Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

A Hybrid Systems Approach to Closed-Loop Navigation of Electromagnetically Actuated Satellite Formations using Potential Functions

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Abstract: Potential function method has been used extensively to navigate robotic vehicles in the form of a closed-loop control law. One problem associated with the potential function method is the presence of local minima in the navigation space. This paper presents a novel approach of using switching in order to avoid the local minima and hence achieving global convergence to the desired configuration. This method is applied to electromagnetically actuated satellites formations having nonlinear dynamics and shown to achieve collision-free reconfiguration with a low computational burden.

1. INTRODUCTION

The Artificial Potential Function Method (APFM) is an algorithm that allows the guidance and control of autonomous vehicles with very little computational burden and hence is suitable for real-time control of complex systems such as a formation or swarm of robotic vehicles. This method is most popular for the terrestrial robotic trajectory planning and is based on the design of a potential function with a global minimum at the desired configuration of the system and maxima at the location of obstacles [Khatib, McInnes, Rimon]. This potential function is used to formulate a Control Lyapunov Function to generate a feedback control law that can reconfigure the system from a given initial state to a desired terminal state while avoiding collisions. The usefulness of this method stems from the fact that it combines the functions of guidance and control, with a low computational burden, and generates a feedback control law with robustness guarantees.

Generally the motion planning algorithm can be divided into three distinct sections [LaValle]. Given the geometry of the vehicle formation, obstacles, initial state, and terminal state a collision free geometric curve is found in the “path-planning” part of the algorithm that completely ignores the dynamics of the formation. Secondly, given this collision free path, a reference trajectory is determined for the formation that is based on the time parameterization of the collision free path. This reference trajectory is such that the “dynamics” of the formation can actually execute it using trajectory following algorithms, which is the last piece of this algorithm.

The feedback motion planning algorithm, on the other hand, combines all these functions and generates a *feedback plan* which produces control inputs for the vehicles in the formation for each state in the state space such that the formation reaches its desired or goal states. The Artificial Potential Function Method (APFM) generates the *feedback plan* by defining a potential function that has simultaneously the properties of a *navigation function* [LaValle] and a Lyapunov Function [Khalil]. The gradient of such a function defines a vector field that defines the descent directions towards the global minimum. Since we have maxima defined at the locations of obstacles, the vehicles avoid these obstacles automatically as they move along these descent directions.

This formulation has at least three limitations. One well documented limitation is the existence of local minima in the potential function. One way of preventing local minima in the potential function is to construct the potential function in such a way that it has only one unique global minimum. See [LaValle] for one such method. Another method is to generate the potential function as a harmonic function which is the solution of Laplace’s equation [Conner]. It should be noted that all these methods require a-priori computation of the potential function over the geometry of the problem which may become computationally expensive for complex problems. This paper uses the hybrid system approach by adding an *obstruction maneuvering* term to the potential function gradient as explained in a later section.

The second limitation of APFM is that it can result in the saturation of the actuators. Limited control authority implies that the desired acceleration at each point in the state-space must remain less than or equal to what the system can actually achieve. This can be ensured by appropriately shaping the potential function in the regions where the acceleration requirements might be higher due to larger position errors.

The third limitation of the APFM method is that it generates non-optimal trajectories. For space missions, the optimality of the trajectories is very important since non-optimality directly translates into penalties to the mass metric for the propulsion system. Nevertheless, from the point of view of the computational complexity, APFM is much more efficient as compared to optimal trajectory generation and implementation. Moreover, APFM can generate feasible trajectories that can be optimized by nonlinear optimization techniques. Thus feasible trajectories, for quite complex formation maneuvers, can be generated by APFM and used as initial starting guesses for the nonlinear optimizer.

Electromagnetic Formation Flying (EMFF) is a novel concept for the control of satellite formations that uses high temperature superconducting (HTS) wire technology to create magnetic dipoles on each satellite in the formation to generate forces and torques in order to maintain and reconfigure the satellite formation. A steerable magnetic dipole on each satellite can be created by using three orthgonogal coils on each satellite in the formation. Force on each satellite in the formation can be applied in any arbitrary direction by using these steerable magnetic dipoles. Since these forces are internal, the center of mass of the formation cannot be moved (momentum is conserved). This can be easily seen for a two satellite formation in which each satellite experiences equal but opposite force due to magnetic dipoles on each satellite. Since satellite formation control involves controlling the relative positions between the satellites, the inability to move the center of mass of the formation is not a limitation in itself. See [Kong], [Ahsun] for a more detailed introduction to the concept of EMFF.

