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Application of a New Scheme for Adaptive Unfalsified Control to a CSTR

Tanet Wonghong and Sebastian Engell *

* *Process Dynamics and Operations, Department of Biochemical and Chemical Engineering, Technische Universität Dortmund, 44221 Dortmund, Germany*
(e-mail: {t.wonghong, s.engell}@bci.uni-dortmund.de)

Abstract: In this paper, a new scheme for adaptive unfalsified control is demonstrated for a well-known example of a nonlinear plant, a continuous stirred tank reactor (CSTR) with the van-der-Vusse reaction scheme. There are two new elements in our scheme: 1. Instead of switching between a finite number of controllers from a given, fixed set, an adaptation of the controller parameters is performed. For this purpose, a population-based evolutionary algorithm is used. 2. As the cost function that was originally proposed by Safonov is unable to correctly assess the potential performance of a controller that has not been in the loop, we propose a new cost function that employs the fictitious error for the actual reference signal. This error signal is determined by estimating the impulse response of the sensitivity function from the observed data.

Keywords: Adaptive Control; Unfalsified Control; Nonlinear Control, Reactor Control

1. INTRODUCTION

Data-driven adaptive unfalsified control was introduced by Safonov et al. (1997). The basic idea is to switch between different candidate controllers in a predefined set of controllers using only the observed data in order to obtain good performance or at least a stable behaviour of the closed-loop system. No plant model is required. As shown in Stefanovic et al. (2005), safe adaptive control can be achieved if the traditional scheme allows the class of candidate controllers to be infinite. Further developments by Wang et al. (2005) led to the concept of cost detectability, the proposal of a cost-detectable cost function and the ϵ-hysteresis algorithm as a switching mechanism. By this mechanism, the scheme searches for the best controller in the fixed set and cannot adapt the set of controllers.

A natural extension of these ideas is the introduction of an adaptation of the set of controllers in order to find the best controller within the set of all controllers of some predefined structure, but with arbitrary parameters. Adaptation requires that the performance of a candidate controller is evaluated correctly when this controller is not in the loop. Recently Engell et al. (2007), Manuelli et al. (2007), and Dehghani et al. (2007) pointed out that the cost function proposed in Wang et al. (2005) will only detect instability of a controller in the set of candidate controllers if this controller is active, i.e. actually in the loop. A brief derivation of this fact is given in section 3. This renders the original cost function that is based on the computation of a fictitious reference signal unsuitable for adaptive unfalsified control. In order to resolve this issue, in Engell et al. (2007) a new cost function was proposed together with a scheme to compute its values for arbitrary controllers that are not in the loop from measured data only.

In this paper, we describe the new scheme and present results of its application to a well-known example of a nonlinear nonminimum phase system, a continuous stirred tank reactor (CSTR) with the van-der-Vusse reaction scheme. In the next section, we first review the original concept of unfalsified control with the ϵ-hysteresis algorithm. Then we show why instability of a candidate controller cannot be detected using the proposed cost function and introduce our new scheme. Section 5 presents simulation studies of the CSTR example. We compare unfalsified control with a fixed set of controllers and with adaptation of the controller parameters. Finally, some hints for further work are given.

2. THE BASIC IDEA OF UNFALSIFIED CONTROL

We consider a *SISO* adaptive unfalsified control system $\Sigma(P, \hat{K})$ mapping r into (u, y). The system is defined on $\Sigma(P, \hat{K}) : \mathcal{L}_{2e} \to \mathcal{L}_{2e}$. The traditional scheme of an adaptive unfalsified control system $\Sigma(P, \hat{K})$ is shown in Fig. 1.

We assume that the control system $\Sigma(P, \hat{K})$ satisfies the zero-input zero-output property which means that if $r = 0$, then $[u, y]^T = \mathbf{0}$ as well, i.e. the plant is operated around an equilibrium at the origin. The unknown plant $P : \mathcal{U} \to \mathcal{Y}$ is defined by

$$\mathbf{P} = \{(r, u, y) \subset \mathcal{R} \times \mathcal{U} \times \mathcal{Y} \mid y = Pu\}. \qquad (1)$$

A set of finitely many controllers $K : \mathcal{R} \times \mathcal{Y} \to \mathcal{U}$ is defined by

