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Self-Repairing and Adaptive Tracking for MIMO Systems with Sensor Failures

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Abstract: This paper presents a new self-repairing control system (SRCS) for unknown stable multiple-input-multiple-output (MIMO) plants with sensor failures. The proposed method can automatically switch from the faulty sensors to the healthy ones if the sensor failures occur. Only the artificial test signals and the integrators are utilized to detect the sensor failures. Hence, the SRCS requires no mathematical model of the plant. Furthermore, the adaptive λ-tracker is introduced not only to cope with uncertainties in parameters but also to eliminate the bias effects of the injected test signals.

1. INTRODUCTION

Sensor failure accommodation is one of the most difficult problems in fault-tolerant control. A wrong feedback signal from a faulty sensor sometimes makes the control system unstable even if the controlled plant is stable. In addition, if the faultily measured output is stuck on just a desired reference input in the steady state, then it is extremely difficult to know whether the sensor failure occurs or not.

Fundamentally, to recover from the effects of the sensor failures, all the failed sensors have to be exactly detected and repaired. However, most existing fault detectors have been based on observers, multiple models and accurate mathematical models of the plants (e.g. R. Isermann, R. Schwarz, and S. Stölzl [2002], R. Isermann [1997] and P. M. Frank [1990]). Hence, because their structures depend on the structures of the plants, they become excessively complex for the plants with large orders.

As a remedy, this paper presents a new simple self-repairing control system (SRCS) for unknown stable multiple-input-multiple-output (MIMO) plants with sensor failures, which can automatically detect the sensor failures and switch from the failed sensors to the healthy ones. The fault detector in the SRCS exploits only the artificial test signals and the integrators. The test signals are well designed so that the outputs of the integrators grow "large" to hit the thresholds if the measured outputs of the plant are stuck due to the failures. Thus, no mathematical model of the plant is required to detect the failure, and so one can construct the simple fault detector whose structure does not depend on the order of the plant.

Unfortunately, injecting the test signal for fault detection might degrade the tracking performance. To cope with this problem, we introduce the adaptive λ-tracker with the switched feedback gain. The concept of the λ-tracking has been presented by A. Ilchmann and E. P. Ryan [1994] and D. E. Miller and E. J. Davison [1994]. The adaptive λ-tracker forces the tracking error to enter the prescribed ball with arbitrarily small radius λ in the presence of the external disturbance including the test signal.

Throughout this paper, $\mathbf{R}$, $\mathbf{R}^+$, $\mathbf{I}$ and $\mathbf{I}^+$ denote real numbers, non-negative real numbers, integers and non-negative integers, respectively. For each vector $\boldsymbol{v} \in \mathbf{R}^n$, its norm is defined by $\|\boldsymbol{v}\| \triangleq \left(\boldsymbol{v}^T\boldsymbol{v}\right)^{\frac{1}{2}}$, and for any function $\boldsymbol{v}(t) \in \mathbf{R}^n$, its ∞-norm is defined by

$$\|\boldsymbol{v}(t)\|_\infty \triangleq \sup_{s\in[t_0,\ t]} \|\boldsymbol{v}(s)\|.$$

2. PROBLEM STATEMENT

In this paper, we consider the $n \in \mathbf{I}^+$-th order controllable and observable linear time invariant MIMO plant:

$$\Sigma_P\ :\ \dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{w}(t) \qquad \boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) \tag{1}$$

where $\boldsymbol{y}(t) \in \mathbf{R}^m$ and $\boldsymbol{u}(t) \in \mathbf{R}^m$ are the actual output and the control input respectively. Furthermore, $y_i(t) \in \mathbf{R}$ and $u_i(t) \in \mathbf{R}$ are the $i \in \{1, 2, \cdots, m\}$-th elements of $\boldsymbol{y}(t)$ and $\boldsymbol{u}(t)$. The unknown disturbance $\boldsymbol{w}(t) \in \mathbf{R}^n$ is supposed to be bounded. Here, we assume that the plant Σ_P is stable, minimum-phase and its high-frequency gain is positive definite, i.e., $\boldsymbol{K} \triangleq \boldsymbol{CB} > 0$.

To measure each output $y_i(t)$, the two sensors are exploited as shown in Figure 1.

$$\hat{y}_i(t) = \sigma_i(t)y_i^1(t) + (1 - \sigma_i(t))\, y_i^2(t) \tag{2}$$

where $y_i^s(t) \in \mathbf{R},\ s \in \{1, 2\}$ is the output measured by the corresponding sensor $\sharp s$. If there is no sensor failure then we have $y_i^s(t) = y_i(t)$. Unfortunately, for each i, one of the i-th sensors fails in the following way:

$$y_i^{f_i}(t) = y_i(t_{Fi}), \quad t \geq t_{Fi} \tag{3}$$

where $f_i \in \{1, 2\}$ is the index corresponding to the failed sensor and $t_{Fi} \geq t_0$ is the failure time. Suppose that f_i and t_{Fi} are unknown.

The switching function $\sigma_i(t) \in \mathbf{R}^+$ in (2) takes values in a set $\{0, 1\}$. The switching logic for the switching function $\sigma_i(t)$ can be arbitrarily designed by the designers. If the

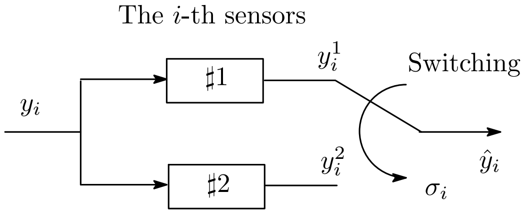

Fig. 1. The two sensors to measure the i-th output $y_i(t)$.

switching function $\sigma_i(t)$ is set as $\sigma_i(t) = 1$ then the first sensor is utilized. Otherwise, the second sensor is utilized.

