\{\mathbf{V}_{\mathbf{f}}\}^{\mathrm{T}}\mathbf{z}_{\mathbf{f}}) + \boldsymbol{\varepsilon}_{\mathbf{f}} \quad (42)$$

Likewise the estimated dynamic terms $\hat{\mathbf{M}}_{\mathbf{x}}(\mathbf{q})$, $\hat{\mathbf{B}}_{\mathbf{x}}(\mathbf{q}, \dot{\mathbf{x}}_0)$, $\hat{\mathbf{g}}_{\mathbf{x}}(\mathbf{q})$, and $\hat{\mathbf{f}}_{\mathbf{x}}(\mathbf{q}, \dot{\hat{\mathbf{q}}})$ are approximated by estimated weights $\{\hat{\mathbf{V}}_i\}, \{\hat{\mathbf{W}}_i\}$ where subscript $i = M, B, g, f$ represents the individual dynamical terms. The following generic NN expressions are then defined for ease of representation such that:

$$\begin{aligned} \mathbf{L}_i &= (\{\mathbf{W}_i\}^{\mathrm{T}}\boldsymbol{\sigma}_i(\{\mathbf{V}_i\}^{\mathrm{T}}\mathbf{z}_i) \\ \hat{\mathbf{L}}_i &= (\{\hat{\mathbf{W}}_i\}^{\mathrm{T}}\boldsymbol{\sigma}_i(\{\hat{\mathbf{V}}_i\}^{\mathrm{T}}\mathbf{z}_i) \\ \tilde{\mathbf{L}}_i &= \mathbf{L}_i - \hat{\mathbf{L}}_i; \end{aligned} \quad (43)$$

where $\mathbf{L}_i$, $\hat{\mathbf{L}}_i$, and $\tilde{\mathbf{L}}_i$ represent the actual, estimated, and error, of the expression accordingly. Hence, using the generic NN expressions, the uncertainties (38) can be written as

$$\begin{aligned} \boldsymbol{\eta} &= (\mathbf{L}_M - \hat{\mathbf{L}}_M)\ddot{\mathbf{x}}_r + (\mathbf{L}_B - \hat{\mathbf{L}}_B)\dot{\mathbf{x}}_r + (\mathbf{L}_g - \hat{\mathbf{L}}_g) \\ &+ (\mathbf{L}_f - \hat{\mathbf{L}}_f) + \boldsymbol{\varepsilon} \end{aligned} \quad (44)$$

where the total approximation error $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_{\mathbf{M}}\ddot{\mathbf{x}}_r + \boldsymbol{\varepsilon}_{\mathbf{B}}\dot{\mathbf{x}}_r + \boldsymbol{\varepsilon}_{\mathbf{g}} + \boldsymbol{\varepsilon}_{\mathbf{f}}$. To compute $\boldsymbol{\eta}$ (44), it is necessary to compute the generic form of

$$\mathbf{L}_i - \hat{\mathbf{L}}_i = \{\mathbf{W}_i\}^{\mathrm{T}}\boldsymbol{\sigma}\left(\{\mathbf{V}_i\}^{\mathrm{T}}\mathbf{z}_i\right) - \{\hat{\mathbf{W}}_i\}^{\mathrm{T}}\boldsymbol{\sigma}\left(\{\hat{\mathbf{V}}_i\}^{\mathrm{T}}\mathbf{z}_i\right), \quad (45)$$

while the error in the sigmoid of the first to second layer weights is calculated as:

$$\tilde{\boldsymbol{\sigma}} = \boldsymbol{\sigma}\left(\{\mathbf{V}\}^{\mathrm{T}}\mathbf{z}\right) - \boldsymbol{\sigma}\left(\{\hat{\mathbf{V}}\}^{\mathrm{T}}\mathbf{z}\right). \quad (46)$$

Utilizing the Taylor series expansion of the term $\boldsymbol{\sigma}(\mathbf{k})$, following the derivation in [Soewandito et al., 2008], equation (45) can be shown that:

$$\begin{aligned} \mathbf{L}_i - \hat{\mathbf{L}}_i &= \{\mathbf{W}_i\}^{\mathrm{T}}\boldsymbol{\sigma}_i - \{\hat{\mathbf{W}}_i\}^{\mathrm{T}}\hat{\boldsymbol{\sigma}}_i - \{\mathbf{W}_i\}^{\mathrm{T}}\hat{\boldsymbol{\sigma}}_i + \{\mathbf{W}_i\}^{\mathrm{T}}\hat{\boldsymbol{\sigma}}_i \\ &= \{\tilde{\mathbf{W}}_i\}^{\mathrm{T}}\hat{\boldsymbol{\sigma}}_i + \{\mathbf{W}_i\}^{\mathrm{T}}\tilde{\boldsymbol{\sigma}}_i \\ &= \{\tilde{\mathbf{W}}_i\}^{\mathrm{T}}\hat{\boldsymbol{\sigma}}_i + \{\mathbf{W}_i\}^{\mathrm{T}}\left[\hat{\boldsymbol{\sigma}}'_i\{\tilde{\mathbf{V}}_i\}^{\mathrm{T}}\mathbf{z}_i + O\left(\{\tilde{\mathbf{V}}_i\}^{\mathrm{T}}\mathbf{z}_i\right)\right] \\ &= \{\tilde{\mathbf{W}}_i\}^{\mathrm{T}}\hat{\boldsymbol{\sigma}}_i \\ &+ (\{\hat{\mathbf{W}}_i\}^{\mathrm{T}} + \{\tilde{\mathbf{W}}_i\}^{\mathrm{T}})\left[\hat{\boldsymbol{\sigma}}'_i\{\tilde{\mathbf{V}}_i\}^{\mathrm{T}}\mathbf{z}_i + O\left(\{\tilde{\mathbf{V}}_i\}^{\mathrm{T}}\mathbf{z}_i\right)\right] \end{aligned} \quad (47)$$