The rest of the paper is arranged as follows. Section 2 presents in detail the reconfiguration algorithm and section 3 presents some simulation results based on the algorithm.

2. RECONFIGURATION OF EM FORMATIONS USING APFM

The purpose of this section is to present an algorithm for the reconfiguration of a fully-actuated EM formation. In the reconfiguration problem, the formation needs to be reconfigured from a known initial state to a known terminal state while avoiding collisions, with the following assumptions:

1. Well-posedness
2. Full actuation
3. Convex satellite shapes
4. Perfect knowledge of full state and dynamics
5. Perfect attitude control

Well-posedness assumption is required since EM actuation cannot change the total linear momentum of the formation. Therefore, any formation reconfiguration problem that requires a net change in the formation linear momentum does not belong to well-posed problems for EMFF. This assumption also excludes formations that require satellites on top of each other and other such scenarios in both the initial and terminal conditions. Full actuation means that, in $\mathbb{R}^3$, each satellite has three orthogonal coils and three orthogonal reaction wheels, therefore enabling the control of all the relative degrees of freedom. Convex shape for the satellites is required since the obstruction functions used in the algorithm are elliposiodal. Although, the satellite itself can be of any shape, the reconfiguration problem assumes that the collision avoidance region is an ellipsoid centered at each satellite in the formation. Perfect knowledge of state and dynamics is assumed, nevertheless the algorithm is based on Lyapunov function which is inherently robust and the method will work for small uncertainties. Lastly, perfect attitude control is assumed to simplify the dipole solutions.

Let the satellites in the formation be numbered from 0 to *N*-1. We can write the translational dynamics of the *N*-satellite EM formation, with respect to the formation center of mass, in an inertial frame as follows [Ahsun]:

$$\begin{aligned} \dot{\mathbf{p}}_i &= \mathbf{v}_i \\ \dot{\mathbf{v}}_i &= \mathbf{f}_i(\mathbf{p}, \mathbf{v}_i, \boldsymbol{\mu}) \end{aligned} \tag{1}$$

where $\mathbf{f}_i : \mathbb{R}^{3N} \times \mathbb{R}^3 \times \mathbb{R}^{3N} \rightarrow \mathbb{R}^{3N}$ represents the nonlinear dynamics of the i[th] satellite in the formation, $\mathbf{p}_i$ is the position vector of satellite-i, $\mathbf{v}_i$ is the velocity vector of satellite-i, $\mathbf{p}$ is the position vector of the whole formation and $\boldsymbol{\mu}$ is the control vector of the whole formation.

Define a scalar potential function for satellite-i with a unique global minimum (at the desired final configuration) as follows:

$$\phi_i(\mathbf{p}) = \tfrac{1}{2}\tilde{\mathbf{p}}_i^{\mathrm{T}}\mathbf{M}_i\tilde{\mathbf{p}}_i + \tfrac{1}{2}\sum_{j=0,\, j\neq i}^{N-1}\left(\lambda_1 \exp\{-\lambda_2(\mathbf{p}_i - \mathbf{p}_j)^T \mathbf{N}(\mathbf{p}_i - \mathbf{p}_j)\}\right) \tag{2}$$

where: $\tilde{\mathbf{p}}_i = \mathbf{p}_i - \mathbf{p}_{i0}$ is the position error for the i[th] satellite

$\mathbf{p}_{i0}$ = desired terminal location, $\mathbf{M}_i$ = a constant symmetric positive definite scaling matrix, λ_1, λ_2 = positive scaling constants for the obstruction function, N = a constant symmetric positive definite matrix for the obstruction function. Such a potential function for a four-satellite formation (in 2D) is shown in Figure 1.

