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FIR Filters for Stationary State Space Signal Models ⋆

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Abstract: A new finite impulse response (FIR) prediction is presented for a state space signal model. The linear predictor proposed in this paper uses the finite number of inputs and outputs on the recent time interval while the infinite impulse response (IIR) predictor with feedback and recursion uses all inputs and outputs from the initial time to the current time. The Yule-Walker equations for the linear forward and backward predictions are obtained from the correlation among states at each time and the orthogonality principle. How to solve these equations without an inverse of a big matrix is discussed. It is shown through simulation that the proposed FIR predictor can be as robust to modelling uncertainties as conventional robust IIR predictors. It is also shown that the good performance is achieved even in the steady state without uncertainties.

1. INTRODUCTION

Prediction problems are mostly associated with how to estimate some unknown variables as accurately as possible from inputs and measured outputs of systems. Among prediction problems, the estimation of the state in the system has been widely investigated since most dynamic systems can be easily represented with state space signal models. "Predictor" is a common terminology for an input/output model while "filter" is often used for a state space signal model. In this paper, "predictor" is used for consistency with input/output models.

In state space signal models, predictors for the state estimation can have the finite impulse response (FIR) or the infinite impulse response (IIR). The conventional IIR predictor is represented as

$$\hat{x}_k = \sum_{i=0}^{k-1} h_{k-i} y_i + \sum_{i=0}^{k-1} l_{k-i} u_i + m_k \hat{x}_0, \tag{1}$$

where 0, $\hat{x}_k$, u_i, and y_i are the initial time, the estimated state, the input, and the output, respectively, and predictor coefficients h_{k-i}, l_{k-i}, and m_k are determined to optimize a given performance criterion. Usually, the predictor (1) is implemented recursively for practical purposes as in the Kalman IIR predictor. On the other hand, the FIR predictor can be represented in the following form:

$$\hat{x}_k = \sum_{i=k-N}^{k-1} h_{k-i} y_i + \sum_{i=k-N}^{k-1} l_{k-i} u_i. \tag{2}$$

It is noted that FIR predictors utilize the finite number of inputs and outputs on the most recent time interval $[k-N, k-1]$, called a moving horizon. It is also observed that the FIR predictor (2) does not include an initial state term on the horizon. It is known that the FIR structure has inherent properties such as a bounded input/bounded output (BIBO) stability (Kalouptsidis [1997]) and robustness. In particular, the FIR predictor has been known to be one of the remedies that avoid the divergence of IIR predictors due to modelling uncertainties and numerical errors (Bruckstein and Kailath [1985], Rao et al. [2001]). Actually, advantages of FIR filters were mentioned in Bruckstein and Kailath [1985] as follows:

> *Limiting the memory span of an estimation algorithm turns out to be a useful practice for solving problems of filter divergence due to mismodeling, for predicting signals with quasi-periodic components, and for detecting sudden and unexpected changes in systems.*

In general, there have been two approaches for the FIR predictor. One approach is to obtain FIR predictors by using only inputs and measured outputs without using *a priori* stochastic information on estimated parameters such as the state in this paper. For the state estimation, the FIR predictors are designed to minimize the conditional expectation of the two norm of the estimation error, *i.e.*, $\mathbf{E}\{\|x_k - \hat{x}_k\|_2 \mid y_{k-N}, \cdots, y_{k-1}\}$. On the basis of this approach, linear predictors without *a priori* initial state information on the moving horizon were developed by a modification from the Kalman filter (Liu and Liu [1994]) and by a solution of a Riccati-type difference equation (Kwon et al. [1989]). The state in this approach is considered to be deterministic, but unknown so that the orthogonality principle can not be used and instead some conditions such as unbiasedness are required. For lack of *a priori* information on the state, these kinds of FIR predictors do not guarantee the good performance in the steady state.

