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Desensitized Model Predictive Control Applied to a Structural Benchmark Problem

Carlos Lana * **Mario Rotea** **

* *School of Aeronautics & Astronautics, Purdue University, West Lafayette, IN 47907.*
** *Department of Mechanical & Industrial Engineering, University of Massachusetts, Amherst, MA 01003*

Abstract: This paper presents a model predictive control formulation that incorporates trajectory sensitivity to improve the robustness of the conventional model predictive control strategy. A structural control benchmark problem is used to illustrate the potential of the approach. The numerical results suggest that the proposed approach may be a viable option to increase the robustness of the conventional model predictive control strategy without increasing the computation requirements.

1. INTRODUCTION

Model predictive control (MPC) is a tool originally developed in industry to solve multivariable control problems with input and state constraints. Basically, MPC solves a constrained optimization problem at each sampling time that provides the current control action to the system [Goodwin et al., 2005]. The importance of MPC resides in its ability to handle multivariable control problems with constraints.

MPC requires knowledge of the exact model of the system. Unfortunately, this ideal situation does not occur in practice as only approximate models are available. When a controller which was designed with an approximate model of the plant is implemented, performance degradation is likely to occur. This may lead not only to an unacceptable behavior but also to system instability.

Several approaches have been proposed to reduce the sensitivity of the controlled system to model inaccuracies and they are known under the name of robust MPC [Michalska and Mayne, 1993, Kothare et al., 1996, Lee and Yu, 1997, Mayne et al., 2000, Lu and Arkun, 2000, Badgwell, 1997, Casavola et al., 2005, Wan and Kothare, 2003, Park and Jeong, 2004]. A common approach to incorporate robustness to MPC is to reformulate it as a minimax problem, where the MPC cost function is maximized over the uncertain parameters and the resulting worst-case cost is minimized over the control. A disadvantage of this approach is that the direct intent of solving the minimax optimization problem is not attractive for online optimization.

A solution to this problem is to minimize an upper bound of the maximum value of the cost function over the uncertain parameters, which leads to a more conservative but simpler formulation. However, the resulting optimization problem is most likely to require a semi-definite program online [Kothare et al., 1996, Lu and Arkun, 2000, Casavola et al., 2005, Cuzzola et al., 2002, Park and Jeong, 2004]. Even though semi-definite programs can be solved in polynomial time, the online computation required to solve these robust MPC problems may still be prohibitive for all but slow/simple dynamical systems. In order to extend the applicability of robust MPC, researchers have developed methods to reduce the online computation by computing a set of controllers offline while leaving online only the selection of the current controller as a function of the measured state [Wan and Kothare, 2003, Munoz de la Pena et al., 2006]. Unfortunately, the online solution of these methods may grow in complexity very quickly with the size of the problem and provide more conservative solutions in the cases where approximations are used. Although the techniques described in this discussion are powerful tools to deal with uncertainty, in practice, conventional MPC is mostly used due to its intuitive design and requirement of solving a simple quadratic program online. In this case, robustness against model inaccuracies is obtained by tuning the MPC design parameters based on simulation of the controlled system for a set of possible uncertainty values. For this reason, robust MPC techniques with simple offline design are highly desirable. A technique that falls into this category is in reference Fukushima and Bitmead [2005].

In this paper we propose to modify the conventional MPC cost function by including a term that penalizes the sensitivity of the state trajectory with respect to changes in the uncertain parameters. This approach, which has been used in optimal control problems in the late sixties [Frank, 1978, Sobral, 1968, Kreindler, 1969], has the potential of desensitizing the controller performance to model inaccuracies, yet it retains the quadratic program architecture of the conventional MPC problem. In this paper we show, by example, that this approach has merit as it produces *robust* control solutions with a real-time computational expense comparable to that of the nominal MPC.

