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Strong Stabilization of a Non-uniform SCOLE Model

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Abstract: The SCOLE (NASA Spacecraft Control Laboratory Experiment) model is considered the best model for the coupled system consisting of a flexible beam with one end clamped and the other end linked to a rigid body. This model has been studied extensively, with most papers assuming that the flexible beam is uniform. In our study we allow the coefficients of the beam equation to vary with position, like in Guo [2002] which has considered the non-homogeneous structure of the beam. It has been proved that the exponential stabilization of the uniform SCOLE model is impossible to achieve by boundary feedback from the natural output signals (the speed and the angular velocity of the rigid body) (see Rao [1995]). Thus the non-uniform SCOLE model is not exponentially stabilizable in general, by using these signals. Although the exponential stabilization of the SCOLE model can be obtained by high order output feedback (see Guo [2002] and Rao [1995]), the corresponding closed-loop system is not well-posed. In addition, such a feedback is difficult to realize in practice. Thus we have to compromise for strong stabilization. Following a recent strong stabilization theorem for passive systems in Curtain and Weiss [2007], we have shown that the non-uniform SCOLE model is strongly stabilizable by static output feedback from either the speed or the angular velocity of the rigid body.

1. INTRODUCTION

The aim of this paper is to investigate the strong stabilization of the non-uniform SCOLE (NASA Spacecraft Control Laboratory Experiment) model. Originally, the SCOLE model was proposed to model a long flexible mast clamped at one end to a massive spaceship and linked at the other end to a rigid antenna reflector. This model consists of the Euler-Bernoulli equation, which describes the vibrations of the flexible mast and the Newton-Euler rigid-body equations, which describe the oscillations of the antenna. The reader is referred to Littman and Markus [1988a, 1988b] for a more detailed description of the SCOLE model.

The SCOLE model is considered the best model for the system consisting of a flexible beam with one end clamped and the other end linked to a rigid body. Therefore it has a big interest for engineering applications. We have been motivated to study the non-uniform SCOLE model by the problem of stabilizing the vibrations of a wind turbine tower. The tower is clamped in the ocean floor and carries at its top the nacelle (containing the electric generator and other equipment) and the turbine, which together have a weight of several hundred tons. The diameter of the tower decreases with height, so that it should be modelled by a non-uniform beam equation. The detailed discussion of this stabilization problem shall be in other papers.

Although the SCOLE model has been studied extensively, most relevant papers assume that the flexible beam is uniform. A non-uniform SCOLE model (the model has variable coefficients depending on height) was studied in Guo [2002] as described below:

$$\begin{cases} \rho(x)w_{tt}(x,t) + (EI(x)w_{xx}(x,t))_{xx} = 0, \\ \qquad 0<x<l, \quad t>0, \\ w(0,t)=0, \quad w_x(0,t)=0, \\ mw_{tt}(l,t) - (EIw_{xx})_x(l,t) = f(t), \\ Jw_{xtt}(l,t) + EI(l)w_{xx}(l,t) = v(t), \end{cases} \tag{1}$$

where the subscripts t and x denote derivatives with respect to the time t and the position x respectively. l is the length of the beam, w stands for the transverse displacement of the beam, and EI and ρ are the flexural rigidity function and the mass density function of the beam. m and J are the mass and the moment of inertia of the rigid body (these are positive constants). $-(EI(x)w_{xx}(x,t))_{xx}\mathrm{d}x$ is the total lateral force acting on a slice of the beam of length $\mathrm{d}x$, located at the position x and the time t. $(EIw_{xx})_x(l,t)$ and $-EI(l)w_{xx}(l,t)$ are the force and the torque acting on the rigid body from the beam at the time t. f and v are the force input and the torque input respectively acting on the rigid body. We assume that $\rho, EI \in C^4[0,l]$, $0<\rho_0 \le \rho(x) < \rho_1$ and $0 < EI_0 \le EI(x) < EI_1$ where $\rho_0, \rho_1, EI_0, EI_1$ are positive constants.

