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Time-optimal trajectory generator under jerk constraints

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Abstract: Step response is widely used as the performance index of controlled systems. Thus, the ideal system would be one that has an output which approaches to the step signal quickly without error or over-shoot. However, if the output of an actual plant converges to the reference signal in a very short period, it can be dangerous to the surrounding environment as well as the operators. Furthermore, the smooth trajectories are required in some cases like vehicle or elevator systems. It is known that some limitations on the jerk of the plant would be necessary in such situations which require ride quality. In this paper, a non-linear feedforward filter for the step signal is proposed. The proposed filter, which has simple structure and requires less computational burden, produces time-optimal trajectories whose jerk guarantees the limitations that are given a priori. The effectiveness is substantiated with numerical examples.

Keywords: Trajectory planning, Optimal trajectory, Feedforward, Nonlinear filters, On-line control, Time-optimal control, Point-to-point control, Step function responses

1. INTRODUCTION

The Point-to-Point control is a fundamental control method, which is based on step responses. Therefore, step responses is one of the most important criteria for controlled systems. The ideal system would be one whose output converges to the given step reference as quick as possible without error or over-shoot. However, it is not preferred that an actual plant moves very speedily. Taking elevators or vehicles for instance, while they should work as fast as possible from the point of working efficiency, it is desired that their jerk are constrained for ride quality. Thus, a moderate trajectory would be necessary even for the Point-to-Point control.

While a generation of the optimal trajectory for any reference signal is difficult, trajectory generations for some limited situations such that the reference is given a priori, for instance, Point-to-Point control, iterative control and so on, are studied in both online and offline frameworks and their effectiveness are verified with real-time experiments. Offline frameworks, for example, a directly generation of trajectory (Tan, 2005) can be expected to achieve higher performance than that of online frameworks. However, it highly depends on the reference signal, therefore, even if the reference is linearly changed, it would require resynthesises. The computational burden or number of experiment to obtain the ideal trajectory is more than those of online frameworks in general. Moreover, calculated trajectories have to be memorised. On the other hand, online frameworks, such as feedforward filter approaches (Tsai, 2004; Jones, 2004) and reference governors (Bemporad, 1997; Vahidi, 2007) provide trajectory generators instead of trajectories themselves. The structure of the generator restricts the achievable performance, which may be worse than that of the offline frameworks in general. However, some of them can deal with any reference signals. Due to its real-time implementation, computational burden has to be considered in another sense of the offline frameworks. The reference governor is well researched theoretically, but its implementation is still developing because of its complexity.

For this issue, trajectory generators under velocity and acceleration constraints are proposed (Schlemmer, 2002; Kim, 2003; Panahi, 2006), while time-optimal trajectory generation itself is not a simple problem, which require a certain computational burden even in the offline frameworks (Agostini, 2003). In fact, the algorithms proposed in (Schlemmer, 2002; Kim, 2003) are not so easily implemented while both methods produce time-optimal trajectories. On the other hand, Panahi et al. (Panahi, 2006) proposed an online generator without multiplication, which requires less computational burden.

In this paper, a new online trajectory generation based on a non-linear feedforward filter, which has a simple structure and requires less computational burden, is proposed. The proposed feedforward filter produces near the time-optimal trajectories under jerk constraints from a given step reference. Then, the effectiveness of the proposed filter and its application with two-degree-of-freedom system (2DOFS) is verified with numerical examples.

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2. MAIN RESULTS

Consider the non-linear filter shown in Fig. 1 such that the third order system with a saturation operator S.

