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Control of a production-inventory system using a PID controller and demand prediction ⋆

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Abstract: A common and important problem in business is the determination of inventory policies for a production system within a changing business environment and market demand. In this paper, an automatic pipeline feedback order-based production control system (APIOBPCS), considering a demand with cyclic and stochastic components, is proposed. The dynamics and delays of the production process are modeled as a pure delay. The control system structure consists of a PID (Proportional, Integrative and Derivative) controller with an Extended Kalman Filter-based demand prediction. The main objective of the this dynamic controller is to stabilize and regulate the inventory levels in function of a desired set-point level. The Extended Kalman Filter (EKF) estimates the parameters of a Volterra time-series model to forecast future values of the demand. A control error analysis is also performed for the proposed inventory control system, in order to obtain bounds for the control errors and to probe its stability. This methodology is useful to make an appropriate decision about the desired inventory level for a given demand prediction error. The inventory control system is evaluated by simulations showing a good performance.

1. INTRODUCTION

An important problem as in business as in manufacturing is the determination of inventory and transportation policies for a physical distribution system within a changing business environment and market demand. Until recently, production and sales managers used to control inventory levels by means of two powerful but limited tools: intuition and experience. However, the size and complexity of modern production and sale operations have grown in such a way that it is not convenient anymore to regulate stock levels without having a quantitative assessment of the involved factors.

Inventories are resources needed for production or commercialization processes, that are kept idle, waiting to be used when necessary. These resources can be of any kind. Inventories are used to compensate or regulate the imbalances of the normal sequence of activities in production and sales processes. In other words, inventories should have a stabilizing effect on material flow patterns [Disney and Towill, 2003]. Due to the fact that demand is usually unknown and stochastic in nature, it is not an easy task to keep inventories on an appropriate and constant level. If the set point of the desired level is set too low, maintenance costs may go down, that is, there is no need for large storage spaces, the insurance costs decrease, and then devaluation costs are lower. However, there persists the risk of losing sales when demand grows beyond the expected figures. On the other hand, if the inventory levels are kept too high, maintenance cost are usually higher due to the larger volume of resources that are kept in stock, the larger space required, and the higher devaluation and maintenance costs. Therefore, an effective supply chain is managed with an aim at keeping a high level of costumer satisfaction while minimizing costs and maximizing profits [Rivera and Pew, 2005]. Results of savings achieved by best-in-class companies, as a result of improving their supply chain operations, amount 5-6% of sales [SimulationDynamics, 2003].

Although research in this area is not novel, it was only recently when the control systems community have paid attention to this topic. This is described thoroughly in an excellent revision of Ortega and Lin [2004]. Previous research works have proposed systems to stabilize the inventory level as is the case of John *et al.* [1994] and Disney and Towill [2003]. More recently, the works of Grubbström and Wikner [1996], Samanta and Al-Araimi [2001], and Rivera and Pew [2005] have explicitly included dynamical controllers, such as PID, on the supply chain, and have obtained promising results.

In this paper we apply to the inventory management field methodologies and tools from the industrial automation and modern control theory. The present work proposes a simple dynamical control system whose main objective is to keep the inventory level at a desired set-point in spite of fluctuations in the demand, and considering lead times

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of the production system. The controller is based on an APIOBPCS model, uses a PID controller and estimator of the demand prediction to keep into a stationary inventory level. Therefore, the inventory level set-point can be lowered without loosing sales opportunities. It is assumed that demand signal is constituted by two components: a cyclic one and a Poisson-like stochastic perturbation. The demand is predicted by a dual joint EKF [Ljung, 1979, Wan and van der Merwe, 2000], which identifies the parameters of a Volterra equation used to model it. Furthermore, a control error analysis is performed for the proposed inventory control system, in order to obtain bounds for the control errors and to probe its stability.

