Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

# An Adaptive Dynamic Matrix Control of a Boiler-Turbine System

**Jae-Du Lee *, Un-Chul Moon*, Seung-Chul Lee*, Kwang Y. Lee****

* School of Electrical and Electronics Engineering, ChungAng University
HukSuk-dong DongJak-Gu, Seoul, Korea, 156-756 (e-mail: ucmoon@cau.ac.kr)
** Department of Electrical and Computer Engineering, Baylor University
Waco, TX 76798-7356, USA (e-mail: Kwang_Y_Lee@baylor.edu)

**Abstract:** This paper proposes an adaptive Dynamic Matrix Control (DMC) and its application to boiler-turbine system. In a conventional DMC, object system is described as a Step Response Model (SRM). However, a nonlinear system is not effectively described as a single SRM. In this paper, nine SRMs at various operating points are prepared. On-line interpolation is performed at every sampling step to find the suitable SRM. Therefore, the proposed adaptive DMC can consider the nonlinearity of boiler-turbine system. The simulation results show satisfactory results with a wide range operation of the boiler-turbine system.

## 1. INTRODUCTION

Model Predictive Control (MPC) refers to a class of control algorithms that compute a sequence of control inputs based on an explicit prediction of outputs within some future horizon (Lee, 1997). The important strengths of MPC is that it can consider the constraints of input and output variables which often exist in real industrial systems. Now, MPC has become a standard tool for process controls. One of the most well-known MPC algorithms for the process control is Dynamic Matrix Control (DMC), which assumes a step-response model (SRM) for the underlying system. The multivariable DMC controller has been discussed extensively in the past by Richalet (1978), Garcia (1986) and Lee (1997). DMC has been successfully applied to numerous industrial processes, and many commercial software have been developed: DMC+, SMC, RMPCT, HIECON, PFC, OPC, etc.

A Boiler-turbine system provides high pressure steam to drive the turbine in thermal electric power generation. Bell and Åström (1987) modeled a boiler-turbine system with a Multi-Input Multi-Output (MIMO) nonlinear system. The severe nonlinearity and wide operation range of the boiler-turbine plant have resulted in many challenges to power system control engineers. Rovnak and Corlis (1991) discussed theoretical and practical aspects of DMC, and presented simulation results of a supercritical boiler. Sanchez and others (1995) presented an application of DMC to steam temperature control of fossil power plants, and showed that the SISO (Single-Input Single-Output) DMC performs better than the PID control. Kim and others (2005) presented the simulation results of DMC to boiler-turbine system. In that paper, they presented simulation results that the SRM obtained from process test data is superior than the SRM from linearization of a mathematical model.

To overcome the nonlinearity of the boiler-turbine plant, many kinds of adaptive and artificial intelligence techniques have also been applied. Hogg and Ei-Rabaie (1991) presented an application of adaptive control, that is, the self-tuning Generalized Predictive Control (GPC) to a boiler system. Prasad, Swidenbank and Hogg (1991) proposed a predictive` control based on a neural network model. Dimeo and Lee (1995) used genetic algorithm to enhance the wide range performance of PI controller or Linear Quadratic Regulator (LQR). Alturki and Abdennour (1999) applied a neural-fuzzy control to a boiler-turbine system. They trained neuro-fuzzy system with the data from five LQRs which are designed for each operating point. Cheung and Wang (1998) presented a comparison of fuzzy and PI controller for drum-boiler system, and concluded that the fuzzy control system has better performance than PID control system especially in setpoint tracking.

In this paper, we proposed an adaptive DMC and its application to a drum-type boiler-turbine system in a fossil power plant. In a conventional DMC, a single SRM describes the dynamics of entire operation range. Therefore, the control performance with a single SRM has a limitation for nonlinear boiler-turbine system. When SRM is updated on-line to consider the present plant condition, the SRM can effectively describe the dynamics of nonlinear boiler-turbine system.

At first, nine SRMs are prepared at typical nine operating points without loss of generality. Interpolation with nine SRMs is performed at every sampling step to find the suitable SRM. Therefore, the proposed adaptive DMC can consider the nonlinearity of boiler-turbine system. The simulation results show satisfactory results with a wide range set point tracking.