Exponential stability is the most desirable kind of stability. However in many situations, it is impossible to achieve exponential stability, so that we must compromise for strong stability. In Rao [1995], using an energy multipliers method, it is proved that the exponential stabilization of the uniform SCOLE model can be obtained by high order

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output feedback. In Guo [2002], it was pointed out that the exponential stabilization of the non-uniform SCOLE model can be achieved by high order output feedback (feedback from the time derivative of the strain at the end) as well. However with high order output feedback, the closed-loop system is not well-posed. In addition, such a feedback is difficult to realize in practice. That's why speed and angular velocity are more natural signals for the stabilization of the SCOLE model. Unfortunately, It is proved in Rao [1995] (using a method of compact perturbation) that the uniform SCOLE model is not exponentially stabilizable by boundary feedback from the speed and the angular velocity of the rigid body. Therefore also the non-uniform SCOLE model cannot be exponentially stabilizable in general, by using these signals. Strong stabilization becomes a good compromise to aim for.

The strong stabilization of the uniform SCOLE model in one specific case was proven in Littman and Markus [1988a]. We are not aware of any strong stabilization result for the non-uniform SCOLE model. In this paper, we use a recent strong stabilization theorem for passive systems developed in Curtain and Weiss [2007]. This theorem is as follows (almost impedance passivity will be defined in Section 2.3):

Theorem 1. If an almost impedance passive system node is either approximately controllable or approximately observable in infinite time, then this system is weakly stabilizable by static output feedback for sufficiently small gain k. Furthermore if the intersection of the spectrum of the semigroup generator of the open-loop system and the imaginary axis is countable, then the closed-loop system and its dual are both strongly stable.

We remark that if the system node is impedance passive (i.e., $E = 0$ in Definition 12) then the gain k can be taken to be any positive number. We also need a result in Guo and Ivanov [2005], which is that the non-uniform SCOLE model with only torque control or only force control is approximately controllable. We shall give the state-space formulation of the non-uniform SCOLE model (1). This will be an impedance passive well-posed system. We show that the intersection of the spectrum of the semigroup generator of the SCOLE model and the imaginary axis is countable. We get the conclusion that the non-uniform SCOLE model is strongly stabilizable using either torque control or force control with colocated feedback.

2. INFINITE-DIMENSIONAL LINEAR SYSTEMS

In this section we introduce some concepts and results on infinite-dimensional linear time invariant systems without proof. For the details we refer to the literature.

2.1 System nodes

As we know that a finite-dimensional linear time invariant system with state space X, input space U and output space Y (all finite-dimensional spaces) has the following state space realization:

$$\begin{cases} \dot{z}(t) = Az(t) + Bu(t), \\ y(t) = Cz(t) + Du(t), \end{cases}$$

where $z(t) \in X$ is called the *system state*, $z(0) = z_0$ is the *initial state* of the system, u is called the system *input* (or *input signal*), $u(t) \in U$, and y is the system *output* (or *output signal*), $y(t) \in Y$. z, u, y are vector-valued and A, B, C, D are matrices of appropriate dimensions.

However in most cases infinite-dimensional linear systems do not have such a nice realization. The most general class of infinite-dimensional linear systems which have a simple realization are system nodes. Before we introduce this concept, we need some preliminaries. Let A be the generator of a strongly continuous semigroup $\mathbb{T}$ on a Hilbert Space X. Then X can determine 2 additional Hilbert space: X_1 which is $\mathcal{D}(A)$ with the norm $\|z\|_1 = \|(\beta I - A)z\|$, and X_{-1} which is the completion of X with respect to the norm $\|z\|_{-1} = \|(\beta I - A)^{-1}z\|$, where $\beta \in \rho(A)$ is fixed. The space X_1 and X_{-1} are independent of the choice of β since different values of β lead to equivalent norms on X_1 and X_{-1}. We have $X_1 \subset X \subset X_{-1}$ densely and with continuous embeddings. We can continually extend A to a bounded operator from X to X_{-1}, still denoted by A. The semigroup generated by this extended A is the extension of $\mathbb{T}$ to X_{-1}, still denoted by $\mathbb{T}$.