The aim of this paper is to design the SRCS which can automatically switch from the faulty sensors to the healthy ones and attain tracking of the output $\boldsymbol{y}(t)$ to an arbitrary bounded reference input $\boldsymbol{r}(t) \in \mathbf{R}^m$ with bounded $\dot{\boldsymbol{r}}(t)$.

In order to explain the concept of the self-repairing control, we define the switching function matrix $\boldsymbol{S}(t) \in \mathbf{R}^{m\times m}$ by

$$\boldsymbol{S}(t) \triangleq \mathrm{diag}[\sigma_1(t),\ \sigma_2(t),\ \cdots,\ \sigma_m(t)]. \tag{4}$$

The switching function matrix $\boldsymbol{S}(t)$ takes matrices in the following set $\mathbb{M}$ of 2^m distinct diagonal matrices whose diagonal elements are either 1 or 0 (see Remark 1).

$$\mathbb{M} \triangleq \{\ \boldsymbol{M}_1,\ \boldsymbol{M}_2,\ \cdots,\ \boldsymbol{M}_L\ \},\quad L = 2^m. \tag{5}$$

Notice that the set $\mathbb{M}$ is known because the number m of the outputs is known.

With the switching function matrix $\boldsymbol{S}(t)$, the vector form of (2) can be represented as

$$\hat{\boldsymbol{y}}(t) = \boldsymbol{S}(t)\boldsymbol{y}_1(t) + (\boldsymbol{I}_m - \boldsymbol{S}(t))\,\boldsymbol{y}_2(t) \tag{6}$$

where $\hat{\boldsymbol{y}}(t) \in \mathbf{R}^m$ and $\boldsymbol{y}_s(t) \in \mathbf{R}^m, s \in \{1,2\}$ are given by

$$\hat{\boldsymbol{y}}(t) = [\hat{y}_1(t),\ \hat{y}_2(t),\ \cdots,\ \hat{y}_m(t)]^T \tag{7}$$

$$\boldsymbol{y}_s(t) = [y_1^s(t),\ y_2^s(t),\ \cdots,\ y_m^s(t)]^T. \tag{8}$$

Therefore, all the failed sensors are replaced if the switching function matrix $\boldsymbol{S}(t)$ can find an appropriate matrix $\boldsymbol{M}^* \in \mathbb{M}$ so that

$$\boldsymbol{y}(t) = \boldsymbol{M}^*\boldsymbol{y}_1(t) + (\boldsymbol{I}_m - \boldsymbol{M}^*)\,\boldsymbol{y}_2(t). \tag{9}$$

In other words, if $\boldsymbol{S}(t) = \boldsymbol{M}^*$ then the actual output $\boldsymbol{y}(t)$ can be successfully measured, i.e., $\hat{\boldsymbol{y}}(t) = \boldsymbol{y}(t)$. This is the key idea of the repairing control against sensor failures.

Remark 1. For example, we consider the case where $m = 2$. Then, the set $\mathbb{M}$ has the four elements.

$$\mathbb{M} = \left\{ \underbrace{\begin{bmatrix}1&0\\0&1\end{bmatrix}}_{\boldsymbol{M}_1},\ \underbrace{\begin{bmatrix}1&0\\0&0\end{bmatrix}}_{\boldsymbol{M}_2},\ \underbrace{\begin{bmatrix}0&0\\0&1\end{bmatrix}}_{\boldsymbol{M}_3},\ \underbrace{\begin{bmatrix}0&0\\0&0\end{bmatrix}}_{\boldsymbol{M}_4} \right\}.$$

If the first sensor for the first element $y_1(t)$ and the second sensor for the second element $y_2(t)$ fail then the matrix $\boldsymbol{M}^*$ to be found is given by $\boldsymbol{M}^* = \boldsymbol{M}_3$.

3. SELF-REPAIRING AND TRACKING

For the plant Σ_P, we construct the SRCS based on the adaptive PI controller as shown in Figure 2.

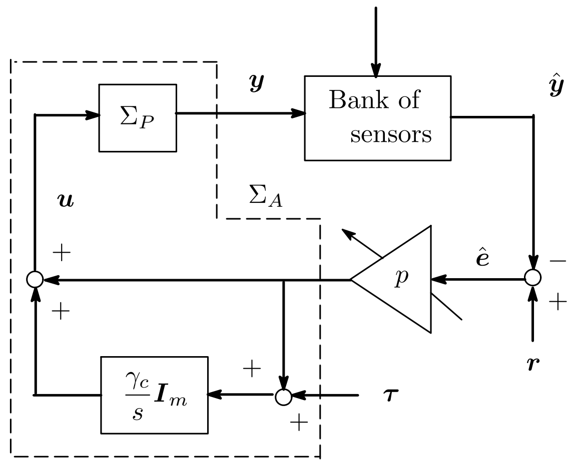

Fig. 2. The proposed SRCS with a PI control structure.

$$\boldsymbol{u}(t) = p(t)\hat{\boldsymbol{e}}(t) + \gamma_c\boldsymbol{v}(t),\quad \gamma_c > 0 \tag{10}$$

$$\dot{\boldsymbol{v}}(t) = p(t)\hat{\boldsymbol{e}}(t) + \boldsymbol{\tau}(t),\quad \boldsymbol{v}(t_0) = \boldsymbol{0} \tag{11}$$

where $\hat{\boldsymbol{e}}(t) \in \mathbf{R}^m$ is the tracking error between the measured output $\hat{\boldsymbol{y}}(t)$ and the reference input $\boldsymbol{r}(t)$, which is defined by

$$\hat{\boldsymbol{e}}(t) \triangleq \boldsymbol{r}(t) - \hat{\boldsymbol{y}}(t) \tag{12}$$