The other approach is to use *a priori* stochastic information on the unknown parameter. In this approach, the parameter can be considered as a random variable and the linear optimal FIR predictor in the steady state is obtained

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through the Yule-Walker equation, which is efficiently solved by the Levinson-Durbin algorithm in the case of input/output models (Lee and Wang [1996], Choi [1997]). In Friedlander et al. [1978], the relationship between the Kalman IIR predictor and the Levinson-Durbin algorithm was discussed over a state space model. To the authors' knowledge, there is no result on an FIR predictor for the state estimation that uses *a priori* stochastic information on the state to achieve the good performance under the steady state. In this paper, we propose the linear FIR predictor for a state space signal model that guarantees the minimum estimation errors in the steady state. Since the state in this paper is regarded as a random variable, its mean and variance are utilized as *a priori* information. The Yule-Walker equations for the linear forward and backward predictions are obtained from the correlation among states at each time and the orthogonality principle. How to solve the corresponding Yule-Walker equations without an inverse of a big matrix will be discussed.

For a long time, robustness for state estimations has been addressed for the analysis and the design of the IIR predictors. In order to build up robustness, robust Kalman and H_∞ IIR predictors were proposed in Xie et al. [1994], de Souza and Xie [1994], Petersen and Savkin [1999], Ishihara et al. [2006] and Wang et al. [2004] at the sacrifice of the performance in the steady state without uncertainties. These robust predictors might lead to the poor performance in the steady state when uncertainties disappear. In this paper, the FIR predictor is required to have the finite memory structure for robustness while the estimation error is minimized in the steady state through the orthogonal principle. The proposed FIR predictor can have the good performance in the steady state while keeping the property of robustness due to the FIR structure. In other words, the good performance can be expected whether uncertainties exist or not.

This paper is organized as follows. In Section II, the Yule-Walker equations for the FIR predictors and their solutions are proposed and their efficient calculations are introduced. In Section III, the proposed FIR predictor is compared with the IIR predictors and the existing FIR predictor through simulation. Finally, conclusions are presented in Section IV.

2. FIR PREDICTORS

Consider a linear discrete-time state space model:

$$\begin{aligned} x_{i+1} &= Ax_i + Bu_i + Gw_i, \quad (3)\\ y_i &= Cx_i + v_i, \quad (4)\end{aligned}$$

where $x_i \in \Re^n$, $u_i \in \Re^r$, and $y_i \in \Re^q$ are the state, the external known input, and the measured output, respectively. The matrix C in (4) is assumed to be of full rank. The system matrix A is Hurwitz, *i.e.*, has only eigenvalues inside the complex unit circle. The system noise $w_i \in \Re^p$ and the output noise $v_i \in \Re^q$ are zero-mean white Gaussian and mutually uncorrelated. The covariances of w_i and v_i are denoted by Q and R, respectively, which are assumed to be positive definite matrices. In addition, $(A, GQ^{\frac{1}{2}})$ is assumed to be observable.

By separating the state x_i into two parts, the system (3)-(4) can be rewritten as

$$\begin{aligned} x_{1,i+1} &= Ax_{1,i} + Bu_i, \quad x_{1,j} = 0, \quad (5)\\ x_{2,i+1} &= Ax_{2,i} + Gw_i, \quad x_{2,j} = x_j, \quad (6)\\ y_i &= C(x_{1,i} + x_{2,i}) + v_i, \quad (7)\end{aligned}$$

for a certain $j < i$, where $x_{1,i}$ is completely determined from past inputs and a zero initial value $x_{1,j} = 0$, and $x_{2,i}$ is a random vector. $y_i = C(x_{1,i} + x_{2,i}) + v_i$ in (7) can be written as $\bar{y}_i \overset{\triangle}{=} y_i - Cx_{1,i} = Cx_{2,i} + v_i$ so that $\bar{y}_i$ can be considered as an output and hence we have only to estimate $x_{2,i}$ from $\bar{y}_i$ and obtain x_i from $x_i = x_{1,i} + x_{2,i}$. From the result based on a following model without an external input:

$$\begin{aligned} x_{i+1} &= Ax_i + Gw_i, \quad (8)\\ y_i &= Cx_i + v_i, \quad (9)\end{aligned}$$

we have a general one with an external input according to the following identifications: $x_i \leftarrow x_{2,i}$ and $y_i \leftarrow \bar{y}_i (= y_i - Cx_{1,i})$.

