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Analysis of networked estimation under contention-based medium access

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Abstract: We investigate the scalability of networked estimation under contention-based medium access. In our set-up, the state of a number of identical first-order linear plants are measured and transmitted over a shared medium. Each sensor transmits its readings to a supervisor node that maintains a continuous-time state estimate for the associated plant. When the medium access delay exceeds the sampling interval, measurements are discarded and replaced by more recent ones. Our analysis of the shared channel determines the probability of packet loss as a function of the sampling interval and the number of contending nodes. We compute the estimation distortion with periodically generated samples as a function of the packet loss rate and sampling interval, and derive a condition for stable estimator performance. We investigate the scalability limits of this stability as a function of the number of nodes. When stable estimation is possible, we provide a procedure that computes the sampling rate that minimizes the average estimation distortion. We reproduce the analysis of estimation performance when the sensors sample asynchronously according to independent Poisson counters.

1. INTRODUCTION

In networked control systems, constraints on the communication between sensors and supervising agents or controllers affect the system design. The communication constraints are sometimes reflected as a trade-off between reliable delivery of individual data packets and the input data load on the communication medium. Such a trade-off occurs when several sensors have to contend over a common wireless channel. When one or more nodes increase the rate of accessing the channel, the contention becomes more intense and the probability of packet collisions increases. On the other hand, for the estimation or control process, the objective is to have packets with measurements arrive at the destination nodes at as high a rate as possible. So, at very low rates of accessing the channel, the probability of collisions is minimized and the portion of packets delivered successfully is the best possible. At very high access rates, the large amount of contention makes the rate of successful delivery of packets very small. This report contains some simple calculations that shed some light on how to choose the rate at which sensors access the shared channel so as to obtain the best estimator performance.'

1.1 Related work

State estimation is an important component in most modern automation systems, with applications in monitoring, fault diagnosis and control. Driven by the strong interest in networked control systems, the problem of Kalman filtering under packet losses Sinopoli et al. [2004] and varying sampling rates Micheli and Jordan [2002] has received considerable attention. This work is related to the research on systems with uncertain observations in the 70's (*e.g.* Hadidi and Schwartz [1979]) but contains new insight into stability properties of the estimation error covariance. The dual problem of control of systems under packet losses has also received some attention, see *e.g.* Gupta et al. [2005] and the references therein.

Papers that shed light on the scalability of networked estimation by studying the interaction between networking and controller co-design are more scarce. Various contributions include Xiao et al. [2003], Adlakha et al. [2007], Branicky et al. [2003], but none of these includes the intricate relationships between channel access rate and the packet loss probability that occur under contention-based communications. Analytical performance studies of delay distributions for contention-based MAC schemes turn out to be non-trivial. Under the assumption of saturated sources, Bianchi [2000] developed effective analysis techniques for 802.11 access points. Sensor networking applications, however, typically do not operate under saturated traffic but with sporadic and correlated traffic. Extensions to this case have recently been proposed in Pollin et al. [2007], Stabellini and Proutiere [2007].

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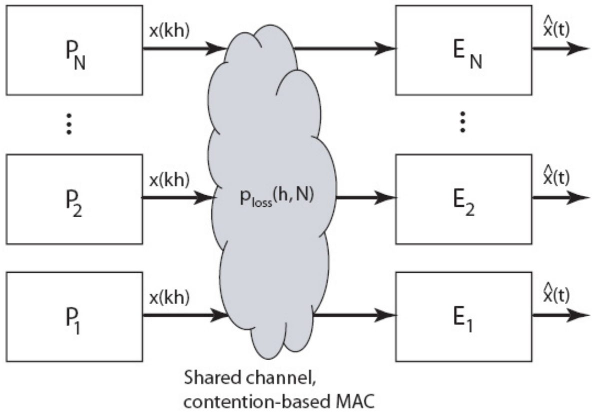

Fig. 1. The estimation problem setup: N identical plants estimated via samples transmitted over a shared channel. Samples could be lost because of contention.

2. ESTIMATION OF A LINEAR SYSTEM

On the time interval $[0, \infty)$, consider the scalar linear system:

$$dx_t = ax_t dt + dW_t, \tag{1}$$

with W_t being a standard one dimensional Wiener process independent of x_0. A sensor measures this process exactly. It samples the state waveform and transmits the samples as data packets over a medium shared with other nodes. Each data packet could be lost due to collisions as well as due to the inherent unreliability of the channel. Here, we focus exclusively on the effect of collisions.