and $p(t) \in \mathbf{R}^+$ is the adaptive gain tuned by

$$p(t) = g_p(\gamma_p)^k,\quad t \in [t_k,\ t_{k+1}) \tag{13}$$

with any positive constants $g_p > 1$ and $\gamma_p > 1$. For every $k \in \mathbf{I}^+$ for which $t_k < \infty$, the switching time t_{k+1} is given by the supremum value of time t satisfying both the following inequalities:

$$\|\hat{\boldsymbol{e}}(t)\| < \lambda + (\gamma_\sigma)^k\pi(t_k)e^{-\frac{1}{(\gamma_\sigma)^k}(t-t_k)} \tag{14}$$

$$\|\boldsymbol{\theta}(t)\| < (\gamma_\sigma)^k\pi(t_k) \tag{15}$$

where $\gamma_\sigma > 1$ is an any constant and $\lambda \in \mathbf{R}^+$ is an arbitrarily small radius of the ball to which the tracking error $\hat{\boldsymbol{e}}(t)$ asymptotically enters. The signals $\boldsymbol{\theta}(t) \in \mathbf{R}^{2m}$ and $\pi(t) \in \mathbf{R}^+$ are defined by

$$\boldsymbol{\theta}(t) \triangleq \left[\ \hat{\boldsymbol{e}}(t)^T,\ \boldsymbol{v}(t)^T\ \right]^T \tag{16}$$

$$\pi(t) \triangleq p(t)\left(\|\boldsymbol{\theta}(t)\|_\infty + 1\right). \tag{17}$$

Furthermore, in (11), $\boldsymbol{\tau}(t) \in \mathbf{R}^m$ is the artificial test signal to find the failed sensors, and each element $\tau_i(t) \in \mathbf{R}$ is identically given by

$$\tau_i(t) = \tau(t) \triangleq \tau\left(\frac{1+(-1)^\kappa}{2}\right),\ t \in [T_\kappa,\ T_{\kappa+1}),$$
$$\kappa = 0, 1, \cdots \tag{18}$$

where $\tau \in \mathbf{R}$ is an any non-zero constant. For every $\kappa \in \mathbf{I}^+$, the switching time T_κ is given by

$$T_\kappa = t_0 + \frac{\kappa(\kappa+1)}{2}. \tag{19}$$

Notice that κ is quite different from k defined in (13).

Finally, the switching function matrix $\boldsymbol{S}(t)$ is given by

$$\boldsymbol{S}(t) = \boldsymbol{M}_l,\quad l = (k \bmod L) + 1,\quad t \in [t_k,\ t_{k+1}). \tag{20}$$

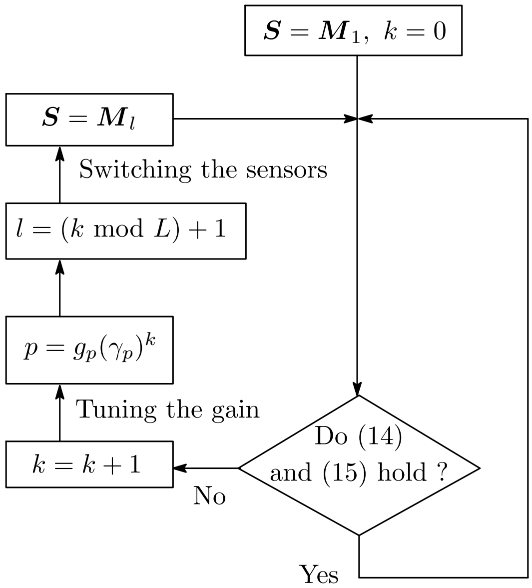

Fig. 3. The computational diagram of the switching logic.

The switching function matrix $\boldsymbol{S}(t)$ changes its values as

$$\cdots \to \boldsymbol{M}_1 \to \boldsymbol{M}_2 \to \cdots \to \boldsymbol{M}_L \to \boldsymbol{M}_1 \to \cdots .$$

The computational diagram of the switching logic is shown in Figure 3. The gain-tuning (13) and the sensor-switching (20) are performed simultaneously if one of the inequalities (14) and (15) does not hold. If the tracking error $\hat{\boldsymbol{e}}(t)$ does not enter the small ball with radius λ asymptotically, then the inequality (14) does not hold. Furthermore, whenever the SRCS becomes the open-loop system by selecting the failed sensors, the use of the test signal $\boldsymbol{\tau}(t)$ and the integrators (the I controllers) makes the signal $\boldsymbol{v}(t)$ "large" (see Lemma 1). Hence, the inequality (15) breaks if the failed sensors are selected ($\boldsymbol{S}(t) \neq \boldsymbol{M}^*$). Thus, the gain-tuning and the sensor-switching will be performed until the healthy sensors are all selected ($\boldsymbol{S}(t) = \boldsymbol{M}^*$) and the tracking error $\hat{\boldsymbol{e}}(t)$ asymptotically enters the prescribed small ball.

As mentioned above, to detect the failures, the I controllers are necessary. This is the reason why we exploit the adaptive λ-tracker with the PI control structure.