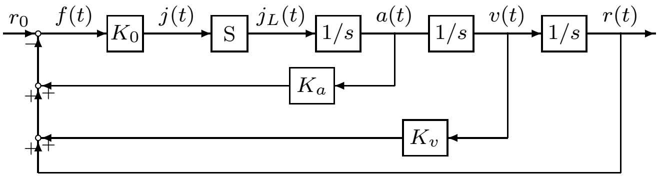

Fig. 1. Proposed feedforward filter A

The non-linear operator S is a static saturation function representing the limitation for jerk, which is given as follows.

$$\mathrm{S}(j) = \begin{cases} -J, \ j < -J \\ \ j\ , \ -J \le j \le J \\ \ J, \ J < j \end{cases} \tag{1}$$

Here, non-negative finite scalar J is the maximum jerk of the plant output or the limitation that should be given for some reason (for example, for ride quality), the reference $r_0 > 0$ is given a priori. When positive scalars K_a, K_v and large enough $K_0 > 0$ are given by

$$K_a := \frac{16r_0}{JT^2}, \ K_v := \frac{5r_0}{3JT}, \ T := \left(\frac{32r_0}{J}\right)^{\frac{1}{3}}, \tag{2}$$

$$K_0 \gg K_a, K_v, \tag{3}$$

then the jerk of the non-linear system's output; $r(t)$ in Fig. 1 guarantee the following constraints.

$$-J \le \frac{d^3}{dt^3} r(t) \le J, \tag{4}$$

Furthermore, when $K_0 \to +\infty$, $r(t)$ becomes the time-optimal trajectory, that means $r(t)$ converges to the reference r_0 at the minimum time under the jerk constraints.

The appropriate values of the gain parameters, K_a, K_v, K_0 are obtained as follows.

Consider simple physics of particles that a stationary particle moves and then stops at the given distance r_0. To move distance r_0 in the minimal time T under the jerk constraints, the particle must accelerate at the positive maximum jerk $+J$ until $T/4$, then it must move at the negative maximum jerk $-J$ until $3T/4$, after that it must move at the positive maximum jerk $+J$ again until T. By simple calculation, we have the following equations for the jerk $j_p(t)$, the acceleration $a_p(t)$, velocity $v_p(t)$, and the time optimal trajectory $r_p(t)$ of the particle.

$$j_p(t) = \begin{cases} +J, \ 0 \le t < \frac{T}{4}, \ \frac{3T}{4} < t \le T \\ -J, \ \frac{T}{4} < t < \frac{3T}{4} \end{cases} \tag{5}$$

$$a_p(t) = \int_0^t j_p(\tau)d\tau, \ v_p(t) = \int_0^t a_p(\tau)d\tau \tag{6}$$

$$r_p(t) = \int_0^t v_p(\tau)d\tau \tag{7}$$

From the terminal condition $r_p(T) = r_0$, we obtain

$$T = \left(\frac{32r_0}{J}\right)^{\frac{1}{3}}. \tag{8}$$

On the other hand, from the block diagram of Fig. 1, the following equations are satisfied.

$$j_L(t) = \mathrm{S}(j(t)), \ j(t) = K_0 f(t) \tag{9}$$

$$f(t) = (r_0 - r(t)) - K_v v(t) - K_a a(t) \tag{10}$$

Since $j_L(t)$ is the jerk of the output signal $r(t)$, the trajectory $r(t)$ is the time optimal, if

$$\begin{array}{ll} j(t) \ge J, & 0 \le t < T/4 \\ j(t) = 0, & t = T/4 \\ j(t) \le -J, & T/4 < t < 3T/4 \\ j(t) = 0, & t = 3T/4 \\ j(t) \ge J, & 3T/4 < t < T \\ j(t) = 0, & t = T \end{array}$$

hold. When letting $K_0 \to +\infty$, K_a and K_v must chosen such that $j(T/4) = 0$, $j(3T/4) = 0$, and $j(T) = 0$ hold. Substituting that $a(t) = a_p(t)$, $v(t) = v_p(t)$, $r(t) = r_p(t)$ hold, we have

$$a_p(T/4) = \frac{1}{4}JT, \ v_p(T/4) = \frac{1}{32}JT^2,$$
$$r_p(T/4) = \frac{r}{12}. \tag{11}$$

Furthermore, by the symmetry of the trajectory, we also have

$$a_p(3T/4) = -\frac{1}{4}JT, \ v_p(3T/4) = \frac{1}{32}JT^2,$$
$$r_p(3T/4) = \frac{11}{12}r. \tag{12}$$