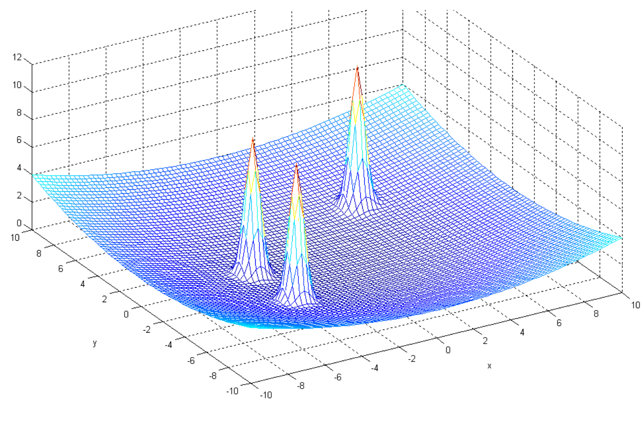

Fig. 1. A potential function for one satellite in a 4-satellite formation

Using these *N* potential functions (one for each satellite in the formation), the formation Lyapunov Function can be written as follows:

$$V(\mathbf{v},\mathbf{p})=\sum_{i=1}^{N-1}\left[\tfrac{1}{2}\mathbf{v}_i^{\mathbf{T}}\mathbf{P}_i\mathbf{v}_i+\tfrac{1}{2}\tilde{\mathbf{p}}_i^{\mathbf{T}}\mathbf{M}_i\tilde{\mathbf{p}}_i+\tfrac{1}{2}\sum_{j=0,j\neq i}^{N-1}\left(\lambda_1\exp\{-\lambda_2(\mathbf{p}_i-\mathbf{p}_j)^T\mathbf{N}(\mathbf{p}_i-\mathbf{p}_j)\}\right)\right] \tag{3}$$

where the $\mathbf{P}_i$'s are symmetric, positive-definite scaling matrices. Note that the Lyapunov function does not include a potential function, of the form given by (2), for satellite-0. This is appropriate for EMFF since total linear momentum of the formation cannot be changed using EM actuation as discussed earlier.

The time derivative of (3) can be written as:

$$\dot{V}(\mathbf{v},\mathbf{p})=\sum_{i=1}^{N-1}\left[\mathbf{v}_i^{\mathbf{T}}\mathbf{P}_i\dot{\mathbf{v}}_i+\tilde{\mathbf{p}}_i^{\mathbf{T}}\mathbf{M}_i\mathbf{v}_i+\sum_{j=0,j\neq i}^{N-1}\nabla O_{ij}(\mathbf{p}_i,\mathbf{p}_j)\left(\mathbf{v}_i-\mathbf{v}_j\right)\right] \tag{4}$$

where:

$$\nabla O_{ij}(\mathbf{p}_i,\mathbf{p}_j)=\\ -\lambda_1\lambda_2\exp\left\{-\lambda_2(\mathbf{p}_i-\mathbf{p}_j)^T\mathbf{N}(\mathbf{p}_i-\mathbf{p}_j)\right\}(\mathbf{p}_i-\mathbf{p}_j)^T\mathbf{N}\quad i\neq j \tag{5}$$

is the gradient of the obstruction function of satellite-i due to satellite-j (note that in the above equation the gradient is defined as a row vector). From (5) it is clear that the gradient of the obstruction function is a skew-symmetric function, i.e.:

$$\nabla O_{ij}(\mathbf{p}_i,\mathbf{p}_j)=-\nabla O_{ji}(\mathbf{p}_j,\mathbf{p}_i)\qquad i\neq j \tag{6}$$

Using the above relationship, (5) can be written as:

$$\dot{V}(\mathbf{v},\mathbf{p})=\sum_{i=1}^{N-1}\left[\mathbf{v}_i^{\mathbf{T}}\mathbf{P}_i\dot{\mathbf{v}}_i+\tilde{\mathbf{p}}_i^{\mathbf{T}}\mathbf{M}_i\mathbf{v}_i+2\sum_{j=0,j\neq i}^{N-1}\nabla O_{ij}\mathbf{v}_i-\nabla O_{i0}(\mathbf{v}_i+\mathbf{v}_0)\right] \tag{7}$$