Remark 2. At the switching time t_{k+1}, at least one of the following equations holds.

$$\|\hat{\boldsymbol{e}}^-(t_{k+1})\| = \lambda + (\gamma_\sigma)^k \pi(t_k) e^{-\frac{1}{(\gamma_\sigma)^k}(t_{k+1}-t_k)}$$
$$\|\boldsymbol{\theta}^-(t_{k+1})\| = (\gamma_\sigma)^k \pi(t_k)$$

where $\hat{\boldsymbol{e}}^-(t_{k+1})$ and $\boldsymbol{\theta}^-(t_{k+1})$ are defined by

$$\hat{\boldsymbol{e}}^-(t_{k+1}) \triangleq \lim_{t \to t_{k+1}-0} \hat{\boldsymbol{e}}(t)$$
$$\boldsymbol{\theta}^-(t_{k+1}) \triangleq \lim_{t \to t_{k+1}-0} \boldsymbol{\theta}(t).$$

Remark 3. From the viewpoint of computational implementation, we cannot directly utilize (19), because $T_\kappa \to \infty$ as $\kappa \to \infty$. To avoid this problem, we find the switching time $T_{\kappa+1}$ with the bounded signals as follows: the switching time $T_{\kappa+1}$ is defined by the minimum time $t \in \mathbf{R}^+$ satisfying

$$\alpha(t) = \beta(\kappa)\alpha(T_\kappa), \quad t \geq T_\kappa$$

where $\alpha(t) \in \mathbf{R}^+$ and $\beta(\kappa) \in \mathbf{R}^+$ are generated by

$$\alpha(t) = e^{-(t-t_0)}, \quad \beta(\kappa) = e^{-(\kappa+1)}.$$

The above switching time T_κ is equivalent to (19).

4. MAIN RESULTS

4.1 Preliminaries

To derive the main results, we need the following lemmas.

Lemma 1. For arbitrary positive constants $\varepsilon \in \mathbf{R}^+$ and $t_S \geq t_0$, we consider a signal $v(t)$ generated by

$$\dot{v}(t) = \varepsilon + \tau(t), \quad v(t_S) = v_S \tag{21}$$

where the signal $\tau(t)$ is the same signal as the test signal given by (18) and (19). Then, we always have

$$\lim_{t\to\infty} |v(t)| = \infty. \tag{22}$$

Proof. From (21), it can be easily verified that

$$\begin{aligned} v(t) &= v_S + \int_{t_S}^{t} \varepsilon ds + \int_{t_0}^{t} \tau(s)ds - \int_{t_0}^{t_S} \tau(s)ds \\ &= \varepsilon t + \tilde{\tau}(t) + \varphi(t_S) \end{aligned} \tag{23}$$

where $\varphi(t_S) \triangleq v_S - \varepsilon t_S - \tilde{\tau}(t_S)$, and for every κ and $t \in [T_\kappa,\ T_{\kappa+1})$, $\tilde{\tau}(t) \in \mathbf{R}$ is successively given by

$$\begin{aligned} \tilde{\tau}(t) &= \int_{T_\kappa}^{t} \tau(s)ds + \int_{t_0}^{T_\kappa} \tau(s)ds \\ &= \tau(t)(t - T_\kappa) + \tilde{\tau}(T_\kappa). \end{aligned} \tag{24}$$

From (18) and (19), the signal $v(t)$ at the switching time $t = T_\kappa$ can be expressed as follows: for $\kappa = 2\iota$, $\forall \iota \in \mathbf{I}^+$,

$$v(T_\kappa) = (\tau + 2\varepsilon)\,\iota^2 + \varepsilon\iota + \varepsilon t_0 + \varphi(t_S) \tag{25}$$

and for $\kappa = 2\iota + 1$, $\forall \iota \in \mathbf{I}^+$,

$$\begin{aligned} v(T_\kappa) &= (\tau + 2\varepsilon)\,\iota^2 + (2\tau + 3\varepsilon)\,\iota \\ &\quad + \tau + \varepsilon(t_0 + 1) + \varphi(t_S). \end{aligned} \tag{26}$$

Clearly, in both (25) and (26), the coefficients of ι^2 and ι do not become zero simultaneously for any ε. Therefore, we have $|v(T_\kappa)| \to \infty$ as $\kappa \to \infty$.

Thus, we can conclude that Lemma 1 is true. ■

Let $\boldsymbol{G}(s)$ be the transfer function matrix of the plant Σ_P, and let $\boldsymbol{G}_a(s)$ denote the transfer function matrix of the augmented system Σ_A (the part indicated by the dashed line in Figure 2). Then we have

$$\boldsymbol{G}_a(s) = \boldsymbol{G}(s)\left(1 + \frac{\gamma_c}{s}\right). \tag{27}$$

Therefore, the augmented system Σ_A has the same properties as the plant Σ_P, that is, the augmented system is minimum-phase and its high frequency gain is the same as the plant Σ_P. Hence, we obtain the following result.

Lemma 2. For the augmented system Σ_A, there exists a non-singular matrix $\boldsymbol{T} \in \mathbf{R}^{(n_p+m)\times(n_p+m)}$ such that

$$\begin{bmatrix} \boldsymbol{y}(t) \\ \boldsymbol{z}(t) \end{bmatrix} = \boldsymbol{T} \begin{bmatrix} \boldsymbol{x}_p(t) \\ \boldsymbol{v}(t) \end{bmatrix} \tag{28}$$

and we have the following representation of the augmented system Σ_A.

$$\dot{\boldsymbol{y}}(t) = \boldsymbol{A}_y\boldsymbol{y}(t) + \boldsymbol{K}p(t)\hat{\boldsymbol{e}}(t) + \boldsymbol{C}_z\boldsymbol{z}(t) + \boldsymbol{w}_y(t)$$
$$\dot{\boldsymbol{z}}(t) = \boldsymbol{A}_z\boldsymbol{z}(t) + \boldsymbol{B}_z\boldsymbol{y}(t) + \boldsymbol{w}_z(t) \quad (29)$$

where $\boldsymbol{A}_z \in \mathbf{R}^{n_p\times n_p}$ is the stable matrix, and $\boldsymbol{w}_y(t) \in \mathbf{R}^m$ and $\boldsymbol{w}_z(t) \in \mathbf{R}^{n_p}$ are the bounded disturbances which include the disturbance $\boldsymbol{w}(t)$ and the test signal $\boldsymbol{\tau}(t)$.