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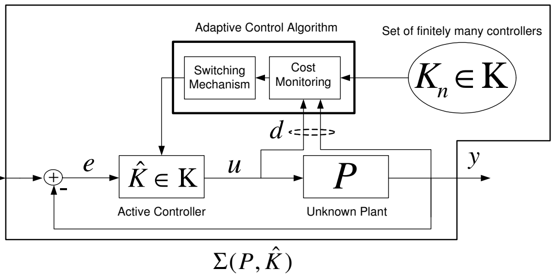

Fig. 1. Traditional Adaptive Unfalsified Control Scheme

$$\mathbf{K} = \{(r, u, y) \subset \mathcal{R} \times \mathcal{U} \times \mathcal{Y} \mid u = K_n \begin{bmatrix} r \\ y \end{bmatrix}\}. \quad (2)$$

The signals $r(t), u(t), y(t)$ are assumed to be square-integrable over every bounded interval $[0, \tau], \tau \in \mathbb{R}_+$. The adaptive control algorithm maps vector signals $d = [u(t), y(t)]^T$ into a choice of a controller $K_n \in \mathbf{K}$, $\mathbf{K} = \{K_n\}, n \in \{1, 2, 3, ..., N\}$.

The adaptive control law has the form:

$$u(t) = \hat{K}(t) \begin{bmatrix} r(t) \\ y(t) \end{bmatrix}$$

where $\hat{K} = K_{n(t)}$ denotes the active controller. $n(t)$ is a piecewise constant function with a finite number of switchings in any finite interval.

Let $d = (u(t), y(t)), 0 \leq t \leq T$ denote experimental plant data collected over the time interval T, and let $\mathbf{D}$ denote the set of all possible such vector signals d. d_τ denotes the truncation of d e.g., all past plant data up to current time τ. The data set $\mathbf{D}_\tau$ is defined by

$$\mathbf{D}_\tau = \{(r, u, y) \subset \mathcal{R} \times \mathcal{U} \times \mathcal{Y} \mid d_\tau = (u_\tau, y_\tau)\}.$$

The $\mathcal{L}_{2e}$ norm of a scalar function of time $x(t)$ at time τ is defined by

$$\|x\|_\tau^2 = \int_0^\tau |x(t)|^2 dt$$

and the $\mathcal{L}_{2e}$ norm of a scalar function of time $x(t)$ in the time interval (a, b) is defined by

$$\|x\|_{(a,b)}^2 = \int_a^b |x(t)|^2 dt.$$

We consider linear time-invariant control laws of the form:

$$K_n = \{(r, u, y) \subset \mathcal{R} \times \mathcal{U} \times \mathcal{Y} \mid u = c_n * (r - y)\} \quad (3)$$

where $*$ denotes the convolution integral, c_n is the impulse response of the n-th controller. $C_n(s)$ denotes the Laplace transform of c_n. We assume that we have observed the input data u_τ and the output data y_τ.

In the original unfalsified control concept (Safonov et al. (1997)), these data are used to evaluate whether the controller K_n meets a specified closed-loop performance criterion

$$J(r_\tau, u_\tau, y_\tau) \leq \alpha \quad (4)$$

where α is called the unfalsification threshold. If this condition is not met, the control law switches to a different controller and the previous controller is discarded. After at most n switchings, a suitable controller is found, if there is such a controller in the set.

The key idea of unfalsified control is to compute the cost $J(K_n, d_\tau, \tau)$ based upon the available measurements. For this purpose,

$$\tilde{r}_n = c_n^{-1} * u + y \quad (5)$$

and

$$\tilde{e}_n = \tilde{r}_n - y \quad (6)$$

where c_n^{-1} is the impulse response of the inverse controller transfer function $C_n^{-1}(s)$. These signals are called the *fictitious reference signal* and the *fictitious error signal.* The fictitious reference signal of controller K_n is the same as the true reference signal if the controller K_n is an active controller $\hat{K}(t), \forall t \in [0, \tau)$. When $J(\tilde{r}_{n_\tau}, u_\tau, y_\tau) > \alpha$, this implies that if the controller K_n were in the loop, it would not satisfy the performance criterion (4). In this case, it is said that the controller K_n is a *falsified* controller. Otherwise the controller K_n is an *unfalsified* controller.