Definition 2. Let Σ be an infinite-dimensional linear system with state space X, input space U, output space Y (all Hilbert spaces) and transfer function $G(s)$. Let A be the generator of a strongly continuous semigroup $\mathbb{T}$ on X, $B \in \mathcal{L}(U, X_{-1})$ and $C \in \mathcal{L}(X_1, Y)$. Σ is called a *system node* if, for all $s, \beta \in \rho(A)$, it has the following realization:

$$\begin{cases} \dot{x} = Ax + Bu, & (2) \\ y = C\&D \begin{bmatrix} x \\ u \end{bmatrix}, & (3) \end{cases}$$

$$G(s) = C\&D \begin{bmatrix} (sI - A)^{-1}B \\ I \end{bmatrix}, \tag{4}$$

where

$$C\&D \begin{bmatrix} x \\ u \end{bmatrix} = C[x - (\beta I - A)^{-1}Bu] + G(\beta)u,$$

$$\mathcal{D}(C\&D) = \left\{ \begin{bmatrix} x \\ u \end{bmatrix} \in X \times U \;\middle|\; Ax + Bu \in X \right\}.$$

A is called the *semigroup generator* (or just the *generator*), B is called the *control operator*, C is called the *observation operator*, $C\&D$ is called the *combined observation/feedthrough operator*, and (A, B, C) is called the *generating triple.*

It is clear that a system node Σ can be determined either by its operators, denoted by $\Sigma = \left[\begin{smallmatrix} A & B \\ -C\&D \end{smallmatrix}\right]$, or by its generating triple (A, B, C) and its transfer function $G(s)$, denoted by $\Sigma = (A, B, C, G)$. For more details about system nodes we refer to Malinen and Staffans [2007] and Staffans [2002].

A well-known sub-class of system nodes are the well-posed systems.

Definition 3. Using the notions in Definition 2, a system node Σ is called a *well-posed linear system* if for some (hence for any) $\tau > 0$ there is a $c_\tau \geq 0$, the following inequality holds

$$\|z(\tau)\|^2 + \|y\|^2_{L^2([0,\tau],\ Y)} \leq c_\tau^2 \left(\|z(0)\|^2 + \|u\|^2_{L^2([0,\tau],\ U)} \right).$$

Well-posedness means that the output signal and final state depend continuously on the input signal and the initial state. Through some extensions of operators, well-posed systems can get the similar realization as the finite-dimensional systems. For more detailed background about well-posed systems we refer to Salamon [1987], Staffans [2002], Staffans and Weiss [2002], Weiss [1994], Weiss et al. [2001] and Weiss and Tucsnak [2003]. Necessary and sufficient conditions for well-posedness have been given in Curtain and Weiss [1989].

2.2 Controllability and observability

As Hilbert spaces have dense linear subspaces, the controllability and observability problems become much more complex in infinite-dimensional systems. There are many different controllability and observability concepts. We at least have 3 important concepts for each of them, namely exact controllability (observability), approximate controllability (observability) and null-controllability (final state observability).

Let Σ be an infinite-dimensional linear time-invariant system with state space X, input space U and output space Y (all Hilbert spaces). Let $\mathbb{T}$ be the strongly continuous semigroup of Σ on X with semigroup generator A. Hence X determines 2 additional Hilbert spaces X_1 and X_{-1}. $B \in \mathcal{L}(U, X_{-1})$ is the control operator of Σ. $C \in \mathcal{L}(X_1, Y)$ is its observation operator. z is its state trajectory and $z(0) = z_0 \in X$. $u(t) \in L^2_{loc}([0,\infty); U)$ $(t \geq 0)$ is its input signal at the time t. We define an operator Φ_τ by

$$\Phi_\tau u = \int_0^\tau \mathbb{T}_{\tau - t} B u(t) \mathrm{d}t \qquad \tau \geq 0.$$

It is clear that $\Phi_\tau \in \mathcal{L}(L^2([0,\infty); U), X_{-1})$. We define another operator Ψ_τ by

$$(\Psi_\tau z_0)(t) = \begin{cases} C\mathbb{T}_t z_0 & \text{for } t \in [0,\tau] \\ 0 & \text{for } t > \tau \end{cases}$$

It is clear that $\Psi_\tau \in \mathcal{L}\left(X_1, L^2([0,\infty); Y)\right)$. For the system Σ, we have the following results:

Definition 4. B is said to be an *admissible control operator* for the system Σ (or for the semigroup $\mathbb{T}$) if $\mathrm{Ran}\Phi_\tau \subset X$ for some $\tau > 0$.