Using (47), the uncertainties $\boldsymbol{\eta}$ (44) can be divided into

$$\boldsymbol{\eta} = \boldsymbol{\xi} + \mathbf{w} \quad (48)$$

where

$$\begin{aligned} \boldsymbol{\xi} &= \left(\{\tilde{\mathbf{W}}_{\mathbf{M}}\}^{\mathrm{T}}\hat{\boldsymbol{\sigma}}_{\mathbf{M}}\right)\ddot{\mathbf{x}}_r + \left(\{\tilde{\mathbf{W}}_{\mathbf{B}}\}^{\mathrm{T}}\hat{\boldsymbol{\sigma}}_{\mathbf{B}}\right)\dot{\mathbf{x}}_r \\ &+ \{\tilde{\mathbf{W}}_{\mathbf{g}}\}^{\mathrm{T}}\hat{\boldsymbol{\sigma}}_{\mathbf{g}} + \{\tilde{\mathbf{W}}_{\mathbf{f}}\}^{\mathrm{T}}\hat{\boldsymbol{\sigma}}_{\mathbf{f}} + \left(\{\hat{\mathbf{W}}_{\mathbf{M}}\}^{\mathrm{T}}\boldsymbol{\sigma}'_{\mathbf{M}}\{\tilde{\mathbf{V}}_{\mathbf{M}}\}^{\mathrm{T}}\mathbf{z}_{\mathbf{M}}\right)\ddot{\mathbf{x}}_r \\ &+ \left(\{\hat{\mathbf{W}}_{\mathbf{B}}\}^{\mathrm{T}}\boldsymbol{\sigma}'_{\mathbf{B}}\{\tilde{\mathbf{V}}_{\mathbf{B}}\}^{\mathrm{T}}\mathbf{z}_{\mathbf{B}}\right)\dot{\mathbf{x}}_r \\ &+ \{\hat{\mathbf{W}}_{\mathbf{g}}\}^{\mathrm{T}}\boldsymbol{\sigma}'_{\mathbf{g}}\{\tilde{\mathbf{V}}_{\mathbf{g}}\}^{\mathrm{T}}\mathbf{z}_{\mathbf{g}} + \{\hat{\mathbf{W}}_{\mathbf{f}}\}^{\mathrm{T}}\boldsymbol{\sigma}'_{\mathbf{f}}\{\tilde{\mathbf{V}}_{\mathbf{f}}\}^{\mathrm{T}}\mathbf{z}_{\mathbf{f}} \end{aligned} \quad (49)$$

and the "whole"errors $\mathbf{w}$

$$\begin{aligned}\mathbf{w} =& \left(\{\tilde{\mathbf{W}}_\mathbf{M}\}^\mathrm{T}\boldsymbol{\sigma}'_\mathbf{M}\{\tilde{\mathbf{V}}_\mathbf{M}\}^\mathrm{T}\mathbf{z}_\mathbf{M}\right)\ddot{\mathbf{x}}_r + \left(\{\tilde{\mathbf{W}}_\mathbf{B}\}^\mathrm{T}\boldsymbol{\sigma}'_\mathbf{B}\{\tilde{\mathbf{V}}_\mathbf{B}\}^\mathrm{T}\mathbf{z}_\mathbf{B}\right)\dot{\mathbf{x}}_r \\ &+ \{\tilde{\mathbf{W}}_\mathbf{g}\}^\mathrm{T}\boldsymbol{\sigma}'_\mathbf{g}\{\tilde{\mathbf{V}}_\mathbf{g}\}^\mathrm{T}\mathbf{z}_\mathbf{g} + \{\tilde{\mathbf{W}}_\mathbf{f}\}^\mathrm{T}\boldsymbol{\sigma}'_\mathbf{f}\{\tilde{\mathbf{V}}_\mathbf{f}\}^\mathrm{T}\mathbf{z}_\mathbf{f} \\ &+ \left(\{\mathbf{W}_\mathbf{M}\}^\mathrm{T}O(\{\tilde{\mathbf{V}}_\mathbf{M}\}^\mathrm{T}\mathbf{z}_\mathbf{M}\right)\ddot{\mathbf{x}}_r + \left(\{\mathbf{W}_\mathbf{B}\}^\mathrm{T}O(\{\tilde{\mathbf{V}}_\mathbf{B}\}^\mathrm{T}\mathbf{z}_\mathbf{B}\right)\dot{\mathbf{x}}_r \\ &+ \{\mathbf{W}_\mathbf{g}\}^\mathrm{T}O(\{\tilde{\mathbf{V}}_\mathbf{g}\}^\mathrm{T}\mathbf{z}_\mathbf{g} + \{\mathbf{W}_\mathbf{f}\}^\mathrm{T}O(\{\tilde{\mathbf{V}}_\mathbf{f}\}^\mathrm{T}\mathbf{z}_\mathbf{f} + \boldsymbol{\varepsilon}\end{aligned} \tag{50}$$

In the Lyapunov analysis in Section 3.4, it becomes evident that it is only $\boldsymbol{\xi}$ that can be canceled by the weight updates. A non-strict assumption that $\|\mathbf{w}\| \leq w_M$ and $\|\boldsymbol{\xi}\| \leq \xi_M$ are defined, following the reason that:

- $\{\mathbf{W}\}, \{\mathbf{V}\}$ and $\boldsymbol{\varepsilon}$ are the actual dynamics, hence bounded.
- $\hat{\boldsymbol{\sigma}}$, its derivative and $O(\{\tilde{\mathbf{V}}\}^\mathrm{T}\mathbf{z})$ are bounded for differentiable functions such as sigmoid and tanh.
- The desired trajectories $\|\mathbf{x}_d, \dot{\mathbf{x}}_d, \ddot{\mathbf{x}}_d\| \leq x_{d,M}$ are bounded.
- $\{\tilde{\mathbf{W}}\}$ and $\{\tilde{\mathbf{V}}\}$ are assumed to bounded, implying that $\{\hat{\mathbf{W}}\}$ and $\{\hat{\mathbf{V}}\}$ are also bounded. (This assumption will be valid when $\{\tilde{\mathbf{W}}\}$ and $\{\tilde{\mathbf{V}}\}$ are shown to be bounded in Section 3.4.)
- $\dot{\mathbf{e}}$ and $\mathbf{e}$ are assumed to be bounded, implying $\dot{\mathbf{x}}, \mathbf{x}$ and $\dot{\mathbf{q}}, \mathbf{q}$, as well as $\ddot{\mathbf{x}}_r$ and $\dot{\mathbf{x}}_r$ are also bounded. (This assumption will be valid when $\dot{\mathbf{e}}$ and $\mathbf{e}$ are shown to be bounded in Section 3.4.)