## 2. BOILER-TURBINE SYSTEM

The model of Bell and Åström (1987) is assumed as a real plant among various nonlinear models for the boiler-turbine system. The model represents a 160 MW oil fired drum-type boiler-turbine-generator for overall wide-range simulations and is described by a third order MIMO nonlinear state equation as follows:

$$\dot{x}_1 = \left(-0.0018u_2 x_1^{9/8} + 0.9u_1 - 0.15u_3\right)/10 \quad (1)$$

$$\dot{x}_2 = [(0.73u_2 - 0.16)x_1^{9/8} - x_2]/10 \quad (2)$$

$$\dot{x}_3 = [141u_3 - (1.1u_2 - 0.19)x_1]/85 \quad (3)$$

$$y_1 = x_1 \quad (4)$$

$$y_2 = x_2 \quad (5)$$

$$y_3 = 0.05(0.13073x_3 + 100a_{cs} + q_e/9 - 67.975) \quad (6)$$

where,

$$\alpha_{cs} = \frac{(1 - 0.001538x_3)(0.8x_1 - 25.6)}{x_3(1.0394 - 0.0012304x_1)} \quad (7)$$

$$q_e = (0.854u_2 - 0.147)x_1 + 45.59u_1 - 2.514u_3 - 2.096 \quad (8)$$

The three state variables $x_1$, $x_2$ and $x_3$ are drum steam pressure ($P$ in MPa), electric power ($E$ in MW) and steam-water fluid density in the drum ($\rho_f$ in kg/m$^2$), respectively. The three outputs $y_1$, $y_2$ and $y_3$ are drum steam pressure ($x_1$), electric power ($x_2$) and drum water level deviation ($L$ in m), respectively. The $y_3$, drum water level $L$, is calculated using two algebraic equations for $\alpha_{cs}$ and $q_e$ which are the steam quality (mass ratio) and the evaporation rate (kg/sec), respectively.

The three inputs $u_1$, $u_2$ and $u_3$ are normalized positions of valve actuators that control the mass flow rates of fuel, steam to the turbine, and feed water to the drum, respectively. Positions of valve actuators are constrained to [0,1], and their rates of change per second are limited to:

$$-0.007 \le du_1/dt \le 0.007 \quad (9)$$

$$-2.0 \le du_2/dt \le 0.02 \quad (10)$$

$$-0.05 \le du_3/dt \le 0.05 \quad (11)$$

## 3. ADAPTIVE DMC WITH INTERPOLATION

### 3.*1 DMC Algorithm*

For a Single-Input Single-Output (SISO) system, the prediction equation is in the following form:

$$Y_{k+1|k} = Y_{k+1|k-1} + S\Delta U_k + Y^d_{k+1|k} \quad (12)$$

where, $Y_{k+1|k}$ is a $p\times 1$ vector representing a prediction of future output trajectory, $[y_{k+1|k},\ldots, y_{k+p|k}]^T$ at $t$=$k$, and $p$ is the prediction horizon; $Y_{k+1|k-1}$ is a $p\times 1$ vector representing the unforced output trajectory $[y_{k+1|k-1},\ldots, y_{k+p|k-1}]^T$, which means the open-loop prediction while the input u remains constant at the previous value $u_{k-1}$; $\Delta U_k$ is an $m\times 1$ input adjustments vector $[\Delta u_k,\ldots,\Delta u_{k+m-1}]^T$ and $m$ is the control horizon; $Y^d_{k+1|k}$ is a $p\times 1$ vector representing an estimate of unmeasured disturbance on the future output; and, $S$ is a $p\times m$ *dynamic matrix* containing the step-response coefficients as follows:

$$S = \begin{bmatrix} s_1 & 0 & \cdots & 0 \\ s_2 & s_1 & \ddots & \vdots \\ s_3 & s_2 & \ddots & 0 \\ \vdots & \vdots & \ddots & s_1 \\ \vdots & \vdots & \cdots & \vdots \\ s_p & s_{p-1} & \cdots & s_{p-m+1} \end{bmatrix} \quad (13)$$

where $s_i$ is the amplitude of step response at the $i$-th sampling step.