Assume that for the adaptive unfalsified control system $\Sigma(P, \hat{K})$ in Fig. 1, d_τ is the resulting plant input-output data collected while $\hat{K}(d_\tau, t)$ is in the loop. A cost function $J(K_n, d_\tau, \tau), K_n \in \mathbf{K}$ satisfies the criterion of *cost detectability* of the adaptive unfalsified control system $\Sigma(P, \hat{K})$ for $\hat{K} \in \mathbf{K}$ with finitely many switching times if with K^f being the final controller that is used in the closed-loop, finiteness of $J(K^f, d_\tau, \tau), K^f \in \mathbf{K}$ as $\tau \to \infty$, is equivalent to stability of the adaptive unfalsified control system $\Sigma(P, \hat{K})$.

The adaptive control algorithm proposed by Wang et al. (2005) consists of two main components:

1. Switching Mechanism
As the switching mechanism, the ϵ-hysteresis algorithm is used:

(1) Initialize: Let $k = 0, \tau_0 = 0$; choose $\epsilon > 0$. Let $\hat{K}(0) = K_1, K_1 \in \mathbf{K}$, be the first active controller in the loop.
(2) $k = k + 1, \tau_k = \tau_{k+1}$
If $J(\hat{K}(k-1), d_{\tau_k}, \tau_k) > \min_{K_n \in \mathbf{K}} J(K_n, d_{\tau_k}, \tau_k) + \epsilon$ then $\hat{K}(k) \leftarrow \arg\min_{K_n \in \mathbf{K}} J(K_n, d_{\tau_k}, \tau_k)$, else $\hat{K}(k) \leftarrow \hat{K}(k-1)$.
(3) go to 2.

2. Cost Monitoring
The fictitious control performances of the candidate controllers are computed as:

$$\tilde{J}(K_n, d_{\tau_k}, \tau_k) = \max_{\tau_j \leq \tau_k} \frac{\sum_{i=0}^{j} |\tilde{e}(K_n, d_{\tau_i})|^2 + \gamma \sum_{i=0}^{j} |u(i)|^2}{\sum_{i=0}^{j} |\tilde{r}(K_n, d_{\tau_i})|^2 + \rho} \quad (7)$$

where γ and ρ are positive numbers. The max function is used to ensure that the cost function $\tilde{J}$ is non-decreasing as time progresses.

In Wang et al. (2005), it was proven that this algorithm leads to a stable closed-loop system if the set of controllers contains at least one stabilizing controller, i.e. at least the terminal controller stabilizes the loop.

3. PROBLEMS WITH SAFONOV'S COST FUNCTION

Recently, Manuelli et al. (2007), Dehghani et al. (2007), and Engell et al. (2007) pointed out deficiencies of the cost function proposed in Wang et al. (2005). The key point is that *cost detectability* only refers to the situation where a destabilizing controller is actually in the loop. When a destabilizing controller is not in the loop, its associated (fictitious) fitness can be bounded even for long observation horizons and be lower than the cost of a stabilizing controller. This may cause switching to an unstable controller even in an ideal situation without measurement noise or disturbances, what clearly is not desirable. And this property makes the original cost function unsuitable for its use in a parameter optimization algorithm because the shape of the fictitious cost function drastically differs from the true cost function such that the algorithm converges to destabilizing controllers.

The pitfall of the cost function $\tilde{J}$ (7) results from the fact that the unstable dynamics of the loop with the non-active destabilizing controller are not excited due to a *rhp*-zero in the transform of the fictitious reference signal at the location of the unstable pole.

Let us assume that a controller C_1 was in the loop up to time τ and the controller C_1 gives a stable closed-loop system with an unknown plant P(s). We want to compute the performance of a controller C_2 which is destabilizing for this unknown plant.

From (5) and (6), the relationship between $\tilde{E}_2(s)$ and $\tilde{R}_2(s)$ is

$$\tilde{E}_2(s) = \frac{1}{1 + C_2(s)P(s)}\tilde{R}_2(s) = S_2(s)\tilde{R}_2(s)$$

and the unknown sensitivity function $S_2(s)$ is unstable with (at least) one (unknown) *rhp* pole $\tilde{p}$. However, as

$$\tilde{R}_2(s) = (1 + (C_2(s)P(s))^{-1})Y(s) = \frac{1 + C_2(s)P(s)}{C_2(s)P(s)}Y(s),$$

the transfer function from $Y(s)$ (the true signal) to $\tilde{R}_2(s)$ (the fictitious input) has the same *rhp* zeros at $\tilde{p}$, so the unknown unstable dynamics are not excited. Hence, as long as the destabilizing controller is not in the loop, the corresponding cost function does not indicate the instability of the closed-loop system with this controller because the fictitious error signal remains small. This also holds when the computations are performed numerically.

