Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Output attitude tracking of a formation of spacecraft

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Abstract: This paper will consider the control of spacecraft in a leader-follower formation using attitude measurements only. To analyze the formation under non vanishing disturbances, the concept of uniform practical exponential stability is defined. To ease the Lyapunov analysis a new theorem is provided, giving sufficient conditions for systems that present a cascaded structure to satisfy this definition. Finally, output control is applied to both the leader and follower spacecraft and the stability of the overall system is analyzed through the application of this new result.

1. INTRODUCTION

1.1 Background

There are several reasons for spacecraft formations gaining so much interest from the research community in the last decade. The most important is the desire to place measuring equipment further apart than what is possible on a single spacecraft. This is desirable because the resolution of measurements are often inversely proportional to the baseline length, meaning that either a large spacecraft, or a formation of smaller, but accurately controlled spacecraft may be used. Large spacecraft that satisfy the demand of resolution are often impractical and are both costly to develop and costly to launch. Smaller spacecraft on the other hand, may be standardized and have a lower developmental cost. In addition, they may be of a lower collective weight and/or smaller size such that cheaper launch vehicles can be used. This also allows for the possibility of them to piggy-back with other commercial spacecraft.

1.2 Previous work

The following is a presentation of some of the works done on output control of spacecraft using quaternion measurements. A globally convergent angular velocity observer can be found in Salcudean [1991] and is highly referenced in the later works on output control of spacecraft. In Lizarrald and Wen [1995] a nonlinear filter is used to compensate for missing velocity measurements. The passivity properties of the system are exploited in an output controller so as to achieve asymptotic stabilization of the closed-loop system. A nonlinear quaternion based feedback control law is used in Joshi et al. [1995] to achieve similar stability results. The controller does not depend on system parameters, and therefore robustness to modeling errors and parametric uncertainties are ensured. Two schemes for output attitude tracking are presented in Caccavale and Villani [1999]. The schemes are based on results achieved for output control of robot manipulators, see Berghuis and Nijmeijer [1993], but as mentioned in Caccavale and Villani [1999] the extension is not straight forward due to the nonlinear mapping between the orientation variables, the unit quaternions. In Bondhus et al. [2005] output control is applied to the synchronization of a leader/follower formation of spacecraft. Nonlinear observers are used to estimate the angular velocities based on quaternion estimates, and the rotation matrices representing the attitude error between the reference trajectory and the leader and the follower spacecraft are shown to converge to the identity matrix from any initial condition. The tracking control problem of a follower spacecraft with coupled rotational and translational motion is addressed in Wong et al. [2005]. Convergence of the position and tracking errors are proven, using only position and attitude orientation measurements. In Tayebi [2006] a spacecraft is stabilized without the use of velocity measurements. A unit quaternion observer is used together with linear feedback in terms of the vector parts of the actual unit quaternion and the estimation error quaternion. Asymptotic stability is proven through Lyapunov analysis. The model of the relative dynamics used in this paper has also been treated in Kristiansen et al. [2006] and Krogstad et al. [2007].

1.3 Contribution

The contribution of this paper is twofold. First, we present a theoretical contribution consisting of a new theorem for a system to be uniformly practically exponentially stable (UPES), provided that the system is of cascaded structure. This theorem assumes each subsystem to be UPES and a specific growth of the interconnection term.

Second, the stability of a leader/follower formation is analyzed taking into account external disturbances, and using a controller observer scheme originally designed for the control of robot manipulators. As opposed to most other papers on the topic, the control of both the leader and follower spacecraft are considered, and the solutions of the system are proved to be exponential convergent to zero, up to a steady-state error that can be arbitrarily reduced by a convenient tuning of the control gains.

2. MATHEMATICAL PRELIMINARIES

2.1 Notation

We use the notation $\dot{x}$ for the time derivative of a vector x, i.e. $\dot{x} = \mathrm{d}x/\mathrm{d}t$. Moreover $\ddot{x} = \mathrm{d}^2x/\mathrm{d}t^2$. The identity matrix in $\mathbb{R}^{n\times n}$ is written $I_{n\times n}$. We use $|\cdot|$ for the Euclidean norm of vectors. We define $\mathcal{B}_\delta := \{x \in \mathbb{R}^n \ : \ |x| \leq \delta\}$. The minimum and maximum eigenvalue of a matrix A are denoted by $\lambda_m(A)$ and $\lambda_M(A)$, respectively.

2.2 Rotation Matrices and Unit Quaternions

We use the rotation matrix R_b^a, to transform vectors represented in coordinate frame $\mathcal{F}_a$ to $\mathcal{F}_b$, while preserving the length of the vectors. Rotation matrices are special orthogonal matrices in $\mathbb{R}^{3\times 3}$, that is, they belong to the space

$$SO(3) = \left\{R \in \mathbb{R}^{3\times 3} \quad | \quad R^\top R = I_{3\times 3}, \det(R) = 1\right\}.$$

We will repeatedly use the fact that $(R_b^a)^\top = (R_b^a)^{-1} = R_a^b$ (where R_a^b is equivalent to the opposite rotation of R_b^a), that the rotation matrix of a composite rotation is given by the product of the rotation matrices (i.e. $R_c^a = R_b^a R_c^b$), and that

$$\dot{R}_b^a = S(\omega_{ab}^a) R_b^a.$$

The vector ω_{ab}^a is the angular velocity vector. The subscript denotes the angular velocity of reference frame $\mathcal{F}_b$ relative to frame $\mathcal{F}_a$, where as the superscript shows that the vector is decomposed in frame $\mathcal{F}_a$. Given a vector $\omega = (\omega_x, \omega_y, \omega_z)$, the matrix S is the skew-symmetric operator defined as

$$S(\omega) = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

i.e. $S(\omega) = -S^\top(\omega)$. Two important properties of the indexed angular velocity representation are $\omega_{ab}^a = -\omega_{ba}^a$ and $\omega_{ac}^a = \omega_{ab}^a + \omega_{bc}^a$.

The quaternions are a generalization of the complex numbers, and the set of quaternions, denoted by $\mathbb{H}$, is defined as, see Ma et al. [2004]:

$$\mathbb{H} = \mathbb{C} + \mathbb{C}j, \quad \text{with } j^2 = -1$$

and where the set of complex numbers is defined as $\mathbb{C} = \mathbb{R} + \mathbb{R}i$ with $i^2 = -1$. Furthermore, an element of $\mathbb{H}$, that is a quaternion, is of the form

$$Q = \eta + \epsilon_1 i + \epsilon_2 j + \epsilon_3 k$$

with $\eta, \epsilon_1, \epsilon_2, \epsilon_3 \in \mathbb{R}$ and $k = ij = -ji$. In this paper we will focus on a subgroup of $\mathbb{H}$, the unit quaternions:

$$\mathbb{S}^3 = \left\{Q \in \mathbb{H} \quad | \quad |Q|^2 = 1\right\}. \tag{1}$$

The unit quaternions (or Euler parameters) can be used to represent rotation matrices, and this representation has the advantage of avoiding singularities (as opposed to rotation matrices represented with Euler angles). We will in the following use the vector q to represent the quaternions, with its elements being the real elements of Q, i.e. $q = (\eta, \epsilon)$ where $\epsilon = (\epsilon_1, \epsilon_2, \epsilon_3)$. The rotation matrix for the unit quaternions is (see Hughes [1986])

$$R(q) = I_{3\times 3} + 2\eta S(\epsilon) + 2S^2(\epsilon).$$

Therefore, q and $-q$ represents the same orientation. We use $\bar{q}$ to denote the complex conjugate of q, i.e. $\bar{q} = (\eta, -\epsilon)$.