We assume that the binary process of losses or successful deliveries of packets from a sensor node is an IID process. The probability of a packet being lost depends on the channel access rates of the nodes, and on the amounts of noise and fading in the channel. In most of the expressions that follow, we will suppress this dependence and use the plain symbol p for the packet loss probability.

3. PERIODIC SAMPLING

When the sensor samples the x-process periodically, we get a periodic stream for input to the shared channel. The stream of packets received at the supervisor on the other hand is not periodic. The sequence of times between arrivals of the packets is an IID process and each member of this sequence has a discrete geometric distribution.

Let the sampling period of sensors be h, and the sequence of times at which packets arrive at the estimator be:

$$\{R_0, R_1, R_2, \dots\},$$

with $R_0 = 0$. Let l_t denote the process that denotes the time of last reception of a packet:

$$l_t = \inf_i \{R_i \,|R_i \le t\}.$$

The periodic sampling at the sensor and the IID loss model for packets mean that $\forall i \ \ge\ 1$,

$$\mathbb{P}\left[R_{i+1} - R_i = nh\right] = (1-p)p^{n-1}, \quad \forall n \ \ge\ 1.$$

The average time-interval (denoted henceforth by the symbol $\bar{\delta R}$) between reception of packets is :

$$\mathbb{E}\left[R_{i+1} - R_i\right] = \sum_{n=1}^{\infty} (1-p)p^{n-1} nh \ = \ \frac{h}{1-p} \quad \forall i.$$

The least-squares estimate $\hat{x}_t$ at the supervisor is given by:

$$\hat{x}_t = x_{l_t} \times e^{a(t-l_t)}. \tag{2}$$

3.1 Quality of estimation with lossy samples

We will measure the *real-time throughput* for the estimation problem directly through the average squared distortion in the supervisor's estimate:

$$J_{\text{Estim}} \triangleq \limsup_{M\to\infty} \frac{1}{M} \int_0^M \mathbb{E}\left[(x_t - \hat{x}_t)^2\right] dt.$$

In order for the estimation distortion to be finite, we need the following condition to be satisfied:

$$2a < \frac{1}{h} \ln\left(\frac{1}{p}\right) \tag{3}$$

When the above condition holds, the distortion can be computed to be:

$$\begin{aligned} J_{\text{Estim}} &= \frac{1}{\bar{\delta R}} \sum_{n=1}^{\infty} (1-p)p^{n-1} \left\{ \frac{e^{2anh}-1}{4a^2} - \frac{nh}{2a} \right\}, \\ &= \frac{1}{4a^2 h} \frac{(1-p)^2 p e^{2ah}}{p\left(1-pe^{2ah}\right)} - \frac{1-p}{4a^2 h} - \frac{1}{2a}. \end{aligned}$$

Note that in the above analysis, we have assumed that if a sample is transmitted, it is transmitted practically at the sampling time. This is not quite true as will be clear in the next section. Nevertheless, in adopting this position, the resulting loss of accuracy is tolerable because the intervals between successful receptions of packets at the estimator are dominated by the multiples of sampling periods over which no successful transmission happens; the fraction of a sampling period that is needed in addition can be ignored.

4. PACKET LOSS RATE AND THE SAMPLING RATE

4.1 Description of the MAC protocol

Each sample generated by sampling the sensor is transmitted over the common channel using slotted ALOHA as the MAC protocol. The competition for accessing the channel is from similar nodes. More precisely, there are assumed to be exactly $N-1$ other nodes each measuring an identical linear plant and sampling at the same rate. The N different plants are statistically identical, but their driving noise processes are mutually independent. In addition to sampling at identical rates, the nodes also sample at exactly the same times. Such an assumption allows us to compute the packet loss rate under slotted ALOHA. The rate thus computed is then an upper bound on the loss rate one would get as a result of staggered transmissions. The main point of this assumption is to have a reasonable yet tractable model for the contention phenomenon. Note that, in our model, a packet transmitted in a slot can be lost only due to collision with a packet transmitted by another node during the same slot.

