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Stabilisation of Singular LPV Systems

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Abstract: This paper deals with the class of singular LPV systems. Sufficient conditions on controllers design are developed in the LMI (Linear Matrix Inequality) terms. In order to reduce the conservatism of the developed result using quadratic method, an approach based on polyquadratic Lyapunov functions is proposed. Numerical example is given to illustrate the effectiveness of the obtained results.

Keywords: LPV systems, Singular systems, Linear matrix Inequalities (LMI), Lyapunov method.

1. INTRODUCTION

Singular systems have attracted particular interest in the literature for their various applications such as robotics (Mills and Goldenberg (1989)), circuits (Newcomb and Dziurla (1989)), aircraft modelling (Stevens and Lewis (1991)) and singular perturbation systems (Dai (1989)).

Many works dealing with the control of switched systems, impulsive and switching singular systems and singular LPV systems have been studied recently for the theoretical and practical point of view (see for example Gelig and Churilov (1998), Yaoa et al. (2006), Masubuchi et al. (2004) and references therein). For the switching systems the control techniques based on switching between different controllers have been applied extensively in recent years, due to their advantages in achieving stability (Daafouz et al. (2002), Ge et al. (2001), Mancilla-Aguilar (2003), Yaoa et al. (2006)) whereas gain-scheduling controllers techniques are used for singular LPV systems (Masubuchi et al. (2003)). In this paper, for the considered class of singular LPV systems, we consider controller obtained by interpolation of linear controller. A such controller is used since a single (continuous/discrete) control law cannot be found for many control problems. However, if there exist a lot of works on singular linear systems (see for exampe Darouach and Boutayeb (1995), Darouach (2006) and references therein), to our knowledge there are few studies on singular LPV systems and their corresponding control problems.

In this paper, we study the design of controllers for singular LPV systems in polytopic form. Using LMI technique and polyquadratic Lyapunov approach, we propose a state controller whose gain is obtained by interpolation of linear state controller. Further, extra degree of freedom is introduced and exploited to reduce conservatism.

The paper is organized as follows. In section 2, the considered class of a discrete-time singular LPV systems is described. In section 3, the stabilisation of singular LPV systems is studied. Relaxations are introduced to design controllers in LMI formulation. Example is given in section 4. Finally, section 5 concludes the paper.

Notation. Throughout this paper, R^n and $R^{n\times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript "$^\mathsf{T}$" denotes matrix transposition, the notation $X \geq Y$ (respectively, $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite) and the symbol $(*)$ denotes the transpose elements in the symmetric positions. I is the identity matrices with compatible dimensions and $I_N = \{1, 2, \cdots, N\}$.

2. PROBLEM STATEMENT

The considered singular LPV system is as follows

$$\begin{aligned} Ex(t+1) &= A(\rho(t))x(t) + B(\rho(t))u(t) \\ y(t) &= C(\rho(t))x(t) \end{aligned} \tag{1}$$

Two important classes of LPV systems can be distinguished; the affine LPV where the state space matrices depend affinely on $\rho(t)$ and the polytopic LPV where the parameter $\rho(t)$ varies in polytope of vertices ρ_i such that $\rho(t) \in Co\{\rho_1, \rho_2, ..., \rho_r\}$. In the sequel only the second class is used in the following form

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$$
\begin{aligned}
Ex(t+1) &= \sum_{i=1}^{N} \xi_i(\rho(t))(A_i x(t) + B_i u(t)) \\
y(t) &= \sum_{i=1}^{N} \xi_i(\rho(t)) C_i x(t)
\end{aligned} \tag{2}
$$

where

$$
\xi_i(\rho(t)) \geq 0, \sum_{i=1}^{N} \xi_i(\rho(t)) = 1 \tag{3}
$$

with $x(t) \in R^n$ is the state vector, $y(t) \in R^p$ is the output vector, $u(t) \in R^m$ is the input vector, $A_i \in R^{n\times n}$, $B_i \in R^{n\times m}$ and $C_i \in R^{p\times n}$. The matrix E may be singular with $0 \leq \text{rank}(E) = n_E < n$.