3.4 Stability Analysis

For the stability analysis, $\mathbf{Z} = diag[\mathbf{W}, \mathbf{V}]$ is defined, such that $\|\mathbf{Z}\| \leq Z_M$, where $\mathbf{W} = diag[\mathbf{W}_\mathbf{M}, \mathbf{W}_\mathbf{B}, \mathbf{W}_\mathbf{g}, \mathbf{W}_\mathbf{f}]$ and $\mathbf{V} = diag[\mathbf{V}_\mathbf{M}, \mathbf{V}_\mathbf{B}, \mathbf{V}_\mathbf{g}, \mathbf{V}_\mathbf{f}]$.

Theorem 1. Let $\mathbf{y}^\mathrm{T} = \left[\mathbf{r}^\mathrm{T}\ \mathbf{r}_2^\mathrm{T}\right]$, if satisfied

$$l_{D,m} > K_{v,M}/M_m \tag{51}$$

where $l_{D,m}, M_m$ are the minimum eigenvalues of $\mathbf{l}_D, \mathbf{M}_\mathbf{x}(\mathbf{q})$ respectively, while $K_{v,M}$ is the maximum eigenvalue of $\mathbf{K}_\mathbf{v}$. Let the NN weight updates be provided as

$$\dot{\hat{\mathbf{W}}}_{\mathbf{M}ij} = \mathbf{F}_{\mathbf{M}ij}(\hat{\boldsymbol{\sigma}}_\mathbf{M}\,(r_{1,i} + r_{2,i})\,\ddot{x}_{r,j} - \kappa\hat{\mathbf{W}}_{\mathbf{M}ij}) \tag{52}$$

$$\dot{\hat{\mathbf{V}}}_{\mathbf{M}k} = \mathbf{G}_{\mathbf{M}k}(\mathbf{z}_\mathbf{M}\,\hat{\boldsymbol{\sigma}}'_{\mathbf{M}k}\,(\sum_{i=1}^{n}\sum_{j=1}^{n}\hat{\mathbf{W}}_{\mathbf{M}ijk}\,(r_{1,i} + r_{2,i})\,\ddot{x}_{r,j}) - \kappa\hat{\mathbf{V}}_{\mathbf{M}k}) \tag{53}$$

$$\dot{\hat{\mathbf{W}}}_{\mathbf{B}ij} = \mathbf{F}_{\mathbf{B}ij}(\hat{\boldsymbol{\sigma}}_\mathbf{B}\,(r_{1,i} + r_{2,i})\,\dot{x}_{r,j} - \kappa\hat{\mathbf{W}}_{\mathbf{B}ij}) \tag{54}$$

$$\dot{\hat{\mathbf{V}}}_{\mathbf{B}k} = \mathbf{G}_{\mathbf{B}k}(\mathbf{z}_\mathbf{B}\,\hat{\boldsymbol{\sigma}}'_{\mathbf{B}k}\,(\sum_{i=1}^{n}\sum_{j=1}^{n}\hat{\mathbf{W}}_{\mathbf{B}ijk}\,(r_{1,i} + r_{2,i})\,\dot{x}_{r,j}) - \kappa\hat{\mathbf{V}}_{\mathbf{B}k}) \tag{55}$$

$$\dot{\hat{\mathbf{W}}}_{\mathbf{g}i} = \mathbf{F}_{\mathbf{g}i}(\hat{\boldsymbol{\sigma}}_\mathbf{g}\,(r_{1,i} + r_{2,i}) - \kappa\hat{\mathbf{W}}_{\mathbf{g}i}) \tag{56}$$

$$\dot{\hat{\mathbf{V}}}_{\mathbf{g}k} = \mathbf{G}_{\mathbf{g}k}(\mathbf{z}_\mathbf{g}\,\hat{\boldsymbol{\sigma}}'_{\mathbf{g}k}\,(\sum_{i=1}^{n}\hat{\mathbf{W}}_{\mathbf{g}ik}\,(r_{1,i} + r_{2,i})) - \kappa\hat{\mathbf{V}}_{\mathbf{g}k}) \tag{57}$$

$$\dot{\hat{\mathbf{W}}}_{\mathbf{f}i} = \mathbf{F}_{\mathbf{f}i}(\hat{\boldsymbol{\sigma}}_\mathbf{f}\,(r_{1,i} + r_{2,i}) - \kappa\hat{\mathbf{W}}_{\mathbf{f}i}) \tag{58}$$

$$\dot{\hat{\mathbf{V}}}_{\mathbf{f}k} = \mathbf{G}_{\mathbf{f}k}(\mathbf{z}_\mathbf{f}\,\hat{\boldsymbol{\sigma}}'_{\mathbf{f}k}\,(\sum_{i=1}^{n}\hat{\mathbf{W}}_{\mathbf{f}ik}\,(r_{1,i} + r_{2,i})) - \kappa\hat{\mathbf{V}}_{\mathbf{f}k}) \tag{59}$$

where output nodes $i, j = 1, \ldots, n$ and hidden nodes $k = 1, \ldots, k_i$ (the subscript 'i'$\equiv M, B, g, f$). It is then obtained that

$$\lim_{t\to\infty} \|\mathbf{y}(t)\| = b_\mathbf{y}, \quad \lim_{t\to\infty} \|\tilde{\mathbf{Z}}(t)\| = b_{\tilde{\mathbf{Z}}} \tag{60}$$

where $b_\mathbf{y}$ and $b_{\tilde{\mathbf{Z}}}$ are defined in (74) and (75) respectively.

