Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Nonlinear Output Robust Regulation of Ground Vehicles in Presence of Disturbances and Parameter Uncertainties ⋆

C. Acosta Lua [*], **B. Castillo–Toledo** [*] **and S. Di Gennaro** [**]

[*] *Centro de Investigación y de Estudios Avanzados – CINVESTAV del IPN, Unidad Guadalajara, Av. Cientifíca, Col. El Bajío, Zapopan, 45010, Jalisco, México. E.mail:* {cacosta, toledo}@gdl.cinvestav.mx

[**] *Department of Electrical and Information Engineering and Center of Excellence DEWS, University of L'Aquila, Poggio di Roio, 67040 L'Aquila, Italy. E.mail:* digennar@ing.univaq.it

Abstract: In this paper a controller based on the so–called robust or structurally stable regulation theory is designed. The ground vehicle motion control is reformulated as a tracking problem of a desired reference, generated by an external system. Moreover, the disturbance acting on the vehicle is supposed to be modeled, i.e. unknown but with a known structure, as happens in many typical situations. The use of immersion techniques eliminates the dependence of the controller on parameters, so obtaining a controller ensuring zero tracking error. Since an immersion for the designed control law can not be easily determined, in this paper we consider the immersion of an approximate expression of the control, so obtaining a bounded tracking error.

Keywords: Ground vehicles, Trajectory tracking, Regulator problem.

1. INTRODUCTION

Vehicle motion control has become an important problem in automotive control applications. Such a control is made possible thanks to the introduction of various "by–wire" subsystems, such as steer–by–wire, brake–by–wire, etc. These represent the electronic equivalent of existing mechanical and hydraulic subsystems.

In the brake stand–alone case there are examples in the literature of linear or nonlinear systems. For linear brake–alone systems, the most common control approach is a PD controller which guarantees simplicity of design, affordable tuning and robustness. These controllers, however, are difficult to integrate with other systems, due to their local validation (van Zanten et al. [1998]). There are many types of nonlinear brake–alone systems, such as Adaptive Braking Systems, Anti–lock Braking System (ABS), etc., developed for improving vehicular steerability and stability by preventing wheel lock in critical circumstances such as for slippery road conditions during braking (Mauer [1995]).

In the case of steer–by–wire subsystems, dual servo–motors are used as steering mechanism and drive interface, so eliminating the connection between the driver and the wheel assembly. This decoupling allows the introduction of actuators such as the active front steer (AFS) or steer–by–wire (SBW), which impose to the wheels a steer angle given by the sum of that imposed by the driver and that imposed by a controller, in order to track a desired vehicle reference path. Analogously, the brake–by–wire subsystem allows the active use of brakes in order to impose to the vehicle a negative longitudinal force. This force determine a yaw moment which can be possibly used to improve a reference tracking. Clearly, servomotor–based steering systems may help to improve lateral vehicle responsiveness and, principally, occupant safety.

Various control architectures have been proposed with the purpose of enhancing vehicle steering. In Ackermann et al. [1995] linear and nonlinear controls were developed for the steering system. In Setlur et al. [2006] the problem of tracking a reference trajectory was solved using a Lyapunov–based control design. In Burgio and Zegelaar [2006], input–output linearizing feedback was proposed for the design of a based integrated vehicle controller, with steering (AFS, SBW) and brakes actuator. In Acosta-Lua et al. [2007] it is showed that ground vehicle motion control can be reformulated as a tracking problem of a desired reference, generated by an external system. Referring to this last paper, it must be stressed that the control problem considered in the present work is particularly challenging due to the presence of parameter uncertanties/variations and to the presence of disturbances (wind, etc.) acting on the vehicle. An important example of external disturbance is the crosswind which, in particular cases, can be particularly strong and can deflect the vehicle's trajectory, affecting the vehicle's stability and generate collisions with peripheries (barriers, curbstones, etc.) or other road users (Hanke et al. [2001], Bosch [1996]).

In this paper we design of a controller for tracking a desired yaw reference, while rejecting disturbances like

⋆ Work partially supported by CONACYT (Project 46538) and "Secreteria de Relaciones Exteriores" (S.R.E.), México, by "Consiglio Nazionale delle Ricerche" (C.N.R.) and "Ministero degli Affari Esteri" (M.A.E.), Italy, and by the HYCON Network of Excellence, contract number FP6–IST–511368.

crosswind and the effects of parameter uncertainties. For, we use the so–called robust or structurally stable regulation theory (Isidori and Byrnes [1990], Huang and Rugh [1992]), to tackle the particular problem. The motion control can be naturally recast as a tracking problem of a desired reference, generated by an external system. Moreover, in many typical situations and with a desired order of approximation, the disturbance acting on the vehicle can be assumed to be modeled with a known structure. This allows the use of immersion techniques in order to eliminate the dependence of the controller on parameters which are uncertain or slowly varying. This brings to the design of a controller ensuring zero tracking error. However, the immersion is the weak point of this "design process". In fact, very often this immersion is difficult or even impossible to find. For this reason in this paper we consider the immersion of an approximated expression of the control law ensuring the exact tracking.

The paper is organized as follows. In Section 2 the mathematical model of a ground vehicle and disturbance is recalled, and the control problem is formulated. In Section 3 some aspects of robust regulation are recalled, while in Section 4 the control problem is solved. Simulations are presented in Section 5, and some comments conclude the paper.

2. MATHEMATICAL MODEL OF THE VEHICLE DYNAMICS

The mathematical model of a ground vehicle can be obtained considering a rigid body connected to the ground trough tires. The essence of the vehicle dynamics can be summarized by the yaw and lateral dynamics, as described in the so–called *Single track model* or *Bicycle model* so considering only three degrees of freedom. Following Burgio and Zegelaar [2006], Setlur et al. [2006], we considered as actuator an active front steer (AFS) and steer by wire (SBW), which can force an incremental steer angle δ_c and active brakes, which impose negative longitudinal force, determining a resulting yaw momentum M_b.

In order to explore the application of the regulation theory to vehicle control, we have considered a simple model under some simplifying assumptions, usually considered in the literature (Burgio and Zegelaar [2006])

- ($H.1$) Roll and pitch dynamics are neglected;
- ($H.2$) The motion takes place on an horizontal surface;
- ($H.3$) The longitudinal velocity v_x is piecewise constant;
- ($H.4$) The system is rigid;
- ($H.5$) The force exerted by the tire do not saturate.