4. A NEW APPROACH TO UNFALSIFIED CONTROL

From the above, the key problem with the original cost function is the use of the fictitious reference signal $\tilde{r}$ and of the corresponding error signal $\tilde{e}$ in the cost function. While this approach has the big advantage that these signals can easily be computed from the measured data, the cost function only indicates instability if the controller under consideration is in the loop. The main idea to resolve the problem is to compute the *fictitious* error e^* that would result with the controller that is evaluated for the *real* reference signal r.

4.1 ***A New Cost Function***

With this signal, a new cost function can be defined as proposed by Engell et al. (2007):

$$J^*(K_n, d_\tau, \tau) = \max_{t<\tau} \frac{\|e^*(K_n, d_t, t)\|_t^2 + \gamma\|u(t)\|_t^2}{\|r(t)\|_t^2 + \rho} \qquad (8)$$

where $e^*(K_n)$ is the fictitious error of a candidate controller K_n for the actual reference signal r.

4.2 ***Numerical Computation of*** e^*

The open problem with the new cost function of course is how it can be computed without model information, solely using measured data. In Engell et al. (2007) the following scheme was proposed:

The sensitivity function with the candidate controller C_i is

$$\tilde{S}_i(s) = \frac{E_i^*(s)}{R(s)} = \frac{1}{1 + C_i(s)P(s)} \qquad (9)$$

where $E_i^*(s)$ is the Laplace transform of the fictitious error e_i^* and $R(s)$ is the Laplace transform of the actual reference signal r that was applied to the loop.

However, we cannot compute e_i^* directly since the plant $P(s)$ is unknown. One possible way is to compute it from the fictitious reference signal and the fictitious error signal as defined above for a candidate controller C_i.

$$\tilde{S}_i(s) = \frac{\tilde{E}_i(s)}{\tilde{R}_i(s)} = \frac{1}{1 + C_i(s)P(s)} \qquad (10)$$

or in the time-domain,

$$\tilde{e}_i(t) = \tilde{s}_i(t) * \tilde{r}_i(t). \qquad (11)$$

The signal of interest is

$$e_i^*(t) = \tilde{s}_i(t) * r(t). \qquad (12)$$

Using (11), $\tilde{s}_i(t)$ can be obtained from the measured data $u(t)$ and $y(t)$ via $\tilde{e}(t)$ and $\tilde{r}(t)$. Practically, the deconvolution can be performed using sampled signals:

$$\tilde{\mathbf{R}}_i \cdot \tilde{\mathbf{s}}_i = \tilde{\mathbf{e}}_i$$

with

$$\tilde{\mathbf{R}}_i = \begin{bmatrix} \tilde{r}_i(k-l) & 0 & 0 & \ddots & 0 \\ \vdots & \tilde{r}_i(k-l) & 0 & \ddots & 0 \\ \tilde{r}_i(k-2) & \ddots & \tilde{r}_i(k-l) & 0 & \vdots \\ \tilde{r}_i(k-1) & \tilde{r}_i(k-2) & \ddots & \ddots & 0 \\ \tilde{r}_i(k) & \tilde{r}_i(k-1) & \tilde{r}_i(k-2) & \cdots & \tilde{r}_i(k-l) \end{bmatrix}, \qquad (13)$$

$$\tilde{\mathbf{e}}_i = \begin{bmatrix} \tilde{e}_i(k-l) \\ \vdots \\ \tilde{e}_i(k) \end{bmatrix} \text{ and } \tilde{\mathbf{s}}_i = \tilde{\mathbf{R}}_i^{-1} \cdot \tilde{\mathbf{e}}_i = \begin{bmatrix} \tilde{s}_i(0) \\ \vdots \\ \tilde{s}_i(l) \end{bmatrix}$$

where l is the estimated settling time of the closed-loop system with the previous controller. Thus

$$\mathbf{e}_i^* = \begin{bmatrix} e_i^*(k-l) \\ \vdots \\ e_i^*(k) \end{bmatrix} = \mathbf{R} \cdot \tilde{\mathbf{R}}_i^{-1} \cdot \tilde{\mathbf{e}}_i \qquad (14)$$

where $\mathbf{R}$ is defined similar to $\tilde{\mathbf{R}}_i$. Similar to the original cost function (7), we define the new cost function of the controller C_i as

$$J^*(C_i, k) = max_{H \le k} \frac{\sum_{j=k-l}^{H} |e_i^*(j)|^2 + \gamma \sum_{j=k-l}^{H} |u(j)|^2}{\sum_{j=k-l}^{H} |r(j)|^2 + \rho}. \qquad (15)$$