Having defined the error dynamics (27), (34) and the uncertainties $\boldsymbol{\eta}$ (48), the Lyapunov function is

$$\begin{aligned}V(\mathbf{r}_1, \mathbf{r}_2, \tilde{\mathbf{Z}}) =& \frac{1}{2}\mathbf{r}_1^\mathrm{T}\mathbf{M}_\mathbf{x}(\mathbf{q})\mathbf{r}_1 + \frac{1}{2}\mathbf{r}_2^\mathrm{T}\mathbf{M}_\mathbf{x}(\mathbf{q})\mathbf{r}_2 \\ &+ \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{\mathbf{W}}_{\mathbf{M}_{ij}}^\mathrm{T}\,\mathbf{F}_{\mathbf{M}_{ij}}^{-1}\,\tilde{\mathbf{W}}_{\mathbf{M}_{ij}} + \ldots + \frac{1}{2}\sum_{k=1}^{k_f}\tilde{\mathbf{V}}_{\mathbf{f}_k}^\mathrm{T}\,\mathbf{G}_{\mathbf{f}_k}^{-1}\,\tilde{\mathbf{V}}_{\mathbf{f}_k}\end{aligned} \tag{61}$$

where $\tilde{\mathbf{W}}_{\mathbf{M}_{ij}} \in \Re^{k_M}, \ldots, \tilde{\mathbf{W}}_{\mathbf{f}_i} \in \Re^{k_f}$ and $\tilde{\mathbf{V}}_{\mathbf{M}_k} \in \Re^{l_M}, \ldots, \hat{\mathbf{V}}_{\mathbf{f}_k} \in \Re^{l_f}$ and $\mathbf{F}_{\mathbf{M}_{ij}}^{-1} \in \Re^{k_M\times k_M}, \ldots, \mathbf{F}_{\mathbf{f}_i}^{-1} \in \Re^{k_f\times k_f}$ and $\mathbf{G}_{\mathbf{M}_k}^{-1} \in \Re^{l_M\times l_M}, \ldots, \mathbf{G}_{\mathbf{f}_k}^{-1} \in \Re^{l_f\times l_f}$ are positive diagonal matrices. With l_i is the input nodes size (the subscript 'i'$\equiv M, B, g, f$).

Then the error dynamics (27), (34), property 2 and the uncertainties $\boldsymbol{\eta}$ (48) are substituted into $\dot{V}(\mathbf{r}_1, \mathbf{r}_2, \tilde{\mathbf{Z}})$ to yield:

$$\begin{aligned}\dot{V} =& -\mathbf{r}_1^\mathrm{T}\mathbf{K}_\mathbf{v}\mathbf{r}_1 - \mathbf{r}_2^\mathrm{T}(\mathbf{M}_\mathbf{x}(\mathbf{q})\mathbf{l}_D - \mathbf{K}_\mathbf{v})\mathbf{r}_2 \\ &+ \mathbf{r}_1^\mathrm{T}\mathbf{B}_\mathbf{x}(\mathbf{q}, \dot{\mathbf{x}}_r)\mathbf{r}_2 + \mathbf{r}_2^\mathrm{T}\mathbf{B}_\mathbf{x}(\mathbf{q}, \dot{\mathbf{x}}_0)\mathbf{r}_1 \\ &+ (\mathbf{r}_1^\mathrm{T} - \mathbf{r}_2^\mathrm{T})(\mathbf{f}_\mathbf{x}(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{f}_\mathbf{x}(\mathbf{q}, \dot{\hat{\mathbf{q}}})) \\ &+ (\mathbf{r}_1^\mathrm{T} - \mathbf{r}_2^\mathrm{T})\mathbf{w} + \boldsymbol{\nu}\end{aligned} \tag{62}$$