2. MODELING THE DYNAMIC OF PRODUCTION-INVENTORY SYSTEM

The underlaying theory of the open-loop model to describe the dynamic of inventory system is explained as follow. The dynamics of an inventory system can be represented by a difference equation:

$$I(k+1) = I(k) + O(k-\tau) - D(k) \tag{1}$$

where, $I_{(k)}$ is the net inventory level, τ represents the order fulfilment time, $O_{(k-\tau)}$ is the prior orders made τ-days before, and $D_{(k)}$ the demand signal. $O_{(k)}$ is generated by a reorder policy.

Traditionally, reorder policies have been based on Economic Order Quantity (EOQ) approaches, such as the $(\hat{s}, \hat{S})$ policy (when the inventory level becomes equal to or less than $\hat{s}$, order up to the level $\hat{S}$). EOQ approaches are widely used but they are not efficient enough, mainly because they are static laws and do not have into account the demand fluctuations.

On the other hand, APIOBPCS models have shown to perform well, stabilizing the dynamic system and reducing the bullwhip effect. Bullwhip effect refers to the scenario where orders to the suppliers tends to have larger fluctuations than sales to the buyer and this distortion propagates and amplifies itself when going upstream [Disney and Towill, 2003, Warburton, 2004]. A basic production-inventory system based on the APIOBPCS scheme has four main components: the *inventory*, that can be modeled as an integrator, the *production process*, that has been modeled in this paper as a finite time delay, the *reorder policy*, and the *demand predictor*. In addition, there are four fundamental information flows [Grubbstöm and Wikner, 1996], namely *demand*, *inventory level*, *work-in-progress* (WIP), and *demand prediction*. Most of the order decision rules are based on one or more of these flows. That is:

$$O(k) = f[I(k), \hat{d}, WIP]. \tag{2}$$

In this work, the demand is supposed to be cyclic, modeling a seasonal demand, adding a stochastic component given by a Poisson noise. For simplicity, a fixed order fulfillment time is assumed.

In contrast to the APIOBPCS analyzed by Disney and Towill [2003], our approach includes in the control loop a PID controller and the demand prediction is generated by a joint dual EKF. We call this approach a PID-APIOBPCS model.

3. DEMAND PREDICTION

3.1 Volterra Models

The demand over time can be thought as a time-series, represented by a nonlinear autoregressive model. One way to model it is by means of a Volterra equation. The finite-dimensional discrete-time Volterra model used in this paper is a single-input, single-output model, relating an input sequence $\{u(k)\}$, to an output sequence $\{y(k)\}$ [Doyle et al., 2002]. This relationship is defined by the equations:

$$y(k) = y_0 + \sum_{n=1}^{N} \gamma_M^n(k) \tag{3}$$

$$\gamma_M^n(k) = \sum_{i_1=0}^{M} \cdots \sum_{i_n=0}^{M} \theta_n(i_1, \ldots, i_n) u(k-i_1) \cdots u(k-i_n)$$

where y_0 and θ_i are the model parameters. It will be convenient to introduce the notation $V_{(N,M)}$, where N denotes the *nonlinear degree* of the model and M denotes its *dynamic order*. In our particular case a $V_{(1,30)}$ model is used. Changing the name of the input and output signals, and taking the mentioned values for N and M, equation (3) is reduced to:

$$\hat{d}(k) = d_0 + \sum_{i=1}^{30} \theta_i d(k-i) \tag{4}$$

where d_0 and θ_i are the model parameters, $\hat{d}_{(k)}$ is the actual estimated demand and $D_{(k-1)}$ are past values of the demand. The values of the unknown parameters will be found by a Kalman Filter, as shown in the next subsection.