As discussed in Section 1, the potential function defined by (7) can possibly have a thin set in the state-space where the satellites can get "stuck" in the local minima. Define a set T_i for each satellite (except satellite-0) comprising of the neighborhoods of all such local minima as follows:

$$T_i=\left\{B_\varepsilon(\mathbf{p}_i)\left|\mathbf{M}_i\tilde{\mathbf{p}}_i+2\sum_{\substack{j=0\\ j\neq i}}^{N-1}\left(\nabla O_{ij}^T\right)-\nabla O_{i0}^T+\frac{m_i}{m_0}\boldsymbol{\kappa}=0,\ \ \mathbf{p}_i\neq\mathbf{p}_{i0}\right.\right\} \tag{8}$$

where $B_\varepsilon(\mathbf{p}_i)$ defines a sphere of radius $0<\varepsilon\ll 1$ centered at $\mathbf{p}_i$, $\boldsymbol{\kappa}\in\mathbb{R}^3$ is defined below, and m_i is mass of the ith satellite. In order to avoid the local minima, a hybrid switching[1] control law for each satellite (except satellite-0) is defined as follows:

$$\dot{\mathbf{v}}_i=-\mathbf{P}_i^{-1}\left\{\mathbf{K}_i(\mathbf{p}_i,\mathbf{v}_i)\mathbf{v}_i+\mathbf{M}_i\tilde{\mathbf{p}}_i+2\sum_{\substack{j=0\\ j\neq i}}^{N-1}\left(\nabla O_{ij}^T\right)-\nabla O_{i0}^T+\frac{m_i}{m_0}\boldsymbol{\kappa}\right\}\ \text{if}\ \mathbf{p}_i\notin T_i \tag{9}$$

$$\dot{\mathbf{v}}_i=-\mathbf{P}_i^{-1}\left\{\mathbf{K}_i(\mathbf{p}_i,\mathbf{v}_i)\mathbf{v}_i+\mathbf{M}_i\tilde{\mathbf{p}}_i+2\sum_{\substack{j=0\\ j\neq i}}^{N-1}\left(\nabla O_{ij}^T\right)-\nabla O_{i0}^T+\frac{m_i}{m_0}\boldsymbol{\kappa}+\gamma_i\mathbf{g}_{imt}\right\}\ \text{if}\ \mathbf{p}_i\in T_i \tag{10}$$

where $\mathbf{K}_i$ is a positive-definite control gain scaling matrix that may be dependent on the current state to tailor the descent rate according to the position and velocity of the satellite, γ_i is a positive scaling constant, and $\mathbf{g}_{imt}\in\mathbb{R}^3$ is a unit vector orthogonal to the velocity of satellite-i:

[1] See [Branicky] and [Haddad] for an introduction to hybrid and switched system stability theory.

$$\mathbf{g}_{imt}\bullet\mathbf{v}_i=0,\qquad\qquad \left|\mathbf{g}_{imt}\right|=1 \tag{11}$$

The switching control law is defined such that if the satellite-i is not in the neighborhood of the local minima, i.e. not in the set T_i, then control is computed according to (9), and if the satellite-i is in the set T_i then (10) is used. Note that if velocity of satellite-i is zero then the term $\mathbf{g}_{imt}$ is not well-defined. This is the reason that the formations with their initial or terminal conditions at or near to the local minima of the potential function are not considered as well-posed for this algorithm.

With this selection of the accelerations, the time derivative of the Lyapunov function can be written as:

$$\dot{V}(\mathbf{v},\mathbf{p})=-\sum_{i=1}^{N-1}\mathbf{v}_i^{\mathbf{T}}\mathbf{K}_i\mathbf{v}_i-\sum_{i=1}^{N-1}\nabla O_{i0}\mathbf{v}_0-\frac{1}{m_0}\sum_{i=1}^{N-1}m_i\mathbf{v}_i^T\boldsymbol{\kappa}\quad\text{if}\ \mathbf{p}_i\notin T_i,\ \ i=1\ldots N-1 \tag{12}$$

