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MIMO Frequency Domain Iterative Tuning for Tracking Control ⋆

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Abstract: A new 'model-free' iterative controller tuning method is presented for multiple-input multiple output control systems based estimation algorithms in the frequency domain. The method relies on efficient computation of the negative gradient of the controller cost function in the frequency domain. Only one experiment is used per iteration and the method is therefore suitable for realtime implementation by periodic adjustments of the controller. Both feedback and/or feed-forward controllers can be tuned. Primary target application areas can be self-tuning feedforward/feedback controllers in industry where the reference signals are periodic.

Keywords: Self tuning control; Feedback/Feedforward control; Iterative methods; Tracking control; Frequency domains; Multi-input/multi-output systems.

1. INTRODUCTION

The 'model-free' method of Iterative feedback tuning (IFT) has been the subject of intensive research effort during the past decade (Hjalmarsson and Gevers M, 1998; Hjalmarsson, 1999; Hjalmarsson, 2002). A most important advantage of IFT is that it is a model-free method, but it requires additional signal injection path and extra manual experiments in each tuning iteration. In (Luo and Veres, 2007*b*), a new iterative tuning method, i.e. the frequency domain iterative tuning (FD-IT) was developed for active noise and vibration control (ANVC) problems with periodic disturbances, which requires no additional signal injection path or extra manual experiments in each tuning iteration.

This paper extends FD-IT method to more general Multi-Input Multi-Output tracking control problems that rely on handling of frequency response of dynamics and the signals' spectra. The approach is applicable to a variety of controller structures, including FIR and frequency selective filter (FSF) based controllers. Apart from initial experiments, it only requires one experiment per iteration while the iterative feedback/feedforward tuning in earlier publications had to perform multiple experiments for feedback and feedforward controllers. This is an essential step forward that makes our method truly applicable as a multi-variable adaptive controller.

Although the new approach is suitable to solve general control problems theoretically, it is particularly suitable for control problems with finite frequency spectrum signals in practice, such as ANVC (Luo and Veres, 2007*b*; Luo and Veres, 2007*a*), and some tracking control problems in industry.

The remainder of this paper is organized as follows. In Section 2 the problem of gradient-based tuning control for tracking is briefly reviewed in the time domain. In Section 3 the idea of frequency domain iterative tuning (FD-IT) is introduced and some implementation topics are discussed. Section 4 compares FD-IT to the time domain iterative feedback tuning (TD-IFT) method. In Section 5 a series of MIMO simulation examples are presented, two different implementations are compared and the robustness of the algorithm is also discussed. Availability of the algorithms for engineers and intelligent physical agents is pointed out in Section 6. The conclusions sum up the results and points to future research directions.

2. GRADIENT BASED TUNING FOR CONTROL

In this section the control problem is outlined and the basic notations, definitions and performance functions are provided. The following symbols will be frequently used in the paper.

∇	Gradient vector of functions
$\mapsto$	Map to
$:=$	Define or denote
$\{\cdot\}^{\mathrm{T}}$	Transpose
$\{\cdot\}^{*}$	Conjugate and transpose
$\phi_{\{\cdot\}}$	Discrete spectrum of a signal
$\phi_{\{\cdot\}}\|_{\Omega}$	Discrete spectrum of a signal over frequency subset ω
$\Phi_{\{\cdot\}}$	Discrete frequency response function of a dynamics
$\Phi_{\{\cdot\}}\|_{\Omega}$	Discrete frequency response over frequency subset ω
DFT	Discrete Fourier transform
diag($\boldsymbol{x}$)	Diagonal matrix with diagonal vector $\boldsymbol{x}$
FRF	Frequency Response Function
LTI	Linear Time Invariant System

Fig. 1 provides a schematic description of the control system considered. Generally, the system input $\boldsymbol{r}_0$ is assumed periodic. The measured output is represented by $\boldsymbol{z} \in \mathbb{R}^{n_z}$. The desired output signal $\boldsymbol{r} \in \mathbb{R}^{n_r}$ is produced by dynamics R as $\boldsymbol{r} = R(\boldsymbol{r}_0)$. P is the unknown plant dynamics with inputs $\boldsymbol{r}$ and control action $\boldsymbol{u}$, and produce $\boldsymbol{z}$. It can be described as

$$\boldsymbol{z} = P(\boldsymbol{r}, \boldsymbol{u}) \tag{1}$$

⋆ This paper was not presented at any IFAC meeting. Corresponding author S. M. Veres. Email: s.m.veres@soton.ac.uk.