3.2 Joint Extended Kalman Filter

The Kalman filter is characterized by a set of equations that synthesizes an optimal estimator of predictor-corrector type in the sense of minimizing the estimate error covariance $\mathbf{P}(k)$. In this particular case, a Joint Extend Kalman Filter [Ljung, 1979, Wan and van der Merwe, 2000] was used to solve the dual problem of simultaneously estimating the state and the model parameters θ from the noisy demand signal. To make the Volterra time-serie into a Markovian process its necessary to model the demand given by a the Volterra equation (4) as that given by the nonlinear auto-regression system (5).

$$\begin{aligned} \mathbf{x}_{(k)} &= \mathbf{F}(\mathbf{x}_{(k-1)}, \theta_{(k-1)}) + B\upsilon_{(k-1)} \\ y_{(k)} &= \mathbf{C}\mathbf{x}_{(k)} + \eta_{(k)} \end{aligned} \tag{5}$$

Then the model is then rewritten as the state space system given by (6):

$$\begin{bmatrix} x_{(k)} \\ x_{(k-1)} \\ \vdots \\ x_{(k-M+1)} \end{bmatrix} = \begin{bmatrix} f(x_{(k-1)} \ldots, x_{(k-M)}, \theta) \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ & & & \\ 0 & \ddots & 0 & \vdots \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_{(k-1)} \\ \vdots \\ x_{(k-M)} \end{bmatrix} \end{bmatrix} + \begin{bmatrix} \upsilon_{(k-1)} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{6}$$

$$y_{(k)} = [1 \; 0 \; \cdots \; 0]\, \mathbf{x}_{(k)} + \eta_{(k)}$$

where $f(x_{(k-1)} \ldots, x_{(k-M)}, \theta_{(k-1)})$ is the Volterra model, and υ and η are the process and measurement noises respectively. The joint EKF approach to determine the unknown parameters θ consists in augmenting the state vector $\mathbf{x}$ with the parameters vector $\theta_{(k)}$. By doing this, a new state vector $\mathbf{z}_{(k)} = [\mathbf{x}_{(k)}^T, \theta_{(k)}^T]^T$ is obtained. Then, estimation is done recursively by writing the state-space equations for the *joint* state as

$$\begin{bmatrix} \mathbf{x}_{(k)} \\ \theta_{(k)} \end{bmatrix} = \begin{bmatrix} \mathbf{F}(\mathbf{x}_{(k-1)}, \theta_{(k-1)}) \\ \mathbf{I}\theta_{(k-1)} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \upsilon_{(k-1)}$$
$$y_{(k)} = [\,1 \; 0 \; \cdots \; 0\,] \begin{bmatrix} \mathbf{x}_{(k)} \\ \theta_{(k)} \end{bmatrix} + \eta_{(k)} \qquad (7)$$

and running a EKF on the joint state-space to produce the simultaneous estimates of the states $\mathbf{x}_{(k)}$ and θ. The EKF equations can be synthesized as follow [Wan and Nelson, 2000]. Initialize with:

$$\begin{aligned} \hat{\mathbf{z}}_{(0)} &= E[\mathbf{z}_{(0)}] \\ \mathbf{P}_{\mathbf{z}_{(0)}} &= E[(\mathbf{z}_{(0)} - \hat{\mathbf{z}_{(0)}})(\mathbf{z}_{(0)} - \hat{\mathbf{z}_{(0)}})^T], \end{aligned} \qquad (8)$$

where E means the expected value. Then, for $k = 1, \ldots, \infty$, the time update equations of the EKF are:

$$\begin{aligned} \hat{\mathbf{z}}_{(k)}^{-} &= \mathbf{F}(\hat{\mathbf{z}}_{(k-1)}, \theta_{(k-1)}, \upsilon_{(k-1)}) \\ \mathbf{P}_{\mathbf{z}_{(k)}}^{-} &= \mathbf{A}_{(k-1)} \; \mathbf{P}_{z_{(k-1)}} \; \mathbf{A}_{(k-1)}^T + \mathbf{R}^{\upsilon} \end{aligned} \qquad (9)$$

and the measurement update equations are:

$$\begin{aligned} \mathbf{K}_{(k)}^{\mathbf{z}} &= \mathbf{P}_{\mathbf{z}_{(k)}}^{-} \mathbf{C}_{(k)}^T (\mathbf{C}_{(k)} \mathbf{P}_{\mathbf{z}_{(k)}}^{-} \mathbf{C}_{(k)}^T + \mathbf{R}^{n})^{-1} \\ \hat{\mathbf{z}}_{(k)} &= \hat{\mathbf{z}}_{(k)}^{-} + \mathbf{K}_{(k)}^{\mathbf{z}} (y_{(k)} - \mathbf{C}_{(k)} \hat{\mathbf{z}}_{(k)}^{-}) \\ \mathbf{P}_{\mathbf{z}_{(k)}} &= (\mathbf{I} - \mathbf{K}_{(k)}^{\mathbf{z}} \mathbf{C}_{(k)}) \mathbf{P}_{\mathbf{z}_{(k)}}^{-} \\ \text{with} \quad \mathbf{A} &= \left. \frac{\partial \mathbf{F}(\mathbf{z}, \theta, \upsilon)}{\partial \mathbf{z}} \right|_{\hat{\mathbf{z}}_{(k)}} \end{aligned} \qquad (10)$$

In the EKF equations, $\mathbf{P_z}$ is the estimate error covariance, $\mathbf{K}_{(k)}^{\mathbf{z}}$ is the Kalman gain and $\mathbf{R}^{\upsilon}$ and $\mathbf{R}^{\eta}$ are the process and measurement noise covariance respectively.

Once the model parameters d_0 and θ have been estimated, they are used together with the model to get a prediction of on step ahead. This predicted state vector is then used for the PID-APIOBPCS reorder policy.

4. PID-APIOBPCS-BASED INVENTORY LEVEL CONTROL

The proposed control law is based on the APIOBPCS, namely in our case PID-APIOBPCS. APIOBPCS has the main advantage over the other reorder policies of including in the decision rule the value of the WIP. A scheme of the APIOBPCS is shown in Fig. 1, and the reorder policy equations are given by (11),

$$O(k) = \hat{d}(k) + \frac{[I_{ref}(k) - I(k)]}{T_i} + \frac{[dWIP(k) - WIP(k)]}{T_w} \qquad (11)$$
$$WIP(k) = WIP(k-1) + O(k) - O(k-\tau)$$
$$dWIP(k) = T_p \hat{d}(k)$$

where, $\hat{d}(k)$ is the estimated demand, and $I_{ref}(k)$ is the inventory level reference. Constants T_i is related to the time to adjust the inventory level, $\hat{T}_P$ is the estimate of the production lead time, and T_w is the time needed to adjust the WIP.

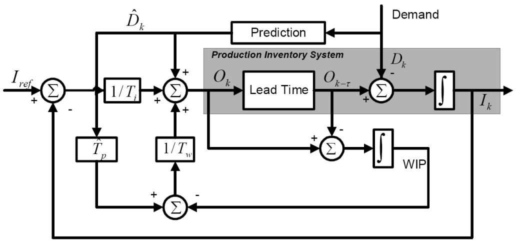

Fig. 1. Ordering system incorporating WIP feedback.

On the other hand, the approach of using only a PID as suggested in Grubbstöm and Wikner [1996], and in Rivera and Pew [2005] to model an order decision rule does not involve an explicit forecasting unit to estimate demand. So, fusing both controllers, it is possible to obtain a new structure and control law. The proposed control schema can be seen in Fig. 2.

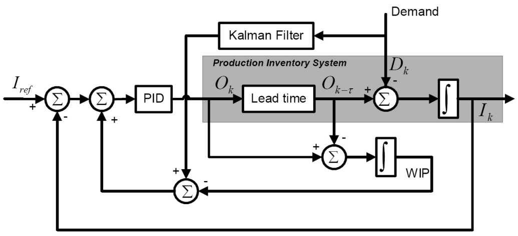

Fig. 2. Proposed PID-APIOBPCS controller.