The vehicle dynamics are hence given by the following model (Burgio and Zegelaar [2006])

$$\begin{aligned} \dot{\psi} &= \omega_\psi \\ J\dot{\omega}_\psi &= \mu\Big[F_f(\alpha_f, N_f, k_f)l_f - F_r(\alpha_r, N_r, k_r)l_r\Big] + M_b + d \\ m(\dot{v}_y + v_x\omega_\psi) &= \mu\Big[F_f(\alpha_f, N_f, k_f) + F_r(\alpha_r, N_r, k_r)\Big] \end{aligned} \tag{1}$$

where

m	Vehicle mass
J	Vehicle Inertia momentum
l_f, l_r	Front and rear vehicle lengths
v_x, v_y	Vehicle longitudinal and lateral velocities
ψ, ω_ψ	Yaw angle and yaw rate
μ	Maximum tire-road friction coefficient
k_f, k_r	Vectors of the left and right tire longitudinal slips
N_f, N_r	Vectors of the left and right tire vertical forces
δ_d, δ_r	Road wheel angles due to the driver and to controller
M_b	Yaw moment
d	External disturbance (typically due to the wind).

Finally, the tire front and rear lateral forces F_f, F_r depend on the longitudinal slips, the tire slip angles (α_f, α_r), and the tire vertical forces

$$\alpha_f = \delta_d + \delta_c - \frac{v_y + l_f\omega_\psi}{v_x}, \quad \alpha_r = -\frac{v_y - l_f\omega_\psi}{v_x}.$$

It is common to assume that $F_f(x, \delta_d + \delta_r, F_{zf}, k_f)$ is invertible with respect to δ_c (Burgio and Zegelaar [2006]), namely the solution of $F_f(x, \delta_d + \delta_r, F_{zf}, k_f) = \bar{F}_f$ for a fixed $\bar{F}_f$ is unique and given by

$$\delta_c = -\delta_d + \frac{v_y + l_f w_\psi}{v_x} + F_f^{-1}\big(F_{f0}, N_f, k_f\big).$$

Under this hypothesis of invertibility (Burgio and Zegelaar [2006]), F_f can be regarded as an input, since it is possible to determine the value of δ_c necessary to impose a desired force $\bar{F}_f$.

Here x, y, z denote the axes of a reference frame fixed with the vehicle. In order to consider the external disturbance d, we will introduce a reference frame fixed with respect to the road. Let X, Y, Z denote the axes of this frame. The yaw angle ψ determines the attitude of the reference frame fixed with the vehicle with respect to that fixed with the road.

The external disturbance d is typically due to the wind. Blasts of lateral wind, or crosswind, can determine dangerous situations (Hanke et al. [2001]). It is usual to consider 12 ranges of wind force, depending on the wind velocity, according to the so–called Beaufort scale (Bft). The occurrence of wind blasts can be dangerous with regard to automobile safety, and the consequent lateral offset has to be reduced by the control system in order to reduce the possibility of accidents. Let us consider the occurrence of wind with respect to the ground, with constant velocity $v_W = \begin{pmatrix} v_{w,X} & v_{w,Y} & 0 \end{pmatrix}^T$ in the (X, Y, Z) frame. In the frame (x, y, z) fixed with the vehicle the wind velocity components are

$$\begin{aligned} v_{w,x} &= v_{w,X}\,\mathrm{c}\psi + v_{w,Y}\,\mathrm{s}\psi \\ v_{w,y} &= -v_{w,X}\,\mathrm{s}\psi + v_{w,Y}\,\mathrm{c}\psi \\ v_{w,z} &= 0 \end{aligned}$$

where $\mathrm{c}(\cdot)$, $\mathrm{s}(\cdot)$ stand for $\cos(\cdot)$ and $\sin(\cdot)$. The resulting wind velocity v_w is a combination of the apparent wind velocity v_x due to the vehicle forward motion plus the component $v_{w,x}$, and the apparent wind velocity v_y due to the vehicle lateral motion plus the crosswind velocity $v_{w,y}$, namely

$$v_w^2 = (v_x + v_{w,x})^2 + (v_y + v_{w,y})^2.$$

The crosswind induces a pitching moment, around the y direction, a roll moment in the x direction, and a yaw moment d (Bosch [1996], Hanke et al. [2001], Rajamani [2006]). Since we suppose that the roll and pitch dynamics can be neglected (assumption $(H.1)$), we will considered only the disturbance d. In terms of the front surface of the vehicle A_s, the overall length $l_f + l_r$ of the vehicle, the air density ρ, and the aerodynamic coefficient c_ψ, the expression of d is

$$d = A_s(l_f + l_r)\rho c_\psi v_w^2/2.$$

In the following, it will be useful to consider a change of coordinates, where in the place of v_y one considers the lateral velocity

$$v_n = v_y - l_{ns}\omega_\psi, \quad l_{ns} = \frac{J}{ml_f} \tag{2}$$

with l_{ns} the distance between the vehicle center of mass and the neutral steer point. Typically, this point is close to rear axle. Hence,

$$\begin{aligned} d = \alpha_0 v_w^2 &= \alpha_1 + \alpha_2 V_y^2 + \alpha_3 \mathrm{s}\psi + \alpha_4 \mathrm{c}\psi \\ &- \alpha_5 V_y \mathrm{s}\psi + \alpha_6 V_y \mathrm{c}\psi \end{aligned}$$

$$\begin{aligned} V_y &= \omega_\psi + v_n/l_{ns} & \alpha_0 &= A_s(l_f + l_r)\rho c_\psi/2 \\ \alpha_1 &= \alpha_0(v_x^2 + v_{w,X}^2 + v_{w,Y}^2) & \alpha_2 &= \alpha_0 l_{ns}^2 \\ \alpha_3 &= 2\alpha_0 v_x v_{w,Y} & \alpha_4 &= 2\alpha_0 v_x v_{w,X} \\ \alpha_5 &= 2\alpha_0 v_{w,X} l_{ns} & \alpha_6 &= 2\alpha_0 l_{ns} v_{w,Y}. \end{aligned}$$