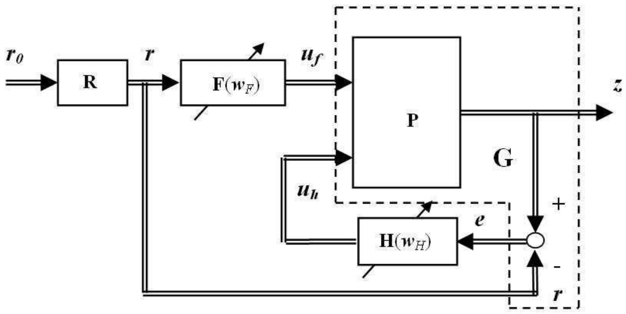

Fig. 1. Block diagram of a linear feedforward feedback system.

The error $\boldsymbol{e} \in \mathbb{R}^{n_e}$ between the system output $\boldsymbol{z}$ and desired output $\boldsymbol{r}$ can be written as

$$\boldsymbol{e} = \boldsymbol{z} - \boldsymbol{r} \tag{2}$$

The control signals from the feedforward controller F and feedback controller H are denoted by $\boldsymbol{u}_f \in \mathbb{R}^{n_u}$ and $\boldsymbol{u}_h \in \mathbb{R}^{n_u}$, respectively. The tunable control system C comprises the parameterized feedforward controller F and the feedback controller H:

$$C(\boldsymbol{w},\boldsymbol{r},\boldsymbol{e}): \quad \begin{aligned} F: \boldsymbol{u}_f &= F(\boldsymbol{w}_F,\boldsymbol{r}) \\ H: \boldsymbol{u}_h &= H(\boldsymbol{w}_H,\boldsymbol{e}) \\ \boldsymbol{u} &= \boldsymbol{u}_f + \boldsymbol{u}_h \end{aligned} \tag{3}$$

which can be tuned by adjusting their parameter vectors in $\boldsymbol{w} := \{\boldsymbol{w}_F, \boldsymbol{w}_H\} \in \mathbb{R}^{n_w}$.

The feedforward reference signal $\boldsymbol{r} \in \mathbb{R}^{n_r}$ is obtained through an unknown but time-invariant dynamics R from $\boldsymbol{r}_0$. In servo control problems, $\boldsymbol{r}_0$ is often assumed stationary and known.

Considering a generalized plant G including P and path of $\boldsymbol{r}$, the above two types of control problems can be described by one system as:

$$\boldsymbol{e} = G(\boldsymbol{r},\boldsymbol{u}) \tag{4}$$

which is framed with the dotted line in Fig. 1.

Assuming a stable LTI system with periodic input $\boldsymbol{r}_0$, the steady output $\boldsymbol{e}$ of G is also periodic. If the system has steady output $\boldsymbol{e}$ with period N then the control performance criterion is defined as the average quadratic performance of a length N output sequence:

$$J(\boldsymbol{w}) := \frac{1}{N}\sum_{t=0}^{N-1} \boldsymbol{e}^{\mathrm{T}}(t) Q \boldsymbol{e}(t) \tag{5}$$

where Q is a priori known weighting matrix.

The objective of iterative tuning control is to adjust the controller parameters $\boldsymbol{w}_F$ and $\boldsymbol{w}_H$ to minimize performance (5).