To compute the inputs, the following on-line optimization is performed at every sampling time:

$$\min_{\Delta U_k} \left\| E_{k+1|k} \right\|_\Lambda + \left\| \Delta U_k \right\|_\Gamma \quad (14)$$

where, $E_{k+1|k} = Y_{k+1|k} - R_{k+1|k} = [e_{k+1},\ldots, e_{k+p}]^T$ is a $p\times 1$ error vector, $R_{k+1|k} = [r_{k+1},\ldots, r_{k+p}]^T$ is a $p\times 1$ vector containing the desired trajectory of the future output, $\Lambda$ and $\Gamma$ are the weights for the weighted Euclidean norm of the corresponding vectors. To the above, the following additional constraints are added:

$$Y_{\min} \le Y_{k+1|k} \le Y_{\max} \quad (15)$$

$$\Delta U_{\min} \le \Delta U_k \le \Delta U_{\max} \quad (16)$$

$$U_{\min} \le U_k \le U_{\max} \quad (17)$$

where $U_k$ is an $m\times 1$ input vector, $[u_k,\ldots,u_{k+m-1}]^T$.

The resulting problem is a Quadratic Programming (QP) problem with the inequality constraints (15)-(17). Once the optimal inputs $[\Delta u_k,\ldots,\Delta u_{k+m-1}]$ are computed, only the first input $\Delta u_k$ is implemented and the rest is discarded. The procedure is repeated at the next sampling time.

In this study, the boiler-turbine system is a Multi-Input Multi-Output (MIMO) system which has three inputs and three outputs. Therefore, the vectors $Y_{k+1|k}$, $Y_{k+1|k-1}$, $Y^d_{k+1|k}$, $R_{k+1|k}$ and $E_{k+1|k}$ are extended to $3p\times 1$ vectors and $\Delta U_k$ is a $3m\times 1$ vector in (12)-(17). The prediction equation of the boiler-

*turbine system is then in the following form:*

$$\bar{Y}_{k+1|k} = \bar{Y}_{k+1|k-1} + \bar{S}\Delta\bar{U}_k + \bar{Y}^d{}_{k+1|k} \quad (18)$$

*where,*

$$\bar{Y}_{k+1|k} = \begin{bmatrix} \bar{y}_{k+1|k} & \bar{y}_{k+2|k} & \cdots & \bar{y}_{k+p|k} \end{bmatrix}^T \quad (19)$$

$$= [(y_{1(k+1|k)}, \quad y_{2(k+1|k)}, \quad y_{3(k+1|k)}), \quad \cdots (y_{1(k+p|k)}, \quad y_{2(k+p|k)}, \quad y_{3(k+p|k)})]^T, \quad (20)$$

$$\Delta\bar{U}_k = \begin{bmatrix} \Delta\bar{u}_k & \Delta\bar{u}_{k+1} & \cdots & \Delta\bar{u}_{k+m-1} \end{bmatrix}^T \quad (21)$$

$$= [(\Delta u_{1(k)}, \quad \Delta u_{2(k)}, \quad \Delta u_{3(k)}), \quad \cdots (\Delta u_{1(k+m-1)}, \quad \Delta u_{2(k+m-1)}, \quad \Delta u_{3(k+m-1)})]^T. \quad (22)$$

The subscripts 1, 2 and 3 in (20) and (22) are the indices for the three outputs and three inputs, and $\bar{S}$ is a $3p \times 3m$ dynamic matrix containing nine step responses as follows:

$$\bar{S} = \begin{bmatrix} \bar{s}_1 & \bar{0} & \cdots & \bar{0} \\ \bar{s}_2 & \bar{s}_1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \bar{s}_1 \\ \vdots & \vdots & \ddots & \vdots \\ \bar{s}_p & \bar{s}_{p-1} & \cdots & \bar{s}_{p-m+1} \end{bmatrix} \quad (23)$$

where, every matrix element $\bar{s}_i$ is a $3 \times 3$ vector containing nine amplitudes of the step response at the $i$-th sampling step. The optimization problem (12) is also extended as follows:

$$\min_{\Delta\bar{U}_k} \left\| \bar{E}_{k+1|k} \right\|_{\Lambda} + \left\| \Delta\bar{U}_k \right\|_{\Gamma} \quad (24)$$

where, $\bar{E}_{k+1|k} = \bar{Y}_{k+1|k} - \bar{R}_{k+1|k}$ . The constraints vectors in (15)-(17) are extended to $3p \times 1$ and $3m \times 1$ vectors respectively, and considered in optimization (24).