In this paper we are interested in developing sufficient conditions for controller design. Our methodology will be mainly based on the Lyapunov theory and LMI formulation.

3. STABILISATION OF DISCRETE-TIME MODELS

3.1 Stability analysis

Consider the Lyapunov dependant parameter function of the form:

$$
V(x(t), \rho(t)) = x(t)^\top \mathscr{P}(\rho(t)) x(t) \tag{4}
$$

with

$$
\mathscr{P}(\rho(t)) = \sum_{i=1}^{N} \xi_i(\rho(t)) E^\top P_i E, E^\top P_i E \geq 0, i \in I_N \tag{5}
$$

The difference of (4) along the solution of the unforced system of (2) is given by

$$
\begin{aligned}
&\Delta V = V(x(t+1), \rho(t+1)) - V(x(t), \rho(t)) \\
&= x(t+1)^\top \mathscr{P}(\rho(t+1)) x(t+1) - x(t)^\top \mathscr{P}(\rho(t)) x(t)
\end{aligned} \tag{6}
$$

Thus, the unforced singular system of (2) is stable if there exist nonsingular symmetric matrices P_i such that the following hold for all $(i,j) \in I_N^2$:

$$
E^\top P_i E \geq 0 \tag{7}
$$

$$
A_i^\top P_j A_i - E^\top P_i E < 0 \tag{8}
$$

In the following, our objective is to introduce extra degree of freedom. This will be very helpful for the development of an LMI-based conditions for state feedback design. Thus the unforced singular system of (2) is stable if there exist nonsingular symmetric matrices P_i, matrices F_i and G_i such that the following LMI hold for all $(i,j) \in I_N^2$:

$$
EP_iE^\top \geq 0 \tag{9}
$$

$$
\begin{pmatrix} -EP_iE^\top + A_iF_i + (A_iF_i)^\top & -F_i^\top + A_iG_i \\ (*) & P_j - (G_i + G_i^\top) \end{pmatrix} < 0 \tag{10}
$$

It is easy to prove that conditions (10) and (8) are equivalents. Thus by multiplying left (10) by the matrix (I, A_i) and right by its transpose we obtain (8).

Notice that when $E = I$ (regular LPV systems), the conditions (9) can be replaced by $P_i > 0$. This result coincides with the result of (Peaucelle et al. (2000)) and also the result of (Daafouz et al. (2002)) when setting $F_i = 0$.

3.2 Controller design

Consider the following control law obtained by interpolation of linear controller:

$$
u(t) = \sum_{i=1}^{N} \xi_i(\rho(t)) K_i x(t) \tag{11}
$$

where $K_i \in R^{m\times n}$. The closed loop system is

$$
Ex(t+1) = \sum_{i=1}^{N}\sum_{j=1}^{N} \xi_i(\rho(t))\xi_j(\rho(t)) \overline{A}_{ij} x(t) \tag{12}
$$

with

$$
\overline{A}_{ij} = A_i + B_i K_j \tag{13}
$$

To derive stability conditions of (12) it is possible to substitute A_i by $A_i + B_iK_j$ in conditions (10). However the obtained conditions are Bilinear Matrix Inequalities (BMI) in K_i, F_i and G_i. A way to get LMI conditions is to chose $F_i = G_i = G$.