The notation used is fairly standard. We denote with $\mathrm{diag}(M_1, \ldots, M_n)$ the matrix with block diagonal entries $M_1, \ldots, M_n$. The symbol $\otimes$ is reserved for the Kronecker product of two matrices. The identity of order n is denoted by I_n. We define the Q-weighted Euclidian vector norm by $\|x\|_Q = \sqrt{x^T Q x}$. If M is a matrix function of the parameter vector $\mu \in \mathbb{R}^m$, we define

$$\nabla_\mu M := \begin{bmatrix} \dfrac{\partial M}{\partial \mu_1}^T & \dots & \dfrac{\partial M}{\partial \mu_m}^T \end{bmatrix}^T \tag{1}$$

where $\mu = [\mu_1 \dots \mu_m]^T$. Finally, we say that matrix M is stable if it has all its eigenvalues inside the open unit circle.

2. DESENSITIZED MODEL PREDICTIVE CONTROL

Let the plant model be defined as follows:

$$x(k+1) = A(\mu)x(k) + B(\mu)u(k), \quad x(0) = x_0 \tag{2}$$

where x is the state and u is the control input. The state space matrices A and B are C^1 functions of the parameter vector $\mu \in \mathbb{R}^m$. The parameter μ, with nominal value μ_0, will be used to represent the model uncertainty. In addition, the control input and the state satisfy $u \in \mathbb{U}$ and $x \in \mathbb{X} \subseteq \mathbb{R}^{n_x}$, where $\mathbb{U}$ and $\mathbb{X}$ are polytopic sets that contain the origin.

Let the *trajectory sensitivity* p, evaluated at $\mu = \mu_0$, be defined as follows:

$$p(k) := \nabla_\mu x(k)\,|_{\mu=\mu_0} \tag{3}$$

From (2) and (3), the trajectory sensitivity p can be represented with the following linear difference equation:

$$p(k+1) = A_\mu x(k) + A_b p(k) + B_\mu u(k) + B_b u_\mu(k) \tag{4}$$

where $p(0) = p_0$ and

$$\begin{aligned} A_\mu &= \nabla_\mu A(\mu)|_{\mu=\mu_0} & A_b &= I_m \otimes A(\mu_0) \\ B_\mu &= \nabla_\mu B(\mu)|_{\mu=\mu_0} & B_b &= I_m \otimes B(\mu_0) \\ u_\mu(k) &= \nabla_\mu u(k)\,|_{\mu=\mu_0} \end{aligned}$$

Since model (2) will be used for open loop prediction, the last term of (4) will be assumed to be zero.

Trajectory sensitivity provides information about how the state trajectories change with infinitesimal variations of μ around its nominal value μ_0. This will be used to reduce the state prediction sensitivity to the model parameter value in model predictive control (MPC).

MPC is a control technique in which the control input at each time is the solution of a constrained optimal control problem with initial condition set to the measured state.

The infinite horizon MPC cost J^∞_{MPC} at time k is defined as

$$J^\infty_{\mathrm{MPC}} = \sum_{j=k}^{\infty} \|x(j)\|^2_{Q_M} + \|u(j)\|^2_R \tag{5}$$

where Q_M and R are positive definite matrices of appropriate dimensions, $x(j)$ is from (2) and the sequence u is the optimization variable.

In order to reduce the sensitivity of the state prediction from the choice of μ in the model, we incorporate a term in the cost that penalizes the energy of the trajectory sensitivity. The cost to be used in the desensitized model predictive control (DMPC) strategy is defined as follows:

$$J^\infty_{\mathrm{DMPC}} = J^\infty_{\mathrm{MPC}} + \sum_{j=k}^{\infty} \|p(j)\|^2_{Q_s} \tag{6}$$

where Q_s is a matrix of appropriate dimensions that weights the relative importance of the trajectory sensitivity p with respect to the MPC cost J^∞_{MPC}.

The use of an infinite horizon cost, (5) and (6), is highly desirable because closed loop stability follows from feasibility of the optimization problem [Goodwin et al., 2005].

Because the constrained optimal control problem is solved numerically, the costs (5) and (6) need to be replaced by finite horizon ones.

The standard cost formulation consists of a truncation of the infinite horizon one with the addition of a terminal cost and a terminal constraint. This terminal cost is often chosen to approximate (in general) the infinite horizon cost.