Given the finite measurements $y_{k-N}, y_{k-N+1}, \cdots, y_{k-1}$ from the system (8)-(9), we want to estimate x_k in a form of

$$\hat{x}_k = -\sum_{i=0}^{N-1} h_{N,N-i} y_{k-N+i}, \quad (10)$$

where N is the size of the predictor horizon and h_{N-i} in (2) is replaced with $-h_{N,N-i}$ for denoting the horizon size and making the derivation more compact. It is noted that $x_{1,i}$ in (5) on $i \in [k-N, k-1]$ can be exactly and uniquely determined from inputs on $[k-N, k-1]$ and a zero initial value $x_{1,k-N} = 0$ on the horizon. Once the FIR predictor of the form (10) is obtained, it is easy to extend to the general one (2) with an external input.

In order to make the FIR predictor (10) optimal for a mean square error criterion, *i.e.*, $\mathbf{E}[(\hat{x}_k - x_k)^T(\hat{x}_k - x_k)]$, the orthogonality principle $\mathbf{E}\{[x_k - \hat{x}_k]y_{k-N+j}^T\} = 0$ for $j = 0, 1, \cdots, N-1$ should be satisfied as follows:

$$\begin{aligned} \mathbf{E}[x_k y_{k-N+j}^T] &= -\sum_{i=0}^{N-1} h_{N,N-i} \mathbf{E}[y_{k-N+i} y_{k-N+j}^T],\\ \Sigma_{k,k-N+j} C^T &= -\sum_{i=0}^{N-1} h_{N,N-i} \big(C\Sigma_{k-N+i,k-N+j}C^T\\ &\quad + \delta_{i-j} R\big), \quad (11)\end{aligned}$$

where $\Sigma_{k-N+i,k-N+j} = \mathbf{E}[x_{k-N+i} x_{k-N+j}^T]$ and δ_{i-j} is a unit impulse.

Under the steady state, $\Sigma_{k-N+i,k-N+j}$ can be represented as Σ_{i-j} and the state covariance Σ_0 is obtained from

$$\Sigma_0 = A\Sigma_0 A^T + GQG^T. \quad (12)$$

Since A is Hurwitz and $(A, GQ^{\frac{1}{2}})$ is observable, the equation (12) is just the Lyapunov equation where there exist the closed-form positive definite solution Σ_0 and many efficient numerical algorithms to solve it. Once Σ_0 is computed, the state covariance Σ_{i-j} is easily obtained as

$$\Sigma_{i-j} = \mathbf{E}[x_{k-N+i}\ x_{k-N+j}^T] = \begin{cases} A^{i-j}\Sigma_0\ ,\ i \geq j, \\ \Sigma_0(A^T)^{j-i}\ ,\quad i < j. \end{cases} \tag{13}$$

The covariance matrix $P_{f,N} = \mathbf{E}\{[x_k - \hat{x}_k][x_k - \hat{x}_k]^T\}$ of the estimation error is

$$\begin{aligned} P_{f,N} &= \mathbf{E}\{[x_k - \hat{x}_k]x_k^T\} \\ &= \mathbf{E}[x_k x_k^T] + \sum_{i=0}^{N-1} h_{N,N-i}\mathbf{E}[y_{k-N+i}x_k^T] \\ &= \Sigma_0 + \sum_{i=0}^{N-1} h_{N,N-i}C\Sigma_{i-N} \\ &= \Sigma_0 + H_N \left[\, \Sigma_{-1}^T C^T\ \cdots\ \Sigma_{-N}^T C^T \,\right]^T, \end{aligned} \tag{14}$$

where H_N is given by

$$H_N \triangleq [\, h_{N,1}\ h_{N,2}\ \cdots\ h_{N,N} \,], \tag{15}$$

the first equality comes from the orthogonality principle and f in $P_{f,N}$ stands for "forward".

By combining relations (11), we have the so-called Yule-Walker equation:

$$H_N \Xi_N = \Gamma_N, \tag{16}$$

where Γ_N and Ξ_N are given by

$$\Gamma_N \triangleq [\, -\Sigma_1 C^T\ -\Sigma_2 C^T\ \cdots\ -\Sigma_N C^T \,], \tag{17}$$