Proof. This lemma can be directly derived from the result by A. Ilchmann and E. P. Ryan [1994] ■

Here, define the actual tracking error as

$$\boldsymbol{e}(t) \triangleq \boldsymbol{r}(t) - \boldsymbol{y}(t). \quad (30)$$

Furthermore, define the set $\mathbb{K}^* \subset \mathbf{I}^+$ as

$$\mathbb{K}^* \triangleq \{\ k \mid \boldsymbol{M}_l = \boldsymbol{M}^*,\ l = (k \bmod L) + 1\ \}. \quad (31)$$

From (20), for all defined number $k \in \mathbb{K}^*$ of switches, we have $\hat{\boldsymbol{e}}(t) = \boldsymbol{e}(t),\ t \in [t_k,\ t_{k+1})$.

Because $\boldsymbol{A}_z$ is the stable matrix, a positive definite matrix $\boldsymbol{P}_z \in \mathbf{R}^{n_p\times n_p}$ exists so that

$$\boldsymbol{A}_z^T\boldsymbol{P}_z + \boldsymbol{P}_z\boldsymbol{A}_z = -\boldsymbol{Q}_z \quad (32)$$

for an arbitrary positive definite matrix $\boldsymbol{Q}_z \in \mathbf{R}^{n_p\times n_p}$. Then, for a number $k \in \mathbb{K}^*$ of switches, we construct the following positive function $W(t) \in \mathbf{R}^+$ with the matrix $\boldsymbol{P}_z$ and a constant $\gamma_z \in \mathbf{R}^+$.

$$W(t) \triangleq \|\boldsymbol{e}(t)\|^2 + \gamma_z\boldsymbol{z}(t)^T\boldsymbol{P}_z\boldsymbol{z}(t), \quad t \in [t_k,\ t_{k+1}). \quad (33)$$

From (29), (30) and (32), the time derivative of $W(t)$ can be expressed as

$$\begin{aligned}\dot{W}(t) = &-2\boldsymbol{e}(t)^T\boldsymbol{K}p(t)\boldsymbol{e}(t) - 2\boldsymbol{e}(t)^T\boldsymbol{A}_y\boldsymbol{e}(t)\\ &-2\boldsymbol{e}(t)^T\boldsymbol{C}_z\boldsymbol{z}(t) + 2\boldsymbol{e}(t)^T\dot{\boldsymbol{r}}(t)\\ &-2\boldsymbol{e}(t)^T\boldsymbol{A}_y\boldsymbol{r}(t) - 2\boldsymbol{e}(t)^T\boldsymbol{w}_y(t)\\ &-\gamma_z\boldsymbol{z}(t)^T\boldsymbol{Q}_z\boldsymbol{z}(t) - 2\gamma_z\boldsymbol{z}(t)^T\boldsymbol{P}_z\boldsymbol{B}_z\boldsymbol{e}(t)\\ &+2\gamma_z\boldsymbol{z}(t)^T\boldsymbol{P}_z\boldsymbol{B}_z\boldsymbol{r}(t) + 2\gamma_z\boldsymbol{z}(t)^T\boldsymbol{P}_z\boldsymbol{w}_z(t). \quad (34)\end{aligned}$$

Recall that $p(t) = g_p(\gamma_p)^k,\ t \in [t_k,\ t_{k+1})$. By introducing a constant $\delta \in \mathbf{R}^+$, $\dot{W}(t)$ can be evaluated as

$$\begin{aligned}\dot{W}(t) \le &-\left(2\lambda_{\min}[\boldsymbol{K}]\ g_p(\gamma_p)^k - p\right)\|\boldsymbol{e}(t)\|^2 + \delta\beta_1(t)\\ &-\gamma_z\left(\lambda_{\min}[\boldsymbol{Q}_z] - 4\right)\|\boldsymbol{z}(t)\|^2 + \gamma_z\beta_2(t) \quad (35)\end{aligned}$$

where

$$p = 3\delta^{-1} + \|\boldsymbol{A}_y\| + \gamma_z^{-1}\|\boldsymbol{C}_z\|^2 + \gamma_z\|\boldsymbol{P}_z\boldsymbol{B}_z\|^2$$
$$\beta_1(t) = \|\dot{\boldsymbol{r}}(t)\| + \|\boldsymbol{A}_z\|^2\|\boldsymbol{r}(t)\|^2 + \|\boldsymbol{w}_y(t)\|^2$$
$$\beta_2(t) = \|\boldsymbol{P}_z\|^2\|\boldsymbol{w}_z(t)\|^2 + \|\boldsymbol{P}_z\boldsymbol{B}_z\|^2\|\boldsymbol{r}(t)\|^2.$$

Because $\dot{\boldsymbol{r}}(t)$, $\boldsymbol{r}(t)$, $\boldsymbol{w}_y(t)$ and $\boldsymbol{w}_z(t)$ are all bounded, there exist constants $\beta_1 \in \mathbf{R}^+$ and $\beta_2 \in \mathbf{R}^+$ such that $\beta_1(t) < \beta_1$ and $\beta_2(t) < \beta_2$.

Now, we choose the constants, γ_z, δ and the positive definite matrix $\boldsymbol{Q}_z$ so that

$$\alpha = \frac{(\lambda_{\min}[Q_z] - 4)}{\lambda_{\max}[P_z]} > 0 \quad (36)$$

$$\frac{\beta}{\alpha} = \lambda^2, \quad \beta = \delta\beta_1 + \gamma_z\beta_2. \quad (37)$$

Note that it is not necessary to know these constants when the controller is designed.

Trivially, if the number k of switches satisfies

$$2\lambda_{\min}[\boldsymbol{K}]\ g_p(\gamma_p)^k - p > \alpha \quad (38)$$

then we have

$$\dot{W}(t) \le -\alpha W(t) + \beta, \quad t \in [t_k,\ t_{k+1}) \quad (39)$$

which implies

$$W(t) \le \lambda^2 + W(t_k)e^{-\alpha(t-t_k)}, \quad t \in [t_k,\ t_{k+1}). \quad (40)$$

From this result, we can obtain the following lemma.