Define the augmented model as

$$z(k+1) = \tilde{A}z(k) + \tilde{B}u(k) \tag{7}$$

where $z(k) = [x(k)^T \;\; p(k)^T]^T$ is the augmented state. In the sequel, the nominal system matrices $A(\mu_0)$ and $B(\mu_0)$ will be denoted with A and B, respectively. The matrices $\tilde{A}$ and $\tilde{B}$ are defined as follows:

$$\tilde{A} = \begin{bmatrix} A & 0 \\ A_\mu & A_b \end{bmatrix}, \qquad \tilde{B} = \begin{bmatrix} B \\ B_\mu \end{bmatrix}. \tag{8}$$

The finite horizon costs at time k are defined as

$$J^N_{\mathrm{MPC}} = \|x(k+N)\|_{P_M} + \sum_{j=k}^{k+N-1} \|x(j)\|^2_{Q_M} + \|u(j)\|^2_R \tag{9}$$

$$J^N_{\mathrm{DMPC}} = \|z(k+N)\|_{P_D} + \sum_{j=k}^{k+N-1} \|z(j)\|^2_{Q_D} + \|u(j)\|^2_R \tag{10}$$

where $Q_D = \mathrm{diag}(Q_M, Q_s)$ and P_D is a positive definite matrix of appropriate dimensions.

The MPC and DMPC optimization problems at time k are defined as

$$\min_u \; J^N_{\mathrm{MPC}} \tag{11a}$$

subject to:

$$x(j+1) = Ax(j) + Bu(j) \tag{11b}$$

$$x(k) = x^m_k,\; x(j) \in \mathbb{X},\; x(k+N) \in \mathbb{X}_f \tag{11c}$$

$$u(j) \in \mathbb{U}, \;\; j = k, \dots, k+N-1 \tag{11d}$$

and

$$\min_u \; J^N_{\mathrm{DMPC}} \tag{12a}$$

subject to:

$$z(j+1) = \tilde{A}z(j) + \tilde{B}u(j) \tag{12b}$$

$$z(k) = z^m_k,\; z(j) \in \mathbb{Z},\; z(k+N) \in \mathbb{Z}_f \tag{12c}$$

$$u(j) \in \mathbb{U}, \;\; j = k, \dots, k+N-1 \tag{12d}$$

where $\mathbb{Z} = \mathbb{X} \times \mathbb{R}^{mn_x}$, $z^m_k = [x^{mT}_k \;\; p^{mT}_k]^T$, and $\mathbb{X}_f$ and $\mathbb{Z}_f$ are the terminal sets. The vector x^m_k is the state of the plant (2) at time k and it is measured. The trajectory sensitivity p is a fictitious state that is available to the controller. For this reason, p^m_k is considered a design variable. Two possible choices are: (i) p^m_k obtained from the sensitivity model (4) and (ii) p^m_k set to zero.

Once the optimization problem (12) is solved, the optimal control input $u(k)$ is fed to the plant and the process repeated.

3. BENCHMARK PROBLEM

In this section we use MPC and DMPC to control a civil engineering structure under the presence of environmental disturbances. In particular, we will use the benchmark problem presented in Spencer et al. [1997], which consists of a high fidelity dynamical model of a three story building subject to ground acceleration disturbances generated by an earthquake. The actuator is an active mass driver (AMD) placed on the top of the third floor which is manipulated through the command input u. A schematic of the building is shown in Figure 1. Controllers will be designed using five available measurements: the absolute acceleration of each of the three floors, $\ddot{x}_{a1}$, $\ddot{x}_{a2}$ and $\ddot{x}_{a3}$, the absolute acceleration $\ddot{x}_{am}$ of the AMD, and the ground acceleration $\ddot{x}_g$.

The objective is to design a controller which is robust against changes in the damping ratio ζ_1 of the first natural frequency. We will use two control techniques: MPC and DMPC, both based on the nominal model.