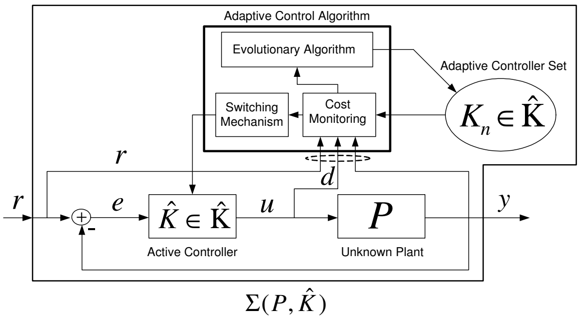

Fig. 2. New Adaptive Unfalsified Control Scheme

4.3 Instability Detection by the New Cost Function

Let us first examine the cost function if the controller under consideration is actually in the loop. So $\hat{K}$ was active from time $(\tau - l)$ up to time τ. From $\tilde{r}_{\hat{K}} = r$ and $\tilde{e}_{\hat{K}} = e$,

$$\tilde{\mathbf{s}}_{\hat{K}} = \tilde{\mathbf{R}}_{\hat{K}}^{-1} \cdot \tilde{\mathbf{e}}_{\hat{K}} = \mathbf{R}^{-1} \cdot \mathbf{e}$$

and we obtain $e^*_{\hat{K}}(t) = e(t)$ and

$$J^*(\hat{K}, \tau) = \max_{(\tau-l)<\tau} \frac{\|e(t)\|^2_{(\tau-l)} + \gamma \|u(t)\|^2_{(\tau-l)}}{\|r(t)\|^2_{(\tau-l)} + \rho}.$$

Thus, $J^*(\hat{K}, \tau)$ will detect instability of an active controller $\hat{K}$.

Now we assume that the controller of interest is not in the loop. Since the controller C_2 is destabilizing, $\frac{\tilde{R}_2(s)}{Y(s)} = \frac{1+C_2(s)P(s)}{C_2(s)P(s)}$ contains (at least) one *rhp* zero. Its reciprocal $\frac{Y(s)}{\tilde{R}_2(s)} = \frac{C_2(s)P(s)}{1+C_2(s)P(s)}$ contains (at least) one unknown *rhp* pole. $Y(s)$ can be computed from (6), hence

$$1 - \frac{\tilde{E}_2(s)}{\tilde{R}_2(s)} = 1 - \tilde{S}_2(s) = \frac{C_2(s)P(s)}{1 + C_2(s)P(s)}$$

$$\tilde{S}_2(s) = \frac{1}{1 + C_2(s)P(s)} = \frac{\tilde{E}_2(s)}{\tilde{R}_2(s)} = \frac{C_2^{-1}(s)U(s)}{C_2^{-1}(s)U(s) + Y(s)}.$$

In the case without measurement noise and disturbances and as l goes to ∞, the sensitivity function $\tilde{s}_2(t)$ can be estimated perfectly from $u(t)$ and $y(t)$ Then the estimated sensitivity function $\tilde{S}_2(s)$ contains (at least) one *rhp* pole. and the fictitious error will tend to infinity for almost all signals $r(t)$ as $\tau \to \infty$ and $l \to \infty$.

4.4 A New Scheme for Unfalsified Control

Similar to the traditional scheme, the new scheme of adaptive unfalsified control $\Sigma(P, \hat{K})$ is presented in Fig. 2.

Using the new cost function defined above, the set of controllers can now be adapted by an evolutionary algorithm.

The truncated window data set $\mathbf{D}_{(\tau-l,\tau)}$ is defined by

$$\begin{aligned} \mathbf{D}_{(\tau-l,\tau)} &= \{(r, u, y) \subset \mathcal{R} \times \mathcal{U} \times \mathcal{Y} \mid \\ d_{(\tau-l,\tau)} &= (r_{(\tau-l,\tau)}, u_{(\tau-l,\tau)}, y_{(\tau-l,\tau)})\} \end{aligned} \quad (16)$$

In our approach, the adaptive control algorithm consists of two components:

1. Switching of the Active Controller
The ϵ-hysteresis algorithm is applied for the switching of the active controller as in the traditional approach. The algorithm scans whether the performance of the currently active controller is inferior to the performance of other controllers in the set. In contrast to the original approach, the modified cost function is used, computed for the windowed data set $\mathbf{D}_{(\tau-l,\tau)}$. The choice of l is discussed below.