Definition 5. C is said to be an *admissible observation operator* for the system Σ (or for the semigroup $\mathbb{T}$) if Ψ_τ has a continuous extension to X for some $\tau > 0$.

Proposition 6. B is an admissible control operator for the system Σ (or for the semigroup $\mathbb{T}$) if and only if its adjoint, B^*, is an admissible observation operator for the dual of Σ, denoted by Σ^* (or for the adjoint of $\mathbb{T}$, denoted by $\mathbb{T}^*$).

Definition 7. The system Σ (or the pair (A, B)) is said to be *exactly controllable in time* $\tau > 0$ if $\mathrm{Ran}\,\Phi_\tau = X$; Σ (or (A, B)) is said to be *approximately controllable in time* $\tau > 0$ if $\mathrm{Ran}\,\Phi_\tau$ is dense in X; Σ (or (A, B)) is said to be *null-controllable in time* $\tau > 0$ if $\mathrm{Ran}\,\Phi_\tau \supset \mathrm{Ran}\,\mathbb{T}_\tau$.

Note: In finite dimensions, the only dense subspace of X is X itself. Therefore in finite dimensions, exact and approximate controllability are equivalent notions. However in infinite dimensions, there exist strict dense subspaces in X. In infinite dimensions, exact controllability seems to have too strong constraint on the final state while approximate controllability is much more relevant to application.

Definition 8. The system Σ (or the pair (A, C)) is said to be *exactly observable in time* $\tau > 0$ if Ψ_τ is bounded from below. Σ (or (A, C)) is said to be *approximately observable in time* $\tau > 0$ if $\mathrm{Ker}\,\Psi_\tau = \{0\}$. Σ (or (A, C)) is said to be *final state observable in time* $\tau > 0$ if there is a $k_\tau > 0$ which satisfies $\|\Psi_\tau z_0\| \geq k_\tau \|\mathbb{T}_\tau z_0\|$ for all $z_0 \in X$.

We often need the controllability concepts without specifying a time τ. Therefore the following definition is introduced.

Definition 9. The system Σ (or the pair (A, B)) is said to be *exactly controllable* if it is exactly controllable in some finite time $\tau > 0$. Σ (or (A, B)) is said to be *approximately controllable* if it is approximately controllable in some finite time $\tau > 0$. Σ (or (A, B)) is said to be *null-controllable* if it is null-controllable in some finite time $\tau > 0$.

Observability concepts without a specified time are introduced in a similar way.

2.3 Passive and conservative linear systems

Passive systems are a class of dynamical systems that can only dissipate energy and cannot produce energy. In passive systems, the energy dissipated by some components in the system equals the difference between the absorbed energy and the increased stored energy. Conservative systems are a special case of passive systems. A passive system is conservative if neither this system nor its dual have any energy dissipation. There are many types of passive and conservative systems. We focus on impedance passive and conservative systems. For more details about passive and conservative systems we refer to Staffans [2002].

Let H be a Hilbert space, $P \in \mathcal{L}(H)$ and $P > 0$. We define an inner product as $< q, \vartheta >_P = < Pq, \vartheta >$ $(\forall q, \vartheta \in H)$ which induces the norm $\|q\|_P = \sqrt{< q, q >_P}$.

Definition 10. A linear system Σ is *impedance P-passive* if, $Y = U$ and for any input signal $u \in L^2_{loc}([0,\infty); U)$, any initial state $z(0) \in X$ and any time $\tau \geq 0$, the following inequality holds

$$\|z(\tau)\|_P^2 - \|z(0)\|_P^2 \leq 2 \int_0^\tau \mathrm{Re} < u(t), y(t) > \mathrm{d}t. \qquad (5)$$

Σ is called *impedance P-energy-preserving* if the above inequality always holds in the form of an equality. Σ is called *impedance P-conservative* if it is impedance P-energy-preserving and its dual system Σ^* is impedance P^{-1}-energy-preserving.