$$\begin{aligned} O(k) = O(k-1) + K_P\,[e(k) - e(k-1)] + K_I e(k-1) + \\ + K_D\,[e(k) - 2e(k-1) + e(k-2)] \end{aligned} \qquad (12)$$
$$e(k) = (I_{ref}(k) - I(k)) + (dWIP(k) - WIP(k))$$
$$WIP(k) = WIP(k-1) + O(k) - O(k-\tau)$$
$$dWIP(k) = \hat{d}(k)$$

Equations (12) represent the reorder policy for the PID-APIOBPCS controller. As it can be seen the reorder policy involves the same variables as the APIOBPCS method, but in this case with the advantages of using a PID controller. The inclusion of a PID is not a capricious choice; according to Kunreuther [1969], top level managers are found to act in a three-terms-control mode, similarly to a PID controller, using memory of past results (integral term), anticipating trends (derivative term), and as well as a proportional term for their future decisions.

Therefore, as shown in Fig. 2, the proposed controller has the basic elements of the APIOBPCS, demand forecast, and WIP compensation, and the PID controller is used as a decision rule maker. It is worth to note that, in this case, the PID actions are physically limited, that is, actions should not take values above 200 and below to 0. That is because we assume that the production system saturates when orders are greater than 200, and orders with negative values do not have a real meaning for inventory systems.

5. STABILITY AND CONTROL ERROR ANALYSIS

Stability analysis is a very important goal that must be satisfied by any control system. One approach to test this issue is by means of a control error analysis. If control errors of the closed-loop control system are *ultimately bounded* [Salle and Lefschetz, 1961], then the entire system has stability under a certain perturbation as demand estimate error.

The analysis is performed by using the Input-Output transfer function model of the system. Considering the demand as an input signal, the inventory system is a Multiple-Input Single-Output (MISO). For the stability analysis we first consider the control system presented in Disney and Towill [2003], which structure is shown in Fig. 1, and modeling the production process as a first order dynamic system instead of a pure delay.

Therefore, the transfer functions of the closed-loop model are obtained by applying the superposition theorem [Ogata, 1997, Kuo, 1980]. That is,

$$G_{I,I_{ref}} = \frac{1/T_i G_1 G_i}{1+G_1G_i1/T_i};\quad G_{I,D} = \frac{G_i}{1+G_1G_i1/T_i};\quad (13)$$
$$G_{I,\hat{D}} = \left(1+\frac{\bar{T}_p}{T_w}\right)\frac{G_1G_i}{1+G_1G_i1/T_i};$$

where,

$$G_1 = \frac{1}{T_p s + (1+T_p/T_w)} \qquad \text{and} \qquad G_i = \frac{1}{s}.$$

In equation (13), $G_{I,I_{ref}}$, $G_{I,\hat{D}}$ and $G_{I,D}$ represents the transfer functions relating the inventory output (I) to the desired inventory level (I_{ref}), the output to the estimated demand ($\hat{D}$), and output to the demand (D) respectively. G_1 is just an intermediate auxiliary transfer function. Then, the system output can be expressed as,

$$I = \frac{1/T_iG_1G_i}{1+G_1G_i1/T_i}I_{ref} - \frac{G_i}{1+G_1G_i1/T_i}D + \frac{(1+\bar{T}_p/T_w)G_1G_i}{1+G_1G_i1/T_i}\hat{D}. \quad (14)$$

Therefore, by using (14), and after some mathematical manipulation it is possible to obtain an expression of the inventory control errors as a function of the demand estimate error as is shown in (15).

$$E_{Inv} = \frac{G_i}{1+G_1G_i1/T_i}E_{Dem} - \frac{G_i}{1+G_1G_i1/T_i}\hat{D} + \left(1+\frac{\bar{T}_p}{T_w}\right)\frac{G_1G_i}{1+G_1G_i1/T_i}\hat{D} - \frac{1}{1+G_1G_i1/T_i}I_{ref}. \quad (15)$$