Table 1. Nine Operating Points

| Operating points | $[y_{1O}, y_{2O}, y_{3O}, u_{1O}, u_{2O}, u_{3O}, x_{3O}]$ |
|---|---|
| OP1 | [10, 50, 0, 0.271, 0.604, 0.336, 449.5] |
| OP2 | [10, 85, 0, 0.402, 0.874, 0.547, 417.5] |
| OP3 | [10, 120, 0, 0.533, 1.144, 0.757, 383.7] |
| OP4 | [11.5, 50, 0, 0.284, 0.548, 0.337, 437.9] |
| OP5 | [11.5, 85, 0, 0.415, 0.779, 0.544, 402.8] |
| OP6 | [11.5, 120, 0, 0.545, 1.009, 0.750, 363.8] |
| OP7 | [13, 50, 0, 0.298, 0.506, 0.338, 423.2] |
| OP8 | [13, 85, 0, 0,0.428, 0.707, 0.541,382.5] |
| OP9 | [13, 120, 0, 0.558, 0.907, 0.745, 331.6] |

### *3.2 Nine Step-Response Models with Process Test*

In a conventional DMC, a single Step Response Model (SRM) describes the dynamics of entire range. The SRM plays a key role to the control performance of DMC. However, the boiler-turbine system (1)-(8) shows severe nonlinearity. Therefore, the control performance with single SRM has a limitation.

The basic idea of this paper is the interpolation of SRMs. When SRM is updated on-line to consider the present plant condition, the SRM can effectively describe the dynamics of nonlinear boiler-turbine system. Without loss of generality, several operating points are selected as base cases in this paper. The values of 10, 11.5 and 13 [*MPa*] are selected for typical values of drum steam pressure ($y_1$). For electric power ($y_2$), 50, 85 and 120 [*MW*] are selected for typical values, while drum water level ($y_3$) is zero [*m*]. Therefore, nine operating points are selected as base cases. Using (1)-(8), the steady state values of inputs and states can be calculated with given output, $y_1$, $y_2$ and $y_3$. Table 1 shows selected 9 operating points.

Kim and others (2005) presented simulation results that the SRM obtained from process test data shows better performance than the SRM from linearization of mathematical model. From this perspective, in this paper, SRMs are developed off-line with process test data. A virtual experiment was performed to develop the SRM by applying step inputs to the plant described by the *nonlinear* model (1)-(8). Fig. 1 shows the nine SRMs of operating points given in Table 1.

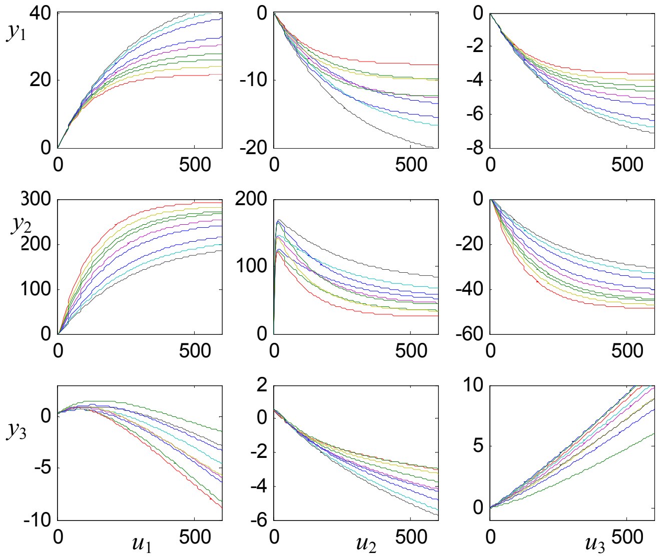


Fig. 1. Nine step-response models corresponding to the operating points given in Table 1.