Substituting them to the necessary conditions $j(T/4) = 0$ and $j(3T/4) = 0$, we obtain the candidates of K_a and K_v as follows,

$$K_a := \frac{16r_0}{JT^2}, \ K_v := \frac{5r_0}{3JT}. \tag{13}$$

Now, we analyse the behaviour of the filter in Fig. 1 with these K_a, K_v, and $K_0 \to +\infty$. Since $j_p(t)$, $v_p(t)$ and $r_p(t)$ are the time optimal (accelerated at the maximum jerk $+J$ until $t = T/4$),

$$a(t) \le a_p(t) = Jt < a_p(T/4) \tag{14}$$

$$v(t) \le v_p(t) = \frac{1}{2}Jt^2 < v_p(T/4) \tag{15}$$

$$r(t) \le r_p(t) = \frac{1}{6}Jt^3 < r_p(T/4) \tag{16}$$

$$\forall t < T/4 \tag{17}$$

hold. Then

$$f(t) > r_0 - r_p(T/4) - K_v v_p(T/4) - K_a a_p(T/4)$$
$$= 0 \tag{18}$$

is obtained. Since K_0 is large enough ($K_0 \gg 0$), we have $j(t) > J > 0, \forall t < T/4$. This means that $r(t)$ is accelerated at the maximum jerk $+J$, therefore $a(t) = a_p(t)$, $v(t) = v_p(t)$ and $r_L(t) = r_p(t)$ hold for $t < T/4$. Furthermore,

$$\lim_{t \to T/4-0} a(t) = a_p(T/4), \tag{19}$$
$$\lim_{t \to T/4-0} v(t) = v_p(T/4), \tag{20}$$
$$\lim_{t \to T/4-0} r(t) = r_p(T/4) \tag{21}$$

are obtained. As a result, $a(t) = a_p(t)$, $v(t) = v_p(t)$ and $r(t) = r_p(t)$ hold for $t \leq T/4$. Note that $j(T/4) = 0$ also holds.

After the time $T/4$, it is hold that

$$a(t) \geq a_p(t) = -J(t - T/2), \tag{22}$$
$$v(t) \geq v_p(t) = -\frac{J}{2}(t - T/2)^2 + \frac{JT^2}{16}, \tag{23}$$
$$r(t) \geq r_p(t) = -\frac{J}{6}(t - T/2)^3 + \frac{JT^2 t}{16} - \frac{JT^3}{64}, \tag{24}$$

since $a_p(t)$, $v_p(t)$ and $r_p(t)$ are the time optimal (moving at the negative maximum jerk $-J$ after the time $T/4$). Hence

$$\begin{aligned} f(t) &\leq r_0 - r_p(t) - K_v v_p(t) - K_a a_p(t) \\ &= -\frac{1}{6}J(t - T/4)(t - 3T/4)(t - 3T) \\ &< 0 \ (T/4 < t < 3T/4) \end{aligned} \tag{25}$$

is satisfied. Since $K_0 \gg 0$ is assumed, $j(t) < -J < 0$ holds for $T/4 < t < 3T/4$. It shows that $r(t)$ moves at the negative maximum jerk $-J$. As a result, $a(t) = a_p(t)$, $v(t) = v_p(t)$ and $r_L(t) = r_p(t)$ hold for $T/4 < t < 3T/4$. In addtion,

$$\lim_{t \to 3T/4-0} a(t) = a_p(3T/4), \tag{26}$$
$$\lim_{t \to 3T/4-0} v(t) = v_p(3T/4), \tag{27}$$
$$\lim_{t \to 3T/4-0} r(t) = r_p(3T/4) \tag{28}$$

are held. Consequently, $a(t) = a_p(t)$, $v(t) = v_p(t)$ and $r(t) = r_p(t)$ hold for $t \leq 3T/4$ and $j(3T/4) = 0$ is also satisfied.