2. Adaptation of the Controller Set
An evolutionary algorithm is used for the optimization because an evolutionary algorithm manipulates a population of controllers. The EA will be executed only at certain instances in time. In our work, the evolutionary algorithm is a so-called evolution strategy where each individual is represented by the vector of controller parameters and by a vector of strategy parameters that control the mutation strength. For details of the algorithm see Engell et al. (2007).

In the implementation of the algorithm, two issues have to be taken care of: window length and excitation. For the computation of the cost function, a sufficient amount of data is necessary, and for accurate results, the window length l should roughly match the settling time of the controlled system. The data in the window must be generated by one fixed controller in the loop, so the evolutionary algorithm can only start after a certain period of time after the last switching. Secondly, the computation will fail if insufficient excitation leads to an ill-conditioned matrix $\tilde{\mathbf{R}}_i$. Therefore we restricted the activation of the evolutionary algorithm to a suitable interval after an excitation of the system by the change of the reference signal.

5. CONTROL OF A CSTR WITH NONMINIMUM PHASE BEHAVIOR

As an example of the application to a nonlinear process we investigate the well-known case study of the control of a CSTR with the van-der-Vusse reaction scheme. The parameters of the model are taken from Engell et al. (1993) and Klatt et al. (1998).

A sketch of the reactor is shown in Fig. 3. The reaction scheme is

$$A \overset{k_1}{\rightarrow} B \overset{k_2}{\rightarrow} C,$$
$$2A \overset{k_3}{\rightarrow} D.$$

The reactor is operated at a constant holdup, i.e. the volume of the contents is constant. The manipulated input $u(t)$ is the flow through the reactor, represented by the inverse of the residence time ($f = F_{in}/V_R$). u is in the range $0 \leq u(t) \leq 30h^{-1}$ We assume that the temperature control is tight so that the dependency of the kinetic parameters on the reactor temperature can be neglected. Under these assumptions, a *SISO* nonlinear model results from mass balances for the components A and B:

$$\begin{aligned} \dot{x}_1 &= -k_1 x_1 - k_3 x_1^2 + (x_{1,in} - x_1)u \\ \dot{x}_2 &= k_1 x_1 - k_2 x_2 - x_2 u \\ y &= x_2 \end{aligned} \quad (17)$$

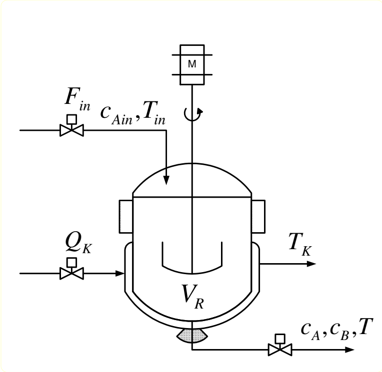

Fig. 3. Continuous Stirred Tank Reactor

where x_1 is the concentration of component A, x_2 is the concentration of component B and $x_{1,in}$ is the feed concentration of A, assumed to be constant. The parameter values are $k_1 = 15.0345h^{-1}, k_2 = 15.0345h^{-1}, k_3 = 2.324l \cdot mol^{-1} \cdot h^{-1}, x_{1,in} = 5.1mol \cdot l^{-1}$.

We assume that the plant P consists of the continuous stirred tank reactor as described by the above model plus a delay of $0.02h$ for the analytic instrument. The controller structure is a PI-controller defined by

$$C(s) = k_p(1 + \frac{1}{T_n s}).$$

First we consider the case of a fixed set of controllers. The initial set of the controller parameters is given by the proportional gains $\mathbf{K}_p = \{10, 50, 100\}$ and the integral times $\mathbf{T}_n = \{0.1, 0.5, 1\}$. The set of controllers thus comprises 9 controllers with parameter vectors $\Theta = \{\theta_i = [k_{p_i}, T_{n_i}]^T, i \in \{1, ..., 9\}\}$. The first active controller assigned to the loop is $\theta_1 = [10, 0.1]^T \in \Theta$. All initial conditions at $\tau = 0$ are zero and the simulation horizon is $t_f = 3.5h$. The constant ϵ in the ϵ-hysteresis algorithm is 0.1 and $\gamma = 10^{-9}$ and $\rho = 0.01$ in the new cost function J^*.