### 3.3 Interpolation for On-Line Step Response Model

In Fig. 1, the nine SRMs show similar patterns although the time constants and steady state gains are different. Therefore, the interpolation with the SRMs can be applied effectively to develop a suitable SRM for on-line application.

The distance $d_i$ between output of $k$-th sampling time ($y_{1(k)}$, $y_{2(k)}$) and $i$-th operating points are defined as follows

$$d_i = \sqrt{\frac{(y_{1(k)} - y_{1O(i)})^2}{1.5} + \frac{(y_{2(k)} - y_{2O(i)})^2}{35}} \quad i = 1,...,9 \tag{25}$$

where, $y_{1O(i)}$ and $y_{2O(i)}$ are the $y_1$ and $y_2$ of the $i$-th operating point, respectively. The two constants 1.5 and 35 in (25) are added to normalize the scales of the two outputs.

Then, the three smallest $d_i$ in (25) are selected, that is, the three close operating points are selected for interpolation in this paper. Three "weights" or "firing strengths" are determined as reciprocals of $d_i$ as follows:

$$\omega_i = \frac{1}{d_i} \quad i=1, 2, 3 \tag{26}$$

Therefore, when ($y_{1(k)}$, $y_{2(k)}$) matches well with the $i$-th operating point, the weight $\omega_i$ has larger value. The interpolated SRM at the $k$-th sampling time, $SRM_{(k)}$, is calculated as the weighted average of three SRMs as follows:

$$SRM_{(k)} = \frac{\sum_{i=1}^{3} \omega_i \, SRM_i}{\sum_{i=1}^{3} \omega_i} \quad \text{at the } k\text{-th step,} \tag{27}$$

where, $SRM_i$ is the step response model for corresponding $\omega_i$.

Therefore, the $SRM_{(k)}$ can cope with the plant nonlinearity based on the given nine SRMs. Fig. 2 shows the overall configuration of the proposed control system.

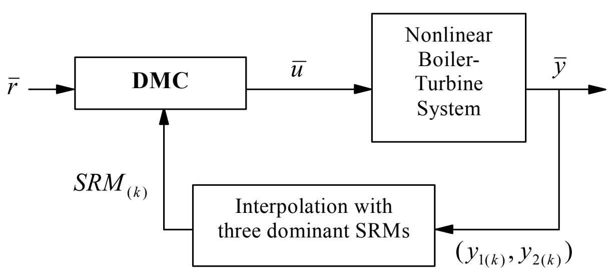


Fig. 2. System configuration of the adaptive DMC.

## 4. SIMULATION RESULTS

The control system and process model were developed with Matlab in a personal computer environment. The sampling time is determined as 5 [sec]. The prediction $p$ is 600 [sec] and control horizons $m$ is 100 [sec], and $\overline{R}_{k+1|k}$ is fixed with three constant setpoint values. In (24), error and input change are weighted for the three outputs and three inputs as follows:

$$\left\| \overline{e}_{k+1|k} \right\| = \begin{bmatrix} e_{1(k+1|k)} \\ e_{2(k+1|k)} \\ e_{3(k+1|k)} \end{bmatrix}^T \begin{bmatrix} 10 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 100 \end{bmatrix} \begin{bmatrix} e_{1(k+1|k)} \\ e_{2(k+1|k)} \\ e_{3(k+1|k)} \end{bmatrix} \tag{28}$$

$$\left\| \Delta\overline{u}_k \right\| = \begin{bmatrix} \Delta u_{1(k)} \\ \Delta u_{2(k)} \\ \Delta u_{3(k)} \end{bmatrix}^T \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta u_{1(k)} \\ \Delta u_{2(k)} \\ \Delta u_{3(k)} \end{bmatrix} \tag{29}$$

In (28), the weights are determined to consider the nominal values of three outputs. The three control actions are equally weighted as ones. More extensive analysis to tuning the DMC is discussed by Dougherty and Cooper (2003).