After the time $3T/4$, since $j_p(t)$, $v_p(t)$ and $r_p(t)$ are the time optimal (moving at the maximum jerk $+J$ after $t = 3T/4$),

$$a(t) \leq a_p(t) = J(t - T) \tag{29}$$
$$v(t) \leq v_p(t) = \frac{1}{2}J(t - T)^2 \tag{30}$$
$$r(t) \leq r_p(t) = \frac{1}{6}J(t - T)^3 + r_0 \tag{31}$$

hold. Then

$$\begin{aligned} f(t) &\geq r_0 - r_p(t) - K_v v_p(t) - K_a a_p(t) \\ &= -\frac{1}{6}J(t + T/4)(t - 3T/4)(t - T) \\ &> 0, \ (3T/4 < t < T) \end{aligned} \tag{32}$$

is obtained. Since K_0 is large enough ($K_0 \gg 0$), we have $j(t) > J > 0$ ($3T/4 < t < T$). This means that $r(t)$ moves at the maximum jerk $+J$, therefore $a(t) = a_p(t)$, $v(t) = v_p(t)$ and $r_L(t) = r_p(t)$ hold for $t < T$. Furthermore,

$$\lim_{t \to T-0} a(t) = a_p(T) = 0, \tag{33}$$
$$\lim_{t \to T-0} v(t) = v_p(T) = 0, \tag{34}$$
$$\lim_{t \to T-0} r(t) = r_p(T) = r_0 \tag{35}$$

are obtained, i.e., $a(t) = a_p(t)$, $v(t) = v_p(t)$ and $r(t) = r_p(t)$ hold for $t \leq T$. Note that $j(T) = 0$, $a(T) = 0$, $v(T) = 0$, and $r(t) = r_0$ also hold, and consequently the output $r(t)$ remains at r_0 for $t \geq T$. This means that the output signal $r(t)$ of the filter is exactly the same as the optimal trajectory $r_p(t)$.

3. IMPLEMENTATION

When the proposed filter is implemented, the gain K_0 should be chosen as a finite real number to avoid numerically ill conditions while we can let $K_0 \to +\infty$ theoretically. In the case of finite K_0, when t is close to T, the filter acts linear system, which might have complex conjugate poles ($|j(t)| \leq J$ holds when t is nearly equal to T). This means that the output of the filter $r(t)$ might overshoot or be oscillatory. To avoid the situation, consider the following filter shown in Fig. 2, here, p_0, p_1, $p_2 > 0$.

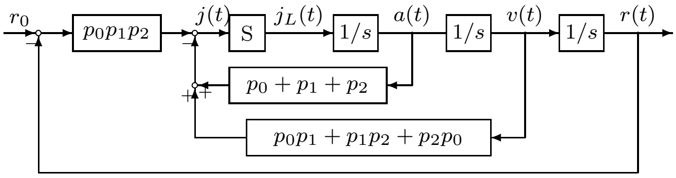

Fig. 2. Proposed feedforward filter B

If J is large enough ($J \to +\infty$), this filter is equivalent to the following linear system;

$$\frac{p_0 p_1 p_2}{(s + p_0)(s + p_1)(s + p_2)} \tag{36}$$

which only has real stable poles $-p_0$, $-p_1$ and $-p_2$. Therefore, the output of the system is not oscillatory. Furthermore, the system shown in Fig. 2 is equivalent to that shown in Fig. 3.

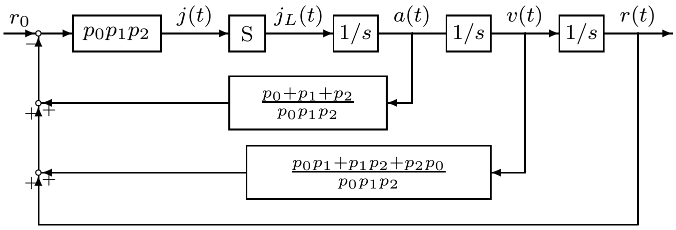

Fig. 3. Equivalent system to filter B

By comparing the systems shown in Fig. 1 and in Fig. 3, we have the following equations;

$$K_0 = p_0 p_1 p_2 \tag{37}$$
$$K_a = \frac{p_0 + p_1 + p_2}{p_0 p_1 p_2} \simeq \frac{1}{p_1 p_2} \tag{38}$$
$$K_v = \frac{p_0 p_1 + p_1 p_2 + p_2 p_0}{p_0 p_1 p_2} \simeq \frac{p_1 + p_2}{p_1 p_2} \tag{39}$$