$$\begin{aligned} \Xi_N &\triangleq \begin{bmatrix} \hat{\Sigma}_0 + R & \hat{\Sigma}_1 & \cdots & \hat{\Sigma}_{N-1} \\ \hat{\Sigma}_{-1} & \hat{\Sigma}_0 + R & \cdots & \hat{\Sigma}_{N-2} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{\Sigma}_{-N+1} & \hat{\Sigma}_{-N+2} & \cdots & \hat{\Sigma}_0 + R \end{bmatrix} \\ &= \begin{bmatrix} \hat{\Sigma}_0 + R & \cdots \\ \vdots & \Xi_{N-1} \end{bmatrix} = \begin{bmatrix} \Xi_{N-1} & \vdots \\ \cdots & \hat{\Sigma}_0 + R \end{bmatrix}, \end{aligned} \tag{18}$$

$$\hat{\Sigma}_i \triangleq C\Sigma_i C^T.$$

From (16), H_N is simply obtained, *i.e.*, $H_N = \Gamma_N \Xi_N^{-1}$. However, the inverse operation of the big matrix such as Ξ_N requires a heavy computation load and may bring out numerical errors so that we need a recursive scheme to compute H_N.

Before obtaining a recursion for H_N, we consider the estimation of x_{k-N}

$$\hat{x}_{k-N} = -\sum_{i=1}^{N} b_{N,i} y_{k-N+i},$$

where, in parallel with the estimation of x_k, we have

$$\Sigma_{-j}C^T = -\sum_{i=1}^{N} b_{N,i}\big(C\Sigma_{i-j}C^T + \delta_{i-j}R\big), \tag{19}$$

$$\begin{aligned} P_{b,N} &\triangleq \mathbf{E}\{[x_{k-N} - \hat{x}_{k-N}][x_{k-N} - \hat{x}_{k-N}]^T\} \\ &= \Sigma_0 + \sum_{i=1}^{N} b_{N,i} C\Sigma_i, \end{aligned} \tag{20}$$

for $1 \leq j \leq N$. As in (14), $P_{b,N}$ can be represented as

$$P_{b,N} = \Sigma_0 + B_N \left[\, \Sigma_N^T C^T\ \cdots\cdots\ \Sigma_1^T C^T \,\right]^T, \tag{21}$$

where B_N is given by

$$B_N \triangleq [\, b_{N,N}\ \cdots\cdots\ b_{N,1} \,]. \tag{22}$$

Note that relations (19) and (21) will give help to obtain H_N. Augmenting relations (19) yields the Yule-Walker equation for the backward predictor

$$B_N \Xi_N = \Omega_N, \tag{23}$$

where Ω_N are given by

$$\Omega_N \triangleq [\, -\Sigma_{-N}C^T\ \cdots\cdots\ -\Sigma_{-1}C^T \,], \tag{24}$$

and Ξ_N is defined in (18).

Now, we are in a position to get recursions for H_N and B_N. Increasing N to $N+1$ and letting $D_N^{(f)} = [D_{N,1}^{(f)}\ D_{N,2}^{(f)}\ \cdots\ D_{N,N}^{(f)}]$ and $D_N^{(b)} = [D_{N,1}^{(b)}\ D_{N,2}^{(b)}\ \cdots\ D_{N,N}^{(b)}]$, we have

$$\Big([\, H_N\ 0 \,] + \Big[\, D_N^{(f)}\ K_{N+1}^{(f)} \,\Big]\Big)\Xi_{N+1} = \Gamma_{N+1}, \tag{25}$$

$$\Big([\, 0\ B_N \,] + \Big[\, K_{N+1}^{(b)}\ D_N^{(b)} \,\Big]\Big)\Xi_{N+1} = \Omega_{N+1}, \tag{26}$$

where $D_N^{(f)}$, $D_N^{(b)}$, $K_{N+1}^{(f)}$, and $K_{N+1}^{(b)}$ will be determined later so that Γ_{N+1} and Ω_{N+1} preserve the form (17) and (24), respectively. If Ξ_{N+1} is represented with Ξ_N according to (18), (25) and (26) can be written as

$$\begin{aligned} &\Big([\, H_N\ 0 \,] + \Big[\, D_N^{(f)}\ K_{N+1}^{(f)} \,\Big]\Big)\begin{bmatrix} \Xi_N & \vdots \\ \cdots & \hat{\Sigma}_0 + R \end{bmatrix} \\ &\qquad = \left[\, \Gamma_N\ -\Sigma_{N+1}C^T \,\right], \end{aligned} \tag{27}$$

$$\begin{aligned} &\Big([\, 0\ B_N \,] + \Big[\, K_{N+1}^{(b)}\ D_N^{(b)} \,\Big]\Big)\begin{bmatrix} \hat{\Sigma}_0 + R & \cdots \\ \vdots & \Xi_N \end{bmatrix} \\ &\qquad = \left[\, -\Sigma_{-N-1}C^T\ \Omega_N \,\right]. \end{aligned} \tag{28}$$