Considering $x = \begin{pmatrix}\psi & \omega_\psi & v_n\end{pmatrix}^T$, $u = \begin{pmatrix}F_f & M_b\end{pmatrix}^T$ as state and input vectors, from (1),(2) one obtains the mathematical model of a vehicle

$$\begin{aligned} \dot{\psi} &= \omega_\psi \\ \dot{\omega}_\psi &= -a_1\omega_\psi + a_2 v_n + a_3\omega_\psi^3 - a_4 v_n^3 - a_5\omega_\psi^2 v_n \\ &+ a_6\omega_\psi v_n^2 + a_{13} + a_{14}V_y^2 + a_{15}\mathrm{s}\psi + a_{16}\mathrm{c}\psi \\ &+ a_{17}V_y\mathrm{s}\psi + a_{18}V_y\mathrm{c}\psi + b_1F_f + b_2M_b + b_4\varphi_r \\ \dot{v}_n &= a_7\omega_\psi - a_8 v_n - a_9\omega_\psi^3 + a_{10}v_n^3 + a_{11}\omega_\psi^2 v_n \\ &- a_{12}\omega_\psi v_n^2 - a_{19} - a_{20}V_y^2 - a_{21}\mathrm{s}\psi - a_{22}\mathrm{c}\psi \\ &- a_{23}V_y\mathrm{s}\psi - a_{24}V_y\mathrm{c}\psi - b_3M_b - b_5\varphi_r. \end{aligned} \tag{3}$$

The parameter expressions are give in Appendix. In (3) the rear tire lateral force F_r has been expanded up to the third order

$$\begin{aligned} F_r &= -C_\alpha \tanh\mathrm{atan}\frac{v_y - l_r\omega_\psi}{v_x} = -C_\alpha \tanh\mathrm{atan}\beta \\ &= -C_\alpha\beta + \frac{2}{3}C_\alpha\beta^3 + C_\alpha\varphi_r, \quad \beta = \frac{v_n - (l_r - l_{ns})\omega_\psi}{v_x} \end{aligned}$$

with C_α the lateral tire stiffness and φ_r the higher order terms in the expansion of the function $\tanh\mathrm{atan}(\cdot)$. However, note that no approximations of F_r has been considered. In the following we suppose that these parameters are uncertain, and their nominal values will be denoted by a_{i0}, b_{j0}, $i = 1, \cdots, 24$, $j = 1, \cdots, 5$.

The output to be controlled is the yaw angle $y = \psi$. The Robust Output Regulation Problem (RORP) for Ground Vehicles can be formulated as in Isidori [1995], and consists of having the output ψ asymptotically tracking the desired reference ψ_r, with a desired yaw rate $\omega_{\psi,r}$, and asymptotically rejecting the perturbation d, despite variations in the parameters of the system. At the same time, we will require that the lateral velocity v_n will tend to zero asymptotically. This has a clear physical interpretation. In the context of the regulator theory, this means to consider the tracking error $e = \psi - \psi_r$ and to determine a controller which force this error to zero.

Note that if the parameters which appear in the definition (2) can be considered known, it is possible to suppose $v_n - v_{r,n}$ a further output of the system, with $v_{r,n}$ a function tending asymptotically to zero. This would simplify the following developments.

3. THE ROBUST REGULATION OUTPUT PROBLEM FOR GROUND VEHICLES

As usual in the regulation theory, the reference signal is generated by a so–called exosystem

$$\begin{aligned} \dot{w} &= s(w) \\ \psi_r &= \psi_r(w). \end{aligned}$$

Equations (3) are in the form $\dot{x} = f(x, w, u, \mu)$, with μ the system parameter vector, having $e = \psi - \psi_r(w) = h(x, w, \mu)$ as output. Hereinafter we assume that the matrices

$$A_0 = \left[\frac{\partial f}{\partial x}\right]_{(0,0,0,0)} \quad B_0 = \left[\frac{\partial f}{\partial u}\right]_{(0,0,0,0)} \quad C_0 = \left[\frac{\partial h}{\partial x}\right]_{(0,0,0)}$$

stand for the nominal values of the linear part of the system, assumed at $\mu = 0$. The following result gives sufficient conditions for the existence of a solution to the RORP, in terms of the existence of a linear immersion (Isidori [1995]).

Proposition. The RORP is solvable by means of a linear controller if the pair (A_0, B_0) is stabilizable, the pair (A_0, C_0) is detectable, there exist mappings $x_{ss} = \pi(w, \mu)$ and $u_{ss} = \gamma(w, \mu)$, with $\pi(0, \mu) = 0$ and $\gamma(0, \mu) = 0$, both defined in a neighborhood $W^\circ \times \mathcal{P}$ of the origin, satisfying for all $(w, \mu) \in W^\circ \times \mathcal{P}$ the so–called regulator equations

$$\begin{aligned} \mathcal{L}_s\pi(w, \mu) &= f(\pi(w, \mu), w, \gamma(w, \mu), \mu) \\ 0 &= h(\pi(w, \mu), w, \mu) \end{aligned} \tag{4}$$

("$\mathcal{L}_s\pi$" represents the Lie derivative of π in the direction of s), and, for some set of q real numbers $a_0, a_1, \cdots, a_{q-1}$,

$$\begin{aligned} \mathcal{L}_s^q\gamma(w, \mu) &= a_0\gamma(w, \mu) + a_1\mathcal{L}_s\gamma(w, \mu) \\ &+ \cdots + a_{q-1}\mathcal{L}_s^{q-1}\gamma(w, \mu) \end{aligned} \tag{5}$$

and moreover the matrix

$$\begin{pmatrix} A_0 - \lambda I & B_0 \\ C_0 & 0 \end{pmatrix}$$

is nonsingular for every λ which is a root of the polynomial $p(\lambda) = \lambda^q + a_{q-1}\lambda^{q-1} + \cdots + a_1\lambda + a_0$ having non–negative real part. ◇