where the lump parameter

$$\begin{aligned}\boldsymbol{\nu} =& \sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{\mathbf{W}}_{\mathbf{M}_{ij}}^\mathrm{T}\left(\mathbf{F}_{\mathbf{M}_{ij}}^{-1}\,\dot{\hat{\mathbf{W}}}_{\mathbf{M}ij} + \hat{\boldsymbol{\sigma}}_\mathbf{M}(r_{1,i} - r_{2,i})\ddot{x}_{r,j}\right) \\ &+ \sum_{i=1}^{n}\sum_{j=1}^{n}\tilde{\mathbf{W}}_{\mathbf{B}_{ij}}^\mathrm{T}\left(\mathbf{F}_{\mathbf{B}_{ij}}^{-1}\,\dot{\hat{\mathbf{W}}}_{\mathbf{B}ij} + \hat{\boldsymbol{\sigma}}_\mathbf{B}(r_{1,i} - r_{2,i})\dot{x}_{r,j}\right) \\ &+ \sum_{i=1}^{n}\tilde{\mathbf{W}}_{\mathbf{g}_i}^\mathrm{T}\left(\mathbf{F}_{\mathbf{g}_i}^{-1}\,\dot{\hat{\mathbf{W}}}_{\mathbf{g}i} + \hat{\boldsymbol{\sigma}}_\mathbf{g}(r_{1,i} - r_{2,i})\right) \\ &+ \sum_{i=1}^{n}\tilde{\mathbf{W}}_{\mathbf{f}_i}^\mathrm{T}\left(\mathbf{F}_{\mathbf{f}_i}^{-1}\,\dot{\hat{\mathbf{W}}}_{\mathbf{f}i} + \hat{\boldsymbol{\sigma}}_\mathbf{f}(r_{1,i} - r_{2,i})\right) \\ &+ \sum_{k=1}^{k_M}\tilde{\mathbf{V}}_{\mathbf{M}_k}^\mathrm{T}\left(\mathbf{G}_{\mathbf{M}_k}^{-1}\,\dot{\hat{\mathbf{V}}}_{\mathbf{M}k} + \mathbf{z}_\mathbf{M}\hat{\boldsymbol{\sigma}}'_{\mathbf{M}k}\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\hat{\mathbf{W}}_{\mathbf{M}ijk}(r_{1,i} - r_{2,i})\ddot{x}_{r,j})\right)\right) \\ &+ \sum_{k=1}^{k_B}\tilde{\mathbf{V}}_{\mathbf{B}_k}^\mathrm{T}\left(\mathbf{G}_{\mathbf{B}_k}^{-1}\,\dot{\hat{\mathbf{V}}}_{\mathbf{B}k} + \mathbf{z}_\mathbf{B}\hat{\boldsymbol{\sigma}}'_{\mathbf{B}k}\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\hat{\mathbf{W}}_{\mathbf{B}ijk}(r_{1,i} - r_{2,i})\dot{x}_{r,j})\right)\right) \\ &+ \sum_{k=1}^{k_g}\tilde{\mathbf{V}}_{\mathbf{g}_k}^\mathrm{T}\left(\mathbf{G}_{\mathbf{g}_k}^{-1}\,\dot{\hat{\mathbf{V}}}_{\mathbf{g}k} + \mathbf{z}_\mathbf{g}\hat{\boldsymbol{\sigma}}'_{\mathbf{g}k}(\sum_{i=1}^{n}\hat{\mathbf{W}}_{\mathbf{g}ik}(r_{1,i} - r_{2,i}))\right) \\ &+ \sum_{k=1}^{k_f}\tilde{\mathbf{V}}_{\mathbf{f}_k}^\mathrm{T}\left(\mathbf{G}_{\mathbf{f}_k}^{-1}\,\dot{\hat{\mathbf{V}}}_{\mathbf{f}k} + \mathbf{z}_\mathbf{f}\hat{\boldsymbol{\sigma}}'_{\mathbf{f}k}(\sum_{i=1}^{n}\hat{\mathbf{W}}_{\mathbf{f}ik}(r_{1,i} - r_{2,i}))\right)\end{aligned} \tag{63}$$

The terms in (62) can be analyzed for its boundedness. Firstly, the second line of (62) can be shown bounded

$$\begin{aligned}&\|\mathbf{r}_1^\mathrm{T}\mathbf{B}_\mathbf{x}(\mathbf{q}, \dot{\mathbf{x}}_r)\mathbf{r}_2 + \mathbf{r}_2^\mathrm{T}\mathbf{B}_\mathbf{x}(\mathbf{q}, \dot{\mathbf{x}}_0)\mathbf{r}_1\| \\ \leq& \|\mathbf{r}_1\|\|\mathbf{r}_2\|B_{\mathbf{x},M}(\|\mathbf{r}_1\| + \|\mathbf{r}_2\| + 2\dot{x}_M).\end{aligned} \tag{64}$$

this is due to the facts $\dot{\mathbf{x}}_r = \mathbf{r}_1 + \dot{\mathbf{x}}$ and $\dot{\mathbf{x}}_0 = \dot{\mathbf{x}} - \mathbf{r}_2$, and by taking into account property 6 and assuming $\|\dot{\mathbf{x}}\| \leq \dot{x}_M$.

Secondly, the third line of (62) can be shown bounded

$$\|\mathbf{f}_\mathbf{x}(\mathbf{q}, \dot{\mathbf{q}}) - \mathbf{f}_\mathbf{x}(\mathbf{q}, \dot{\hat{\mathbf{q}}})\| \leq \zeta_{\mathrm{friction}} \tag{65}$$

which by using (26) and property 3 the followings hold:

(1) $\mathbf{J}$ is full-rank (non-singular) hence $\mathbf{J}^{-1}$ exists.
(2) $\boldsymbol{\tau}_{vis}\dot{\hat{\mathbf{q}}}$ is bounded: because $\boldsymbol{\tau}_{vis}$ is bounded and assuming $\dot{\mathbf{q}}$ is bounded, hence $\dot{\hat{\mathbf{q}}}$ is also bounded. The boundedness of $\dot{\mathbf{q}}$ can be fairly assumed if $\mathbf{r_{x2}}$ can be shown bounded (in Section 3.4) – which implies $\dot{\mathbf{x}}$ is bounded, and further showing $\dot{\mathbf{q}} = \mathbf{J}^{-1}\dot{\mathbf{x}}$ is bounded.
(3) $\boldsymbol{\tau}_{cou}(\text{sgn}(\dot{\mathbf{q}}) - \text{sgn}(\dot{\hat{\mathbf{q}}}))$ is bounded: because $\boldsymbol{\tau}_{cou}$ is bounded (11) and $(\text{sgn}(\dot{q}_i) - \text{sgn}(\dot{\hat{q}}_i)) \leq 2$ is bounded.
(4) $\boldsymbol{\tau}_{sti}(\exp^{(-\boldsymbol{\tau}_{dec}\dot{\mathbf{q}}^2)}\text{sgn}(\dot{\mathbf{q}}) - \exp^{(-\boldsymbol{\tau}_{dec}\dot{\hat{\mathbf{q}}}^2)}\text{sgn}(\dot{\hat{\mathbf{q}}}))$ is bounded: because $\boldsymbol{\tau}_{sti}$ is bounded (12) and $\text{sgn}(\cdot)$ and $\exp^{-|a|}$ are bounded.