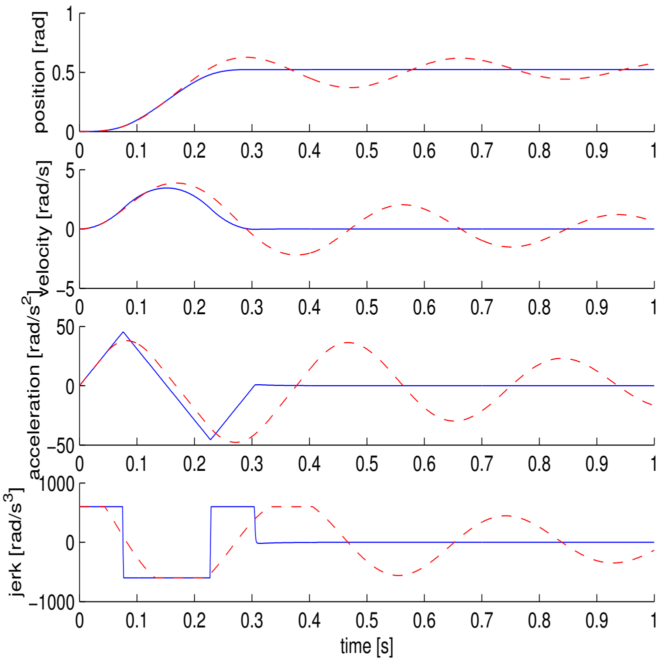

Fig. 4. Step responses of proposed filter A

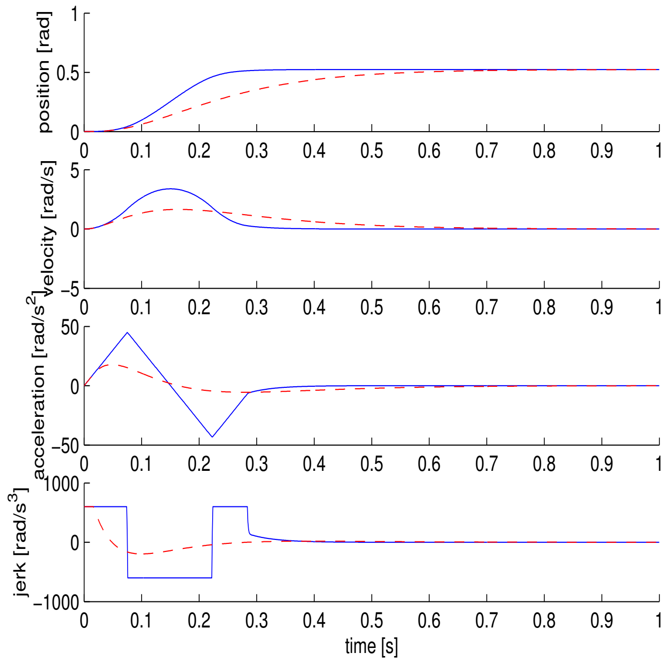

Fig. 5. Step responses of proposed filter B

here, $p_0 \gg p_1, p_2$ is assumed. Letting $\frac{1}{p_1 p_2} = K_a$ and $\frac{p_1+p_2}{p_1 p_2} = K_v$, we obtain

$$K_0 = p_0 p_1 p_2 = p_0/K_a \tag{40}$$

$$\begin{aligned} p_0 + p_1 + p_2 &= p_0 + (p_1 + p_2) \\ &= K_0 K_a + K_v/K_a \end{aligned} \tag{41}$$

$$\begin{aligned} p_0 p_1 + p_1 p_2 + p_2 p_0 &= p_0(p_1 + p_2) + p_1 p_2 \\ &= K_0 K_v/K_a^2 + 1/K_a. \end{aligned} \tag{42}$$

It is confirmed that the poles p_1 and p_2 can not be complex conjugate as follows. When $(p_1 - p_2)^2 \geq 0$ holds, the poles are not complex conjugate. and the system shown in Fig. 2 with (40) - (42) produces non-oscillatory trajectories even though K_0 is not large enough. It is easy to confirm that

$$\begin{aligned} (p_1 - p_2)^2 = (p_1 + p_2)^2 - 4p_1 p_2 &= (\frac{K_v}{K_a})^2 - \frac{4}{K_a} \\ = \frac{(\frac{K_v^2}{K_a} - 4)}{K_a} &= \frac{4}{5K_a} > 0 \end{aligned} \tag{43}$$

holds. Hence both poles p_1 and p_2 are real.