If $P = I$, we say *"impedance passive"* instead of "impedance I-passive". The concepts of *impedance energy preserving* and *impedance conservative* are defined similarly. From the energy point of view, (5) is an energy balance inequality. $E(t) = \frac{1}{2}\|z(t)\|_P^2 = \frac{1}{2} < Pz(t), z(t) >$ stands for the energy stored in the system at the time t, and Re$< u(t), y(t) >$ means the incoming power of the system at the time t.

It is well known that, in finite-dimensional systems, the Kalman-Yakubovich-Popov (KYP) lemma provides equivalence between the positive real property of a system in the frequency domain, the passivity of the system in the time domain, and existence of solution to a linear matrix inequality decided by the operators of system's state-space representation (see Geest and Trentelman [1997], Lozano et al. [2000]). In infinite-dimensional case, we have a similar result. The following lemma is an extension of the KYP lemma to system nodes:

Lemma 11. The system node (2)-(4) is impedance passive if and only if the operator

$$N = \begin{bmatrix} A & B \\ -C\&D \end{bmatrix}, \quad \mathcal{D}(N) = \mathcal{D}(C\&D)$$

is maximal dissipative.

The fact that N is dissipative means that

$$\mathrm{Re}\left\langle \begin{bmatrix} A & B \\ -C\&D \end{bmatrix} \begin{bmatrix} z \\ u \end{bmatrix}, \begin{bmatrix} z \\ u \end{bmatrix} \right\rangle \leq 0 \quad \forall \begin{bmatrix} z \\ u \end{bmatrix} \in \mathcal{D}(N).$$

Now we define *almost impedance passivity* following Curtain and Weiss [2007].

Definition 12. Let Σ be a system node with generating triple (A, B, C), transfer function G, state space X, input space U, and output space Y. Let $E = E^* \in \mathcal{L}(U)$. If we replace G with $G + E$ (and keep A, B, C unchanged), we get a modified system node Σ_E. If there exists E such that Σ_E is impedance passive, we call the original system node Σ *almost impedance passive.*

2.4 Stability and stabilization

Unlike the finite-dimensional linear systems, there are at least three kinds of different asymptotic stability of state space in infinite-dimensional linear systems: weak, strong and exponential stability. The stability of a system is equivalent to the stability of the semigroup of this system.

Definition 13. Let Σ be an infinite-dimensional linear system with strongly continuous semigroup $\mathbb{T}$ and state space X.
(1) The system Σ (or the semigroup $\mathbb{T}$) is called *weakly stable* if $< \mathbb{T}_t z, y > \rightarrow 0$ as $t \rightarrow \infty$, for all $z, y \in X$.
(2) The system Σ (or the semigroup $\mathbb{T}$) is called *strongly stable* if $\mathbb{T}_t z \rightarrow 0$ as $t \rightarrow \infty$, for all $z \in X$.
(3) The system Σ (or the semigroup $\mathbb{T}$) is called *exponentially stable* if its growth bound is negative.

Output feedback stabilization of a system means finding a feedback operator K, to make the system stable with the input $u = Ky$. The closed-loop system must have a strongly continuous semigroup describing the evolution of its state. If the resulting closed-loop system is exponentially, or strongly, or weakly stable, then we call the original system *exponentially, or strongly,* or *weakly stabilizable* (by output feedback) respectively.

The following is a simple but useful proposition, taken from Benchimol [1978].

Proposition 14. If a system is weakly stable and its generator has compact resolvents, this system is strongly stable.

3. STRONG STABILIZATION OF THE SCOLE MODEL

In order to formulate the SCOLE model as a state space system, we introduce the following auxiliary functions: $z_1(x,t) = w(x,t)$, $z_2(x,t) = w_t(x,t)$, $z_3(t) = w_t(l,t)$, $z_4(t) = w_{xt}(l,t)$. Here $z_1(\cdot,t)$ and $z_2(\cdot,t)$ are the states of the beam at the time t $\big(z_1(x,t)$ is the transverse displacement of the beam at the position x and the time t, and $z_2(x,t)$ is the transverse movement speed of the beam at the position x and the time t $\big)$, and $z_3(t)$ and $z_4(t)$ are the states of the rigid body at the time t ($z_3(t)$ is the speed of the rigid body at the time t and $z_4(t)$ is the angular velocity of the rigid body at the time t).