By computing the partial blocks of (27) and (28), we have the following relations:

$$\begin{aligned} H_N\Xi_N + D_N^{(f)}\Xi_N + K_{N+1}^{(f)}\left[\, \hat{\Sigma}_{-N}\ \cdots\ \hat{\Sigma}_{-1} \,\right] &= \Gamma_N, \\ B_N\Xi_N + D_N^{(b)}\Xi_N + K_{N+1}^{(b)}\left[\, \hat{\Sigma}_1\ \cdots\ \hat{\Sigma}_N \,\right] &= \Omega_N, \end{aligned}$$

where, using (16) and (23), we obtain

$$\begin{aligned} D_N^{(f)} &= -K_{N+1}^{(f)}\left[\, \hat{\Sigma}_{-N}\ \cdots\ \hat{\Sigma}_{-1} \,\right]\Xi_N^{-1} \\ &= K_{N+1}^{(f)} C B_N, \end{aligned} \tag{29}$$

$$\begin{aligned} D_N^{(b)} &= -K_{N+1}^{(b)}\left[\, \hat{\Sigma}_1\ \cdots\ \hat{\Sigma}_N \,\right]\Xi_N^{-1} \\ &= K_{N+1}^{(b)} C H_N. \end{aligned} \tag{30}$$

By computing the remaining blocks of (27) and (28) and taking similar steps, we have another following relations:

$$H_N \begin{bmatrix} \hat{\Sigma}_N \\ \vdots \\ \hat{\Sigma}_1 \end{bmatrix} + D_N^{(f)} \begin{bmatrix} \hat{\Sigma}_N \\ \vdots \\ \hat{\Sigma}_1 \end{bmatrix} + K_{N+1}^{(f)}(\hat{\Sigma}_0 + R)$$

$$= H_N \begin{bmatrix} \hat{\Sigma}_N \\ \vdots \\ \hat{\Sigma}_1 \end{bmatrix} + K^{(f)}_{N+1} C(P_{b,N} - \Sigma_0)C^T$$

$$+ K^{(f)}_{N+1}(\hat{\Sigma}_0 + R) = -\Sigma_{N+1}C^T, \tag{31}$$

$$B_N \begin{bmatrix} \hat{\Sigma}_{-1} \\ \vdots \\ \hat{\Sigma}_{-N} \end{bmatrix} + D^{(b)}_N \begin{bmatrix} \hat{\Sigma}_{-1} \\ \vdots \\ \hat{\Sigma}_{-N} \end{bmatrix} + K^{(b)}_{N+1}(\hat{\Sigma}_0 + R)$$

$$= B_N \begin{bmatrix} \hat{\Sigma}_{-1} \\ \vdots \\ \hat{\Sigma}_{-N} \end{bmatrix} + K^{(b)}_{N+1} C(P_{f,N} - \Sigma_0)C^T$$

$$+ K^{(b)}_{N+1}(\hat{\Sigma}_0 + R) = -\Sigma_{-N-1}C^T, \tag{32}$$

where the relations (21) and (29) are used in the first equation, and the relation (14) and (30) are used in the second equation. Solving for $K^{(f)}_{N+1}$ and $K^{(b)}_{N+1}$ by using (29) and (30) gives us

$$K^{(f)}_{N+1} = \left(-\Sigma_{N+1}C^T - H_N \begin{bmatrix} \hat{\Sigma}_N \\ \vdots \\ \hat{\Sigma}_1 \end{bmatrix} \right) \hat{P}^{-1}_{b,N}, \tag{33}$$

$$K^{(b)}_{N+1} = \left(-\Sigma_{-N-1}C^T - B_N \begin{bmatrix} \hat{\Sigma}_{-1} \\ \vdots \\ \hat{\Sigma}_{-N} \end{bmatrix} \right) \hat{P}^{-1}_{f,N}, \tag{34}$$