4. NUMERICAL EXAMPLES

The effectiveness of the proposed methods are verified by numerical examples. Firstly, the response for a given step reference is considered. The step response of the proposed filter A; $r(t)$, $v(t)$, $a(t)$, and $j_L(t)$ are shown in Fig. 4, here, $r_0 = \pi/6$, $J = 600$ $K_0 = 2000, 200000$. Fig. 5 shows the step response of the proposed filter B with (40) - (42), here, $r_0 = \pi/6$, $J = 600$ $K_0 = 2000, 200000$. In both figures, the solid curve shows the case of $K_0 = 200000$ and the dashed curve shows the case of $K_0 = 2000$. As can be seen from both figures, when K_0 is large enough (in the case of $K_0 = 200000$), the trajectories of both filters A, B are almost the same. Furthermore, the output $r(t)$ accelerates at the maximum jerk $+J$ until $t = T/4$, then it moves at the negative maximum jerk $-J$. After that it moves again at the maximum jerk $+J$ until it enters the linear region. The results show that the proposed filter produces a trajectory that guarantees a convergence time near the minimum using the maximum jerk when K_0 can be considered as $+\infty$.

On the other hand, when K_0 is not large enough, the behaviour of filters A and B are different from each other. As discussed in the previous section, the output of filter A is oscillatory, while that of filter B does not overshoot. It shows the effectiveness of filter B with (40) - (42).

Secondly, the effectiveness of the 2DOFS with filter B is verified with a mathematical model of a rotary flexible link system. Consider the simplified system shown in Fig. 6. It is known that the system can be approximated by the

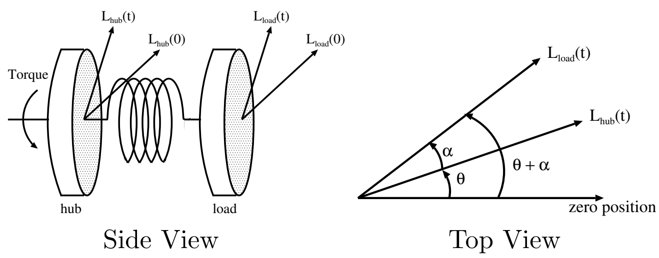

Fig. 6. Simplified model of the flexible link

following kinetic equations(Quanser, 2003).

$$\begin{cases} J_{hub}\ddot{\theta} + J_{load}(\ddot{\theta} + \ddot{\alpha}) + B_{eq}\dot{\theta} = \tau \\ J_{load}(\ddot{\theta} + \ddot{\alpha}) + K_{Stiff}\alpha = 0 \end{cases} \tag{44}$$

Here, J_{hub}[kgm^2] and J_{load}[kgm^2] are inertia moment of the hub and the load, respectively, B_{eq}[Nm/rad] is viscous modulus of the hub, τ[Nm] is input torque of the motor, K_{stiff}[N/m] is stiffness of the link. Letting output y be angle of load ($y = \theta + \alpha$) and state vector x_p be as follows,

$$x_p = [\ \theta\ \alpha\ \dot{\theta}\ \dot{\alpha}\ ]^T \tag{45}$$

a mathematical model of the plant is given by the following state equation.