The quaternion product between two vectors $q_a = (\eta_a, \epsilon_a)$ and $q_b = (\eta_b, \epsilon_b)$ is defined, see Egeland and Gravdahl [2002], as

$$q_a \otimes q_b = \begin{bmatrix} \eta_a\eta_b - \epsilon_a^\top \epsilon_b \\ \eta_a\epsilon_b + \eta_b\epsilon_a + S(\epsilon_a)\epsilon_b \end{bmatrix}.$$

We define the matrix

$$E(q) = \eta I_{3\times 3} + S(\epsilon).$$

The kinematic differential equation can now be derived as

$$\dot{q} = \frac{1}{2}\begin{bmatrix} -\epsilon^\top \\ E(q) \end{bmatrix}\omega,$$

relating the time derivative of the quaternion to the angular velocity. We will use the notation q_{ab} for the quaternion describing the orientation of a frame $\mathcal{F}_b$ relative to a frame $\mathcal{F}_a$.

Perfect tracking in terms of the quaternion error $q_{dl} = \bar{q}_{id} \otimes q_{il}$, where $q_{id}(t)$ represents a possibly time varying reference orientation and q_{id} represents the actual orientation, is achived when $q_{dl} = (\pm 1, 0, 0, 0)$.

2.3 Stability Definition

Practical exponential stability properties pertain to parameterized nonlinear time-varying systems of the form

$$\dot{x} = f(t, x, \theta), \tag{2}$$

where $x \in \mathbb{R}^n$, $t \in \mathbb{R}_{\geq 0}$, $\theta \in \mathbb{R}^m$ is a constant parameter and $f : \mathbb{R}_{\geq 0} \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is locally Lipschitz in x and satisfies Carathéodory conditions for any parameter θ under consideration. θ is a free tuning parameter, and can for instance be a control gain.

Definition 1. Let Δ be a positive constant, and let $\Theta \subset \mathbb{R}^m$ be a set of parameters. The system (2) is said to be *uniformly practically exponentially stable on* Θ if, given any $\delta > 0$, there exists a parameter $\theta^\star(\delta) \in \Theta$ and positive constants $k(\delta)$ and $\gamma(\delta)$ such that, for any $x_0 \in \mathcal{B}_\Delta$ and any $t_0 \in \mathbb{R}_{\geq 0}$ the solutions of (2) satisfies, for all $t \geq t_0$,

$$|x(t, t_0, x_0, \theta^\star)| \leq \delta + k(\delta)\,|x_0|\, e^{-\gamma(\delta)(t-t_0)}.$$

This property is strongly related to its *asymptotic* homologous introduced (and commented in detail) in Chaillet and Loría [2008, 2006]. It is however a stronger property (though only locally), as it imposes an exponential behavior of the solutions in the considered domain of the state-space. We will also stress that *ultimate boundedness* is a weaker property than practical stability. For a system posessing the latter property, the vicinity of the origin to which the solutions converge may be made arbitrary small by convenient tuning of some parameters of the system, typically the control gains.

2.4 Lyapunov Sufficient Conditions

Sufficient conditions for UPES are given in the following theorem:

Theorem 1. Let Θ be a subset of $\mathbb{R}^m$, $\Delta > 0$ and suppose that, given any $\delta > 0$, there exist a parameter $\theta^\star(\delta) \in \Theta$, a continuously differentiable Lyapunov function $V_\delta : \mathbb{R}_{\geq 0} \times \mathcal{B}_\Delta \to \mathbb{R}_{\geq 0}$ and positive constants $\kappa(\delta)$, $\underline{\kappa}(\delta)$, $\overline{\kappa}(\delta)$ such that, for all $x \in \mathcal{B}_\Delta \setminus \mathcal{B}_\delta$ and all $t \in \mathbb{R}_{\geq 0}$,

$$\underline{\kappa}(\delta)\,|x|^p \leq V_\delta(t, x) \leq \overline{\kappa}(\delta)\,|x|^p \tag{3}$$

$$\frac{\partial V_\delta}{\partial t}(t,x) + \frac{\partial V_\delta}{\partial x}(t,x) f(t,x,\theta^\star) \leq -\kappa(\delta)\,|x|^p \;, \quad (4)$$

where p denotes a positive constant. Assume also that

$$\lim_{\delta\to 0} \frac{\overline{\kappa}(\delta)\delta^p}{\underline{\kappa}(\delta)} = 0.$$

Then the system $\dot{x} = f(t,x,\theta)$ introduced in (2) is UPES on the parameter set Θ.

The following result establishes UPES for systems presenting a cascaded structure:

Theorem 2. Under Assumptions 1–3 below, the cascaded system

$$\dot{x}_1 = f_1(t,x_1,\theta_1) + g(t,x,\theta) \quad (5)$$

$$\dot{x}_2 = f_2(t,x_2,\theta_2) \quad (6)$$

is UPES on $\Theta_1 \times \Theta_2$.

Assumption 1. There exists a continuous function $g_0 : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ and, for any $\theta = (\theta_1^\top, \theta_2^\top)^\top \in \Theta$, there exists a class $\mathcal{K}$ function G_{θ_1} independent of θ_2 and such that, for all $x = (x_1^\top, x_2^\top)^\top \in \mathcal{B}_{\Delta_1} \times \mathcal{B}_{\Delta_2}$ and all $t \in \mathbb{R}_{\geq 0}$,

$$|g(t,x,\theta)| \leq g_0(|x_1|) G_{\theta_1}(|x_2|)\,.$$

Assumption 2. Let Δ be a positive number. Given any $\delta_1 > 0$, there exists a parameter $\theta_1^\star(\delta_1) \in \Theta_1$, a continuously differentiable Lyapunov function $V_{\delta_1} : \mathbb{R}_{\geq 0} \times \mathcal{B}_{\Delta_1} \to \mathbb{R}_{\geq 0}$ and positive constants $\kappa(\delta_1)$, $\underline{\kappa}(\delta_1)$, $\overline{\kappa}(\delta_1)$, $c(\delta_1)$, $\eta(\delta_1)$ such that, for all $x_1 \in \mathcal{B}_{\Delta_1} \backslash \mathcal{B}_{\delta_1}$ and all $t \in \mathbb{R}_{\geq 0}$,

$$\underline{\kappa}(\delta_1)\,|x_1|^p \leq V_{\delta_1}(t,x_1) \leq \overline{\kappa}(\delta_1)\,|x_1|^p \quad (7)$$

$$\frac{\partial V_{\delta_1}}{\partial t}(t,x_1) + \frac{\partial V_{\delta_1}}{\partial x_1}(t,x_1) f_1(t,x_1,\theta_1^\star) \leq -\kappa(\delta_1)\,|x_1|^p \;, \quad (8)$$

$$\left| \frac{\partial V_{\delta_1}}{\partial x_1}(t,x_1) \right| g_0(|x_1|) \leq c(\delta_1) + \eta(\delta_1)\,|x_1|^p \;, \quad (9)$$

$$\lim_{\delta_1\to 0} \frac{\overline{\kappa}(\delta_1)\delta_1^p}{\underline{\kappa}(\delta_1)} = 0 \quad (10)$$

where p denotes a positive constant.