where $\hat{P}_{f,N} = CP_{f,N}C^T + R$, $\hat{P}_{b,N} = CP_{b,N}C^T + R$, and $P_{f,N}$ and $P_{b,N}$ are given by (14) and (20). Note that we have

$$CK^{(f)}_{N+1} = -\left[\, I \;\; CH_N \,\right] \left[\, \hat{\Sigma}^T_{N+1} \;\cdots\; \hat{\Sigma}^T_1 \,\right]^T \hat{P}^{-1}_{b,N}, \tag{35}$$

$$CK^{(b)}_{N+1} = -\left[\, I \;\; CB_N \,\right] \left[\, \hat{\Sigma}^T_{-1} \;\cdots\; \hat{\Sigma}^T_{-N-1} \,\right]^T \hat{P}^{-1}_{f,N}, \tag{36}$$

by multiplying both sides of (33) and (34) by C.

Now, what remains to do is to compute $CP_{b,N}C^T$ and $CP_{f,N}C^T$ in (33) and (34). Recalling two relations $H_{N+1} = [H_N \;\; 0] + [D^{(f)}_N \;\; K^{(f)}_{N+1}]$ and $B_{N+1} = [0 \;\; B_N] + [K^{(b)}_{N+1} \;\; D^{(b)}_N]$ in (25) and (26), we can represent $CP_{b,N}C^T$ and $CP_{f,N}C^T$, recursively, *i.e.*,

$$CP_{f,N+1}C^T$$

$$= C\Sigma_0C^T + CH_{N+1} \begin{bmatrix} C\Sigma_{-1} \\ \vdots \\ C\Sigma_{-N-1} \end{bmatrix} C^T$$

$$= CP_{f,N}C^T + C \left[\, D^{(f)}_N \;\; K^{(f)}_{N+1} \,\right] \begin{bmatrix} C\Sigma_{-1} \\ \vdots \\ C\Sigma_{-N-1} \end{bmatrix} C^T$$

$$= CP_{f,N}C^T - CK^{(f)}_{N+1}CK^{(b)}_{N+1}\hat{P}_{f,N},$$

and

$$CP_{b,N+1}C^T$$

$$= C\Sigma_0C^T + CB_{N+1} \begin{bmatrix} C\Sigma_{N+1} \\ \vdots \\ C\Sigma_1 \end{bmatrix} C^T$$

$$= CP_{b,N}C^T + C \left[\, K^{(b)}_{N+1} \;\; D^{(b)}_N \,\right] \begin{bmatrix} C\Sigma_{N+1} \\ \vdots \\ C\Sigma_1 \end{bmatrix} C^T$$

$$= CP_{b,N}C^T - CK^{(b)}_{N+1}CK^{(f)}_{N+1}\hat{P}_{b,N},$$

where first equalities come from (14) and (21), and third ones come from (35) and (36).

What has been done so far is summarized in the following table and theorem.

Recursive algorithm

FIR predictor :

$$\hat{x}_k = -\sum_{i=0}^{N-1} h_{N,N-i}\, y_{k-N+i},$$

Iterative computation :

Initialization :

$$H_1 = -A\Sigma_0C^T(\hat{\Sigma}_0 + R)^{-1}$$

$$B_1 = -\Sigma_0A^TC^T(\hat{\Sigma}_0 + R)^{-1}$$

$$\hat{P}_{f,1} = \hat{P}_{b,1} = \hat{\Sigma}_0 + R$$

Loop $N = 1, 2 \cdots$

Compute $K^{(f)}_{N+1}$ from (33).

Compute $K^{(b)}_{N+1}$ from (34).

$$H_{N+1} = \left[\, H_N \;\; 0 \,\right] + K^{(f)}_{N+1} \left[\, CB_N \;\; I \,\right]$$

$$B_{N+1} = \left[\, 0 \;\; B_N \,\right] + K^{(b)}_{N+1} \left[\, I \;\; CH_N \,\right]$$

$$\hat{P}_{f,N+1} = \hat{P}_{f,N} - CK^{(f)}_{N+1}CK^{(b)}_{N+1}\hat{P}_{f,N}$$

$$\hat{P}_{b,N+1} = \hat{P}_{b,N} - CK^{(b)}_{N+1}CK^{(f)}_{N+1}\hat{P}_{b,N}$$

Theorem 1. The coefficients H_N (15) of the optimal FIR predictor (10) is given by $H_N = \Gamma_N\Xi^{-1}_N$, where Γ_N and Ξ_N are defined in (17) and (18), respectively. H_N can be computed from recursion shown in the above table to avoid an inverse operation of a big matrix. By using a deterministic smoother (Han et al. [2002]), the FIR predictor (10) can be easily extended to the general one (2) with an external input.