3.1 The model

The model used for design and analysis is the evaluation model described in Spencer et al. [1997]. This model has 28 states and it is given by the following equations

$$\dot{\eta} = A\eta + E\ddot{x}_g + Bu \tag{13a}$$
$$y = C_y\eta + F_y\ddot{x}_g + D_yu + v \tag{13b}$$
$$z = C_z\eta + F_z\ddot{x}_g + D_zu \tag{13c}$$

where η is the state vector, $\ddot{x}_g$ is the ground acceleration, u is a scalar control input. The vector $z = [x_{a1}\ x_{a2}\ x_{a3}\ x_m\ \dot{x}_{a1}\ \dot{x}_{a2}\ \dot{x}_{a3}\ \dot{x}_m\ \ddot{x}_{a1}\ \ddot{x}_{a2}\ \ddot{x}_{a3}\ \dot{x}_{am}]^T$ is the vector of of all system outputs and $y = [\ddot{x}_{a1}\ \ddot{x}_{a2}\ \ddot{x}_{a3}\ \ddot{x}_{am}\ \ddot{x}_g]^T$ $+v^T$ is the vector of measured accelerations used to compute the control input. The variable x_i is the displacement of the ith floor relative to the ground, x_m is the displacement of the AMD relative to the third floor, $\ddot{x}_{ai}$ is the absolute acceleration of the ith floor, $\ddot{x}_{am}$ is the absolute acceleration of the AMD and v is the measurement noise.

We want to evaluate the robustness of the controllers to changes in the damping ratio ζ_1 of the first natural frequency. The available model is an input-output model that does not have an explicit dependence on the damping ratio. In this paper we replace the state space model given in Spencer et al. [1997] with a realization that explicitly depends on the parameter ζ_1. This model is obtained following the approach in D'Amato and Rotea [1998]. The nominal value for ζ_1 was found to be 0.0033. This model is used to determine the sensitivity with respect to the damping ratio ζ_1 and to conduct robustness studies by varying the damping ratio.

3.2 Evaluation criteria and implementation constraints

Reference Spencer et al. [1997] provides ten evaluation criteria $J_1 - J_{10}$ to compare the performance of the closed loop system. Here we consider only those that directly measure the effect of the controller on the building.

Stochastic evaluation criteria The ground acceleration $\ddot{x}_g$ is a stationary stochastic process with power spectral density

$$S_{\ddot{x}_g\ddot{x}_g}(\omega_g, \zeta_g) = S_0(\omega_g, \zeta_g)\frac{4\zeta_g^2\omega_g^2 + \omega^4}{(\omega^2 - \omega_g^2) + 4\zeta_g^2\omega_g^2\omega^2} \tag{14}$$

where the natural frequency ω_g and the damping ratio ζ_g are set to 37.3 and 0.3, respectively. The scaling factor S_0 is chosen such that the rms value of the ground acceleration takes the value of $\sigma_{\ddot{x}_g} = 0.12$ g's.

When both the random ground disturbance and the measurement noise are applied to the structure, we use the following criteria to evaluate the controller performance:

$$J_1 = \max\left\{\frac{\sigma_{d1}}{\sigma_{x_{30}}}, \frac{\sigma_{d2}}{\sigma_{x_{30}}}, \frac{\sigma_{d3}}{\sigma_{x_{30}}}\right\} \tag{15}$$
$$J_2 = \max\left\{\frac{\sigma_{\ddot{x}_{a1}}}{\sigma_{\ddot{x}_{30}}}, \frac{\sigma_{\ddot{x}_{a2}}}{\sigma_{\ddot{x}_{30}}}, \frac{\sigma_{\ddot{x}_{a3}}}{\sigma_{\ddot{x}_{30}}}\right\} \tag{16}$$

where the notation used is as follows: the interstory drift d_i is the relative lateral displacement between consecutive floors ($d_1 = x_1, d_2 = x_2 - x_1, d_3 = x_3 - x_2$) and the symbol σ_x denotes the rms value of the stochastic variable x. The normalization constants $\sigma_{x_{30}}$ and $\sigma_{\ddot{x}_{30}}$ are defined in Spencer et al. [1997].

The computation of J_1 and J_2 will be done numerically by simulating the closed loop system during 300 seconds.