Hence, we need to check the stabilizability of the pair (A_0, B_0) and detectability of the pair (A_0, C_0) where in our case

$$A_0 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -a_{10} & a_{20} \\ 0 & a_{70} & -a_{80} \end{pmatrix}, \quad B_0 = \begin{pmatrix} 0 & 0 \\ b_{10} & b_{20} \\ 0 & -b_{30} \end{pmatrix}$$
$$C_0 = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}.$$

It is easy to check that the reachability and observability matrices have full rank, and then matrices K, G_1 can be designed so that $\big(A_0 + B_0K\big)$ and $\big(A_0 - G_1C_0\big)$ are Hurwitz. The next step is to determine the center manifold $x_{ss} = \pi(w, \mu) = \big(\pi_\psi(w, \mu) \quad \pi_{\omega_\psi}(w, \mu) \quad \pi_{v_n}(w, \mu)\big)^T$ and the steady–state control $u_{ss} = \gamma(w, \mu) = \big(\gamma_{F_f}(w, \mu) \quad \gamma_{M_b}(w, \mu)\big)^T$ solutions of the regulation equation (4), which in our case become

$$\begin{aligned}
\mathcal{L}_s\pi_\psi &= \pi_{\omega_\psi} \\
\mathcal{L}_s\pi_{\omega_\psi} &= -a_1\pi_{\omega_\psi} + a_2\pi_{v_n} + a_3\pi^3_{\omega_\psi} - a_4\pi^3_{v_n} - a_5\pi^2_{\omega_\psi}\pi_{v_n} \\
&+ a_6\pi_{\omega_\psi}\pi^2_{v_n} + a_{13} + a_{14}\pi^2_{V_y} + a_{15}\mathrm{s}\pi_\psi + a_{16}\mathrm{c}\pi_\psi \\
&+ a_{17}\pi_{V_y}\mathrm{s}\pi_\psi + a_{18}\pi_{V_y}\mathrm{c}\pi_\psi + b_1\gamma_{F_f} + b_2\gamma_{M_b} + b_4\bar{\varphi}_r \\
\mathcal{L}_s\pi_{v_n} &= a_7\pi_{\omega_\psi} - a_8\pi_{v_n} - a_9\pi^3_{\omega_\psi} + a_{10}\pi^3_{v_n} \\
&+ a_{11}\pi^2_{\omega_\psi}\pi_{v_n} - a_{12}\pi_{\omega_\psi}\pi^2_{v_n} - a_{19} - a_{20}\pi^2_{V_y} - a_{21}\mathrm{s}\pi_\psi \\
&- a_{22}\mathrm{c}\pi_\psi - a_{23}\pi_{V_y}\mathrm{s}\pi_\psi - a_{24}\pi_{V_y}\mathrm{c}\pi_\psi - b_3\gamma_{M_b} - b_5\bar{\varphi}_r \\
0 &= \pi_\psi - \psi_r
\end{aligned} \tag{6}$$

with $\pi_{V_y} = \pi_{\omega_\psi} + \pi_{v_n}/l_{ns}$, and $\bar{\varphi}_r = \varphi_r\big|_{\substack{\omega_\psi = \pi_{\omega_\psi} \\ v_n = \pi_{v_n}}}$. From the last and the first equations of (6) one easily gets $\pi_\psi = \psi_r$, $\pi_{\omega_\psi} = \mathcal{L}_s\psi_r = (\partial\psi_r/\partial w)s(w)$. From the remaining equations

$$\begin{aligned}
\mathcal{L}^2_s\psi_r &= -a_1\mathcal{L}_s\psi_r + a_2\pi_{v_n} + a_3(\mathcal{L}_s\psi_r)^3 - a_4\pi^3_{v_n} \\
&- a_5(\mathcal{L}_s\psi_r)^2\pi_{v_n} + a_6(\mathcal{L}_s\psi_r)\pi^2_{v_n} + a_{13} + a_{14}\tilde{\pi}^2_{V_y} \\
&+ a_{15}\mathrm{s}\psi_r + a_{16}\mathrm{c}\psi_r + a_{17}\tilde{\pi}_{V_y}\mathrm{s}\psi_r + a_{18}\tilde{\pi}_{V_y}\mathrm{c}\psi_r \\
&+ b_1\gamma_{F_f} + b_2\gamma_{M_b} + b_4\tilde{\varphi}_r \\
\mathcal{L}_s\pi_{v_n} &= a_7\mathcal{L}_s\psi_r - a_8\pi_{v_n} - a_9(\mathcal{L}_s\psi_r)^3 + a_{10}\pi^3_{v_n} \\
&+ a_{11}(\mathcal{L}_s\psi_r)^2\pi_{v_n} - a_{12}(\mathcal{L}_s\psi_r)\pi^2_{v_n} - a_{19} - a_{20}\tilde{\pi}^2_{V_y} \\
&- a_{21}\mathrm{s}\psi_r - a_{22}\mathrm{c}\psi_r - a_{23}\tilde{\pi}_{V_y}\mathrm{s}\psi_r - a_{24}\tilde{\pi}_{V_y}\mathrm{c}\psi_r \\
&- b_3\gamma_{M_b} - b_5\tilde{\varphi}_r
\end{aligned}$$

where $\tilde{\pi}_{V_y} = \mathcal{L}_s\psi_r + \pi_{v_n}/l_{ns}$ and $\tilde{\varphi}_r = \varphi_r\big|_{\substack{\omega_\psi = \mathcal{L}_s\psi_r \\ v_n = \pi_{v_n}}}$ one easily works out the steady state control components γ_{F_f}, γ_{M_b}. For, from the second equation note first that π_{v_n} always exists since $a_8 > 0$. Then, consider that the control requirements are fulfilled considering a function $\pi_{v_n}(w)$ such that $\lim_{t\to 0} \pi_{v_n}(w(t)) = 0$ for every initial condition $w(0)$. Once π_{v_n} has been fixed, one gets

$$\gamma(w, \mu) = \begin{pmatrix} b_1 & b_2 \\ 0 & -b_3 \end{pmatrix}^{-1} \begin{pmatrix} \kappa_1 \\ \kappa_2 \end{pmatrix} = \begin{pmatrix} \dfrac{1}{b_1}\kappa_1 + \dfrac{b_2}{b_1b_3}\kappa_2 \\ -\dfrac{1}{b_3}\kappa_2 \end{pmatrix}$$