Assumption 3. The system $\dot{x}_2 = f_2(t,x_2,\theta_2)$ is UPES on Θ_2.

The proofs are omitted due to the lack of space.

3. MODEL

3.1 Model of Leader Spacecraft

The model for the leader spacecraft is (Hughes [1986]):

$$\dot{q}_{il} = \frac{1}{2}\begin{bmatrix} -\epsilon_{il}^\top \\ E\,(q_{il}) \end{bmatrix} \omega_{il}^l \quad (11)$$

$$J_l \dot{\omega}_{il}^l + C_l\left(\omega_{il}^l\right)\omega_{il}^l = \tau_l + d_l \quad (12)$$

with $J_l \in \mathbb{R}^{3\times3}$ being the leader spacecraft inertia matrix, ω_{il}^l the angular velocity of the spacecraft relative to the inertial frame, $C_l\left(\omega_{il}^l\right) = -S\left(J_l\omega_{il}^l\right)$ and τ_l and d_l the input and disturbance moments on the leader spacecraft, respectively.

3.2 Model of Follower Spacecraft

The model used for the follower spacecraft is similar to the one found in Kristiansen et al. [2007], where the model of the relative attitude in a leader-follower formation can be written as

$$\dot{q}_{lf} = \frac{1}{2}\begin{bmatrix} -\epsilon_{lf}^\top \\ E\,(q_{lf}) \end{bmatrix} \omega_{lf}^f \quad (13)$$

$$J_f \dot{\omega}_{lf}^f + C_f(\omega_{lf}^f)\omega_{lf}^f + n_f(\omega_{il}^l, \omega_{lf}^f) = \Gamma_a + \Gamma_d \quad (14)$$

with $J_f \in \mathbb{R}^{3\times3}$ being the follower spacecraft inertia matrix, ω_{lf}^f the angular velocity of the follower spacecraft relative to the leader spacecraft, $C_f = -S(J_f\omega_{lf}^f)$ and

$$\begin{aligned} n_f =& (S(R_l^f\omega_{il}^l)J_fR_l^f - J_fR_l^fJ_l^{-1}S(\omega_{il}^l)J_l)\omega_{il}^l \\ & (-S(J_fR_l^f\omega_{il}^l) + J_fS(R_l^f\omega_{il}^l) + S(R_l^f\omega_{il}^l)J_f)\omega_{lf}^f \end{aligned}$$

Furthermore,

$$\Gamma_a = \tau_f^f - J_fR_l^fJ_l^{-1}\tau_l^l \quad (15)$$

and

$$\Gamma_d = d_f^f - J_fR_l^fJ_l^{-1}d_l^l \quad (16)$$

with τ_f and d_f as the input and disturbance moments on the follower spacecraft, respectively.

Remark 1. Note that the matrices C_i, $i \in \{l,f\}$ satisfy the inequalities

$$|C_i\,(a)\,b| \leq |J_i|\,|a|\,|b|$$

and are linear in their arguments, i.e.

$$C_i\,(\phi_1 a + \phi_2 b) = \phi C_i\,(a) + \phi C_i\,(b)$$

for any vectors $a, b \in \mathbb{R}^3$ and any constants $\phi_1, \phi_2 \in \mathbb{R}$.

3.3 Model Assumptions

We pose the following assumption on the spacecraft models:

Assumption 4. The inertia matrices J_i, $i \in \{l,f\}$ are symmetric and positive definite, and satisfy the inequalities

$$\alpha_{J_i} \leq |J_i| \leq \beta_{J_i}$$

with $\alpha_{J_i}, \beta_{J_i} \in \mathbb{R}$ being positive constants.

Assumption 5. The disturbance moments d_i, $i \in \{l,f\}$ are bounded

$$|d_i| \leq \beta_{d_i}$$

with $\beta_{d_i} \in \mathbb{R}$ being positive constants. Furthermore, we assume that

$$|\tau_l + d_l| \leq \beta_{(\tau_l+d_l)}$$

with $\beta_{(\tau_l+d_l)}$ being a positive constant.

4. CONTROLLER-OBSERVER DESIGN

4.1 Leader Spacecraft

The desired angular velocity of the leader spacecraft is usually given with reference to the inertial frame as ω_{id}^i. In the leader spacecraft frame, it is

$$\omega_{id}^l = R_i^l \omega_{id}^i$$

where as its time derivative is

$$\begin{aligned} \dot{\omega}_{id}^l &= \dot{R}_i^l\omega_{id}^i + R_i^l\dot{\omega}_{id}^i \\ &= -S\left(\omega_{il}^l\right)\omega_{id}^l + R_i^l\dot{\omega}_{id}^i \end{aligned}$$

We see that to evaluate the derivative we need to know the actual velocity of the leader spacecraft ω_{il}^l, so we will therefore use the modified acceleration vector

$$\begin{aligned} a_d &= -S\left(\omega_{id}^l\right)\omega_{id}^l + R_i^l\dot{\omega}_{id}^i \\ &= R_i^l\dot{\omega}_{id}^i \end{aligned} \quad (17)$$

Let us assume the following:

Assumption 6. The desired angular velocity and the desired angular acceleration of the leader spacecraft are bounded, i.e. $\left|\omega_{id}^{l}\right| \leq \beta_{\omega_{id}^{l}}$ and $\left|\dot{\omega}_{id}^{l}\right| \leq \beta_{\dot{\omega}_{id}^{l}}$.