Deterministic evaluation criteria In this case, the ground acceleration is one of the two historical earthquake records: 1940 El Centro NS and 1968 Hachinohe NS. The controllers are evaluated according to the following criteria

$$J_6 = \max_{\substack{ElCentro\\Hachinohe}} \max_t \left\{\frac{|d_1(t)|}{x_{30}}, \frac{|d_2(t)|}{x_{30}}, \frac{|d_3(t)|}{x_{30}}\right\} \tag{17}$$
$$J_7 = \max_{\substack{ElCentro\\Hachinohe}} \max_t \left\{\frac{|\ddot{x}_{a1}(t)|}{\ddot{x}_{30}}, \frac{|\ddot{x}_{a2}(t)|}{\ddot{x}_{30}}, \frac{|\ddot{x}_{a3}(t)|}{\ddot{x}_{30}}\right\} \tag{18}$$

where the normalization constants x_{30} and $\ddot{x}_{30}$ are defined in Spencer et al. [1997]. In addition, the peak values of the control input u, AMD displacement x_m and AMD absolute acceleration $\ddot{x}_{am}$ must satisfy the following hard constraints.

$$\max_{\substack{ElCentro\\Hachinohe}} \max_t |u(t)| \leq 3\text{v} \tag{19}$$
$$\max_{\substack{ElCentro\\Hachinohe}} \max_t |\ddot{x}_{am}(t)| \leq 6\text{g} \tag{20}$$
$$\max_{\substack{ElCentro\\Hachinohe}} \max_t |x_m(t)| \leq 9\text{cm}. \tag{21}$$

3.3 Controller design

In MPC and DMPC, a model of the plant is used to determine the future values of the system state. Unfortunately, this calculation cannot be performed as the future values of the ground acceleration $\ddot{x}_g$ are unknown. In this paper we use a model to estimate the future values of the ground acceleration. The model is obtained from the Tanai-Tajimi

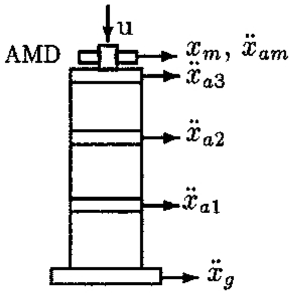

Fig. 1. Schematic of the three-story building

spectral description given in (14) with the following state space realization [Mei et al., 2001]:

$$\dot{\theta} = A_g\theta + B_g w \tag{22a}$$

$$\ddot{x}_g = C_g\theta \tag{22b}$$

where w is a white noise process and the system matrices are given by

$$A_g = \begin{bmatrix} 0 & 1 \\ -\omega_g^2 & -2\zeta_g\omega_g \end{bmatrix} \qquad B_g = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{23a}$$

$$C_g = \begin{bmatrix} -\omega_g^2 & -2\zeta_g\omega_g \end{bmatrix} \sqrt{S_0} \tag{23b}$$

We now define the model that will be used for design as the evaluation model (13) augmented with the ground acceleration model (22) as follows

$$\dot{\eta}_a = A_a\eta_a + E_a w + B_a u \tag{24a}$$

$$y = C_{ya}\eta_a + D_{ya}u + v \tag{24b}$$

where

$$A_a = \begin{bmatrix} A & EC_g \\ 0 & A_g \end{bmatrix} \quad E_a = \begin{bmatrix} 0 \\ B_g \end{bmatrix} \quad B_a = \begin{bmatrix} B \\ 0 \end{bmatrix} \tag{25a}$$

$$C_{ya} = [\, C_y \;\; FC_g \,] \quad D_{ya} = D_y \tag{25b}$$

Because only certain outputs are measured, the state of the system is estimated with an observer. In this paper we implement a Kalman filter designed for the model in (24) where y is measured and u is known.