4.2 Computing the packet loss rate

We will now proceed with the mission of finding the dependence of the packet loss rate p on the sampling period h. We assume that the nodes can detect collisions of data packets either by some collision sensing mechanism or by receiving acknowledgements upon successful transmissions. We denote by L, the duration which includes the length of one data packet and some addition time needed for any acknowledgement packets. In the numerical calculations which we present later, we have assumed the nominal value of 1ms for L. Then, over the interval between two successive sampling instants at sensor nodes, the number of slots available for attempted transmissions equals

$$S(h) = \left\lfloor \frac{h}{L} \right\rfloor.$$

We number the slots from 0 up to $S(h) - 1$. Under slotted ALOHA, each of the nodes employs an identical randomized strategy for packet transmission during every slot until either the number of slots runs out or the packet has been successfully transmitted. During each slot, if the packet at a node is still to be transmitted, then, with probability q_{tr} the node attempts transmission. The event that one node decides to attempt transmission is independent of a similar decision at any other node or whether any other node has already successfully transmitted its packet.

If at the beginning of any slot, there are n nodes that are yet to successfully transmit their packets, they are the only potential contenders for the channel. The probability that a successful transmission happens during this slot is exactly equal to

$$n q_{tr} (1 - q_{tr})^{n-1}.$$

Let us use the symbol a_n to denote this probability. For $1 \le k \le N$, the number of slots required for exactly k of the nodes to have successfully transmitted their packets is the sum of independent geometric random variables:

$$\sum_{i=1}^{k} 1 + \tau_i,$$

where, τ_i is a geometric random variable taking values on the set of non-negative integers with its PMF given by:

$$\mathbb{P}[\tau_i = j] = a_{N-i+1} (1 - a_{N-i+1})^j \quad \forall j \ \ge 0.$$

Let π_k denote the probability that during a sampling period, exactly k nodes succeeded in transmitting their packets. We have $\forall k$ such that $1 \le k \le N-1$,

$$\pi_k = \mathbb{P}\left[\left(\sum_{i=1}^{k} \tau_i\right) \le S(h) - k\right] - \mathbb{P}\left[\left(\sum_{i=1}^{k+1} \tau_i\right) \le S(h) - k - 1\right]. \quad (4)$$

We have appropriate formulas for the special cases when $k = 0, 1, N$:

$$\begin{aligned}
\pi_0 &= \mathbb{P}[\tau_1 > S(h) - 1] \ = \ (1 - a_N)^{S(h)}, \\
\pi_1 &= \mathbb{P}[\tau_1 \le S(h) - 1] - \mathbb{P}[\tau_1 + \tau_2 > S(h) - 2], \\
&= 1 - \pi_0 - \mathbb{P}[\tau_1 + \tau_2 > S(h) - 2], \\
\pi_N &= \mathbb{P}\left[\left(\sum_{i=1}^{N} \tau_i\right) \le S(h) - N\right].
\end{aligned}$$

We also have the following formula for $2 \le k \le N-1$:

$$\begin{aligned}
&\mathbb{P}\left[\left(\sum_{i=1}^{k} \tau_i\right) \le S(h) - k\right] \\
&= \left(\prod_{m=0}^{k-1} a_{N-m}\right) \times \sum_{i=0}^{k-1} \frac{(1 - a_{N-i})^{k-1} - (1 - a_{N-i})^{S(h)}}{a_{N-i} \prod_{\substack{j \ne i \\ 0 \le j \le k-1}} (a_{N-j} - a_{N-i})}.
\end{aligned} \quad (5)$$

The last formula is obtained by first writing down the $\mathcal{Z}$-transform of the sum of independent Geometric random variables, then using a partial fraction expansion for the transform and finally inverting it back to obtain the desired PMF. Now we are ready to compute the average packet loss rate $p(h)$ for each of the nodes:

$$\begin{aligned}
p(h) &= \frac{1}{N} \mathbb{E}[\# \text{ of Nodes failing to transmit in } S(h) \text{ slots}] \\
&= \frac{1}{N} (N - \mathbb{E}[\# \text{ of Nodes succeeding}]) \\
&= \sum_{k=0}^{N} \frac{N-k}{N} \pi_k.
\end{aligned} \quad (6)$$

The upshot of the analysis in this section is the fact that we have an explicit expression for the average packet loss rate. The downside is that we now have to choose the best value of attempted transmit probability during each slot, namely, q_{tr} and this is computationally intensive. This choice depends on the number of nodes N and on the number of slots $S(h)$. Once this calculation is performed, we have the desired relationship between the average sampling rate and the average packet loss rate offered by slotted ALOHA with the matching transmit probability.