$$\dot{V}(\mathbf{v},\mathbf{p})=-\sum_{i=1}^{N-1}\mathbf{v}_i^{\mathbf{T}}\mathbf{K}_i\mathbf{v}_i-\sum_{i=1}^{N-1}\nabla O_{i0}\mathbf{v}_0-\frac{1}{m_0}\sum_{i=1}^{N-1}m_i\mathbf{v}_i^T\boldsymbol{\kappa}+\sum_{k\in I}\gamma_k\mathbf{v}_k^{\mathbf{T}}\mathbf{g}_{kmt}\ \ \forall\ \mathbf{p}_k\in T_k \tag{13}$$

where the index set I is defined such that it contains the indices of all the satellites for which the control law given by (10) is used. By construction, the *obstruction maneuvering* term $\mathbf{g}_{imt}$ is always orthogonal to the velocity of satellite $\mathbf{v}_i$, therefore the last term in (13) is zero. Thus, in both the cases, the time derivative of the Lyapunov function becomes:

$$\dot{V}(\mathbf{v},\mathbf{p})=-\sum_{i=1}^{N-1}\mathbf{v}_i^{\mathbf{T}}\mathbf{K}_i\mathbf{v}_i-\sum_{i=1}^{N-1}\nabla O_{i0}\mathbf{v}_0-\frac{1}{m_0}\sum_{i=1}^{N-1}m_i\mathbf{v}_i^T\boldsymbol{\kappa} \tag{14}$$

Since the linear momentum of the EM formation is conserved in the absence of external perturbations, we have:

$$\sum_{i=1}^{N-1}m_i\mathbf{v}_i=-m_0\mathbf{v}_0 \tag{15}$$

Using this relation, (14) can be written as:

$$\dot{V}(\mathbf{v},\mathbf{p})=-\sum_{i=1}^{N-1}\mathbf{v}_i^{\mathbf{T}}\mathbf{K}_i\mathbf{v}_i-\sum_{i=1}^{N-1}\nabla O_{i0}\mathbf{v}_0-\mathbf{v}_0^T\boldsymbol{\kappa} \tag{16}$$

In order to make the Lyapunov function negative semi-definite, the parameter $\boldsymbol{\kappa}$ is chosen according to the following relation:

$$\boldsymbol{\kappa}=-\sum_{i=1}^{N-1}\nabla O_{i0}^T \tag{17}$$

With this choice of $\boldsymbol{\kappa}$, the time rate of change of the formation Lyapunov function becomes:

$$\dot{V}(\mathbf{v},\mathbf{p})=-\sum_{i=1}^{N}\mathbf{v}_i^{\mathbf{T}}\mathbf{K}_i\mathbf{v}_i\leq 0 \tag{18}$$

Note that if the controlled system were not a switched system then Lyapunov stability would follow from the negative semi-definiteness of (18). However, since the controlled system is switched, Lyapunov stability does not follow from the negative semi-definiteness of the individual Lyapunov functions (see Example 2.1 in [Branicky]). To prove stability for switched systems, an additional condition is required such that each individual Lyapunov function is strictly monotonically non-increasing on the sequences of all the switching times (see Theorem 2.3 in [Branicky]). For the case of EMFF, the same Lyapunov function is used for both instances of the switched controlled system, i.e. with and without the *obstruction maneuvering* term. Equation (18) shows that this Lyapunov function is negative semi-definite for all times and in particular at any switching times that could occur. Therefore, the switched control system is stable in the sense of Lyapunov. To show asymptotic stability for well-posed formations, the Invariance Principle or LaSalle's Theorem [Khalil], with additional conditions for asymptotic stability of switched dynamical systems [Li], [Bacciotti], will be used.

The basic argument in LaSalle's theorem is that if the global minimum is asymptotically stable then the formation cannot get "stuck" at any configuration other than the global minimum due to the inherent dynamics and control law formulation. Let $\mathbf{x} = [\tilde{\mathbf{p}} \quad \mathbf{v}]^T \in \mathbb{R}^{6N}$ be a point in the state-space $X = \left\{ \forall \mathbf{x} \in \mathbb{R}^{6N} \right\}$. Then by construction the Lyapunov function given by (3) is positive-definite in X. Define the set:

$$S = \left\{ \mathbf{x} \in X \mid \dot{V}(\mathbf{v}, \mathbf{p}) = 0 \right\} \tag{19}$$

as the set of points where the time derivative of the Lyapunov function is zero. According to LaSalle's theorem, assuming that there are no accumulation points of switching times, all solutions in the state-space converge to the *largest invariant set* in the set of points where the time derivative of the Lyapunov function is zero, i.e. set S as defined above. To determine the *largest invariant set* in S, it can be seen from (18) that:

$$\dot{V}(\mathbf{v}, \mathbf{p}) = 0 \quad \Rightarrow \quad \mathbf{v}_i(t) = 0 \quad \Rightarrow \quad \dot{\mathbf{v}}_i(t) = 0 \qquad \forall i \tag{20}$$