Substituting the results into (62) and taking into account (1), (8) and $\|\mathbf{w}\| \leq w_M$, $\dot{V}(\mathbf{r}_1, \mathbf{r}_2, \tilde{\mathbf{Z}})$ becomes

$$\begin{aligned} \dot{V} \leq &- K_{v,m}\|\mathbf{r}_1\|^2 - (M_m l_{D,m} - K_{v,M})\|\mathbf{r}_2\|^2 \\ &+ \|\mathbf{r}_1\|\|\mathbf{r}_2\| B_{\mathbf{x},M}(\|\mathbf{r}_1\| + \|\mathbf{r}_2\| + 2\dot{x}_M) \\ &+ (\|\mathbf{r}_1\| + \|\mathbf{r}_2\|)(\zeta_{\text{friction}} + w_M) + \boldsymbol{\nu} \end{aligned} \tag{66}$$

It can be seen that the lump parameter ν is made up of the derivatives of the NN weights $\dot{\tilde{m}}$ and $(\mathbf{r}_1^{\mathrm{T}} - \mathbf{r}_2^{\mathrm{T}})\boldsymbol{\xi}$. The idea is to cancel $(\mathbf{r}_1^{\mathrm{T}} - \mathbf{r}_2^{\mathrm{T}})\boldsymbol{\xi}$ with $\dot{\tilde{m}}$. Since only $(\mathbf{r}_1^{\mathrm{T}} + \mathbf{r}_2^{\mathrm{T}})$ term is available, then only $\mathbf{r}_1^{\mathrm{T}}\boldsymbol{\xi}$ can be accommodated by the updates of the weights. Furthermore, $-\dot{\tilde{m}} = \dot{\hat{m}}$, since $\tilde{m} = m - \hat{m}$ and m is constant. With the weight updates (52) – (59), and taking into consideration $\boldsymbol{\xi} \leq \xi_M$, the lump parameter $\boldsymbol{\nu}$ becomes

$$\begin{aligned} \boldsymbol{\nu} =& \kappa \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{\mathbf{W}}_{\mathbf{M}_{ij}}^{\mathrm{T}} \hat{\mathbf{W}}_{\mathbf{M}ij} + \ldots + \kappa \sum_{k=1}^{k_f} \tilde{\mathbf{V}}_{\mathbf{f}_k}^{\mathrm{T}} \hat{\mathbf{V}}_{\mathbf{f}k} + 2\|\mathbf{r}_2\|\boldsymbol{\xi} \\ \leq& - \kappa\|\tilde{\mathbf{Z}}\|^2 + \kappa\|\tilde{\mathbf{Z}}\| Z_M + 2\|\mathbf{r}_2\|\xi_M \end{aligned} \tag{67}$$

Equation (67) is obtained by making use of

$$\langle \tilde{\mathbf{W}}, \hat{\mathbf{W}} \rangle = \sum_{i=1}^{n}\sum_{j=1}^{n} \tilde{\mathbf{W}}_{\mathbf{M}_{ij}}^{\mathrm{T}} \hat{\mathbf{W}}_{\mathbf{M}ij} + \ldots + \sum_{i=1}^{n} \tilde{\mathbf{W}}_{\mathbf{f}_i}^{\mathrm{T}} \hat{\mathbf{W}}_{\mathbf{f}i} \tag{68}$$

$$\langle \tilde{\mathbf{V}}, \hat{\mathbf{V}} \rangle = \sum_{k=1}^{k_M} \tilde{\mathbf{V}}_{\mathbf{M}_k}^{\mathrm{T}} \hat{\mathbf{V}}_{\mathbf{M}k} + \ldots + \sum_{k=1}^{k_f} \tilde{\mathbf{V}}_{\mathbf{f}_k}^{\mathrm{T}} \hat{\mathbf{V}}_{\mathbf{f}k} \tag{69}$$

$$\langle \tilde{\mathbf{Z}}, \hat{\mathbf{Z}} \rangle = \langle \tilde{\mathbf{V}}, \hat{\mathbf{V}} \rangle + \langle \tilde{\mathbf{W}}, \hat{\mathbf{W}} \rangle. \tag{70}$$

where $\hat{\mathbf{Z}} = \mathbf{Z} - \tilde{\mathbf{Z}}$, and therefore $\langle \tilde{\mathbf{Z}}, \hat{\mathbf{Z}} \rangle = \langle \tilde{\mathbf{Z}}, \mathbf{Z} \rangle - \|\tilde{\mathbf{Z}}\|^2 \leq \|\tilde{\mathbf{Z}}\|\|\mathbf{Z}\| - \|\tilde{\mathbf{Z}}\|^2 \leq \|\tilde{\mathbf{Z}}\| Z_M - \|\tilde{\mathbf{Z}}\|^2$.

Substituting $\boldsymbol{\nu}$ (67) into (66) and defining $\mathbf{y}^{\mathrm{T}} = \begin{bmatrix}\mathbf{r}_1^{\mathrm{T}} & \mathbf{r}_2^{\mathrm{T}}\end{bmatrix}$, $\dot{V}(\mathbf{r}_1, \mathbf{r}_2, \tilde{\mathbf{Z}})$ can be expressed as

$$\begin{aligned} \dot{V}(\mathbf{y}, \tilde{\mathbf{Z}}) \leq& -\mathbf{y}^{\mathrm{T}} \begin{bmatrix} K_{v,m} & -\frac{1}{2}p \\ -\frac{1}{2}p & (M_m l_{D,m} - K_{v,M}) \end{bmatrix} \mathbf{y} \\ &+ \begin{bmatrix} \zeta_{\text{friction}} + w_M & 0 \\ 0 & \zeta_M \end{bmatrix} \mathbf{y} - \kappa\|\tilde{\mathbf{Z}}\|^2 + \kappa\|\tilde{\mathbf{Z}}\| Z_M \end{aligned} \tag{71}$$

where $p = B_{\mathbf{x},M}(\|\mathbf{r}_1\| + \|\mathbf{r}_2\| + 2\dot{x}_M)$ and $\zeta_M = \zeta_{\text{friction}} + w_M + 2\xi_M$. The matrix

$$\boldsymbol{\Psi} = \begin{bmatrix} K_{v,m} & -\frac{1}{2}p \\ -\frac{1}{2}p & (M_m l_{D,m} - K_{v,M}) \end{bmatrix} \tag{72}$$

is positive definite if $p < 2\sqrt{K_{v,m}(M_m l_{D,m} - K_{v,M})}$ by virtue of (51) the term $2\sqrt{K_{v,m}(M_m l_{D,m} - K_{v,M})}$ is positive. Equation (71) can therefore be written as

$$\begin{aligned} \dot{V}(\mathbf{y}, \tilde{\mathbf{Z}}) \leq& -\Psi_m \left[\|\mathbf{y}\| - \frac{\zeta_M}{2\Psi_m}\right]^2 - \kappa \left[\|\tilde{\mathbf{Z}}\| - \frac{Z_M}{2}\right]^2 \\ &+ \frac{\zeta_M^2}{4\Psi_m} + \frac{\kappa Z_M^2}{4} \end{aligned} \tag{73}$$