Lemma 3. If a defined number $k \in \mathbb{K}^*$ of switches satisfies (38) then for such $k \in \mathbb{K}^*$ and $t \in [t_k,\ t_{k+1})$

$$\|\boldsymbol{e}(t)\| \le \lambda + c_1\pi(t_k)e^{-\frac{\alpha}{2}(t-t_k)} \quad (41)$$
$$\|\boldsymbol{\theta}(t)\| \le c_2\pi(t_k) \quad (42)$$

where the positive constants $c_1, c_2 \in \mathbf{R}^+$ and $\alpha \in \mathbf{R}^+$ are independent of the number k.

Proof. Clearly, because the number k satisfies (38), we can verify that (40) holds.

From (28) and (33) it follows that

$$W(t) \le \|\boldsymbol{e}(t)\|^2 + \gamma_z\|\boldsymbol{P}_z\|\|\boldsymbol{T}\|\left(\|\boldsymbol{x}_p(t)\|^2 + \boldsymbol{v}(t)^2\right). \quad (43)$$

Because A_p is the stable matrix, we can see that

$$\|\boldsymbol{x}_p(t)\| \le c_p\left(p(t)\|\boldsymbol{e}(t)\| + \|\boldsymbol{v}(t)\|\right), \quad t \ge t_0 \quad (44)$$

where $c_p \in \mathbf{R}^+$ is an appropriately defined constant. Substituting (44) into (43) gives

$$W(t) \le c_1'\pi(t)^2, \quad t \in [t_k,\ t_{k+1}) \quad (45)$$

where $c_1' \in \mathbf{R}^+$ is a suitably defined constant.

Thus, from (40) and (45), we obtain

$$\|\boldsymbol{e}(t)\| \le \lambda + (c_1')^{\frac{1}{2}}\pi(t_k)e^{-\frac{\alpha}{2}(t-t_k)} \quad (46)$$

which yields (41).

On the other hand, from (45) there exists a constant $c_2' \in \mathbf{R}^+$ such that

$$\|\boldsymbol{\theta}(t)\|^2 \le c_2'\left(1 + W(t_k)\right), \quad t \in [t_k,\ t_{k+1}). \quad (47)$$

Taking $\pi(t) > 1$ into account, we have from (43)

$$\|\boldsymbol{\theta}(t)\|^2 \le 2(c_2)^2\pi(t_k)^2, \quad t \in [t_k,\ t_{k+1}) \quad (48)$$

with $c_2 = \left(c_2'(1 + c_1')\right)^{\frac{1}{2}}$. This implies that (42) holds. ■

4.2 Main results

The main results of this paper can be summarised in the following theorem.

Theorem 1. For the plant Σ_P, we construct the SRCS by (10)-(20). Then, the following three properties hold: **(P1)** the gain-tuning (13) and the sensor-switching (20) cease within a finite number of switches, **(P2)** at the last switching time, all the healthy sensors are successfully selected, i.e., $\boldsymbol{S}(t) = \boldsymbol{M}^*$, and **(P3)** the λ-tracking can be achieved, that is,

$$\lim_{t\to\infty}\|\boldsymbol{e}(t)\| \le \lambda. \quad (49)$$

Proof. First of all, we consider the case where there exists a finite number $k^* \in \mathbf{I}^+$ of switches such that the following inequality and (38) hold simultaneously.

$$(\gamma_\sigma)^{k^*} > \max\left[\frac{2}{\alpha},\ c_1,\ c_2\right]. \quad (50)$$

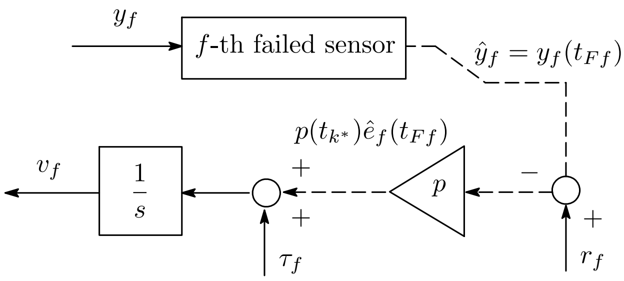

Fig. 4. The f-th failed sub-system with the stuck signals.

If $k^* \in \mathbb{K}^*$ then we can verify from Lemma 3 that for $t \in [t_{k^*},\ t_{k^*+1})$

$$\|\boldsymbol{e}(t)\| \le \lambda + c_1\pi(t_{k^*})e^{-\frac{\alpha}{2}(t-t_{k^*})} \quad (51)$$

$$\|\boldsymbol{\theta}(t)\| \le c_2\pi(t_{k^*}). \quad (52)$$

Here, we assume that the next switching time $t = t_{k^*+1}$ exists. Define constants $\rho_1(\xi_1) \in \mathbf{R}^+$ and $\rho_2(\xi_2) \in \mathbf{R}^+$ which are determined by positive constants $\xi_1 \in \mathbf{R}^+$ and $\xi_2 \in \mathbf{R}^+$ respectively.

$$\rho_1(\xi_1) \triangleq \|\boldsymbol{e}^-(t_{k^*+1})\| - \|\boldsymbol{e}(t_{k^*+1}-\xi_1)\| > 0 \quad (53)$$

$$\rho_2(\xi_2) \triangleq \|\boldsymbol{\theta}^-(t_{k^*+1})\| - \|\boldsymbol{\theta}(t_{k^*+1}-\xi_2)\| > 0 \quad (54)$$

where $\boldsymbol{e}^-(t_{k^*+1}) = \hat{\boldsymbol{e}}^-(t_{k^*+1})$.