The MPC controller is designed with a discretization of the nominal model (24). In the cost we have chosen to penalize the energy associated with the acceleration of each floor. That is,

$$J^N_{\mathrm{MPC}} = \|\eta_a(k+N)\|^2_{P_M} + \sum_{j=k}^{k+N-1} \|y(j)\|^2_{Q_M} + \|u(j)\|^2_R \tag{26}$$

where $Q_M = \mathrm{diag}(1, 1, 1, 0, 0)$ and the terminal weight P_M is selected to represent an infinite horizon cost with control input chosen to be $u(j) = -Kx(j)$ for $j > k + N - 1$. The matrix K is the solution of an unconstrained infinite horizon optimal control problem with cost $\sum_{j=0}^{\infty} \|y(j)\|^2_{\hat{Q}} + \|u(j)\|^2_{\hat{R}}$. That is, K and P_M are the solution to

$$P_M = A_a^T P_M A_a - K^T(\hat{R} + B_a^T P_M B_a)K + C_{ya}^T \hat{Q} C_{ya} \tag{27a}$$

$$K = (D_{ya}^T \hat{Q} D_{ya} + \hat{R} + B_a^T P_M B_a)^{-1} (B_a^T P_M A_a + D_{ya}^T \hat{Q} C_{ya}) \tag{27b}$$

where $\hat{Q} = Q_M$ and $\hat{R} = 50$. With this linear controller the closed loop system satisfies the constraints (19)-(21) for both earthquake records. The horizon length is $N = 30$. No constraints for the terminal set were used. The constraints (19)-(21) are enforced over the prediction horizon.

Similarly, DMPC is designed with the discretization of the nominal model (24) augmented with the corresponding sensitivity equation

$$p(j+1) = \frac{dA_a}{d\zeta_1}\eta_a(j) + A_a p(j) \tag{28}$$

as it was explained in Section 2. The initial condition p_k^m, for the trajectory sensitivity in (12), is chosen to be zero. The weighting matrices Q_M and R, the horizon length N, and the constraints are the ones used in MPC.

The sensitivity weight Q_s is designed to penalize the trajectory sensitivity corresponding to the acceleration of each floor, that is, $Q_s = 10^{-6} q_s C_{ya}^T Q_M C_{ya}$. Three DMPC controllers are tested. DMPC1 is designed with $q_s = 1$, DMPC2 with $q_s = 3$ and DMPC3 with $q_s = 5$. The terminal weight P_D, for each controller, is obtained following the same approach taken in MPC but using, instead, the augmented system matrices and the weighting matrix $\hat{Q} = Q_D = \mathrm{diag}\{Q_M, Q_s\}$.

All the controllers are implemented with a sampling time equal to 0.001s and a computation delay of 200μs as specified in Spencer et al. [1997]. The optimization problems were solved with the optimization toolbox from Matlab.

3.4 Results

MPC and DMPC were tested on nine plants with damping ratio ζ_1 ranging from -80% to 80% of its nominal value. To compare the robustness of the controllers we calculate the mean and the standard deviation as percentage of the mean, for each evaluation criteria. Tables 1 and 2 show the numerical results.

DMPC penalizes the energy of the sensitivity of the floor accelerations to changes in the damping ratio. Thus, one would expect the standard deviation associated with the floor accelerations (criteria J_2 and J_7) to decrease as q_s increases. Notice from the Table 2 that the standard deviations for J_2 and J_7 follow this pattern. DMPC is able to decrease the standard deviation, compared to MPC, from 18% up to 9.3% for J_2 and from 3.7% to 2.5% for J_7. This effect can also be seen in Figure 2 and Figure 3, which show the variation of J_2 and J_7 with damping ratio for the four controllers. In these plots, the computed evaluation criteria is shown with markers while the connecting lines are interpolated values. The criterion J_2 exhibits the well-known tradeoff between performance and robustness; while J_2 increases with q_s, its variation with damping ratio decreases with q_s.

The evaluation criteria J_1 and J_6 are associated with the interstory drift. The sensitivity of those outputs were not penalized in DMPC. Yet, the standard deviation using DMPC, compared to that using MPC, has decreased from 44.8% up to 31.5% for J_1 and remained practically unchanged for J_6.