The maximum error E_{Inv}, independently of the values of E_{Dem}, $\hat{D}$ and I_{ref}, will be achieved when the transfer function operators have their maximum values. These maximum values can be obtained by using ∞-Norm ($\|\cdot\|_\infty$), defined as $\|H(s)\|_\infty = \max_\omega |H(j\omega|)$, [Vidyasagar, 1993] in (15). Then, applying norm properties, and taking into account the values used in the model ($T_p = \hat{T}_p = T_w = T_i = 1$), (15) can be reduced to (16)

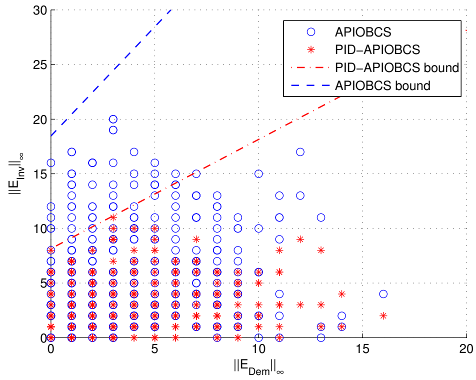

Fig. 3. Errors are bounded

$$\|E_{Inv}\|_\infty \le \left\|\frac{G_i}{1+G_1G_i1/T_i}\right\|_\infty \|E_{Dem}\|_\infty - \left\|\frac{G_i}{1+G_1G_i1/T_i}\right\|_\infty \|\hat{D}\|_\infty + \quad (16)$$
$$\left\|\left(1+\frac{\bar{T}_p}{T_w}\right)\frac{G_1G_i}{1+G_1G_i1/T_i}\right\|_\infty \|\hat{D}\|_\infty + \left\|\frac{1}{1+G_1G_i1/T_i}\right\|_\infty \|I_{ref}\|_\infty.$$

Finally, we can obtain a boundary for the inventory control errors for the APIOBPCS control system as

$$\|E_{Inv}\|_\infty \le 2\,\|E_{Dem}\|_\infty + 1.1533\,\|I_{ref}\|_\infty. \quad (17)$$

A similar procedure can be performed for the proposed PID-APIOBPCS presented in this work. Resulting,

$$G_{I,I_{ref}} = \frac{G_{1PID}G_i}{1+G_{1PID}G_i};\quad G_{I,\hat{D}} = \frac{G_{1PID}G_i}{1+G_{1PID}G_i} \quad (18)$$
$$G_{I,D} = \frac{G_i}{1+G_{1PID}G_i};$$

where,

$$G_{1PID} = \frac{G_{PID}G_p}{1+G_{PID}G_pT_p};\quad G_{PID} = \frac{K_ds^2+K_ps+K_i}{s};$$
$$G_p = \frac{1}{T_ps+1} \qquad \text{and} \qquad G_i = \frac{1}{s}. \quad (19)$$

Then, the expression of the system output as a function of signals I_{ref}, D and $\hat{D}$ is,

$$I = \frac{G_{1PID}G_i}{1+G_{1PID}G_i}I_{ref} - \frac{G_i}{1+G_{1PID}G_i}D + \frac{G_{1PID}G_i}{1+G_{1PID}G_i}\hat{D}. \quad (20)$$

Once again, by taking ∞-Norm and the norm properties, using typical values for $K_p = (30); K_d = (1); K_i = (10)$; $T_p = 1$, and performing the same steps as in the API-OBPCS case, the equation that relates the demand estimate errors to the inventory level errors can be obtained by

$$\|E_{Inv}\|_\infty \le 1\,\|E_{Dem}\|_\infty + 1.0185\,\|I_{ref}\|_\infty. \quad (21)$$

Then, evaluating the expression given by (17) and (21)in a graphical interpretation is possible to analyze the stability problem for the inventory control system.