We assume that the reference signal is

$$r(t) = \begin{cases} 0mol \cdot l^{-1} & : \quad 0 \le t < 0.15h; \\ 0.7mol \cdot l^{-1} & : \quad 0.15h \le t < 1.15h; \\ 0.9mol \cdot l^{-1} & : \quad 1.15h \le t < 2.15h; \\ 1.09mol \cdot l^{-1} & : \quad 2.15h \le t < 3.5h. \end{cases}$$

The window length for the computation of the fictitious error signal is $l = 0.3h$.

5.1 CSTR without adaptation of the controller set

Since the first active controller is a stabilizing controller for all three operating points, the ϵ-hysteresis algorithm keeps the first controller with parameters θ_1 in the loop also for the third operating point rather than switching to the better controller because the threshold ϵ prevents the switching as can be seen from Fig. 4. As Fig. 5 shows, the plant dynamics change significantly at the third operating point and an adaptation of the controller would be beneficial.

5.2 CSTR with adaptation of the controller set

The evolutionary algorithm is executed three times at $0.45h$, $1.45h$, and $2.45h$. The evolutionary algorithm is a standard evolution strategy with adaptation of the search parameters according to Schwefel (1995) and Quagliarella et al. (1998). In this application, the size of the population is equal to the number of candidate controllers $\mu = n$. The $(\mu + \lambda)$ selection is chosen with $\mu = 9$ and $\lambda = 63$. This means that the best of controllers kept from the set of the old controllers and 63 offspring. The search space of solutions $\mathbf{K}_p \times \mathbf{T}_n$ is restricted to $[-100, 100] \times [0.01, 1]$ and the initial strategy parameters are set to 10% of the ranges of the variables.