The following controller-observer scheme is the same as in [Caccavale and Villani, 1999, Theorem 1]:

Controller 1. Let the control law be

$$\tau_l^l = J_l a_r + C_l(\omega_o)\omega_r + k_v(\omega_r - \omega_o) - k_p\epsilon_{dl} \tag{18}$$

$$a_r = a_d - \frac{1}{2}\lambda_d E(q_{de})\omega_{de} \tag{19}$$

$$\omega_r = \omega_{id}^l - \lambda_d\epsilon_{de} \tag{20}$$

$$\omega_o = \omega_{ie}^l - \lambda_e\epsilon_{el} \tag{21}$$

with $k_v, k_p, \lambda_e, \lambda_d \in \mathbb{R}$ constants to be defined, ϵ_{dl} as the vector part of the quaternion product $q_{dl} = \bar{q}_{id} \otimes q_{il}$, ϵ_{de} as the vector part of $q_{de} = \bar{q}_{id} \otimes q_{ie}$, ϵ_{el} and η_{el} as the vector and scalar part of $q_{el} = \bar{q}_{ie} \otimes q_{il}$, respectively, $\omega_{de}^l = \omega_{ie}^l - \omega_{id}^l$ and $E(q_{de}) = \eta_{de}I + S(\epsilon_{de})$. Here, q_{id} represents the orientation of the desired frame, q_{ie} the orientation of the estimated frame, and finally q_{il} the actual orientation of the leader spacecraft, all relative to the inertial frame. Let the observer be

$$\dot{z} = a_r + J_l^{-1}(l_p\epsilon_{el} - k_p\epsilon_{dl} + l_v\lambda_e\eta_{el}\epsilon_{el}) \tag{22}$$

$$\omega_{ie}^l = z + \lambda_e\epsilon_{el} + 2J_l^{-1}l_v\epsilon_{el} \tag{23}$$

with $l_v, l_p \in \mathbb{R}$ constants to be defined.

Let us first define the sliding variables

$$\sigma_d = \omega_{il}^l - \omega_r \tag{24}$$

$$= \omega_{dl}^l + \lambda_d\epsilon_{de} \tag{25}$$

and

$$\sigma_e = \omega_{il}^l - \omega_o \tag{26}$$

$$= \omega_{el}^l + \lambda_e\epsilon_{el} \tag{27}$$

Define $\tilde{\eta}_{dl} := 1 - \eta_{dl}$ and $\tilde{\eta}_{el} := 1 - \eta_{el}$. Let $x_2 := (\sigma_d, \tilde{\eta}_{dl}, \epsilon_{dl}, \sigma_e, \tilde{\eta}_{el}, \epsilon_{el})$. The error dynamics can be written on state space form $\dot{x}_2 = f_2(t, x_2, \theta_2)$, where

$$f_2(t, x_2, \theta_2) = \begin{bmatrix} J_l^{-1}\xi_3 \\ \frac{1}{2}\begin{bmatrix}\epsilon_{dl}^\top \\ E(q_{dl})\end{bmatrix}\omega_{dl}^l \\ J_l^{-1}\xi_4 \\ \frac{1}{2}\begin{bmatrix}\epsilon_{el}^\top \\ E(q_{el})\end{bmatrix}\omega_{el}^l \end{bmatrix} \tag{28}$$

with

$$\xi_3 = -C_l(\omega_{il}^l)\sigma_d - k_v\sigma_d - k_p\epsilon_{dl} + k_v\sigma_e - C_l(\sigma_e)\omega_r - J_l S\left(\omega_{ld}^l\right)\omega_{id}^l + d_l \tag{29}$$

$$\xi_4 = -(l_v E(q_{el}) - k_v I)\sigma_e - l_p\epsilon_{el} - k_v\sigma_d - C_l(\sigma_e)\omega_r - C_l(\omega_{il}^l)\sigma_d + d_l \tag{30}$$

Remark 2. Note that we have chosen to characterize perfect tracking in terms of the quaternion error to when $\eta_{dl} = +1$ and $\eta_{el} = +1$, cf. the discussion about perfect tracking in Section 2.2. We could just as well have used $\eta_{dl} = -1$ and $\eta_{el} = -1$, or both - that is, defined tracking error in terms of the scalar part of the quaternion product as $1 - |\eta_{dl}|$ and $1 - |\eta_{el}|$. Throughout the literature it has been common to use the signum function in the control law for efficient maneuvers. Such an approach would not fit our framework, since this would violate the assumption of our system to be locally Lipschitz and satisfy the Charathéodory conditions. A thorough analysis of stability with respect to sets using discontinuous Lyapunov functions can be found in Fragopoulos and Innocenti [2004].

Proposition 3. Let Assumption 4, 6 and 5 hold. Then, the system $\dot{x}_2 = f_2(t, x_2, \theta_2)$ is UPES.

Proof. The proof is mostly similar to the proof of [Caccavale and Villani, 1999, Theorem 1]. Consider the positive definite Lyapunov function candidate

$$V_2 = \frac{1}{2}\sigma_d^\top J_l\sigma_d + k_p((1-\eta_{dl})^2 + \epsilon_{dl}^\top\epsilon_{dl}) + \frac{1}{2}\sigma_e^\top J_l\sigma_e + l_p((1-\eta_{el})^2 + \epsilon_{el}^\top\epsilon_{el})$$

Following the steps of the proof of [Caccavale and Villani, 1999, Theorem 1] we find that the time derivative of the Lyapunov function candidate along the error dynamics are

$$\begin{aligned}\dot{V}_2 = &- k_v\sigma_d^\top\sigma_d - k_p\lambda_d\eta_{el}\epsilon_{dl}^\top\epsilon_{dl} + k_p\lambda_d\eta_{dl}\epsilon_{el}^\top\epsilon_{dl} \\ &- \sigma_d^\top C(\sigma_e)\omega_r + \sigma_d^\top J_l S(\omega_{ld}^l)\omega_{id}^l \\ &- (l_v\eta_{el} - k_v)\sigma_e^\top\sigma_e - l_p\lambda_e\epsilon_{el}^\top\epsilon_{el} \\ &- \sigma_e^\top C(\sigma_e)\omega_r - \sigma_e^\top C(\omega_{il}^l)\sigma_d\end{aligned}$$

From Remark 1 we have that $|C_l(a)b| \leq \beta_{J_l}|\,|a|\,|b|$ and we see that the following inequalities hold:

$$\sigma_d^\top C(\sigma_e)\omega_r \leq \frac{1}{2}\beta_{J_l}(|\sigma_d|^2 + |\sigma_e|^2)(\left|\omega_{dl}^l\right| + \beta_{\omega_{id}^l} + |\sigma_d|)$$

$$\sigma_e^\top C(\omega_{il}^l)\sigma_d \leq \frac{1}{2}\beta_{J_l}(|\sigma_d|^2 + |\sigma_e|^2)(\left|\omega_{dl}^l\right| + \beta_{\omega_{id}^l})$$

$$\sigma_e^\top C(\sigma_e)\omega_r \leq \beta_{J_l}|\sigma_e|^2(\left|\omega_{dl}^l\right| + \beta_{\omega_{id}^l} + |\sigma_d|)$$

$$\sigma_d^\top J_l S(\omega_{ld}^l)\omega_{id}^l \leq \beta_{J_l}\beta_{\omega_{id}^l}|\sigma_d|\left(|\sigma_d| + \lambda_d(|\epsilon_{el}| + |\epsilon_{dl}|)\right)$$