From the switched feedback control law, given by (9) and (10), zero velocity for all times implies that:

$$\mathbf{M}_i \tilde{\mathbf{p}}_i + 2\sum_{\substack{j=0 \\ j \neq i}}^{N-1} \left(\nabla O_{ij}^T \right) - \nabla O_{i0}^T + \frac{m_i}{m_0} \boldsymbol{\kappa} = 0 \quad \text{if } \mathbf{p}_i \notin T_i \tag{21}$$

$$\mathbf{M}_i \tilde{\mathbf{p}}_i + 2\sum_{\substack{j=0 \\ j \neq i}}^{N-1} \left(\nabla O_{ij}^T \right) - \nabla O_{i0}^T + \frac{m_i}{m_0} \boldsymbol{\kappa} + \gamma_i \mathbf{g}_{imt} = 0 \quad \text{if } \mathbf{p}_i \in T_i \tag{22}$$

By construction, (21) is never satisfied except when $\tilde{\mathbf{p}}_i = 0$ (which is the desired equilibrium condition) otherwise if it is true for $\tilde{\mathbf{p}}_i \neq 0$ then $\mathbf{p}_i \in T_i \subset X$ which is a contradiction. In the neighborhood around the local minima, i.e. set T_i, (22) is applicable and no value of $\mathbf{p}_i \in T_i \subset X$ can satisfy this equation due to the *obstruction maneuvering* term $\gamma_i \mathbf{g}_{imt}$ which is added for this specific purpose. Thus, it can be seen that the velocity of all the satellites cannot be zero simultaneously and in particular, in the set T_k the velocity of the kth satellite would be nonzero, forcing the Lyapunov function to be strictly negative-definite and consequently decreasing from one switching time to the next. In summary due to the combination of the two facts namely,

1. $\tilde{\mathbf{p}}_i = 0$ is the only zero velocity solution of (21),
2. Lyapunov function is strictly decreasing in between any possible switching times (see Theorem 2 in [Li]),

the largest invariant set in the set S (19) is:

$$E = \left\{ \mathbf{p}_0 \right\} \tag{23}$$

hence the formation converges to this set asymptotically for almost all initial conditions.

In the above algorithm, it was assumed that the acceleration given by (9) and (10) can actually be realized by the EM actuation system. These equations are a set of 3*N*-3 nonlinear polynomial equations in an unknown dipole strength vector $\boldsymbol{\mu}$. The methods of solving this set of equations are discussed in [Ahsun] where it is shown that these can be solved efficiently in real-time.

3. SIMULATION RESULTS

In order to test the algorithm developed in the last section for the reconfiguration of general EM formations using APFM, a nonlinear simulation was built to simulate EM formations in 2D. In all the results presented in this section, each satellite is assumed to have same mass of 30 kg and each coil is assumed to have a value of 100 turns-m^2.

First simulation result is presented for a five-satellite formation in which some of the satellites are already present at their final positions. Despite this fact, as fig. 2 shows, they move aside to give way to other satellites in such a way that the formation achieves its desired configuration. Figure 3 shows the Lyapunov function values as a function of time for the formation and shows that although the Lyapunov function for an individual satellite may increase during the maneuver, the total formation Lyapunov function monotonically decreases for all times.