$$\begin{cases} \dot{x_p} &= A_p x_p + B_p u \\ y &= C_p x_p \end{cases} \tag{46}$$

$$A_p = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \gamma & -\delta & 0 \\ 0 & -\gamma\dfrac{(J_{hub}+J_{load})}{J_{load}} & \delta & 0 \end{bmatrix}$$

$$\gamma = \frac{K_{stiff}}{J_{hub}},\ \delta = \frac{-\eta_m\eta_g K_t K_m {K_g}^2 + B_{eq}R_m}{J_{hub}R_m}$$

$$B_p = \left[\ 0\ 0\ \frac{\eta_m\eta_g K_t K_g}{J_{hub}R_m}\ -\frac{\eta_m\eta_g K_t K_g}{J_{hub}R_m}\ \right]^T$$

$$C_p = [\ 1\ 1\ 0\ 0\ ]$$

Here, η_m and η_g are the efficiency of the motor and the gear box, respectively, K_t[Nm/A] is the motor torque coefficient, K_m[Vs/rad] is back electromotive force of the motor, K_g is the total gear ratio between the motor shaft and the hub, and $R_m[\Omega]$ is resistance of the motor. For the system, a closed loop system is synthesised by PID controller and a 2DOFS with filter B is synthesised in the same manner as (Chen, 2007).

The step responses of filter B ($r(t)$, $v(t)$ and $a(t)$ and $j_L(t)$) and those of the plant model (angle, angular velocity, angular acceleration, angular jerk, input) are shown in Fig. 7, here, reference steps are $r_0 = \pi/6$[rad] and $\pi/3$[rad], and the jerk limit is $J = 600$. In the figure, the solid curve, dashed curve, and dash-and-dotted curve show the behaviour of the plant model with the proposed 2DOFS with filter B, that with a conventional linear 2DOFS (without considering jerk limit), and the trajectories of the proposed filter B, respectively.

Due to the 2DOFS (delay of feedforward F_r in (Chen, 2007)), the behaviour of the plant model delay slightly behind those of filter B. However, the 2DOFS with proposed filter B;

(1) appropriately and effectively use jerk and input, while the linear 2DOFS requires large jerk/input when t is small and does not use them when $r(t)$ is close to r_0.
(2) whose speed and acceleration are symmetric for time, while linear system tends to move fast when t is small and stop passively when $r(t)$ is close to r_0.

in both case of $r_0 = \pi/3, \pi/6$. As a result, even though the proposed methods' start-up time are slower, their settling time are faster than the linear system. The result shows that the effectiveness of the proposed filter.

5. CONCLUSION

In this paper, a non-linear feedforward filter which produces near the time-optimal trajectory for a given step reference signal under jerk constraint is proposed. The proposed filter has simple structure, which requires less computational burden, and, in fact, it is easy implementable. The effectiveness is substantiated with numerical examples. Issues in the future are synthesis method for reference signals other than step signal, and asymmetric limitation for jerk.