3. SIMULATION

In this section, it is shown via simulation that the proposed FIR predictor can have the good performance compared with other predictors. The F-404 engine model with temporary uncertainties is represented as

$$x_{i+1} = \begin{bmatrix} 0.931 + \delta_i & 0 & 0.111 \\ 0.008 & 0.98 + \delta_i & -0.017 \\ 0.014 & 0 & 0.895 + 0.1\delta_i \end{bmatrix} x_i$$

$$+ \begin{bmatrix} 0.051 \\ 0.049 \\ 0.048 \end{bmatrix} w_i,$$

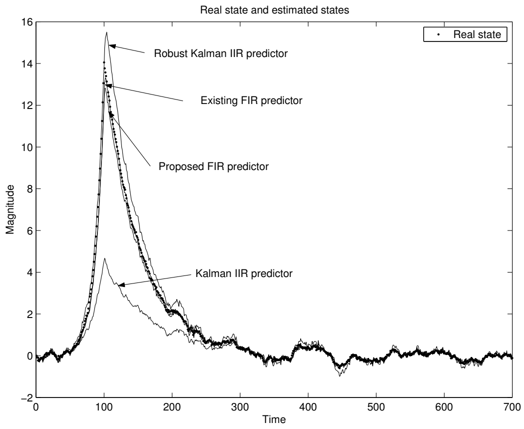

Fig. 1. Real state and estimated states of four predictors

$$y_i = \begin{bmatrix} 1+0.1\delta_i & 0 & 0 \\ 0 & 1+0.1\delta_i & 0 \end{bmatrix} x_i + v_i,$$

where the parameter δ_i is given by

$$\delta_i = \begin{cases} 0.1, & 50 \le i \le 100, \\ 0, & \text{otherwise.} \end{cases} \tag{37}$$

The Kalman IIR predictor, the robust Kalman IIR predictor (Xie et al. [1994]), and the existing FIR predictor (Kwon et al. [1999]) are employed for comparison. As mentioned in Introduction, the existing FIR predictor (Kwon et al. [1999]) does not use *a priori* information on the state to be estimated. The size of the moving horizon N for the proposed and existing FIR predictors is set to 3. Figures 1 and 2 compare how four kinds of predictors behave for temporarily modeling uncertainties. These figures show that, due to finite memory, the estimation errors of the proposed and the existing FIR predictors are much smaller than that of the Kalman IIR predictor on the interval where modeling uncertainties are applied. The robust Kalman IIR predictor also has small estimation errors when uncertainties exist. When uncertainties disappear and estimations are carried out in the steady state, the two norms of estimation errors taken from 200 samples are as follows:

Predictor	RKP	EFP	PFP	KP
Estimation error	9.15	4.17	2.03	1.68

where RKP, EFP, PFP, and KP stand for the robust Kalman IIR predictor, the existing FIR predictor, the proposed FIR predictor, and the Kalman IIR predictor, respectively. As seen in the table, the Kalman IIR predictor is the best in the steady state while the robust Kalman IIR predictor has the poorest performance. The estimation error of the proposed FIR predictor is smaller than that of the existing FIR predictor. It makes sense since the proposed FIR predictor utilizes *a priori* stochastic information on the steady state.

To be summarized, it can be seen that the suggested FIR predictor is as robust as the robust IIR predictors when applied to systems with model parameter uncertainties, but has the better performance than the robust IIR filter and the existing FIR filter when uncertainties do not show up.