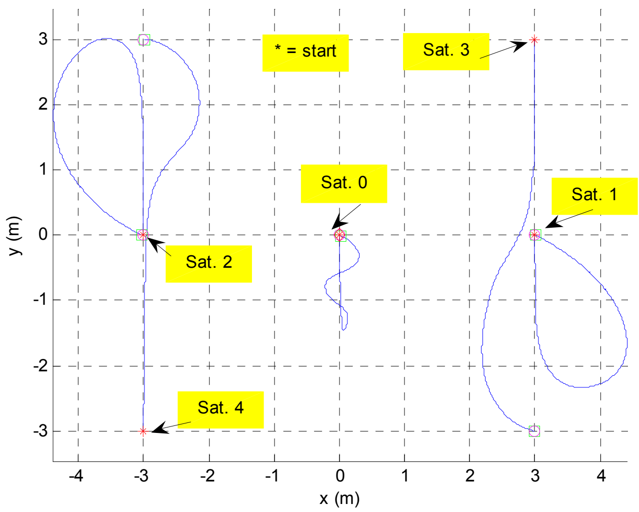

Fig. 2. Trajectories for the five-satellite formation reconfiguration.

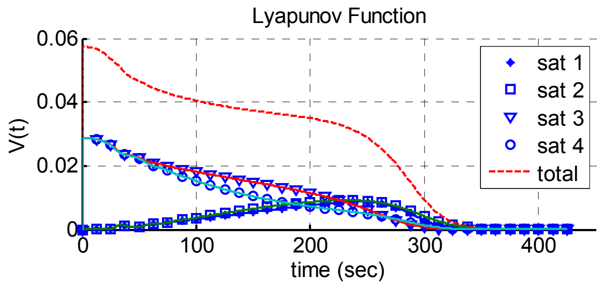

Fig. 3. functions for the five-satellite formation.

The second simulation result is presented for a four-satellite square formation with terminal conditions as mirror image of the initial conditions. The trajectories for the formation are shown in fig. 4, which shows that the satellites initially descend down the potential function and are repelled due to the obstruction functions and as a result approach the neighborhoods of the local minima. Due to the presence of *obstruction maneuvering term* in the controller, a circulation starts as an "emergent behavior" and formation reconfigures itself while avoiding the local minima. The Lyapunov function of the formation is given in fig. 5 and again shows that the total formation Lyapunov function is monotonically decreasing for all times.

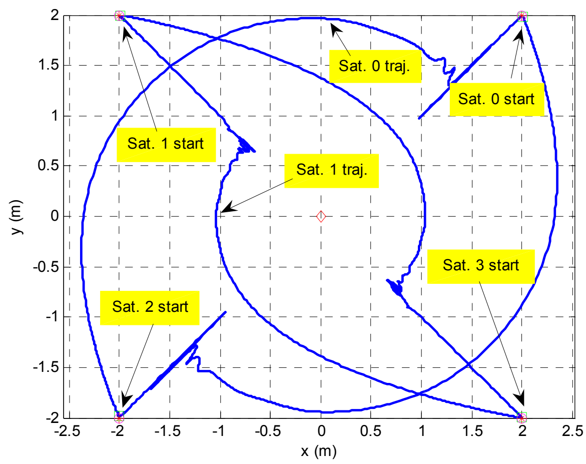

Fig. 4. Trajectories for the four-satellite square formation reconfiguration.

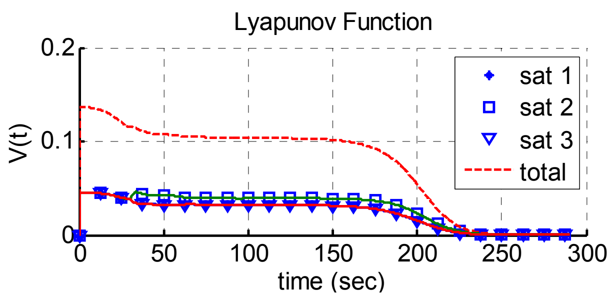

Fig. 5. function for the four-satellite square formation.

4. CONCLUSIONS

This paper has presented a switching control law that can be used for closed-loop navigation of a formation or swarm of vehicles using potential functions. Although the method was used to reconfigure electromagnetic satellite formations, the method is applicable in general to fully-actuated non-holonomic robots having nonlinear dynamics. This method can be used to generate feasible trajectories for reconfiguration of formations and an optimizer can optimize these trajectories. Potential function shaping [Ahsun] can be used to reduce the sub-optimality of the trajectories generated by this algorithm.

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