We define $z(t) = [z_1(\cdot,t), z_2(\cdot,t), z_3(t), z_4(t)]^T$ (the superscript T means transpose) to be the state of the SCOLE model at the time t. For each fixed t, this vector consists of two functions of space x and two complex numbers.

The natural energy state space of the SCOLE model is

$$X = \mathcal{H}_l^2(0,l) \times L^2(0,l) \times \mathbb{C}^2,$$

where

$$\mathcal{H}_l^2(0,l) = \big\{ h \in \mathcal{H}^2(0,l) \mid h(0) = h_x(0) = 0 \big\}$$

and $\mathcal{H}^n$ $(n \in \mathbb{N})$ denotes a Sobolev space. As EI and ρ are bounded positive functions of x, we define the inner product on X as follows

$$< q, \vartheta >= \int_0^l \big(EI(x) q_{1xx}(x) \bar{\vartheta}_{1xx}(x) + \rho(x) q_2(x) \bar{\vartheta}_2(x) \big) \mathrm{d}x + m q_3 \bar{\vartheta}_3 + J q_4 \bar{\vartheta}_4,$$

$$\forall q = [q_1, q_2, q_3, q_4]^T \in X, \;\; \forall \vartheta = [\vartheta_1, \vartheta_2, \vartheta_3, \vartheta_4]^T \in X.$$

It is easy to verify that X is a Hilbert space with respect to the above inner product. The norm induced by this inner product is:

$$\|q\|^2 = \int_0^l EI(x) |q_{1xx}(x)|^2 \mathrm{d}x + \int_0^l \rho(x) |q_2(x)|^2 \mathrm{d}x + m|q_3|^2 + J|q_4|^2.$$

If we replace q with $z(t)$ in the above norm, the first term of this norm equals 2 times the bending energy in the beam at the time t; the second term equals 2 times the kinetic energy in the beam at the time t, the third and forth terms represent 2 times the kinetic energy of the rigid body at the time t. So this norm stands for 2 times the physical energy of the whole system at the time t.

Now we formulate this model as a state space system. We define the generator A as follows:

$$Aq = \begin{bmatrix} q_2 \\ -\rho^{-1}(x) \big(EI(x) q_{1xx}(x) \big)_{xx} \\ m^{-1} \big(EI q_{1xx} \big)_x (l) \\ -J^{-1} EI(l) q_{1xx}(l) \end{bmatrix} \quad \forall q \in \mathcal{D}(A),$$

$$\mathcal{D}(A) = \left\{ q \in \left(\mathcal{H}^4 \cap \mathcal{H}_l^2\right) \times \mathcal{H}_l^2 \times \mathbb{C}^2 \;\middle|\; \begin{matrix} q_3 = q_2(l) \\ q_4 = q_{2x}(l) \end{matrix} \right\}.$$

Let $B = [B_1 | B_2]$ where $B_1 = [0, 0, \frac{1}{m}, 0]^T$ and $B_2 = [0, 0, 0, \frac{1}{J}]^T$. We use collocated sensor and actuator, so

$$C = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = B^* = \begin{bmatrix} 0 & 0 & \frac{1}{m} & 0 \\ 0 & 0 & 0 & \frac{1}{J} \end{bmatrix}.$$

We let $D = 0$.

According to the above setting, the SCOLE model (1) can be formulated into the following state space realization

$$\begin{cases} \dot{z}(t) = A z(t) + B u(t), \\ y(t) = C z(t). \end{cases} \tag{6}$$

It is clear that B and C are bounded i.e., $B \in \mathcal{L}(\mathbb{C}^2, X)$, $C \in \mathcal{L}(X, \mathbb{C}^2)$, so that this system is well-posed. We recall that the system input is $u = \left[\begin{smallmatrix} f \\ v \end{smallmatrix}\right]$ where f is force input and v is torque input. It is clear that $y = \left[\begin{smallmatrix} y_1 \\ y_2 \end{smallmatrix}\right] = \left[\begin{smallmatrix} m^{-1} z_3 \\ J^{-1} z_4 \end{smallmatrix}\right]$ where y_1 is proportional to the transverse speed of the rigid body and y_2 is proportional to the angular velocity of the rigid body. We should explain our terminologies. "Only force control" means that the control input is only force f ($v = 0$). In this case output feedback is only from the speed signal y_1. Only torque control means that the control input is only torque v ($f = 0$). In this case output feedback is only from the angular velocity signal y_2. Now we give a brief proof of our main result.