From the left continuities of the signals $\boldsymbol{e}(t)$ and $\boldsymbol{\theta}(t)$, there exist constant ξ_1 and ξ_2 satisfying

$$\rho_1(\xi_1) < \left((\gamma_\sigma)^{k^*} - c_1 e^{\frac{\alpha}{2}\xi_1}\right)\pi(t_{k^*})e^{-\frac{\alpha}{2}(t_{k^*+1}-t_{k^*})} \quad (55)$$

$$\rho_2(\xi_2) < \left((\gamma_\sigma)^{k^*} - c_2\right)\pi(t_{k^*}). \quad (56)$$

For such constants ξ_1 and ξ_2, we obtain

$$\begin{aligned}\|\boldsymbol{e}^-(t_{k^*+1})\| &= \|\boldsymbol{e}(t_{k^*+1}-\xi_1)\| + \rho_1(\xi_1)\\ &\le \lambda + c_1\pi(t_{k^*})e^{-\frac{\alpha}{2}(t_{k^*+1}-\xi_1-t_{k^*})} + \rho_1(\xi_1)\\ &< \lambda + (\gamma_\sigma)^{k^*}\pi(t_{k^*})e^{-\frac{\alpha}{2}(t_{k^*+1}-t_{k^*})}\\ &< \lambda + (\gamma_\sigma)^{k^*}\pi(t_{k^*})e^{-\frac{1}{(\gamma_\sigma)^{k^*}}(t_{k^*+1}-t_{k^*})} \quad (57)\end{aligned}$$

$$\begin{aligned}\|\boldsymbol{\theta}^-(t_{k^*+1})\| &= \|\boldsymbol{\theta}(t_{k^*+1}-\xi_2)\| + \rho_2(\xi_2)\\ &\le c_2\pi(t_{k^*}) + \rho_2(\xi_2) < (\gamma_\sigma)^{k^*}\pi(t_{k^*}). \quad (58)\end{aligned}$$

This mean that the next switching does not occur, and contradicts the above-mentioned assumption. By contradiction, the next switching time $t = t_{k^*+1}$ does not exist if $k^* \in \mathbb{K}^*$.

If $k^* \notin \mathbb{K}^*$ then at least one failed sensor is selected at the switching time $t = t_{k^*}$. In this case, the SRCS has at least one open-loop sub-system as shown in Figure 4. From Lemma 1 and Figure 4 with setting $v(t) = v_f(t)$ and $\varepsilon = p(t_{k^*})\hat{e}_f(t_{Ff})$ where $f \in \{1, 2, \cdots, m\}$, we can verify that the signal $v_f(t)$ grows "large" and forces the signal $\boldsymbol{\theta}(t)$ to hit the switching threshold (15). So, the next switching time $t = t_{k^*+1}$ exists. Thus, the switching actions occur until all the failed sensors are replaced, i.e., $\boldsymbol{S}(t) = \boldsymbol{M}^*$. Finally, the switching function matrix $\boldsymbol{S}(t)$ can take the solution $\boldsymbol{M}^*$ within at most $k^* + L$ switches, and attain $\boldsymbol{S}(\infty) = \boldsymbol{S}(t_{s^*}) = \boldsymbol{M}^*,\ k^* < s^* \le k^* + L$.

On the other hand, in the case where the finite number k^* satisfying (38) and (50) does not exist, it is clear that the gain-tuning and the sensor-switching cease within a finite number (less than k^*) of switches and the healthy sensors are all selected at the last switching time.

Consequently, the gain-tuning and the sensor-switching cease within the finite number of switches, and the healthy sensor is selected at the last switching time. Thus, the properties (P1) and (P2) hold. Furthermore, after the last switching, from (14), the error $\boldsymbol{e}(t)$ enters the small ball with radius λ. The property (P3) holds.

Also clearly, from (13), (15) and (17), for any finite k, the gain $p(t_k)$ and the signal $\|\boldsymbol{\theta}(t_k)\|_\infty$ are bounded. Notice that the signals $p(t_k)$ and $\boldsymbol{\theta}(t_k)$ are bounded even if the failed sensors are selected. Therefore, the signals $p(t)$ and $\boldsymbol{\theta}(t)$ are bounded because the switching actions cease within the finite number of switches. Hence, taking into account that the plant Σ_P is stable, we can verify that all the signals are bounded.

Thus, we can conclude that Theorem 1 is true. ■

5. SIMULATION RESULTS

To show the effectiveness of the SRCS, we explore two simulations.

Consider the following 2I2O plant Σ_P:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t),\quad \boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t)$$

$$\boldsymbol{A} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix},\ \boldsymbol{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ \boldsymbol{C} = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}.$$

Here, we suppose that the first sensor for $y_1(t)$ and the second sensor for $y_2(t)$ fails at the time 10 sec, that is,

$$t_F = t_{F1} = t_{F2} = 10$$

$$\boldsymbol{M}^* = \boldsymbol{M}_3 \in \mathbb{M}$$

where $\boldsymbol{M}_3$ is given in Remark 1.

For this plant Σ_P, we construct the SRCS as follows.

The reference input $\boldsymbol{r}(t) = [r_1(t),\ r_2(t)]^T$ is given by

$$r_1(t) = \begin{cases} t/2 & (t \le 10) \\ 5 & (10 < t) \end{cases},\quad r_2(t) = \begin{cases} t/5 & (t \le 10) \\ 2 & (10 < t) \end{cases}.$$

The parameters to be designed are given by

$$\gamma_c = 1,\ g_p = 10,\ \gamma_p = 1.5$$

$$\gamma_\sigma = 1.01,\ \lambda = 0.05.$$

To confirm the effect of the test signal $\boldsymbol{\tau}(t)$, we explore the simulations for the following two cases.

Case 1: the test signal $\boldsymbol{\tau}(t)$ is injected, i.e., $\tau = 1$.