$Y^d_{k+1|k}$ in (18) is taken as a constant bias of difference between the actual measurement and the open-loop model output. Output constraint (15) is not considered in this study and input constraints (9)–(11) are implemented in the form of (16), and three inputs are constrained in [0, 1] in (17).

The system is assumed initially to be in steady state with operating point 1 in Table 1, $\overline{y}$=(10, 50, 0), $\overline{u}$=(0.271, 0.604, 0.336), $\overline{x}$ = (10, 50, 449.5). The reference is successively changed to demonstrate the wide range tracking ability of the proposed adaptive DMC as follows:

$$\overline{r} = \begin{cases} (13,120,0), & \text{for } 0 < t < 400 \\ (10,50,0), & \text{for } 400 \le t < 800 \\ (11.5,80,0), & \text{for } 800 \le t \le 1200 \end{cases} \tag{30}$$

That is, the setpoints of pressure and electric load are changed to (13, 120) at $t$= 0, (10, 50) at $t$= 400, and (11.5, 80) at $t$=800 successively, while the drum water level is kept to zero. The first step change represents abrupt increment of reference from operating point 1 to 9 in Table 1, second step change represents abrupt decrement of reference from operating point 9 to 1, and the third reference is around the operating point 5.

Fig. 3 shows the three outputs of the simulation. In the figure, the horizontal axis is time [sec], and the vertical axis is 0.1*[MPa] for $y_1$, [MW] for $y_2$ and [cm] for $y_3$. The $y_1$ and $y_2$ track the references within 100 seconds, and $y_3$ tracks the reference within 150 seconds in every change. The drum water level is increased to 22 when the electric power is abruptly decreased, while within 15 in the other changes. Fig. 3 shows that the proposed adaptive DMC algorithm can successfully applied to the wide range operation of boiler-turbine system. Fig. 4 shows the three inputs of the simulation. The horizontal axis is time [sec] and units for input variables are normalized positions of valve actuators for the three inputs $u_1$, $u_2$ and $u_3$.

Fig. 5 represents the dominant operating point which means the operating point with maximum weight in (26). The horizontal axis is time [sec], and the vertical axis is the operating point in Fig. 5. At $t$= 0, the operating point is 1, because the simulation is started at operating point 1. As outputs are increased, the dominant operating point is moved to operating points 2, 3, 6 and 9 successively. From $t$= 400, the dominant operating point moved to 7, 4, 1 and it moves to operating points 2 and 5 from $t$= 800 successively.

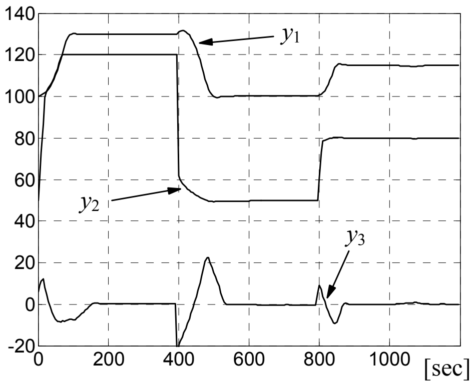


Fig. 3. Outputs of the adaptive DMC.

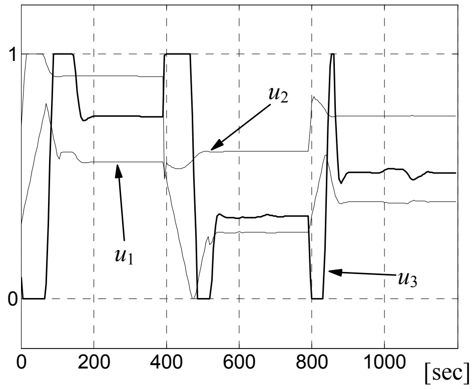


Fig. 4. Inputs of the adaptive DMC.