Proposition 15. The generator A of the system (6) is skew-adjoint.

This can be proved by showing that A is skew-symmetric and onto. We omit the proof.

It is well known that if $A : \mathcal{D}(A) \to X$ is skew-adjoint, then A generates an unitary group. Therefore the semigroup of the system (6) is unitary, and this system is not stable.

Proposition 16. The resovents $(\beta I - A)^{-1}$ ($\beta \in \rho(A)$) of the generator A are compact.

Proof. We know that $I \in \mathcal{L}(\mathcal{D}(A), X)$ is a compact operator and $(\beta I - A)^{-1} \in \mathcal{L}(X, \mathcal{D}(A))$ $\forall \beta \in \rho(A)$. it is clear that $I(\beta I - A)^{-1} : X \to X$. So $I(\beta I - A)^{-1}$ is compact. Thus $(\beta I - A)^{-1}$ is compact.

Proposition 17. The system (6) is impedance passive.

Proof. From Proposition 16, we know that A has compact resolvents, which means that A is maximal dissipative if it is dissipative. Combining this fact with Lemma 11, we know that the system (6) is impedance passive if and only if

$$\operatorname{Re} \left\langle \begin{bmatrix} A & B \\ -C & 0 \end{bmatrix} \begin{bmatrix} z \\ y \end{bmatrix}, \begin{bmatrix} z \\ y \end{bmatrix} \right\rangle \leq 0.$$

As A is skew-adjoint (see Proposition 15), it follows that $< Az, z >= 0$. Using this fact, by a computation, we get

$$\operatorname{Re} \left\langle \begin{bmatrix} A & B \\ -C & 0 \end{bmatrix} \begin{bmatrix} z \\ y \end{bmatrix}, \begin{bmatrix} z \\ y \end{bmatrix} \right\rangle = 0.$$

So the system (6) is impedance passive.

Remark 18. In fact the system (6) is impedance conservative. As the fact that the system (6) is impedance passive is enough for us, we omit the simple proof for conservativity.

To prove our main result, we need a proposition from the operator theory:

Proposition 19. Let X be an infinite-dimensional Hilbert space and let $A : \mathcal{D}(A) \to X$ be a skew-adjoint operator with compact resolvents. Then the spectrum of A, $\sigma(A)$, consists of at most countably many imaginary eigenvalues.

Theorem 20. The system (6) is strongly stabilizable by static output feedback from either the speed or the angular velocity of the rigid body.

Proof. From Guo and Ivanov [2005] we know that the system (6) is approximately controllable with only force control (with input $\left[\begin{smallmatrix} f \\ 0 \end{smallmatrix}\right]$) or only torque control (with input $\left[\begin{smallmatrix} 0 \\ v \end{smallmatrix}\right]$). From Proposition 17 we know that the system (6) is impedance passive (regardless of the choice of input signal). By Theorem 1, it follows that system (6) is weekly stabilizable by static output feedback from either the speed or the angular velocity of the rigid body. Combining Proposition 15, Proposition 16 and Proposition 19, it follows that the spectrum of the generator A of the system (6), $\sigma(A)$, consists of at most countably many imaginary eigenvalues. Using Theorem 1 again, we get Theorem 20.

Remark 21. In the proof of Theorem 20, the step from weak stabilization to strong stabilization can also be proved using Proposition 14 by showing that the generator of the closed-loop system has compact resolvents. We omit this proof. One more thing we need to mention is that as system (6) is impedance passive, then the feedback gain k can be taken to be any positive number in theory.

4. CONCLUSION

In this paper, we have given a strongly stabilizing static output feedback for the non-uniform SCOLE model using measurements of either speed or angular velocity of the rigid body. This work was motivated by the study of the stabilization of a wind turbine tower.

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