where

$$\begin{aligned}
\kappa_1 &= \mathcal{L}^2_s\psi_r + a_1\mathcal{L}_s\psi_r - a_2\pi_{v_n} - a_3(\mathcal{L}_s\psi_r)^3 + a_4\pi^3_{v_n} \\
&+ a_5(\mathcal{L}_s\psi_r)^2\pi_{v_n} - a_6(\mathcal{L}_s\psi_r)\pi^2_{v_n} - a_{13} - a_{14}\tilde{\pi}^2_{V_y} \\
&- a_{15}\mathrm{s}\psi_r - a_{16}\mathrm{c}\psi_r - a_{17}\tilde{\pi}_{V_y}\mathrm{s}\psi_r - a_{18}\tilde{\pi}_{V_y}\mathrm{c}\psi_r - b_4\tilde{\varphi}_r \\
\kappa_2 &= \mathcal{L}_s\pi_{v_n} - a_7\mathcal{L}_s\psi_r + a_8\pi_{v_n} + a_9(\mathcal{L}_s\psi_r)^3 - a_{10}\pi^3_{v_n} \\
&- a_{11}(\mathcal{L}_s\psi_r)^2\pi_{v_n} + a_{12}(\mathcal{L}_s\psi_r)\pi^2_{v_n} + a_{19} + a_{20}\tilde{\pi}^2_{V_y} \\
&+ a_{21}\mathrm{s}\psi_r + a_{22}\mathrm{c}\psi_r + a_{23}\tilde{\pi}_{V_y}\mathrm{s}\psi_r + a_{24}\tilde{\pi}_{V_y}\mathrm{c}\psi_r + b_5\tilde{\varphi}_r.
\end{aligned}$$

It is clear that this control does not ensure the fulfillment of the regulation requirements in presence of parameter perturbations of the parameter vector μ. For, an appropriate immersion of $\gamma(w, \mu)$ has to be determined.

3.1 Approximate Solution to the RORP

Unfortunately, in the case under study the term $\tilde{\varphi}_r$, due to F_r, renders difficult the determination of such an immersion. It is hence natural to consider the following approximation

$$\gamma_a(w, \mu) = \begin{pmatrix} \gamma_{F_f,a} \\ \gamma_{M_b,a} \end{pmatrix} = \begin{pmatrix} \dfrac{1}{b_1}\kappa_{a,1} + \dfrac{b_2}{b_1b_3}\kappa_{a,2} \\ -\dfrac{1}{b_3}\kappa_{a,2} \end{pmatrix} \tag{7}$$

where

$$\begin{aligned}
\kappa_{a,1} &= \mathcal{L}^2_s\psi_r + a_1\mathcal{L}_s\psi_r - a_2\pi_{v_n} - a_3(\mathcal{L}_s\psi_r)^3 + a_4\pi^3_{v_n} \\
&+ a_5(\mathcal{L}_s\psi_r)^2\pi_{v_n} - a_6(\mathcal{L}_s\psi_r)\pi^2_{v_n} - a_{13} - a_{14}\tilde{\pi}^2_{V_y} \\
&- a_{15}\mathrm{s}\psi_r - a_{16}\mathrm{c}\psi_r - a_{17}\tilde{\pi}_{V_y}\mathrm{s}\psi_r - a_{18}\tilde{\pi}_{V_y}\mathrm{c}\psi_r \\
\kappa_{a,2} &= \mathcal{L}_s\pi_{v_n} - a_7\mathcal{L}_s\psi_r + a_8\pi_{v_n} + a_9(\mathcal{L}_s\psi_r)^3 - a_{10}\pi^3_{v_n} \\
&- a_{11}(\mathcal{L}_s\psi_r)^2\pi_{v_n} + a_{12}(\mathcal{L}_s\psi_r)\pi^2_{v_n} + a_{19} + a_{20}\tilde{\pi}^2_{V_y} \\
&+ a_{21}\mathrm{s}\psi_r + a_{22}\mathrm{c}\psi_r + a_{23}\tilde{\pi}_{V_y}\mathrm{s}\psi_r + a_{24}\tilde{\pi}_{V_y}\mathrm{c}\psi_r.
\end{aligned}$$

Using this approximated control, with $\gamma_a(w, \mu) = 0$, equations (4) are not verified anymore, since

$$\frac{\partial\pi(w, \mu)}{\partial w}s(w) \neq f(\pi(w, \mu), w, \gamma_a(w, \mu), \mu)$$
$$0 = h(\pi(w, \mu), w, \mu)$$

namely $\pi(w, \mu)$ is not rendered invariant by $\gamma_a(w, \mu)$. Hence, even if the control u would force the system trajectory on $\pi(w, \mu)$, the flow do not remain on it, and a nonzero error is determined.

3.2 A Case study

For the sake of clarity, in the following we determine an immersion for $\gamma_a(w, \mu)$ for a specific reference path, corresponding to $\psi_r = \lambda_r \mathrm{s}\omega t = w_1$ with $\lambda_r = 1/6$ and the exosystem given by

$$\dot{w}_1 = \omega w_2$$
$$\dot{w}_2 = -\omega w_1.$$

Moreover, it is convenient to choose $\pi_{v_n} = 0$. This is congruent with the constraints previously commented on π_{v_n}. Therefore, (7) becomes

$$\begin{aligned}
\gamma_a &= \begin{pmatrix} \gamma_{F_f,a}(w, \mu) \\ \gamma_{M_b,a}(w, \mu) \end{pmatrix} \\
\gamma_{F_f,a} &= \beta_0 - \beta_1 w_1 + \beta_2 w_2 + \beta_3 w_2^2 + \beta_4 w_2^3 + \beta_5 \mathrm{s}w_1 \\
&+ \beta_6 \mathrm{c}w_1 + \beta_7 w_2 \mathrm{s}w_1 + \beta_8 w_2 \mathrm{c}w_1 \\
\gamma_{M_b,a} &= -\theta_0 + \theta_1 w_2 - \theta_2 w_2^2 - \theta_3 w_2^3 - \theta_4 \mathrm{s}w_1 - \theta_5 \mathrm{c}w_1 \\
&- \theta_6 w_2 \mathrm{s}w_1 - \theta_7 w_2 \mathrm{c}w_1
\end{aligned} \tag{8}$$