4.3 Heuristic choice for q_{tr}

There is a suboptimal approach to choosing the transmit probability q_{tr}. For $1 \le i \le N$, τ_i is a geometric random variable with the success probability parameter being a_{N-i+1}. All moments of a geometric random variable are simultaneously minimized by maximizing the success probability parameter. So, the random variable τ_i is minimized (each of its moments is simultaneously minimized) when a_{N-i+1} is maximized. The latter happens when $q_{tr} = \frac{1}{N-i+1}$. Thus to minimize τ_1, we should set $q_{tr} = \frac{1}{N}$ and similarly, to minimize τ_N, we should set $q_{tr} = 1$. Based on these considerations, we are only able to conclude that the optimal q_{tr} should lie between $\frac{1}{N}$ and 1.

We now outline a heuristic calculation that arrives at the assignment:

$$q_{tr} = \frac{2}{N+2}.$$

We will arrive at such an assignment after some approximations.

Firstly, we consider an approximate scheme to maximize the probability (π_N) that all N nodes succeed in transmitting their packets within a sampling period.

$$\pi_N = \mathbb{P}\left[\left(\sum_{i=1}^{N} \tau_i\right) \le S(h) - N\right]$$

The approximation consists in claiming that π_N is maximized by picking q_{tr} that minimizes

$$\mathbb{E}\left[\left(\sum_{i=1}^{N}\tau_i\right)-\left(S\left(h\right)-N\right)\right].$$

Here, $S(h)-N$ does not depend on q_{tr}. Using this observation and the fact that each τ_i has geometric distribution, we see that q_{tr} should minimize the expression:

$$\sum_{i=1}^{N}\frac{1-a_{N-i+1}}{a_{N-i+1}}.$$

Now we adopt a second approximation, one that lets us approach the calculation of of $p(h)$. To derive an (approximate) expression for $p(h)$, we use the form of the expression in equation (6) and claim that $p(h)$ is minimized when

$$\sum_{i=1}^{N}\left(N-i\right)\frac{1-a_{N-i+1}}{a_{N-i+1}}.$$

is minimized. The difference between the last two expressions is in the weighting factor $N-i$ present in the one for $p(h)$. This factor reflects the fact that τ_i affects the statistics of the timing for the i^{th} successful transmission, but also those of the remaining successful transmissions; we have used a linear factor $i-1$ to model this dependence. The last approximation step is to replace the weighting term $N-i$ with $N-i+1$, so that we can obtain a closed form expression.

Hence the suboptimal q_{tr} we are seeking is the one that minimizes:

$$\sum_{i=1}^{N}\left(N-i+1\right)\frac{1-a_{N-i+1}}{a_{N-i+1}}$$
$$=\sum_{i=1}^{N}\left(N-i+1\right)\frac{1-\left(N-i\right)q_{tr}\left(1-q_{tr}\right)^{N-i}}{\left(N-i+1\right)q_{tr}\left(1-q_{tr}\right)^{N-i}}.$$

Notice that this is the same as minimizing:

$$\sum_{i=1}^{N}\left(N-i+1\right)\frac{1}{\left(N-i+1\right)q_{tr}\left(1-q_{tr}\right)^{N-i}},$$
$$=\sum_{i=1}^{N}\frac{1}{q_{tr}\left(1-q_{tr}\right)^{N-i}},$$
$$=\frac{1-\left(1-q_{tr}\right)^{N+1}}{q_{tr}^2\left(1-q_{tr}\right)^{N}}.$$

The above minimization gives us:

$$q_{tr}^{*}\approx\frac{2}{N+2}.$$

Now, we are in a position to calculate the packet loss rate as a function of the sampling rate h. In figure 2, we have a plot of the packet loss rate when the number of nodes are $2, 5$ and 15.

5. SCALABILITY AND PERFORMANCE OPTIMIZATION

Given a number of sensor nodes, we first need to make sure that the estimation errors stay bounded. This of course rests on the dependence of $p(h)$ on h and this function is influenced by N. Under periodic sampling, the estimate is stable whenever inequality (3) is satisfied. In figure 3, we have, for different choices for the number of competing nodes, the maximum tolerated values for the parameter $2a$. When N is $2, 5$ or 15, the maximum possible values for a are respectively $0.325, 0.15$ and 0.46.