Theorem 1. The singular system (12) is stable if there exist nonsingular matrices G, P_i and N_j such that the following LMI hold for all $(i,j,k) \in I_N^3$:

$$
EP_iE^\top \geq 0 \tag{14}
$$

$$
\begin{pmatrix} -EP_iE^\top + \Phi_{ij} + \Phi_{ij}^\top & -G^\top + \Phi_{ij} \\ (*) & P_k - (G + G^\top) \end{pmatrix} < 0 \tag{15}
$$

with

$$
\Phi_{ij} = A_iG + B_iN_j \tag{16}
$$

The controller gains are defined by:

$$
K_i = N_i G^{-1} \tag{17}
$$

Proof: Multiplying (15) by $\sum_{i=1}^{N}\sum_{j=1}^{N} \xi_i(\rho(t))\xi_j(\rho(t))$ and according to (17), we obtain

$$
\begin{aligned}
&\sum_{i=1}^{N}\sum_{j=1}^{N} \xi_i(\rho(t))\xi_j(\rho(t)). \\
&\begin{pmatrix} -EP_iE^\top + \overline{A}_{ij}G + (\overline{A}_{ij}G)^\top & -G^\top + \overline{A}_{ij}G \\ (*) & P_k - (G + G^\top) \end{pmatrix} < 0
\end{aligned} \tag{18}
$$

Which is only stability conditions (10) by substituting A_i by $A_i + B_iK_j$ and $F_i = G_i$ by G. □

However choosing single matrix G instead of multiple matrices G_i introduces some conservatism. In the sequel another formulation is proposed to overcome these limitations.

3.3 Main result

In order to introduce more relaxation,we propose to reduce the number of synthesis LMI conditions ($N^3 + N$ in theorem 1) and to use different matrices G_i. For this, we propose to write the model (2) and the controller (11) as follows

$$\overline{E}\overline{x}(t+1) = \sum_{i=1}^{N} \xi_i(\rho(t))\overline{A}_i\overline{x}(t) \tag{19}$$

Where

$$\overline{x}(t) = \left(x(t)^\top, u(t)^\top\right), \overline{A}_i = \begin{pmatrix} A_i & B_i \\ K_i & -I \end{pmatrix}, \overline{E} = \begin{pmatrix} E & 0 \\ 0 & 0 \end{pmatrix} \tag{20}$$

Consequently, to design controller it suffices to substitute A_i by $\overline{A}_i$ in conditions (10). However the obtained conditions are BMI in K_i, F_i and G_i. To derive LMI conditions we propose the following result.

Theorem 2. The singular system (19) is stable if there exist nonsingular matrices G_{1i}, and G_{2i}, G_{3i}, P_{1i}, P_{3i}, P_{2i} such that the following LMI hold for all $(i, j) \in I_N^2$.

$$EP_{1i}E^\top \geq 0 \tag{21}$$

$$\begin{pmatrix} \begin{pmatrix} -EP_{1i}E^\top + \\ A_iG_{1i} + B_iG_{2i} + \\ (A_iG_{1i} + B_iG_{2i})^\top \end{pmatrix} & \begin{pmatrix} B_iG_{3i} + \\ N_i^\top - \\ G_{2i}^\top \end{pmatrix} & \begin{pmatrix} -G_{1i}^\top \\ +A_iG_{1i} \\ +B_iG_{2i} \end{pmatrix} & \begin{pmatrix} -G_{2i}^\top \\ +B_iG_{3i} \end{pmatrix} \\ (*) & \begin{pmatrix} -G_{3i} \\ -G_{3i}^\top \end{pmatrix} & \begin{pmatrix} N_i - \\ G_{2i} \end{pmatrix} & \begin{pmatrix} -G_{3i}^\top \\ -G_{3i} \end{pmatrix} \\ (*) & (*) & \begin{pmatrix} P_{1j} - \\ G_{1i} \\ -G_{1i}^\top \end{pmatrix} & \begin{pmatrix} P_{2j} - \\ G_{2i}^\top \end{pmatrix} \\ (*) & (*) & (*) & \begin{pmatrix} P_{3j} - \\ G_{3i} \\ -G_{3i}^\top \end{pmatrix} \end{pmatrix} < 0 \tag{22}$$