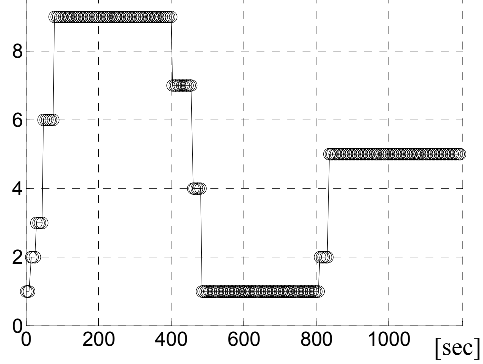


Fig. 5. Dominant operating points.

## 5. CONCLUSION

This paper proposes an adaptive Dynamic Matrix Control (DMC) and its application to a boiler-turbine system. In this paper, nine SRMs are prepared at various operating points covering the operation of the nonlinear plant. On-line interpolation with three dominant SRMs is performed at every sampling time to find a suitable SRM. The simulation shows satisfactory results with a wide range operation of boiler-turbine system. Therefore, the proposed adaptive DMC can effectively consider the nonlinearity of the boiler-turbine system. When the operating points are selected properly, the proposed adaptive DMC algorithm can be widely applied to various nonlinear plant control problem.

## ACKNOWLEDGEMENT

This work has been supported in part by EESRI (R-2005-B-103), which is funded by Ministry of Commerce, Industry and Energy, South Korea.

## REFERENCES

Alturki F. A. and A. B. Abdennour. (1999). Neuro-fuzzy control of a steam boiler-turbine unit. In *Proceedings of IEEE International Conference on Control Applications*, pp. 958-962, Hawai, U.S.A.

Bell R. D. and K. J. Åström. (1987). *Dynamic models for boiler-turbine-alternator units: Data logs and parameter estimation for a 160 MW unit*, Report: TFRT-3192, Lund Institute of Technology, Sweden.

Cheung K. P. and L. X. Wang. (1998). Comparision of fuzzy and pi controllers for a benchmark drum-boiler model. In *Proceedings of IEEE International Conference on Control Applications*, pp. 958-962, Trieste, Italy.

Dimeo R. and K. Y. Lee. (1995). Boiler-turbine control system design using a genetic algorithm. *IEEE Transactions on Energy Conversion*, **Vol. 10**, No. 4, pp. 752-759, December.

Dougherty D. and D. J. Cooper. (2003). Tuning guidelines of a dynamic matrix controller for integrating (non-self-regulating) processes. *Industrial & Engineering Chemistry Research*, **Vol. 42**, pp. 1739-1752.

Garcia C. E. and A.M. Morshedi. (1986). Quadratic programming solution of dynamic matrix control (QDMC). *Chem. Eng. Commun.*, **Vol. 46**, pp. 73-87.

Hogg B. W. and N. M. Ei-Rabaie. (1991). Multivariable generalized predictive control of a boiler system. *IEEE Transactions on Energy Conversion*, **Vol. 6**, No. 2, pp. 282-288, June.

Kim U.-G., U.-C Moon, S.-C. Lee, and K. Y. Lee. (2005). Application of dynamic matrix control to a boiler-turbine system. *2005 General Meeting of IEEE Power Engineering Society*, San Francisco, 12-16 June.

Lee J. H. (1997). Model predictive control in the process industries: Review, current status and future outlook.

*Proceedings of the 2nd Asian Control Conference*, **Vol II**, pp. 435-438, Seoul. July 22-25.

Prasad G., E. Swidenbank, and B. W. Hogg. (1991). A neural net model-based multivariable long-range predictive control strategy applied in thermal power plant control. *IEEE Transactions on Energy Conversion*, **Vol. 13**, No. 2, pp. 176-182, March.

Richalet J., A. Rault, J. L. Testud, and J. Papon. (1978). Model predictive heuristic control: application to industrial processes. *Automatica*, **Vol. 14**, No. 5, pp. 413-428.

Rovnak J. A. and R. Corlis (1991). dynamic matrix based control of fossile power plant. *IEEE Transactions on Energy Conversion*, **Vol. 6**, No. 2, pp. 320-326, June.

Sanchez L. A., F. G. Arroyo, and R. A. Villavicencio. (1995). Dynamic matrix control of steam temperature in fossil power plant. *IFAC Control of Power Plants and Power Systems*, Cancun, Mexico.