with the β_i and θ_i given in Appendix. Finally, the determination of an immersion is easier if $\gamma_{F_f,a}(w, \mu)$, $\gamma_{M_b,a}$, are polynomials in w_1, w_2. Hence, we will assume the approximations $\mathrm{s}w_1 \simeq w_1 - \frac{1}{3!}w_1^3$, $\mathrm{c}w_1 \simeq 1 - \frac{1}{2!}w_1^2$. Therefore, from (8)

$$\gamma_{F_f,a} = \beta_0 - \beta_1 w_1 + \beta_2 w_2 + \beta_3 w_2^2 + \beta_4 w_2^3 + (\beta_5 + \beta_7 w_2)\Big(w_1 - \frac{1}{3!}w_1^3\Big) + (\beta_6 + \beta_8 w_2)\Big(1 - \frac{1}{2!}w_1^2\Big)$$

$$\gamma_{M_b,a} = -\theta_0 + \theta_1 w_2 - \theta_2 w_2^2 - \theta_3 w_2^3 - (\theta_4 + \theta_6 w_2)\Big(w_1 - \frac{1}{3!}w_1^3\Big) - (\theta_5 + \theta_7 w_2)\Big(1 - \frac{1}{2!}w_1^2\Big)$$

and their immersions are given by $\dot\zeta_{12} = \Phi_{a,1}\zeta_1$, $\gamma_{F_f,a} = \Gamma_1\zeta_1$, and $\dot\zeta_2 = \Phi_{a,1}\zeta_2$, $\gamma_{M_b,a} = \Gamma_2\zeta_2$, respectively, with $\Phi_{a,1} = \Phi_{a,2}$, $\Gamma_1 = \Gamma_2$ and

$$\Phi_{a,1} = \begin{pmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ a_0 & a_1 & \cdots & a_9 \end{pmatrix}, \quad \Gamma_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}^T$$

$a_0 = a_1 = a_3 = a_5 = a_7 = a_9 = 0$, $a_2 = -576\omega^8$, $a_4 = -820\omega^6$, $a_6 = -273\omega^4$, $a_8 = -30\omega^2$.

A difficulty arises from the fact that $\Phi_{a,1}$, $\Phi_{a,2}$ have the same eigenvalues. The consequence is that the pair

$$\begin{pmatrix} A_0 & -B_0\Gamma \\ 0 & \Phi \end{pmatrix}, \begin{pmatrix} C_0 & 0 \end{pmatrix}$$

is not observable, and it is not possible to use the classical controller (Isidori [1995]). Therefore, an alternative controller is hereinafter proposed. Following (Acosta-Lua et al. [2007]), we first consider

$$B_{10} = \begin{pmatrix} 0 \\ b_{10} \\ 0 \end{pmatrix}, \quad B_{20} = \begin{pmatrix} 0 \\ b_{20} \\ -b_{30} \end{pmatrix}.$$

Hence, the controller is

$$\begin{aligned} \dot\xi_{11} &= (A_0 + B_{10}K_1 - G_{11}C_0)\xi_{11} + B_{20}u_2 + G_{11}e \\ \dot\xi_{12} &= -G_{12}C_0\xi_{11} + \Phi_1\xi_{12} + G_{12}e \\ \dot\xi_{21} &= (A_0 + B_{20}K_2 - G_{21}C_0)\xi_{21} + B_{10}u_1 + G_{21}e \\ \dot\xi_{22} &= -G_{22}C_0\xi_{21} + \Phi_2\xi_{22} + G_{22}e \\ u_1 &= K_1\xi_{11} + \Gamma_1\xi_{12} \\ u_2 &= K_2\xi_{21} + \Gamma_2\xi_{22} \end{aligned} \tag{9}$$

where K_1, K_2 are such that the matrix

$$\begin{aligned} A_c &= A_0 + B_{10}K_1 + B_{20}K_2 \\ &= A_0 + B_0K \end{aligned} \qquad K = \begin{pmatrix} K_1 \\ K_2 \end{pmatrix} \tag{10}$$

is Hurwitz, and G_{11}, G_{12} G_{21}, G_{22} make stable the matrices

$$A_{d,i} = \begin{pmatrix} A_0 & -B_{i0}\Gamma_i \\ 0 & \Phi_i \end{pmatrix} - \begin{pmatrix} G_{i1} \\ G_{i2} \end{pmatrix} \begin{pmatrix} C_0 & 0 \end{pmatrix} \tag{11}$$

$i = 1, 2$. Let us show that the proposed controller solves the RORP. For, note that the controlled dynamics are

$$\begin{aligned} \dot x &= A_0x + B_{10}u_1 + B_{20}u_2 + f_0(x, u, w, \mu) \\ \dot\xi_{11} &= (A_0 + B_{10}K_1 - G_{11}C_0)\xi_{11} + G_{11}e + B_{20}K_2\xi_{21} + B_{20}\Gamma_2\xi_{22} \\ \dot\xi_{12} &= -G_{12}C_0\xi_{11} + \Phi_1\xi_{12} + G_{12}e \\ \dot\xi_{21} &= (A_0 + B_{20}K_2 - G_{21}C_0)\xi_{21} + G_{21}eB_{10}K_1\xi_{11} + B_{10}\Gamma_1\xi_{12} \\ \dot\xi_{22} &= -G_{22}C_0\xi_{21} + \Phi_2\xi_{22} + G_{22}e \\ u_1 &= K_1\xi_{11} + \Gamma_1\xi_{12} \\ u_2 &= K_2\xi_{21} + \Gamma_2\xi_{22}. \end{aligned}$$