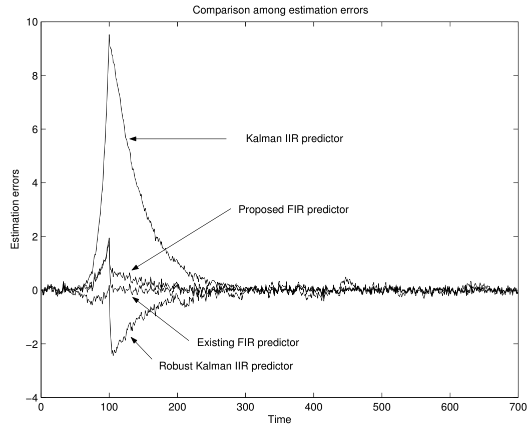

Fig. 2. Estimation errors of four predictors

4. CONCLUSION

In this paper, the FIR predictor was presented for stochastic state space signal models. The FIR predictor proposed in this paper uses the finite number of inputs and outputs on the recent time interval so that it is believed to be more robust than the IIR predictors when applied to systems with temporary model parameter uncertainties. In addition, the good performance is achieved in the steady state since *a priori* stochastic information on the state is utilized. The Yule-Walker equations for the FIR predictor were obtained from forward and backward predictions by the orthogonality principle. We discussed how to solve these equations efficiently. It was shown through simulation that the proposed FIR predictor is as robust as the robust IIR predictor and the existing FIR predictor for state space models. The variance of the estimation error of the proposed FIR predictor was shown to be smaller than that of the existing FIR predictor when uncertainties disappear and estimations are carried out in the steady state.

The proposed FIR predictor would be a good substitute for robust IIR predictors and the existing FIR predictor in order to achieve both robustness against modelling uncertainties and the good performance in the steady state.

REFERENCES

A. M. Bruckstein and T. Kailath. Recursive limited memory filtering and scattering theory. *IEEE Trans. Inform. Theory*, 31:440–443, 1985.

B. Choi. A recursive alrorithm for solivng the normal equations of bilaternal AR models. *IEEE Trans. Singal Processing* , 45(3):786–789, 1997.

C. E. de Souza and L. Xie. Robust H_∞ filtering. *Control and Dynamic Systems*, 65:323–377, 1994.

B. Friedlander, T. Kailath, M. Morf, and L. Ljung. Extended Levinson and Chandrasekhar Equation for General Discrete-Time Linear Estimation Problems. *IEEE Trans. Automat. Contr.* , 23(4):653–659, 1978.

S. Han, W. H. Kwon, and P. S. Kim. Quasi-Deadbeat Minimax Filters for Deterministic State Space Models. *IEEE Trans. Automat. Contr.*, 47(11):1904–1908, 2002.

J. Y. Ishihara, M. H. Terra, and J. C. T. Campos. Robust Kalman Filter for Discriptor Systems. *IEEE Trans. Automat. Contr.*, 51(8):1354–1358, 2006.

N. Kalouptsidis. *Signal processing Systems: Theory and Design.* John Wiley, 1997.

O. K. Kwon, W. H. Kwon, and K. S. Lee. FIR filters and recursive forms for discrete-time state-space models. *Automatica*, 25:715–728, 1989.

W. H. Kwon, P. S. Kim, and P. Park. A receding horizon Kalman FIR filter for discrete time-invariant systems. *IEEE Trans. Automat. Contr.*, 44(9):1787–1791, 1999.

L.-M. Lee and H.-C. Wang. An Extended Levison-Durbin Algorithm for the Analysis of Noisy Autogressive Process. *IEEE Signal Processing Letters* , 3(1):13–15, 1996.

D. Liu and X. Liu. Optimal state estimation without the requirement of *a priori* statistics information of the initial state. *IEEE Trans. Automat. Contr.*, 39(8):2087–2091, 1994.

I. R. Petersen and A. V. Savkin. *Robust Kalman filtering for signals and systems with large uncertainties.* Birkhauser, Boston, 1999.

C. V. Rao, J. B. Rawlings, and J. H. Lee. Contrained linear estimation - A moving horizon approach. *Automatica*, 37(10):1619–1628, 2001.

Z. Wang, J. Lam, and X. Liu. Robust Filtering for Discrete-Time Moarkovian Jump Delay Systems. *IEEE Signal Processing Letters*, 11(8):659–662, 2004.

L. Xie, Y. C. Soh, and C. E. de Souza. Robust Kalman filtering for uncertain discrete-time systems. *IEEE Trans. Automat. Contr.*, 39:1310–1313, 1994.