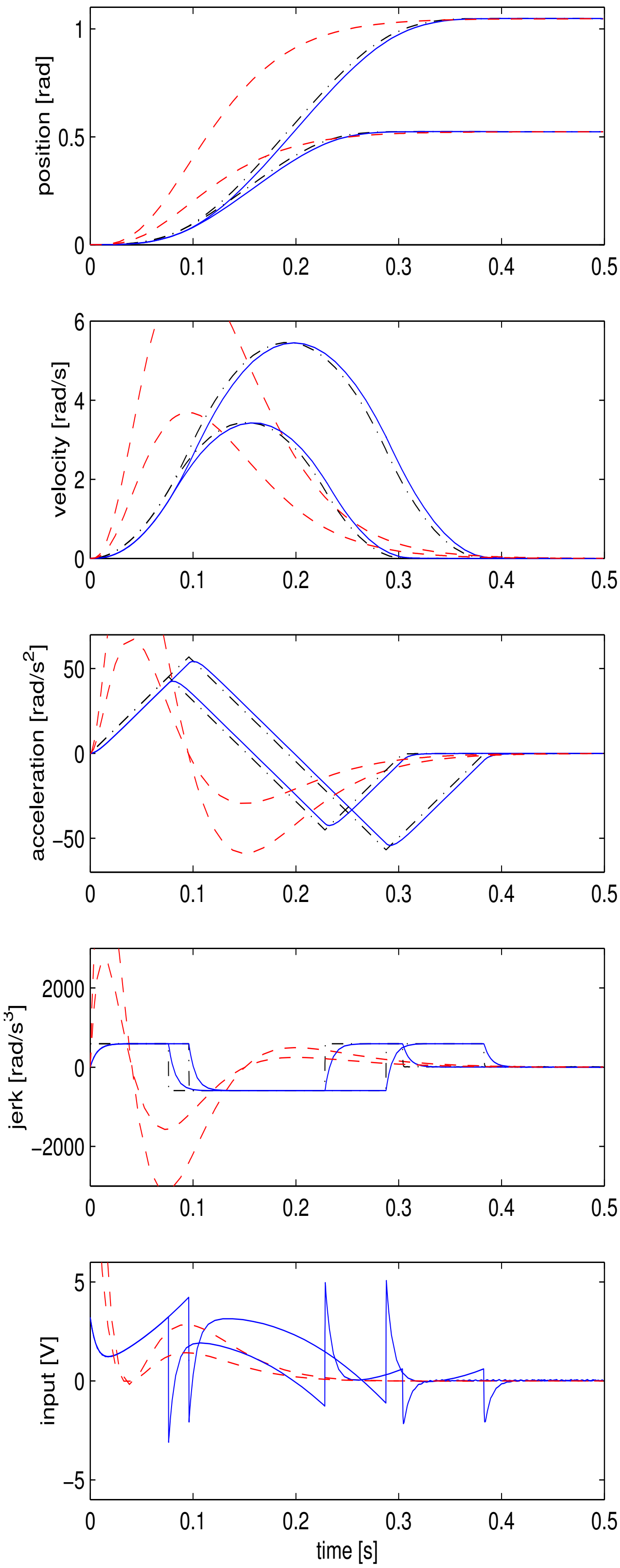

Fig. 7. Step responses of flexible link model

REFERENCES

Agostini, M.J., Parker, G.G., Schaub, H., Groom, K. and Robinett III, R.D. (2003) Generating Swing-Suppressed Maneuvers for Crane Systems With Rate Saturation, IEEE Trans. Cont. Sys. Tech., Vol.11, No.4, 471-481.

Bemporad, A., Casavola, A. and Mosca, E. (1997) Nonlinear Control of Constrained Linear Systems via Predictive Reference Management, IEEE Trans. Automatic Control, Vol.42, No.3, 340-349.

Chen, G, Hayashi, H, Okuda, T and Takami, I. (2007) An Online Generation of Time-Optimal Trajectories via Simple Feedforward Filter and Its Implementation with Two-degree-of-freedom System, Proc. 2007 IEEE Conf. Cont. Application, pp.282-287 (CD-ROM).

Jones, D. and Mansoor, S. (2004) Predictive Feedforward Control for a Hydroelectric Plant, IEEE Trans. Cont. Sys. Tech., Vol.12, No.6, 956-965.

Kim, Y.O., Ha, I.J. (2003) Time-Optimal Control of a Single-DOF Mechanical System Considering Actuator Dynamics, IEEE Trans. Cont. Sys. Tech., Vol.11, No.6, 919-932.

Panahi, I.M.S. and Toliyat, H.A. (2006) An Efficient Processor-Based Online Generation of Time-Optimal Trajectories, IEEE Trans. Cont. Sys. Tech., Vol.14, No.5, 966-973.

Schlemmer, M. and Agrawal, S.K. (2002) A Computational Approach for Time-Optimal Planning of High-Rise Elevators, IEEE Trans. Cont. Sys. Tech., Vol.10, No.1, 105-111.

Tan, K.K., Zhao, S. and Huang, S. (2005) Iterative Reference Adjustment for High-Precision and Repetitive Motion Control Applications, IEEE Trans. Cont. Sys. Tech., Vol.13, No.1, 85-97.

Tsai, K.Y., Schaper, D.D. and Kailath, T. (2004) Design of Feedforward Filters for Improving Tracking Performances of Existing Feedback Control Systems, IEEE Trans. Cont. Sys. Tech., Vol.12, No.5, 742-749.

"Rotary Flexible Link (SRV02) A" (2003), Quanser Consulting, Inc.

Vahidi, A., Kolmanovsky, I. and Stefanopoulou, A (2007) Constraint Handling in a Fuel Cell System: A Fast Reference Governor Approach, IEEE Trans. Cont. Sys. Tech. Vol.15, No.1, 86-98.