We will in the following use that

$$|\epsilon_{de}| \leq |\epsilon_{el}| + |\epsilon_{dl}| \tag{31}$$

and

$$\left|\omega_{dl}^l\right| \leq |\sigma_d| + \lambda_d|\epsilon_{de}| \tag{32}$$

After some intermediate calculations we end up with:

$$\begin{aligned}\dot{V}_2 \leq &-\left(k_v - \beta_{J_l}(\lambda_d|\epsilon_{de}| + \frac{3}{2}|\sigma_d| + (2+\lambda_d)\beta_{\omega_{id}^l})\right)|\sigma_d|^2 \\ &- \left(l_v\eta_{el} - k_v - 2\beta_{J_l}(\lambda_d|\epsilon_{de}| + \beta_{\omega_{id}^l} + \frac{7}{4}|\sigma_d|)\right)|\sigma_e|^2 \\ &- \frac{1}{2}(k_p\lambda_d\eta_{el} - \beta_{J_l}\beta_{\omega_{id}^l}\lambda_d)|\epsilon_{dl}|^2 \\ &- \frac{1}{2}(l_p\lambda_e - \beta_{J_l}\beta_{\omega_{id}^l}\lambda_d)|\epsilon_{el}|^2 \\ &- \frac{1}{2}\begin{bmatrix}|\epsilon_{dl}| \\ |\epsilon_{el}|\end{bmatrix}^\top\begin{bmatrix}k_p\lambda_d\eta_{el} & -k_p\lambda_d \\ -k_p\lambda_d & l_p\lambda_e\end{bmatrix}\begin{bmatrix}|\epsilon_{dl}| \\ |\epsilon_{el}|\end{bmatrix} \\ &+ (|\sigma_d| + |\sigma_e|)\beta_{d_l}\end{aligned}$$

Let $|x_2| \leq \bar{\Delta}_2 < 1$. Then, for any $\delta_2 \leq |x_2|$, we define

$$k_v^\star := \beta_{J_l}\left(\lambda_d + \frac{3}{2}\bar{\Delta}_2 + (2+\lambda_d)\beta_{\omega_{id}^l} + \frac{\beta_{d_l}}{\delta}\right)$$

$$l_v^\star := \frac{1}{\sqrt{1-\bar{\Delta}_2^2}}\left(k_v + 2\beta_{J_l}\left(\lambda_d + \frac{7}{4}\bar{\Delta}_2 + \beta_{\omega_{id}^l}\right) + \frac{\beta_{d_l}}{\delta}\right)$$

$$k_p^\star := \frac{\beta_{J_l}\beta_{\omega_{id}^l}}{\sqrt{1-\bar{\Delta}_2^2}}$$

$$l_p^\star := \max\left\{\frac{\beta_{J_l}\beta_{\omega_{id}^l}\lambda_d}{\lambda_e}, \frac{k_p\lambda_d}{\lambda_e\sqrt{1-\bar{\Delta}_2^2}}\right\}$$

such that with $k_v > k_v^\star$, $l_v > l_v^\star(k_v)$, $k_p > k_p^\star$ and $l_p > l_p^\star(k_p)$ condition (4) of Theorem 1 is satisfied, provided that η_{el} does not change sign. Note that for the considered domain of the state space, namely where $|x_2| \le \bar{\Delta}_2$, V_2 is in fact a proper Lyapunov function, i.e. its time derivative can be bounded as in (4). To see this, let c_1 and c_2 be positive constants. For $|x_2| \le \bar{\Delta}_2$ we have that $\eta_{el}, \eta_{dl} > 0$, so $-c_1|\epsilon_{dl}|^2 \le -1/2c_1(|\epsilon_{dl}|^2 + (1-\eta_{dl})^2)$ and $-c_2|\epsilon_{el}|^2 \le -1/2c_2(|\epsilon_{el}|^2 + (1-\eta_{el})^2)$. Condition (3) is satisfied with $V_\delta = V_2$, $\underline{\kappa}(\delta) = \min\{1/2\alpha_{J_l}, k_p, l_p\}$, $\overline{\kappa}(\delta) = \max\{1/2\beta_{J_l}, 2k_p, 2l_p\}$. Hence, for any $x(0) \in \mathcal{B}_{\Delta_2}$, where $\Delta_2 := \sqrt{\overline{\kappa}(\delta)/\underline{\kappa}(\delta)}\bar{\Delta}_2$, we are ensured that η_{el} does not change sign. Furthermore,

$$\lim_{\delta_2 \to 0} \frac{\overline{\kappa}(\delta_2)\,\delta_2^p}{\underline{\kappa}(\delta_2)} = \lim_{\delta_2 \to 0} \frac{\max\{\frac{1}{2}\beta_{J_l}, 2k_p, 2l_p\}\delta_2^2}{\min\{\frac{1}{2}\alpha_{J_l}, k_p, l_p\}} = 0$$

and we can conclude UPES with $\theta = (k_p, l_p, k_v, l_v)$ as tuning parameter. ■

4.2 Follower spacecraft

In the design and analysis of the follower spacecraft, we will *overline* the subscripts to distinguish vectors from the vectors related to the leader spacecraft. The subscript $\bar{d}$ denote the desired frame and $\bar{e}$ the estimated frame of follower spacecraft. E.g. $\omega^i_{l\bar{d}}$ will be the desired angular velocity of the follower spacecraft relative to the leader spacecraft.

Consider the control law:

Controller 2.

$$\tau^f_f = J_f a_{\bar{r}} + C_f(\omega_{\bar{o}})\omega_{\bar{r}} + k_{\bar{v}}(\omega_{\bar{r}} - \omega_{\bar{o}}) - k_{\bar{p}}\epsilon_{\bar{d}f} \quad (33)$$

$$a_{\bar{r}} = a_{\bar{d}} - \frac{1}{2}\lambda_{\bar{d}} E(q_{\bar{d}\bar{e}})\omega^f_{\bar{d}\bar{e}} \quad (34)$$

$$\omega_{\bar{r}} = \omega^f_{l\bar{d}} - \lambda_{\bar{d}}\epsilon_{\bar{d}\bar{e}} \quad (35)$$

$$\omega_{\bar{o}} = \omega^f_{l\bar{e}} - \lambda_{\bar{e}}\epsilon_{\bar{e}f} \quad (36)$$

with $k_{\bar{v}}, k_{\bar{p}}, \lambda_{\bar{d}}, \lambda_{\bar{e}} \in \mathbb{R}$ positive constants, $\epsilon_{\bar{d}f}$ as the vector part of the quaternion product $q_{\bar{d}f} = \bar{q}_{l\bar{d}} \otimes q_{lf}$, $\epsilon_{\bar{d}\bar{e}}$ as the vector part of $q_{\bar{d}\bar{e}} = \bar{q}_{l\bar{d}} \otimes q_{l\bar{e}}$, $\epsilon_{\bar{e}f}$ as the vector part of $q_{\bar{e}f} = \bar{q}_{l\bar{e}} \otimes q_{lf}$, $\omega^f_{\bar{d}\bar{e}} = \omega^f_{l\bar{e}} - \omega^l_{l\bar{d}}$ and $E(q_{\bar{d}\bar{e}}) = \eta_{\bar{d}\bar{e}} I + S(\epsilon_{\bar{d}\bar{e}})$. Here, the desired orientation of the follower spacecraft relative to the leader is described by $q_{l\bar{d}}$, the actual orientation of the follower spacecraft relative to the leader is q_{lf}, and finally $q_{l\bar{e}}$ is the estimated orientation of the follower spacecraft relative to the leader. Since the states ω^f_{lf} and ω^l_{il} are assumed unknown, we have introduced the acceleration vector $a_{\bar{d}} = R^f_i \dot{\omega}^i_{l\bar{d}}$. Let the observer be