Hence if

$$\|\mathbf{y}\| > \sqrt{\frac{\zeta_M^2}{4\Psi_m^2} + \frac{\kappa Z_M^2}{4\Psi_m}} + \frac{\zeta_M}{2\Psi_m} \equiv b_{\mathbf{y}}, \quad \text{and} \tag{74}$$

$$\|\tilde{\mathbf{Z}}\| > \sqrt{\frac{\zeta_M^2}{4\kappa\Psi_m} + \frac{Z_M^2}{4}} + \frac{Z_M}{2} \equiv b_{\tilde{\mathbf{Z}}} \tag{75}$$

then $\dot{V}(\mathbf{y}, \tilde{\mathbf{Z}}) < 0$. According to Lyapunov's extension theorem [LaSalle, 1960] this demonstrates

$$\lim_{t\to\infty} \|\mathbf{y}(t)\| = b_{\mathbf{y}}, \quad \lim_{t\to\infty} \|\tilde{\mathbf{Z}}(t)\| = b_{\tilde{\mathbf{Z}}} \tag{76}$$

This shows $\|\mathbf{r}_1\|$, $\|\mathbf{r}_2\|$ and $\|\tilde{\mathbf{W}}\|, \|\tilde{\mathbf{V}}\|$ are bounded. It can be shown through classical control that a bounded input $\mathbf{r}_2$, yields bounded outputs $\dot{\tilde{\mathbf{x}}}$ and $\tilde{\mathbf{x}}$. Having bounded inputs $\mathbf{r}_1$ and $\dot{\mathbf{x}}$, yields error signals $\lim_{t\to\infty} \mathbf{e} = 0$ and $\dot{\mathbf{e}}$, $\int_0^{\tau=t} \mathbf{e}\, d\tau$ that are also bounded.

4. PERFORMANCE EVALUATION

The proposed NN-based observer-controller is validated with 6 DOF PUMA 560 manipulator which does not have velocity feedback sensors. In addition to the proposed NN adaptive motion control proposed in this paper, two other types of control strategies are performed for comparison: (i) the model-based inverse dynamics motion controller in Operational Space Formulation [Khatib, 1987] – without friction compensation and (ii) Proportional-plus-Derivative (PD) control with gravity model compensation. A periodic circular trajectory – 75 mm radius and 2 second circular period – for end-effector position with a constant orientation was set as the desired path the experimentations.

The performance of the proposed NN based adaptive control is shown in Fig. 1(c). Without any prior knowledge of the robot dynamics, the controller was initialized with zero weights. The proposed NN adaptive control was shown to effectively learn and reduce the tracking errors. Table 1 shows that the proposed NN control strategy yields comparable performance, although slightly less, to inverse dynamics strategy, without prior knowledge of the robot dynamics. It should be noted that a friction model was not included in the inverse dynamic controller. This is a common practice in the implementation as coefficients of frictions are difficult to identify and it varies with many factors, such as temperature, presence of dust and dirt, humidity, etc. However, the friction model is included in the proposed NN adaptive controller, as described in the derivation hence identification is not an issue. Figure 2 also shows the boundedness and stability of the norms of NN weights.

5. CONCLUSIONS AND FUTURE WORKS

In this paper, the extension of the operational space formulation to NN adaptive motion observer-controller is derived and validated through real-time experimentation. It is shown that even without prior knowledge to the robot dynamics, the proposed neural network is effective in its task and achieved a performance slightly less than inverse

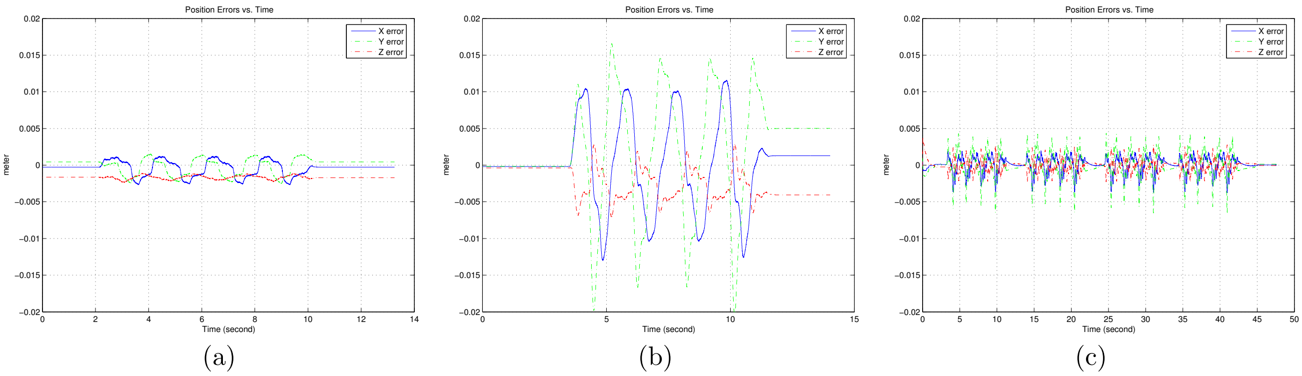

Fig. 1. End-effector tracking error with respect to the reference trajectory for (a) inverse dynamics control (b) PD control with gravity compensation and (c) NN adaptive control strategies – obtained through real-time implementation.