Considering that $e = C_0x + h_0(x, w)$, and setting $w = 0$, $\mu = 0$ (since the solution $\pi(w, \mu)$ exists for every value of μ in a neighborhood of $\mu = 0$) one works out

$$\begin{aligned} \dot x &= A_0x + B_{10}K_1\xi_{11} + B_{10}\Gamma_1\xi_{12} + B_{20}K_2\xi_{21} + B_{20}\Gamma_2\xi_{22} + T_{nl,0} \\ \dot\xi_{11} &= (A_0 + B_{10}K_1 - G_{11}C_0)\xi_{11} + G_{11}C_0x + B_{20}K_2\xi_{21} + B_{20}\Gamma_2\xi_{22} + T_{nl,11} \\ \dot\xi_{12} &= -G_{12}C_0\xi_{11} + \Phi_1\xi_{12} + G_{12}C_0x + T_{nl,21} \\ \dot\xi_{21} &= (A_0 + B_{20}K_2 - G_{21}C_0)\xi_{21} + G_{21}C_0x + B_{10}K_1\xi_{11} + B_{10}\Gamma_1\xi_{12} + T_{nl,12} \\ \dot\xi_{22} &= -G_{22}C_0\xi_{21} + \Phi_2\xi_{22} + G_{22}C_0x + T_{nl,22} \end{aligned}$$

where T_{nl} denotes the nonlinear terms. Considering the new variables

$$e_1 = x - \xi_{11}, \quad e_2 = x - \xi_{21}, \quad \eta_1 = -\xi_{12}, \quad \eta_2 = -\xi_{22}$$

one gets

$$\begin{aligned} \dot x &= A_cx - B_{10}K_1e_1 - B_{10}\Gamma_1\eta_1 - B_{20}K_2e_2 - B_{20}\Gamma_2\eta_2 + T_{nl,0} \\ \dot e_1 &= (A_0 - G_{11}C_0)e_1 - B_{10}\Gamma_1\eta_1 + \bar T_{nl,11} \\ \dot\eta_1 &= -G_{12}C_0e_1 + \Phi_1\eta_1 + \bar T_{nl,21} \\ \dot e_2 &= (A_0 - G_{21}C_0)e_2 - B_{20}\Gamma_2\eta_2 + \bar T_{nl,12} \\ \dot\eta_2 &= -G_{22}C_0e_2 + \Phi_2\eta_2 + \bar T_{nl,22} \end{aligned}$$

with A_c given by (10) and $\bar T_{nl}$ the nonlinear terms in the new coordinates. The dynamic matrix of the linear part is

$$\begin{pmatrix} A_c & -B_{10}K_1 & -B_{10}\Gamma_1 & -B_{20}K_2 & -B_{20}\Gamma_2 \\ 0 & A_0 - G_{11}C_0 & -B_{10}\Gamma_1 & 0 & 0 \\ 0 & -G_{12}C_0 & \Phi_1 & 0 & 0 \\ 0 & 0 & 0 & A_0 - G_{21}C_0 & -B_{20}\Gamma_2 \\ 0 & 0 & 0 & -G_{22}C_0 & \Phi_2 \end{pmatrix}$$

whose eigenvalues are those of A_c $A_{d,1}$ $A_{d,2}$ which are Hurwitz. This proves that the stability property is ensured. It remains to check the regulation property. However, as already mentioned, the center manifold is not rendered invariant by the approximate steady state control, so that the exact tracking can not be ensured, and a steady–state error will appear. In the simulation section it will be shown that in the case under study one can obtain errors reasonably small.

4. SIMULATION RESULTS

We considered simulations based on data from a prototype vehicle (Setlur et al. [2006], Lee et al. [2004], Rajamani [2006]). The nominal parameters are $m_0 = 1500$

Kg,$J_0 = 2830$ Kg m^2, $l_{f0} = 1.3$ m, $l_{r0} = 1.5$ m, $l_0 = l_{r0} + l_{f0}$, $C_{a0} = 6510$ N/rad, $v_{x0} = 28$ m/s, $\mu_{f0} = 0.66$, $l_{ns0} = J_0/\mu_0 l_{f0}$, $c_{\psi 0} = 0.8$, $\rho_0 = 1.2$ Kg/m^3, $A_s = 1.6 + 0.00056(m_0 - 765) = 2.0116$ m^2 (Rajamani [2006]), with while the real ones are $m = 1.1\, m_0$ $J = 1.05\, J_0$, $l_r = l_{r0}$, $l_f = l_{f0}$, $C_a = 0.8\, C_{a0}$, $v_x = v_{x0}$, $\mu_f = 0.6\, \mu_{f0}$, $l = l_r + l_f$, $l_{ns} = J/\mu l_f$, $A_s = 2.0956$ m^2 (calculated with the same formula), $c_\psi = 1.15 c_{\psi 0}$, $\rho = 1.15 \rho_0$. The wind components are

$$v_{w,X} = 0.45 + \mathcal{D}_X(t), \quad v_{w,Y} = 0.45 + \mathcal{D}_Y(t)$$

with $\mathcal{D}_X(t) = \mathcal{D}_Y(t) = 0.1\ \mathcal{N}$ random disturbances, modeled by uniform distributions $\mathcal{N}$.

The results are summarized in Figure 1, which shows the effectiveness of the proposed control scheme. In particular, the tracking error $\psi - \psi_r$ is of the order of 10^{-4} rad, while the absolute lateral velocity $|v_n|$ is less than of 2 m/s. We remind that this last can not be exactly zero due to the fact that the control law γ has been approximated in order to obtain an exact immersion.

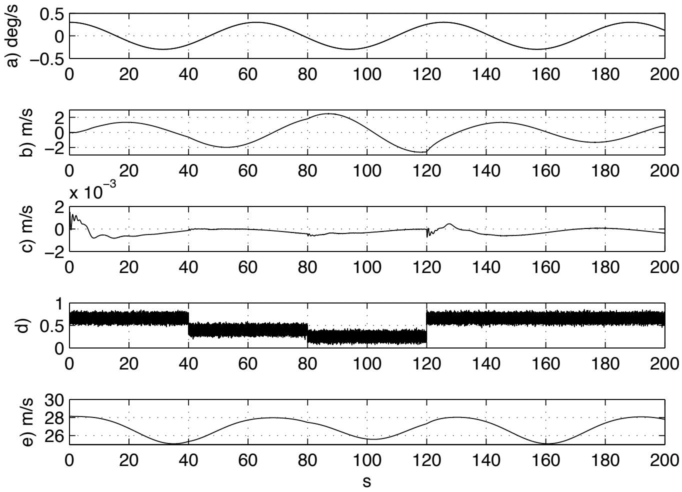

Fig. 1 a) Yaw angle ψ and reference ψ_r; b) Lateral velocity v_n; c) Tracking error $\psi - \psi_r$; d) Tire–road friction coefficient; e) Wind velocity v_w

CONCLUSIONS

This paper presents an approach to the vehicle dynamics control based on the robust, or structurally stable, regulation. Such a controller takes into account the presence of parametric uncertainties in the control law. The dynamic controller is derived considering an approximation of the exact controller. Such an approximated controller ensures a zero tracking error in a practical sense (ultimate boundedness of the trajectories) of the yaw angle reference, and small lateral velocities.