$$\dot{z} = a_{\bar{r}} + J_f^{-1}(l_p\epsilon_{\bar{e}f} - k_p\epsilon_{\bar{d}f} + l_{\bar{v}}\lambda_{\bar{e}}\eta_{\bar{e}f}\epsilon_{\bar{e}f}) \quad (37)$$

$$\omega^f_{l\bar{e}} = z + \lambda_{\bar{e}}\epsilon_{\bar{e}f} + 2J_f^{-1} l_{\bar{v}}\epsilon_{\bar{e}f} \quad (38)$$

with $l_{\bar{v}}$ and $l_{\bar{p}}$ positive constants.

To ease the analysis we will define the variables:

$$\sigma_{\bar{d}} = \omega^f_{lf} - \omega_{\bar{r}} \quad (39)$$

$$= \omega^f_{\bar{d}f} + \lambda_{\bar{d}}\epsilon_{\bar{d}\bar{e}} \quad (40)$$

and

$$\sigma_{\bar{e}} = \omega^f_{lf} - \omega_{\bar{o}} \quad (41)$$

$$= \omega^f_{\bar{e}f} + \lambda_{\bar{e}}\epsilon_{\bar{e}f} \quad (42)$$

Define $\tilde{\eta}_{\bar{d}f} := 1 - \eta_{\bar{d}f}$ and $\tilde{\eta}_{\bar{e}f} := 1 - \eta_{\bar{e}f}$. Let $x_1 := (\sigma_{\bar{d}}, \tilde{\eta}_{\bar{d}f}, \epsilon_{\bar{d}f}, \sigma_{\bar{e}}, \tilde{\eta}_{\bar{e}f}, \epsilon_{\bar{e}f})$. We can write the error dynamics on state space form, as:

$$\dot{x}_1 = \tilde{f}_1(t, x_1, \theta_1) + \tilde{g}(t, x) \quad (43)$$

$$\dot{x}_2 = f_2(t, x_2, \theta_2) \quad (44)$$

where

$$\tilde{f}_1(t, x_1, \theta_1) := \begin{bmatrix} J_f^{-1}\xi_1 \\ \frac{1}{2}\begin{bmatrix} \epsilon^\top_{\bar{d}f} \\ E(q_{\bar{d}f}) \end{bmatrix}\omega^f_{\bar{d}f} \\ J_f^{-1}\xi_2 \\ \frac{1}{2}\begin{bmatrix} \epsilon^\top_{\bar{e}f} \\ E(q_{\bar{e}f}) \end{bmatrix}\omega^f_{\bar{e}f} \end{bmatrix}$$

with

$$\begin{aligned}\xi_1 = &- C_f(\omega^f_{lf})\sigma_{\bar{d}} - k_{\bar{v}}\sigma_{\bar{d}} - k_{\bar{p}}\epsilon_{\bar{d}f} + k_{\bar{v}}\sigma_{\bar{e}} - C_f(\sigma_{\bar{e}})\omega_{\bar{r}} \\ &+ J_f S\left(\omega^f_{lf}\right)\omega^f_{l\bar{d}} + d^f_f - J_f R^f_l J_l^{-1}(\tau^l_l + d^l_l)\end{aligned}$$

$$\begin{aligned}\xi_2 = &- (l_{\bar{v}} E(q_{\bar{e}f}) - k_{\bar{v}} I)\sigma_{\bar{e}} - l_p\epsilon_{\bar{e}f} - k_{\bar{v}}\sigma_{\bar{d}} - C_f(\sigma_{\bar{e}})\omega_{\bar{r}} \\ &- C_f(\omega^f_{lf})\sigma_{\bar{d}} + d_f - J_f R^f_l J_l^{-1}(\tau^l_l + d^l_l)\end{aligned}$$

$$\tilde{g}(t, x) := \begin{bmatrix} -n_f(\omega^l_{il}, \omega^f_{lf}) - J_f S(\omega^f_{l\bar{d}}) R^f_l \omega^l_{il} \\ 0 \\ -n_f(\omega^l_{il}, \omega^f_{lf}) \\ 0 \end{bmatrix}$$

Finally, $f_2(t, x_2, \theta_2)$ is as in (28-30). Since $\omega^l_{il} = \sigma_d - \lambda_d\epsilon_{de} + \omega^l_{id}(t)$ and $\omega^f_{lf} = \sigma_d - \lambda_{\bar{d}}\omega^f_{l\bar{d}}(t)$, it may happen that $\tilde{g}(t, x) \neq 0$ when $x_2 = 0$. For that reason, define $g(t, x) := \tilde{g}(t, x_1, x_2) - \tilde{g}(t, x_1, 0)$ and $f_1(t, x_1, \theta_1) := \tilde{f}(t, x_1, \theta_1) + \tilde{g}(t, x_1, 0)$. Similar arguments as in Remark 2 would apply to this system. We are now ready to state the following proposition:

Proposition 4. Let Assumption 4 and 6 hold. Then, the system $\dot{x}_1 = f_1(t, x_1, \theta) + g(t, x)$, $\dot{x}_2 = f_2(t, x_2, \theta)$ is UPES.

Proof. To prove this proposition we will apply Theorem 2. We will first prove Assumption 1, i.e. boundedness of the interconnection term $g(t, x)$. We will in the following use $|S(\alpha)| = |\alpha|$ and $|R| = 1$. It can be shown that

$$|g(t, x)| \le a_1|x_2| + a_2|x_2|^2 \quad (45)$$

with a_1, a_2 being positive constants, independent of x_1, x_2, θ_1, θ_2 and t. Hence, the function g_0 and G_{θ_1} of Assumption 1 can be chosen as

$$g_0(s) = 1 \quad (46)$$

$$G_{\theta_1}(s) = a_1 s + a_2 s^2 \quad \forall s \ge 0 \quad (47)$$

Therefore, Assumption 1 is satisfied. Now we will prove Assumption 2. Consider the positive-definite Lyapunov function

$$\begin{aligned}V_1 = &\frac{1}{2}\sigma^\top_{\bar{d}} J_f \sigma_{\bar{d}} + k_{\bar{p}}((1 - \eta_{\bar{d}f})^2 + \epsilon^\top_{\bar{d}f}\epsilon_{\bar{d}f}) \\ &+ \frac{1}{2}\sigma^\top_{\bar{e}} J_f \sigma_{\bar{e}} + l_{\bar{p}}((1 - \eta_{\bar{e}f})^2 + \epsilon^\top_{\bar{e}f}\epsilon_{\bar{e}f})\end{aligned}$$