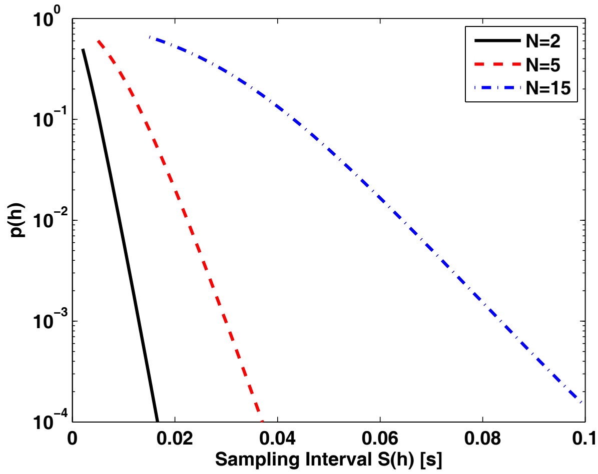

Fig. 2. The average packet loss rate as a function of the sampling rate.

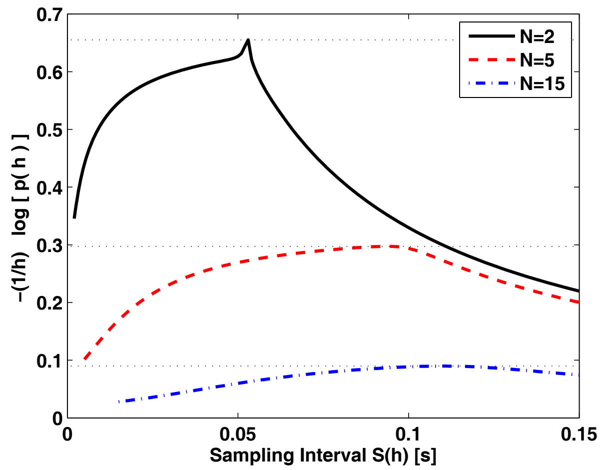

Fig. 3. The limits of bounded estimation distortion. We probe the maximum allowed value of a suggested by inequality (3). For each choice of N, the peak of the plotted curve represents the maximum tolerated value of $2a$.

If the number of nodes is not too large to cause instability, the task then is to find the choice of sampling period h which minimizes the average estimation distortion. We have a computational procedure to perform this optimization. The procedure is essentially to consider different possible reasonable values of h from roughly NL to $500L$ and to pick the one that minimizes the estimation distortion J. This is computationally feasible because it involves search over the range of a single scalar variable. The results of this optimization are shown in figures 4, 5.

6. SAMPLING ACCORDING TO A POISSON PROCESS

Sometimes, in explicit recognition of the fact that a TDMA-based scheme for accessing the channel is not

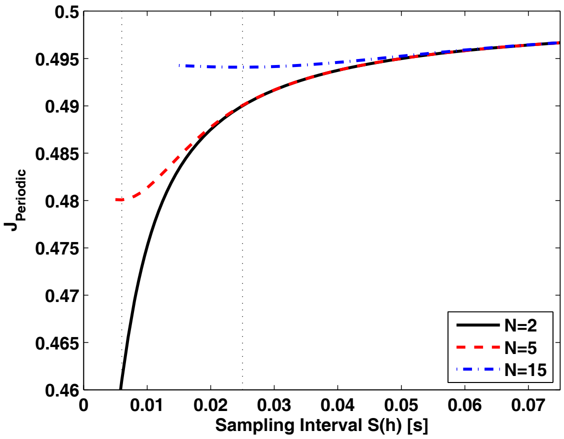

Fig. 4. Optimization of estimation distortion. For each choice of N, we plot the distortion under periodic sampling as a function of h. The above plots are for a stable system: $a = -1$.