5.1 Main results

As it can be seen in Fig. 3, inventory level errors are bounded for both cases by the straight line, given by equations (17) and (21). In inventory models, the desired inventory level is usually arbitrarily chosen, based on demand requirements and storage capabilities. The equations above presented can be used to set the value of the desired inventory level (the value of the abscise) at an arbitrarily low value, provided that the out-of-stocks are avoided. In this figure it is clear that for the APIOBPCS model, the minimum inventory level must be chosen around 16 units, while for the PID-APIOBPCS that value can be as low as 8 units. This is an important aspect in the inventory problem. In addition, for both cases the inventory level error is bounded by the demand prediction error, but in the case of the PID-APIOBPCS, errors in the demand prediction have less effects on the inventory level. In order to prevent out-of-stock situations, the desired inventory level should be used as a design parameter and should be chosen looking at the prediction error, that is, *the higher this error is, the higher the desired inventory level must be chosen.*

6. SIMULATION RESULTS

To show the feasibility and performance of the proposed inventory controller, as well as the stability properties obtained in the preceding theoretical development, a simulation study has been carried out using a Matlab-Simulink model. The joint Dual Extended Kalman filter was implemented in a Matlab S-Function, using the model given in (7), and the EKF equations (8), (9) and (10). Parameters $\mathbf{R}^{\upsilon}$ and $\mathbf{R}^{\eta}$ were used as design parameters, and set to 10 and 40 respectively.

The time-series model used to approximate the demand is given by Eq. (4), and its forecasting is performed by the EKF. The demand signal was generated by a sum of *sin* and *cos* terms, with different amplitudes, phases and frequencies. A Poisson noise, with $\lambda = 10$, was also added to the seasonal signals.

For all simulation runs, the inventory level set-point was set to 20 units, and the PID action limited to a maximum of 200 units, assuming that this is the capacity of the production system. In addition, demand signal has been added with an extra term, representing sudden stochastic changes on the value of demand.

6.1 EOQ reorder policy

In order to evaluate performance and advantages of using the proposed PID-APIOBPCS controller, a simulation of a production-inventory system controlled by the EOQ $(\hat{s}, \hat{S})$ policy is compared. Simulation is run for 365 days (one year), and results are shown in Fig. 4. In general, this is an acceptable policy for controlling and ideal inventory production system, but in presence of delays in the production line and a variable demand, its good performance is degraded. Note that in Fig. 4 the EOQ policy is far from being effective due to inventory level often falls below zero, meaning that there are many lost sales and, although the production system is always working at his maximum capacity, the inventory falls a number of times in stock-out situations.

6.2 APIOBPCS reorder policy

In this point is presented the APIOBPCS control system performance. The gain values were all set to one, due to the fact that those values are related to production and lead times. Desired inventory level was set to 20. Results presented in Fig. 5 shown a good performance for this control system. The inventory level stays stable around 20 units, and compared to the previous case, inventory level seldom falls below 0. The bullwhip effect is appreciated in some peaks, caused by abrupt changes in demand, but they are canceled in around 7 days.