Control type	Inverse-dynamics	PD + gravity	NN with observer
X_{error}	2.5	12.5	2.5
Y_{error}	2.5	20.0	6
Z_{error}	2.5	7.0	1.5

Table 1. Position errors comparison

dynamics strategy. This could be seen as an interesting alternative to the inverse-dynamics strategy, in term of the cost of finding the robot dynamics. Future work would be to extend this work into force-motion controller in the operational space framework.

REFERENCES

B. Armstrong, O. Khatib, and J. Burdick. The explicit dynamic model and inertial parameters of the puma 560 arm. In *Proc. IEEE Conf. Robot. Autom.*, pages 510–518, San Francisco, CA, April7–10, 1986.

H. Berghuis and H. Nijmeijer. A passivity approach to controller-observer design for robots. volume 9, pages 740–754, December, 1993.

J.J. Craig, P. Hsu, and S.S. Sastry. Adaptive control of mechanical manipulators. In *Proc. IEEE Conf. Robot. Autom.*, pages 190–195, San Francisco, CA, April7–10, 1986.

S.S. Ge, C.C. Hang, and L.C. Woon. Adaptive neural network control of robot manipulators in task space. *IEEE Trans. Ind. Electron.*, 44(6):746–752, December 1997.

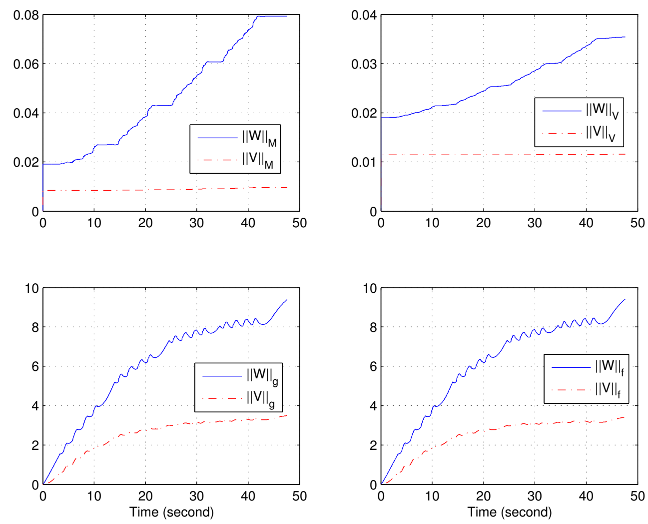

Fig. 2. Real-time: Time history of the NN weights

S. Hu, M.H. Ang, Jr, and H. Krishnan. Neural network controller for constrained robot manipulators. In *Proc. IEEE Conf. Robot. Autom.*, pages 1906–1911, San Francisco, CA, May 2000.

R.S. Jamisola, M.H. Ang, Jr., T.M. Lim, O. Khatib, and S.Y. Lim. Dynamics identification and control of an industrial robot. In *Proc. 9th Int. Conf. Adv. Robot.*, pages 323–328, Tokyo, Japan, October25–27, 1999.

R.S. Jamisola, D. Oetomo, M. H. Ang, Jr., O. Khatib T.M. Lim, , and S.Y. Lim. Compliant motion using mobile manipulator: An operational space formulation approach to aircraft canopy polishing. *Adv. Robot.*, 19(5):613–634, Jun 2005.

O. Khatib. A unified approach for motion and force control of robot manipulators: The operational space formulation. *IEEE J. Robot. Automat.*, RA-3(1):43–53, February 1987.

C.M. Kwan, A. Yeşildirek, and F.L. Lewis. Robust force/motion control of constrained robots using neural network. In *Proc. IEEE Conf. Decision and Control*, pages 1862–1867, Lake Buena Vista, Florida, December 1994.

J.P. LaSalle. Some extensions of Liapunov's second method. *IEE Trans. Circuit Theory*, pages 520–527, 1960.

F.L. Lewis, C.T. Abdallah, and D.M. Dawson. *Control of Robot Manipulators*. MacMillan, NY, 1993.

F.L. Lewis, A. Yesildirek, , and K. Liu. Multilayer neural-net robot controller with guaranteed tracking performance. *IEEE Trans. Neural Networks*, 7:388–399, March 1996.

N.H. McClamroch and D. Wang. Feedback stabilization and tracking of constrained robots. *IEEE Trans. Automat. Contr.*, 33(5):419–426, May 1988.

R.H. Middleton and G.C. Goodwin. Adaptive computed torque control for rigid link manipulations. *Syst. Contr. Lett.*, 10:9–16, 1988.

S. Nicosia and P. Tomei. Robot control by using only joint position measurements. *IEEE Trans. Automat. Contr.*, 9:1058–1061, 1990.

R. Ortega and M. Spong. Adaptive motion control of rigid robots: A tutorial. In *Proc. IEEE Conf. Decision and Control*, pages 1576–1584, Austin, TX, December 1988.

J. Russakow, O. Khatib, and S. Rock. Extended operational space formulation for serial-to-parallel chain (branching) manipulators. In *Proc. IEEE Int. Conf. Robot. Automat.*, volume 1, pages 1056–1061, Nagoya, Japan, May 1995.

J.-J.E. Slotine and W. Li. On the adaptive control of robot manipulators. *Int. J. Robot. Res.*, 6(3):49–59, 1987.

D. B. Soewandito, D. N. Oetomo, and M. H. Ang, Jr. The operational space formulation with neural-network adaptive motion control of robot manipulators. In *Proc. Int. Fed. of Automatic Control (IFAC) 2008*, July 2008.

Q.H. Xia, S.Y. Lim, , M.H. Ang, Jr, and T.M. Lim. Adaptive joint friction compensation using a model-based operational space velocity observer. In *Proc. IEEE Conf. Robot. Autom.*, pages 3081–3086, New Orleans, LA, April26 – May1, 2004.