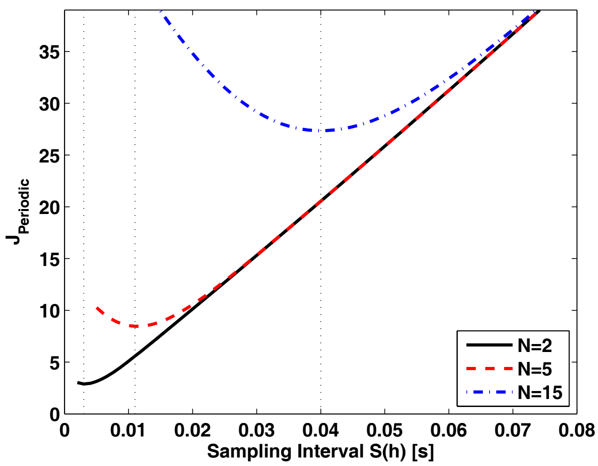

Fig. 5. Optimization of estimation distortion. For each choice of N, we plot the distortion under periodic sampling as a function of h. The above plots are for an unstable system: $a = 0.001$.

scalable, a non-periodic and randomized sampling scheme could be adopted. Sampling the measurement waveforms using independent Poisson counters at different sensors is such an alternative (Micheli and Jordan [2002]).

Let the rate of the Poisson counter be λ. Then, we have $\forall i \geq 1$:

$$\mathbb{P}\left[R_{i+1} - R_i > s\right] = \sum_{n=0}^{\infty} p^n e^{-\lambda s} \frac{(\lambda s)^n}{n!}.$$

This gives a common exponential probability density function for the inter-reception intervals. This PDF (as a function of $s \geq 0$) has the form:

$$\lambda\,(1-p)e^{-\lambda(1-p)s}.$$

Then, the average time-interval ($\delta\bar{R}$) between reception of packets is :

$$\delta\bar{R} = \frac{1}{\lambda\,(1-p)}.$$

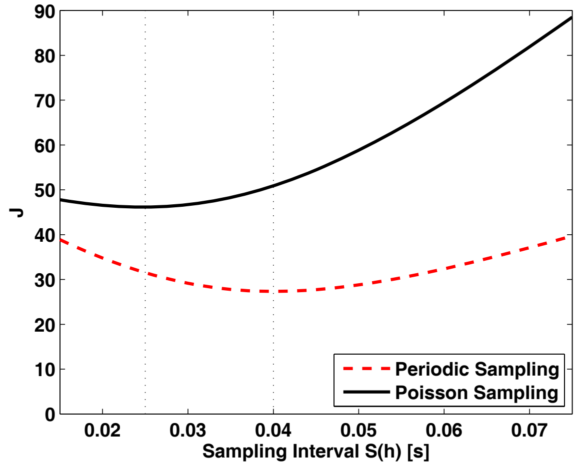

Fig. 6. Comparison of estimation distortion under periodic and Poisson sampling schemes. We have: $N = 15$ and $a = 0.001$.

The average squared distortion in the supervisor's estimate is:

$$J_{\text{Estim}} = \frac{1}{\delta\bar{R}} \int_0^{\infty} \left\{ \frac{e^{2at} - 1}{4a^2} - \frac{t}{2a} \right\} \lambda\,(1-p)e^{-\lambda(1-p)t} dt.$$

If $2a < \lambda(1-p)$, then the distortion is finite and can be computed to be:

$$J_{\text{Estim}} = \frac{1}{4a^2} \left\{ \frac{\lambda^2(1-p)^2}{\lambda(1-p) - 2a} - \lambda(1-p) \right\} - \frac{1}{2a}.$$

When $2a \geq \lambda(1-p)$, the sampling rate is insufficient and the average distortion is unbounded.

6.1 Performance comparison between the two sampling schemes

Given a particular setting, it can sometimes be interesting to compare the performance of the two presented sampling schemes. For this comparison to be fair, the inter-sampling times must be the same. This means letting $\lambda = 1/h$, which gives the same average time between sampling attempts and also the same average time between reception of packets for both sampling schemes. If further, the loss probability, p, is assumed to be either fixed or only dependent on h and the number of sensors, N, the two schemes have the same loss probability in this setting. We will now prove the superiority of periodic sampling which is suggested by figure 6. However, we should bear in mind that TDMA style periodic sampling has the burden of clock synchronization in practice.

Lemma 1. *Under these conditions, periodic sampling will always outperform Poisson sampling, i.e.* $J_{Estim}^{Poisson} > J_{Estim}^{Periodic}$ *will hold for all feasible values of* a, h, *and* p.

Proof. To begin with, the feasible set for the comparison must be determined. The sets for the two sampling schemes are:

$$\text{Periodic sampling:} \quad \left\{(a,h,p) : 2a < -\frac{1}{h}\ln p,\ p \in [0,1)\right\}$$

$$\text{Poisson sampling:} \quad \left\{(a,h,p) : 2a < \frac{1}{h}(1-p),\ p \in [0,1)\right\}$$

Since we have

$$-\ln p > (1-p) \text{ for } p \in [0,1)$$

the feasible set will be determined by:

$$D = \{(a,h,p) : 2ah < 1-p,\ p \in [0,1),\ 2ah \neq 0\}$$

The requirement $2ah \neq 0$ comes from the fact that a step size of zero is not allowed.