REFERENCES

J. Ackermann, J. Guldner, W. Sienel, R. Steinhauser, and V. Utkin. Linear and nonlinear controller design for robust automotive steering. *IEEE Transactions on Control System Technology*, 3:132–143, 1995.

C. Acosta-Lua, B. Castillo-Toledo, S. Di Gennaro, and A. Toro. Nonlinear robust regulation of ground vehicle motion. *Proceedings of the 46th Conference on Decision and Control*, 2007.

R. Bosch. *Automotive Handbook.* Robert Bosch Gmbh, Stuttgart, Germany, 4th edition, 1996.

G. Burgio and P. Zegelaar. Integrated vehicle control using sterring and brakes. *International Journal of Control*, 79, 2: 162–169, 2006.

O. Hanke, T. Bertram, and M. Hiller. Analysis and control of vehicle dynamics under crosswind conditions. *IEEE/ASME Transactions on Mechatronics*, 2001.

J. Huang and W. J. Rugh. An approximation method for the nonlinear servomechanism problem. *IEEE Transactions on Automatic Control*, 37, 9:1395–1398, 1992.

A. Isidori. *Nonlinear Control Systems.* Springer–Verlag, London, 3rd edition, 1995.

A. Isidori and C.I. Byrnes. Output regulation of nonlinear systems. *IEEE Transactions on Automatic Control*, 35:131–140, 1990.

C. Lee, K. Hedrick, and K. Yi. Real–time slip–based estimation of maximum tire–road friction coefficient. *IEEE/ASME Transactions on Mechatronics*, 9, 2:454–458, 2004.

G.F. Mauer. A fuzzy logic controller for an abs braking system. *IEEE Transactions on Fuzzy Systems*, 3:381–388, 1995.

R. Rajamani. *Vehicle Dynamics and Control.* Springer, New York, 2006.

P. Setlur, J.R. Wagner, D. M. Dawson, and D. Braganza. A trajectory tracking steer–by–wire control system for ground vehicles. *IEEE Transactions on Vehicular Technology*, 55, 1: 76–85, 2006.

A. van Zanten, R. Erhardt, K. Landesfeind, and G. Pfaff. Vdc system development and perspective. *SAE*, 1998.

APPENDIX – PARAMETERS IN (3) AND IN (8)

The parameters in (3) are given by

$$a_1 = (l_r - l_{ns}) a_2, \quad a_2 = C_\alpha \frac{l_r \mu_f}{J v_x}, \quad a_3 = \frac{2(l_r - l_{ns})^3}{3 v_x^2} a_2$$

$$a_4 = \frac{2}{3 v_x^2} a_2, \quad a_5 = \frac{2(l_r - l_{ns})^2}{v_x^4} a_2, \quad a_6 = \frac{2(l_r - l_{ns})}{v_x^2} a_2$$

$$a_7 = (l_r - l_{ns}) a_8, \quad a_8 = C_\alpha \frac{\mu_f (l_f + l_r)}{m l_f v_x}, \quad a_9 = \frac{2(l_r - l_{ns})^3}{3 v_x^2}$$

$$a_{10} = \frac{2}{3 v_x^2} a_8 \quad a_{11} = \frac{2(l_r - l_{ns})^2}{v_x^2} a_8, \quad a_{12} = \frac{2(l_r - l_{ns})}{v_x^2} a_8$$

$$b_1 = \frac{\mu_f l_f}{J}, \quad b_2 = \frac{1}{J}, \quad b_3 = \frac{1}{m l_f}$$

$$b_4 = C_\alpha \frac{\mu l_r}{J}, \quad b_5 = C_\alpha \frac{l_f + l_r}{l_f} \frac{\mu}{m}$$

$a_{j+12} = \alpha_j / J$, $a_{j+18} = \alpha_j / (m l_f)$, $j = 1, \cdots, 6$.

The parameters in (8) are given by

$$\beta_0 = \frac{1}{b_1}\left(\frac{b_2}{b_3} a_{19} - a_{13}\right), \quad \beta_1 = \omega^2 \frac{1}{b_1}$$

$$\beta_2 = \omega \frac{1}{b_1}(a_1 - a_7 \frac{b_2}{b_3} a_{19}), \quad \beta_3 = \omega^2 \frac{1}{b_1}(a_{20} \frac{b_2}{b_3} a_{19} - a_{14})$$

$$\beta_4 = \omega^3 \frac{1}{b_1}(a_9 \frac{b_2}{b_3} a_{19} - a_3), \quad \beta_5 = \frac{1}{b_1}(a_{21} \frac{b_2}{b_3} a_{19} - a_{15})$$

$$\beta_6 = \frac{1}{b_1}(a_{22} \frac{b_2}{b_3} a_{19} - a_{16}), \quad \beta_7 = \omega \frac{1}{b_1}(a_{23} \frac{b_2}{b_3} a_{19} - a_{17})$$

$$\beta_8 = \omega \frac{1}{b_1}(a_{24} \frac{b_2}{b_3} a_{19} - a_{18}),$$

$$\theta_0 = \frac{1}{b_3} a_{19}, \quad \theta_1 = \omega a_7 \frac{1}{b_3}, \quad \theta_2 = \omega^2 a_{20} \frac{1}{b_3}$$

$$\theta_3 = \omega^3 a_9 \frac{1}{b_3}, \quad \theta_4 = a_{21} \frac{1}{b_3}, \quad \theta_5 = a_{22} \frac{1}{b_3}$$

$$\theta_6 = \omega a_{23} \frac{1}{b_3}, \quad \theta_7 = \omega a_{24} \frac{1}{b_3}.$$