This function satisfies condition (7) of Theorem 2 with $V_\delta = V_1$, $p = 2$, $\underline{\kappa}(\delta) = \min\{1/2\alpha_{J_f}, k_{\bar{p}}, l_{\bar{p}}\}$ and $\overline{\kappa}(\delta) = \max\{1/2\beta_{J_f}, 2k_{\bar{p}}, 2l_{\bar{p}}\}$. Furthermore,

$$\frac{\partial V_1}{\partial x_1} \le x_1^\top Q$$

where $Q := \mathrm{diag}(J_f, 2k_{\bar{p}}I_{4\times4}, J_f, 2l_{\bar{p}}I_{4\times4})$, so

$$\left|\frac{\partial V_1}{\partial x_1}\right| g_0(|x_1|) \le \lambda_M(Q)|x_1|.$$

It will shortly be shown that $\lambda_M(Q)$ depends on δ_1 through $k_{\bar{p}}$ and $l_{\bar{p}}$. Since any linear function can be upper bounded by the sum of a quadratic function and a constant, we conclude that condition (9) is satisfied for some constants $c(\delta_1)$ and $\eta(\delta_1)$ and $p = 2$. To prove condition (8), it should first be noted that there exist positive constants a_3, a_4 and a_5, independent of x_1, x_2, θ_1, θ_2 and t, such that for $|x_1| \ge \delta_1$,

$$\left|\frac{\partial V_1}{\partial x_1}\tilde{g}(t, x_1, 0)\right| \le a_3\frac{|x_1|^2}{\delta_1} + a_4|\sigma_{\bar{d}}|^2 + a_5|\sigma_{\bar{e}}|^2 \qquad (48)$$

The intermediate calculations have been left out due to space limitations. Let $|x_1| \le \bar{\Delta}_1 \le 1$. For any $\delta_1 \le |x_1|$ the time derivative of the Lyapunov function can be upper bounded by:

$$\begin{aligned}\dot{V}_1 \le &-\Big(k_{\bar{v}} - \beta_{J_f}\Big(\lambda_{\bar{d}} + \frac{3}{2}\bar{\Delta}_1 + (2+\lambda_{\bar{d}})\beta_{\omega_{l\bar{d}}^f}\Big) \\ &-a_4 - \frac{\beta^2_{\omega_{l\bar{d}}^f} + \tilde{\beta} + a_3}{\delta_1}\Big)|\sigma_{\bar{d}}|^2 \\ &-\Big(l_{\bar{u}}\eta_{\bar{e}l} - k_{\bar{v}} - 2\beta_{J_f}\Big(\lambda_{\bar{d}} + \frac{7}{4}\bar{\Delta}_1 + \beta_{\omega_{l\bar{d}}^f}\Big) \\ &-a_5 - \frac{\tilde{\beta} + a_3}{\delta_1}\Big)|\sigma_{\bar{e}}|^2 \\ &-\frac{1}{2}\Big(k_{\bar{p}}\eta_{\bar{e}l}\lambda_{\bar{d}} - \lambda_{\bar{d}}\beta_{J_f}\beta_{\omega_{l\bar{d}}^f} - \frac{a_3}{\delta_1}\Big)|\epsilon_{\bar{d}f}|^2 \\ &-\frac{1}{2}\Big(l_{\bar{p}}\lambda_{\bar{e}} - \lambda_{\bar{d}}\beta_{J_f}\beta_{\omega_{l\bar{d}}^f} - \frac{a_3}{\delta_1}\Big)|\epsilon_{\bar{e}f}|^2 \\ &-\frac{1}{2}\begin{bmatrix}|\epsilon_{\bar{d}f}| \\ |\epsilon_{\bar{e}f}|\end{bmatrix}^\top \begin{bmatrix}k_{\bar{p}}\eta_{\bar{e}l}\lambda\bar{d} & -k_{\bar{p}}\lambda\bar{d} \\ -k_{\bar{p}}\lambda\bar{d} & l_{\bar{p}}\lambda_{\bar{e}}\end{bmatrix}\begin{bmatrix}|\epsilon_{\bar{d}f}| \\ |\epsilon_{\bar{e}f}|\end{bmatrix}\end{aligned}$$

where $\tilde{\beta} := \beta_{d_f} + \beta_{J_f}\alpha_{J_l}\beta_{(\tau_l+d_l)}$. By defining

$$\begin{aligned}k_{\bar{v}}^\star :=&\beta_{J_f}\Big(\lambda_{\bar{d}} + \frac{3}{2}\bar{\Delta}_1 + (2+\lambda_{\bar{d}})\beta_{\omega_{l\bar{d}}^f}\Big) + a_4 + \frac{\beta^2_{\omega_{l\bar{d}}^f} + \tilde{\beta} + a_3}{\delta_1} \\ l_{\bar{v}}^\star :=&\frac{1}{\sqrt{1-\bar{\Delta}_1^2}}\Big(k_{\bar{v}} + 2\beta_{J_f}\Big(\lambda_{\bar{d}}\frac{7}{4}\bar{\Delta}_1 + \beta_{\omega_{l\bar{d}}^f}\Big) + a_5 + \frac{\tilde{\beta}+a_3}{\delta_1}\Big) \\ k_{\bar{p}}^\star :=&\frac{\lambda_{\bar{d}}\beta_{J_f}\beta_{\omega_{l\bar{d}}^f} + \frac{a_3}{\delta_1}}{\lambda_{\bar{d}}\sqrt{1-\bar{\Delta}_1^2}} \\ l_{\bar{p}}^\star :=&\max\left\{\frac{\lambda_{\bar{d}}\beta_{J_f}\beta_{\omega_{l\bar{d}}^f} + \frac{a_3}{\delta_1}}{\lambda_{\bar{e}}}, \frac{k_{\bar{p}}\lambda_{\bar{d}}}{\lambda_{\bar{e}}\sqrt{1-\bar{\Delta}_1^2}}\right\}\end{aligned}$$

and choosing the control gains such that $k_{\bar{v}} > k_{\bar{v}}^\star$, $l_{\bar{v}} > l_{\bar{v}}^\star(k_{\bar{v}})$, $k_{\bar{p}} > k_{\bar{p}}^\star$ and $l_{\bar{p}} > l_{\bar{p}}^\star(k_{\bar{p}})$ we satisfy condition (8), provided that $x(0) \in \mathcal{B}_{\Delta_1}$ where $\Delta_1 := \sqrt{\overline{\kappa}(\delta)/\underline{\kappa}(\delta)}\bar{\Delta}_1$. Since the gains are linearly dependent on $1/\delta_1$

$$\lim_{\delta_1\to0}\frac{\overline{\kappa}(\delta_1)\,\delta_1^p}{\underline{\kappa}(\delta_1)} = \lim_{\delta_1\to0}\frac{\max\{1/2\beta_{J_f}, 2k_{\bar{p}}, 2l_{\bar{p}}\}\delta_1^p}{\min\{1/2\alpha_{J_f}, k_{\bar{p}}, l_{\bar{p}}\}} = 0$$

holds, and condition (10) is satisfied. Hence, by the same arguments as in the proof of Proposition 3, all conditions of Assumption 2 are satisfied. Finally, it is shown in Proposition 3 that the system $\dot{x}_2 = f(t, x_2, \theta_2)$ is UPES and the conclusion of Proposition 4 follows. ■