4. CONCLUSIONS

We have presented a method to desensitize the performance of MPC from model inaccuracies. The method is

	J_1	J_2	J_6	J_7	u_{max}[V]	$x_{m,max}$[cm]	$\ddot{x}_{am,max}$[g]
MPC	0.1687	0.1977	0.3372	0.6533	2.0698	7.3004	5.9993
DMPC1	0.1656	0.1951	0.3381	0.6400	2.0677	7.3294	5.9815
DMPC2	0.1681	0.2133	0.3397	0.6066	2.1447	7.6933	5.9801
DMPC3	0.1774	0.2385	0.3407	0.5903	2.1784	7.8161	5.9837

Table 1. Evaluation criteria comparison: mean.

	J_1 [%]	J_2 [%]	J_6 [%]	J_7 [%]	u_{max} [%]	$x_{m,max}$ [%]	$\ddot{x}_{am,max}$ [%]
MPC	44.8	18.0	0.6	3.7	0.6	0.5	0.7
DMPC1	44.4	13.8	0.6	3.7	0.5	0.5	0.8
DMPC2	38.8	8.7	0.6	3.7	0.7	0.7	0.9
DMPC3	31.5	9.3	0.6	2.5	0.7	0.7	1.0

Table 2. Evaluation criteria comparison: standard deviation as percentage of the mean.

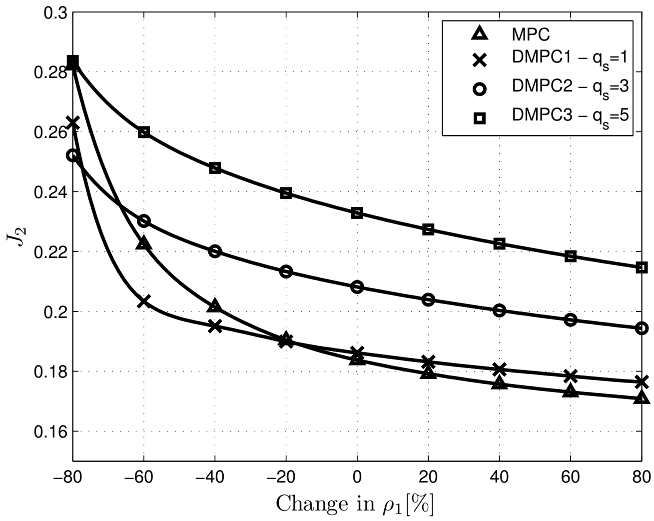

Fig. 2. Changes in criterion J_2 with the damping ratio ζ_1.

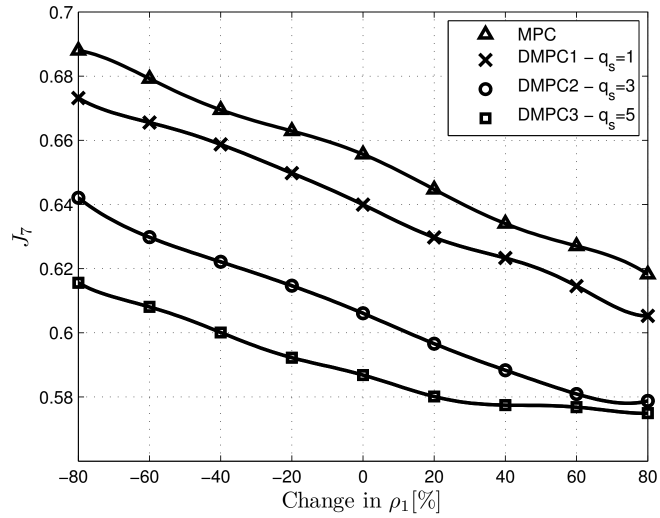

Fig. 3. Changes in criterion J_7 with the damping ratio ζ_1.

based on the simple idea of incorporating the sensitivity of the state trajectory to changes in model parameters in the MPC cost function. The resulting method, which we introduced as DMPC, retains the on-line computational simplicity of the conventional MPC problem.

We have demonstrated by example the effectiveness of this idea using the benchmark problem from Spencer et al. [1997]. Numerical results show that DMPC can be less sensitive against model inaccuracies than MPC. This suggests that the use of the trajectory sensitivity in the MPC cost has potential benefits that have not yet been exploited and therefore it is a research direction worth exploring.

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