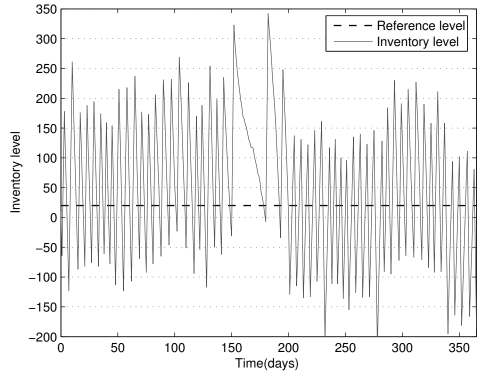

Fig. 4. Inventory level using an $(\hat{s}, \hat{S})$ policy

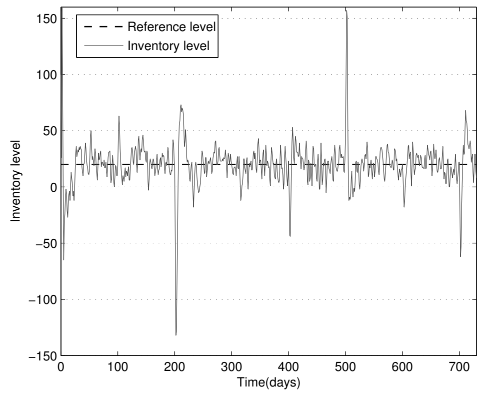

Fig. 5. Inventory level for the APIOBPCS method

6.3 PID-APIOBPCS reorder policy

Finally, the proposed control system is tested under simulation. PID parameters were set to $K_P = 30; K_I = 1$; $K_D = 10$. These values give a good response in terms of dampness and speed. The desired inventory level was, again, set to 20 units. The results are presented in Fig. 6. Note in this case that the inventory level is more stable. Bullwhip effect has diminished a bit compared to the previous PID and APIOBPCS cases: peaks still exist, but they are smaller and are canceled in around 5 days. Although there are some inventory level values below zero, these are not as many as in the previous analysis. In this case it is kept an acceptable level of lost sales, and the desired inventory level should be lower than the EOQ policy, only a PID controller, or with the APIOBPCS approach.

6.4 Simulation results evaluation

To show the advantage of the proposed controller, the results for the different controllers are compared follow-

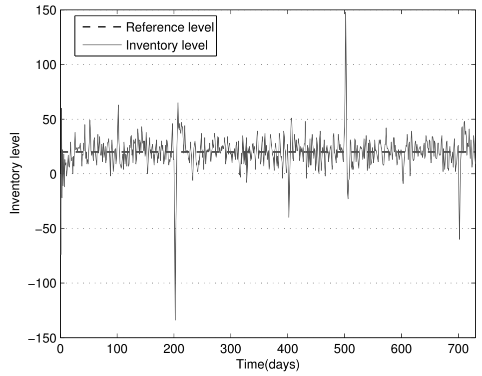

Fig. 6. Inventory for PID-APIOBPCS controller.

Table 1. Inventory Cost System

Policy / Costs	LS costs	IL costs	**Total Cost**
$(\hat{\mathbf{s}}, \hat{\mathbf{S}})$	80517	12994	93511
PID	8082	4475	12560
APIOBPCS	3357	4405	7762
PID-APIOBPCS	2010	4211	6221

ing the next procedure. Suppose that the inventory cost function is defined as,

$$C = \sum_{i=0}^{T_f} \{3LS_i + 0.3IL_i\} \tag{22}$$

where, LS_i means *Lost Sales* and represent the number of units below zero in the inventory level, and IL_i is the inventory level at any sample time. The first term of the functional is the cost of lost sales, and represent not only an economic quantity but also a measure on service quality and customer satisfaction level. The second one, represent the cost of inventory daily maintenance. Table 1 shows the results after applying this cost funcional for each control system approach, with $T_f = 730 = 2\,\text{years}$. Note that proposed controller, PID-APIOBPCS, shows the lowest cost. These results agree with the conclusions obtained from previous experiments, where the PID-APIOBPCS shows a better performance than the other approaches.

7. CONCLUSIONS

In this paper, an approach and a systematic design methodology to obtain a reorder policy for inventory systems based on the APIOBPCS control scheme was presented. The new reorder policy includes a PID controller and an estimate of the demand prediction obtained by using a joint dual EKF. This new approach control is called PID-APIOBPCS. An explicit evaluation of control error in terms of the demand prediction error and design parameters was also performed. To show the practical feasibility and performance of the proposed control algorithm as well as stability properties obtained in the present work, a simulation study was carried out for a simple production-inventory system. The proposed policy was compared to classical reorder policies such as an $(\hat{s}, \hat{S})$ policy, and the APIOBPCS policy. The results show the practical feasibility and good performance of the proposed approach to production-inventory systems.

Future research will include more complex models for the production-inventory systems, such as multiple-echelon and multiple-products production-inventory systems, and the inclusion in the design methodology of technics of optimal control to obtain an optimum operative condition for the controller, as well as, for the planning of the desired inventory level. Controllers should also have the possibility of managing backordering and saturation in a better way. Improvement on the demand prediction is also a pending issue.

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