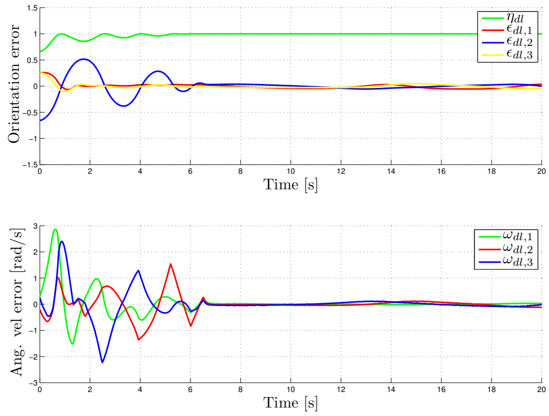

Fig. 1. Orientation and angular velocity tracking error of the leader spacecraft

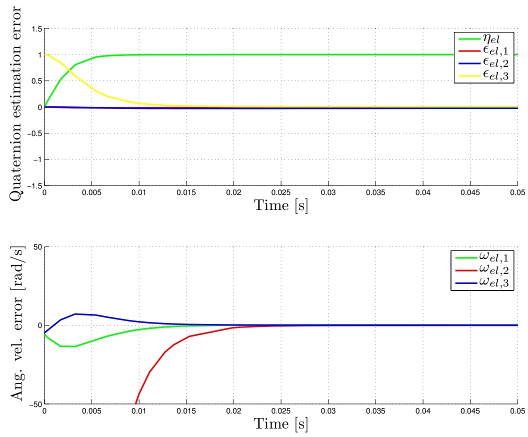

Fig. 2. Orientation and angular velocity estimation error of the leader spacecraft

5. SIMULATION

The spacecraft inertia matrices were chosen to be $J_l = J_f = \mathrm{diag}\{6, 7, 8\}$, where as the input torque were saturated to $\max\{\tau_l\} = \max\{\tau_f\} = 20$. The disturbances acting on the spacecraft, d_l and d_f, were band-limited white noise of power 0.1 and sample time of 0.1 acting about all body frame axis. Examples of disturbances on a spacecraft orbiting Earth are torques due to gravitational, aerodynamic and magnetic forces. The initial conditions for the leader spacecraft model were $q_{il}(0) = (1/2, 1/2, 1/2, 1/2)$ and $\omega_{il}(0) = (0.2, 0.3, -0.2)$, where as the controller had initial conditions $q_{ie}(0) = (1/2, -1/2, 1/2, -1/2)$ and $z(0) = (5, 6, 4)$ and gains $k_p = 24$, $k_v = 426$, $l_v = 2700$, $l_p = 144$, $\lambda_d = 20$ and $\lambda_e = 10$. The reference signal was chosen as $\dot{\omega}_{id}^i = 0.1 \times (\sin\frac{\pi}{32}t + \frac{\pi}{2}, \sin\frac{\pi}{4}t, \sin\frac{\pi}{8}t + \frac{\pi}{4})$. Figure 1 shows the orientation and angular velocity tracking error of the leader spacecraft. Figure 2 shows the estimation errors. The follower spacecraft were chosen to track the orientation and angular velocity of the leader spacecraft. The initial conditions of the follower spacecraft model were $q_{lf}(0) = (1/2, 1/2, 1/2, 1/2)$ and $\omega_{lf}(0) = (0, 0, 0)$. The controller initial conditions and gains were $q_{ie}(0) = (1/2, -1/2, -1/2, 1/2)$ and $z(0) = (5, 6, 4)$ and

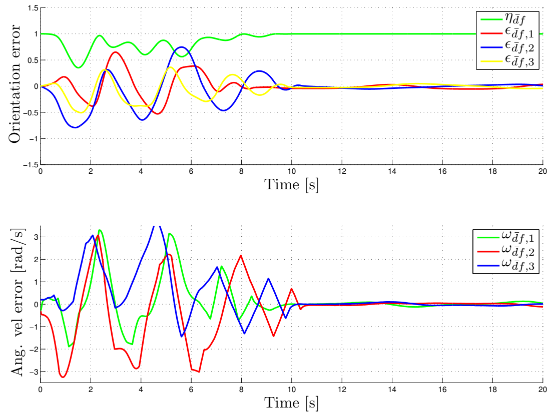

Fig. 3. Orientation and angular velocity tracking error of the follower spacecraft

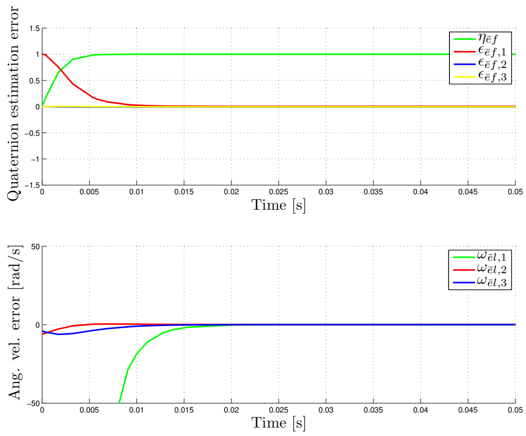

Fig. 4. Orientation and angular velocity estimation error of the follower spacecraft

$k_{\bar{p}} = k_p$, $k_{\bar{v}} = k_v$, $l_{\bar{v}} = l_v$, $l_{\bar{p}} = l_p$, $\lambda_{\bar{d}} = \lambda_d$ and $\lambda_{\bar{e}} = \lambda_e$, respectively. The gains of the controller and observer were chosen based on the outcome of the Lyapunov analysis in the previous sections. Figure 3 and 4 show the simulation results.

6. CONCLUSION

We have stated a definition for UPES and theorems for Lyapunov sufficient conditions for a systems to satisfy the definition. Exponentially stable equilibrium points are well known for their robustness to disturbances vanishing at the equilibrium point and their fast convergence rate. With the new theorems, stability of a neighborhood of such equilibrium points can be adressed under nonvanishing perturbations, and by a convenient tuning of parameters such neighborhoods can be made arbitrarily small, while still ensuring exponential convergence of the solutions.

The theorems were used to analyze the stability of spacecraft in formation using a cascaded reasoning. Both spacecraft were controlled using knowledge of their orientation only. Simulations were performed, which support the robustness results of the stability analysis.

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