The controller gains are defined by:

$$K_i = N_iG_{1i}^{-1} \tag{23}$$

Proof: Substituting A_i by $\overline{A}_i$ in conditions (10) with $F_i = G_i$ we get

$$\overline{E}P_i\overline{E}^\top \geq 0 \tag{24}$$

$$\begin{pmatrix} -\overline{E}P_i\overline{E}^\top + \overline{A}_iG_i + (\overline{A}_iG_i)^\top & -G_i^\top + \overline{A}_iG_i \\ (*) & P_j - (G_i + G_i^\top) \end{pmatrix} < 0 \tag{25}$$

The obtained conditions (25) are BMI in K_i and G_i. To derive LMI conditions we propose to chose matrix G_i with the following structure

$$G_i = \begin{pmatrix} G_{1i} & 0 \\ G_{2i} & G_{3i} \end{pmatrix} \tag{26}$$

Thus

$$\overline{A}_iG_i = \begin{pmatrix} A_iG_{1i} + B_iG_{2i} & B_iG_{3i} \\ N_i - G_{2i} & -G_{3i} \end{pmatrix} \tag{27}$$

with $N_i = K_iG_{1i}$. Writing the symmetric matrices P_i as follows

$$P_i = \begin{pmatrix} P_{1i} & P_{2i} \\ P_{2i}^\top & P_{3i} \end{pmatrix} \tag{28}$$

and according to (27) we obtain (22) from (25). Conditions (21) are obtained directely from (24). $\square$

It is important to note the relaxation introduced by theorem 2 compared with the result of the theorem 1. First, the number of constraint is reduced to $(N^2 + N)$ LMI instead of $(N^3 + N)$. Second, extra degree of freedom is introduced by using different matrices G_i.

4. NUMERICAL EXAMPLE

Consider a numerical example with the following data:

$$A_1 = \begin{bmatrix} 0.2 & 0.5 & 0 \\ -0.1 & 0.2 & 0 \\ 0.2 & 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 & -2 \\ 0 & 0 \\ 2 & 2 \end{bmatrix} \tag{29}$$

$$A_2 = \begin{bmatrix} 0.7 & 0.1 & 0 \\ 0 & 0.1 & -0.1 \\ 0.2 & 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 1 & -1 \\ -1 & -1 \\ 0 & 1 \end{bmatrix} \tag{30}$$

The singular matrix, E, are given by:

$$E = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 10 \end{bmatrix} \tag{31}$$

Solving LMI (21)-(22), we get feasible problem:

$$G_{11} = 10^3 \begin{bmatrix} 5.9046 & 1.7629 & 2.6364 \\ 4.9195 & 0.2476 & 3.9188 \\ 2.8554 & 1.4127 & 5.2596 \end{bmatrix}, \quad G_{12} = 10^3 \begin{bmatrix} 5.2602 & 2.4886 & 2.5751 \\ 3.6801 & 1.5581 & 3.9265 \\ 2.2568 & 2.3790 & 4.4724 \end{bmatrix} \tag{32}$$

$$N_1 = \begin{bmatrix} -292.0268 & -51.7925 & -252.3748 \\ 553.1883 & 310.9225 & 20.9454 \end{bmatrix}, \quad N_2 = 10^3 \begin{bmatrix} 0.1714 & 2.2936 & 0.2577 \\ 0.6544 & 2.3273 & 0.3582 \end{bmatrix} \tag{33}$$

which give the controller gains

$$K_1 = \begin{bmatrix} -0.0122 & -0.0359 & -0.0151 \\ 0.1969 & -0.1213 & -0.0043 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0.9209 & -2.1228 & 1.3911 \\ 0.9698 & -1.9823 & 1.2621 \end{bmatrix} \tag{34}$$

We note that the conditions (14)-(15) fail to prove the stabilisation of the given example.

5. CONCLUSION

In this paper the stabilisation of class of singular LPV systems is studied. Sufficient conditions to design controllers for discrete-time case are developed in the LMI terms. To reduce the conservatism of the existing result using quadratic method, the proposed approach uses polyquadratic Lyapunov functions and introduces extra degree of freedom. Numerical example is given to illustrate the benefit of derived results. These results will be extended to continuous-time case.

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