Now consider the function:

$$\begin{aligned} J_{Estim}^{Diff} &= J_{Estim}^{Poisson} - J_{Estim}^{Periodic} = \\ &= \frac{1}{4a^2}\left\{\frac{\frac{1}{h^2}(1-p)^2}{\frac{1}{h}(1-p)-2a} - \frac{1}{h}(1-p)\right\} - \frac{1}{2a} - \\ &- \left\{\frac{1}{4a^2h}\frac{(1-p)^2e^{2ah}}{1-pe^{2ah}} - \frac{1-p}{4a^2h} - \frac{1}{2a}\right\} = \\ &= \frac{1}{4a^2h}\left\{\frac{(1-p)^2}{1-p-2ah} - \frac{(1-p)^2e^{2ah}}{1-pe^{2ah}}\right\} = \\ &= \frac{(1-p)^2}{4a^2h}\frac{1+(2ah-1)e^{2ah}}{1-p-2ah+e^{2ah}(2ahp+p^2-p)} \end{aligned}$$

Since $h > 0$, the first part of the expression is positive. Let $x = 2ah$ and $y = p$, the feasible set now becomes:

$$D' = \{(x,y) : x < 1-y,\ y \in [0,1),\ x \neq 0\}$$

What we now need to show is that:

$$\frac{1+(x-1)e^x}{1-y-x+e^x(xy+y^2-y)} > 0,\ (x,y) \in D'$$

This will for example hold if both the numerator and the denominator are positive.

Define the following functions:

$$\begin{aligned} f(x) &= 1+(x-1)e^x \\ g(x,y) &= 1-y-x+e^x(xy+y^2-y) \end{aligned}$$

To show that $f(x) > 0,\ (x,y) \in D'$, we use the derivative $f'(x) = xe^x$ and calculate $f'(x) = 0$. We have $f'(x) = 0$ for $x = 0$ and $x \to -\infty$. For $x = 0$, we have $f(x) = 0$ and $f''(x) = 1 > 0$, meaning that it is a local minimum for $f(x)$. Since $\lim_{x\to-\infty} f(x) = 1$, $x = 0$ is a global minimum for $f(x)$. Thus, we have shown that $f(x) > 0$ for $(x,y) \in D'$ (since $x = 0 \notin D'$).

For $g(x,y)$, rewrite the function as:

$$g(x,y) = (1-x-y)(1-ye^x)$$

and use that:

$$x < 1-y \iff 1-x-y > 0$$

This means that we only have to show that $1-ye^x > 0$ for $(x,y) \in D'$. In D', we have $1-ye^x > 1-ye^{1-y} = \gamma(y)$. Since $\gamma(0) = 1$, $\gamma(1) = 0$, and $\gamma'(y) = -(1-y)e^{1-y} < 0$ for $y \in [0,1)$, it holds that $\gamma(y) > 0,\ (x,y) \in D'$ which gives $g(x,y) > 0,\ (x,y) \in D'$. This gives $J_{Estim}^{Diff} > 0$ for $(a,h,p) \in D$, which concludes the proof.

Note that the relation $J_{Estim}^{Poisson} > J_{Estim}^{Periodic}$ also will hold if the packet drop probability is larger for Poisson sampling since $J_{Estim}^{Poisson}$ is an increasing function in p. $\square$

7. CONCLUSIONS

We have, through explicit calculations, suggested a way in which the rate of accessing a shared channel can be chosen mindful of collisions. Our analysis considers how the sampling interval affects packet loss rate in a contention-based medium access protocol, and the combined effect of sampling and packet loss on the achievable estimation error variance. We have also investigated how the achievable performance depends on the time constants of the processes and the number of contending sensors.

The main weakness of this work is the conservative nature of the model for collisions. We have assumed a synchronized sampling scheme and worked out the performance of slotted ALOHA. This provides only a qualitative picture of the contention situation in a network with no good synchronization. The actual intensity of contention under TDMA style sampling with staggered transmissions will be lower than our analysis suggests.

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