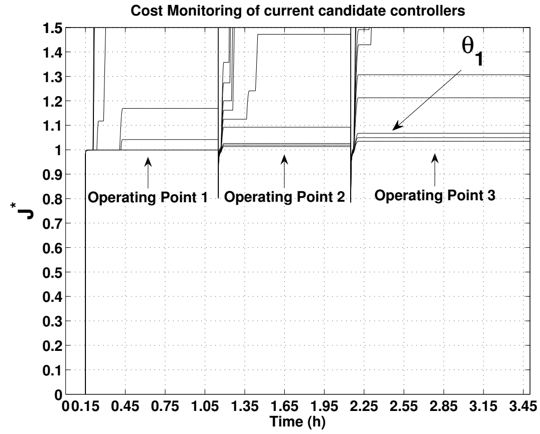

Fig. 4. Control Performance of the candidate controllers

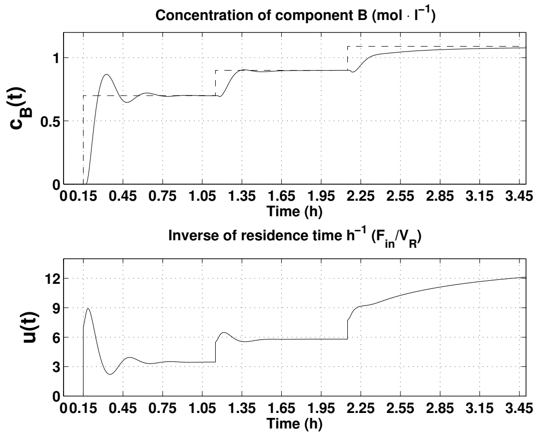

Fig. 5. Output Response and Input Response

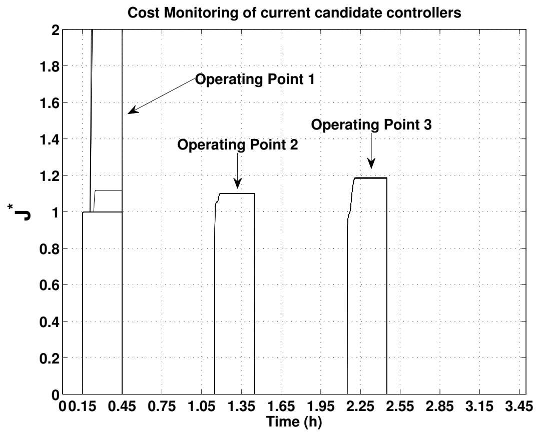

Fig. 6. Control Performance of the candidate controllers

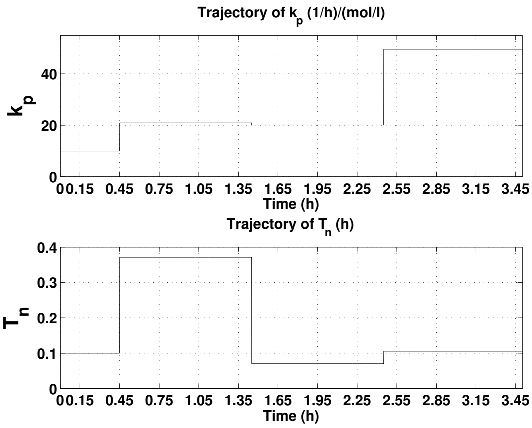

Fig. 7. Evolution of the active controller $\hat{K}$

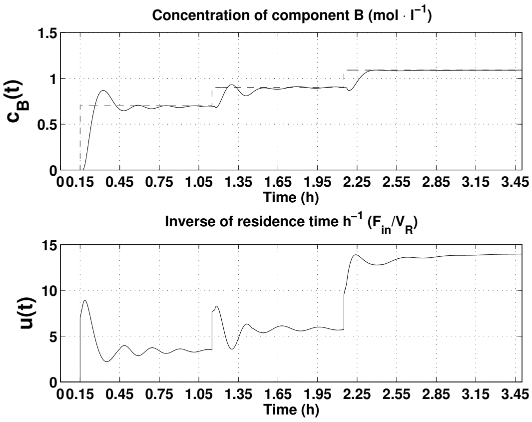

Fig. 8. Output Response and Input Response

The first evolutionary algorithm was executed by using measured data $d_{(0.15h,0.45h)}$ obtained with the first active controller θ_1 that was in the loop during $t \in (0.00h, 0.45h)$. The values of the cost functions of all candidate controllers while the first controller is in the loop are shown in Fig. 6. We can see that costs of the non-active destabilizing controllers reach large values even though were are not in the loop. At the first execution of the ES at $t = 0.45h$, the evolution strategy returns a new set of controllers for the first operating point after 15 generations. As shown in Fig. 7, the new active controller is $\theta^*_{p_1} = [20.875, 0.371]^T$.

The evolutionary algorithm was executed for the second time using the data $d_{(1.15h,1.45h)}$ with the active controller $\theta^*_{p_1}$. After the second execution of the ES at $t = 1.45h$, the evolution strategy returned a new set of controllers for the second operating point after 14 generations. The new active controller is $\theta^*_{p_2} = [20.184, 0.0705]^T$ (see Fig. 7).

The evolutionary algorithm was executed for the third time using the data $d_{(2.15h,2.45h)}$ with the active controller $\theta^*_{p_2}$. After 14 generations the ES returned a new set of controllers for the third operating point. The new active controller is $\theta^*_{p_3} = [49.477, 0.105]^T$ (see Fig. 7).

The output and the manipulated variable are shown in Fig. 8. We can see that the active controller is well adapted to the change of the dynamics of the unknown plant.

6. CONCLUSIONS AND FUTURE WORK

In this paper, a new scheme for unfalsified control with adaptation of the set of candidate controllers was presented. It was explained that the traditional cost function in unfalsified control is not suitable in this context because it cannot distinguish between stabilizing and destabilizing non-active controllers, and a new cost function was introduced. The example of a CSTR with nonminimum phase nonlinear dynamics showed that good performance can be obtained with the new concept.

There are two important issues with the proposed scheme that will be addressed in future work: 1. Automatic triggering of the evolution strategy. This requires to estimate the settling time of the closed-loop system and to monitor the condition number of the matrices involved online. 2. While evolutionary algorithms work with a population of solutions, the diversity of the population is diminished during the optimization. So in the end, the set of controllers will consist of nearly identical controllers, and switching between these controllers will not make sense any more. While this is not a problem if the plant dynamics is fixed, it is not desired if the diversity of the population is seen as a measure to react to changes of the plant dynamics. Either a part of the population can be fixed and excluded from the evolution, or algorithms that maintain diversity must be used.

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