Case 2: the test signal $\boldsymbol{\tau}(t)$ is not injected, i.e., $\tau = 0$.

Figures 5 and 6 indicates the results for the above cases. Each figure indicates the actual tracking error $\boldsymbol{e}(t) = [e_1(t),\ e_2(t)]^T$ and the switching functions $\sigma_1(t)$ and $\sigma_2(t)$.

Figure 5 shows that it takes 0.7 sec to detect the sensor failures, and the SRCS can complete to replace the failed sensors, i.e., $\boldsymbol{S}(t) = \mathrm{diag}[\sigma_1(t),\ \sigma_2(t)] = \boldsymbol{M}^* = \mathrm{diag}[0,\ 1],\ t \ge 10.7$ sec. Furthermore, we can find that

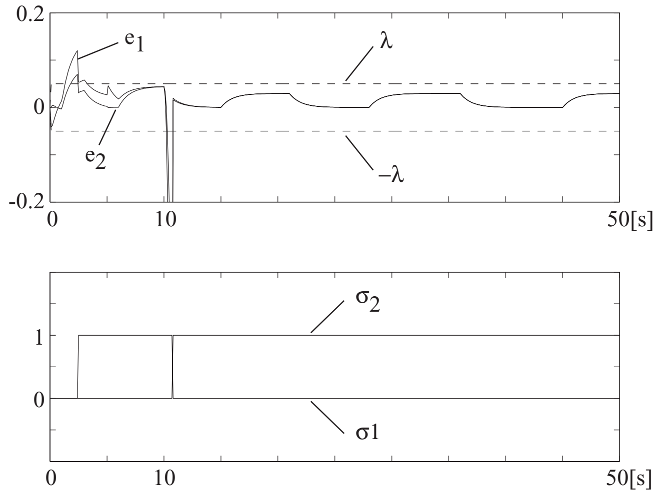

Fig. 5. Simulation results for Case 1.

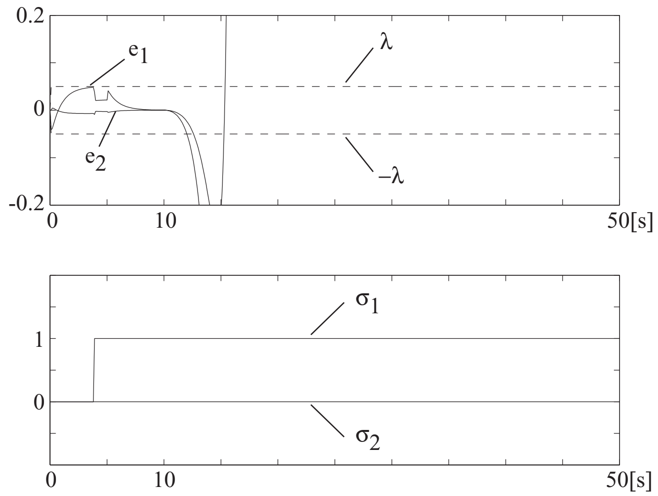

Fig. 6. Simulation results for Case 2.

the actual tracking error $\boldsymbol{e}(t)$ enters the ball with radius $\lambda = 0.05$. Of course, all the signals are bounded.

On the other hand, in Figure 6, the fault detection cannot be achieved with no use of the test signal $\boldsymbol{\tau}(t)$, and the λ tracking cannot be attained. Hence, comparing the two results, we can conclude that the test signal $\boldsymbol{\tau}(t)$ is necessary to detect the failures exactly.

6. CONCLUDING REMARKS

This paper has presented the novel SRCS for unknown MIMO plants with sensor failures, which can automatically replace the failed sensors with the healthy ones and achieve the λ-tracking. No *a priori* information about the plant is required to find the healthy sensors. This is one of the advantages of the SRCS.

In this paper, we have considered only the case where the measured outputs are stuck at the last measured values. Generally, the faulty sensor signal goes to some fixed value and then sticks at some time. Fortunately, this type of the failure also can be accommodated directly by the SRCS. However, there is the other type of the failure – variation in the sensor gain (e.g. H. Wang, Z. J. Huang and S. Daley [1997]). If variation is supposed to be piecewise constant then the SRCS can guarantee boundedness of all the signals.

The switched adaptive λ-tracker exploited in the SRCS is based on high-gain feedback. Hence, the SRCS also can be applied to the nonlinearly-perturbed plants Σ_P with the output-dependent disturbance $\boldsymbol{w}(t) = \boldsymbol{w}(t, \boldsymbol{y}(t))$ under the assumption that $\|\boldsymbol{w}(t, \boldsymbol{y}(t))\| \leq w_1 + w_2\|\boldsymbol{y}(t)\|$ for constants $w_1,\ w_2 \in \mathbf{R}^+$. Indeed, according to A. Ilchmann and E. P. Ryan [1994], the high-gain adaptive λ-tracker of another type (the continuously tuned adaptive controller) provides the robust stability and the λ-tracking for the above-mentioned nonlinearly-perturbed systems.

In the previous work by M. Takahashi [2003], for SISO systems, the SRCS has been developed against the actuator failures that requires no information about the plants. To detect the failures, we have exploited the unstable controller – by using the unstable controller, the signals in the SRCS grow up exponentially to hit the switching threshold when the the SRCS becomes the open-loop system due to the failures. Unfortunately, the exact fault detection is not guaranteed. By contrast, the proposed SRCS utilizes the test signal $\boldsymbol{\tau}(t)$ and the I controllers. From Lemma 1 and Theorem 1, it is theoretically proved that the healthy sensors are all selected.

The assumptions on the plants might be severe restrictions in practical cases. To cope with this problem, we can introduce "a parallel feed-forward compensator" to have the augmented systems satisfying the assumptions (see e.g. H. Kaufman, I. Bar-Kana and K. Sobel [1998]).

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