In general, the problem of minimizing $J(\boldsymbol{w}_F,\boldsymbol{w}_H)$ is not necessarily convex. The tuning method only finds a suboptimal solution at a local minimum. This suboptimal solution of the problem is given by solving $\boldsymbol{w}^o = \{\boldsymbol{w}_F^o, \boldsymbol{w}_H^o\}$ to satisfy

$$\nabla J(\boldsymbol{w}_F^o, \boldsymbol{w}_H^o) = \boldsymbol{0} \tag{6}$$

3. ITERATIVE TUNING IN THE FREQUENCY DOMAIN

In this section a general framework of frequency domain iterative feedback-feedforward tuning is introduced and some implementation issues are also discussed.

3.1 Gradient estimate in the frequency domain

Considering the MIMO system described by Fig.1assume an N-length output data set $\mathscr{E} := \{\boldsymbol{e}(0);\ldots;\boldsymbol{e}(N-1)\}, \boldsymbol{e}(t) := \{e_1(t),\ldots,e_{ny}(t)\} \in \mathrm{R}^{n_e}$, that can be rewritten with the output channels as $\mathscr{E} = \{\boldsymbol{e}_1,\ldots,\boldsymbol{e}_{n_e}\}, \boldsymbol{e}_i = \{e_i(0);\ldots;e_i(N-1)\}, i = 1,\ldots,n_e$.

Using notations $\omega_m := \frac{2\pi}{N}m, m = 0,\ldots,N-1$ the m-th discrete frequency for N-length data, $\boldsymbol{\phi}_e^i := \{\phi_e^i(\omega_0);\ldots;\phi_e^i(\omega_{N-1})\} \in \mathbb{C}^N$ defines the discrete spectrum of N-length $\boldsymbol{y}_i$, which can be estimated by $\boldsymbol{\phi}_e^i \doteq \mathrm{DFT}(\boldsymbol{e}_i)$. Furthermore, the discrete spectrum of $\mathscr{E}$ is described as $\boldsymbol{\phi}_e := \{\boldsymbol{\phi}_e^1;\ldots;\boldsymbol{\phi}_e^{n_e}\} \in \mathbb{C}^{(n_e\times N)\times 1}$. There are similar notations used such as $\boldsymbol{\phi}_r$, $\boldsymbol{\phi}_{uf}$ and $\boldsymbol{\phi}_{uh}$.

In the frequency domain, the plant G is described as a function $\{\boldsymbol{\phi}_d,\boldsymbol{\phi}_u\} \mapsto \boldsymbol{\phi}_e$:

$$\boldsymbol{\phi}_e = \Phi_G(\boldsymbol{\phi}_r,\boldsymbol{\phi}_u) = \Phi_G(\boldsymbol{\phi}_r,\boldsymbol{\phi}_u^1,\ldots,\phi_u^{n_u}) \tag{7}$$

and the controller C is described as a function $\{\boldsymbol{w},\boldsymbol{\phi}_r,\boldsymbol{\phi}_e\} \mapsto \boldsymbol{\phi}_u$:

$$\Phi_C(\boldsymbol{w},\boldsymbol{\phi}_r,\boldsymbol{\phi}_e): \quad \begin{aligned} \Phi_F: \boldsymbol{\phi}_{uf} &= \Phi_F(\boldsymbol{w}_F,\boldsymbol{\phi}_r) \\ \Phi_H: \boldsymbol{\phi}_{uh} &= \Phi_H(\boldsymbol{w}_H,\boldsymbol{\phi}_e) \\ \boldsymbol{\phi}_u &= \boldsymbol{\phi}_{uf} + \boldsymbol{\phi}_{uh} \end{aligned} \tag{8}$$

Note that in LTI systems the frequency response functions (FRF) Φ_G, Φ_H and Φ_F are derivative functions with respect to the inputs inputs' spectrum. Therefore, some notations can be defined as follows: $\Phi_G := \frac{\partial\boldsymbol{\phi}_e}{\partial\boldsymbol{\phi}_u} \in \mathbb{C}^{(n_e\times N)\times(n_u\times N)}$, $\Phi_F := \frac{\partial\boldsymbol{\phi}_{uf}}{\partial\boldsymbol{\phi}_r} \in \mathbb{C}^{(n_u\times N)\times(n_r\times N)}$, $\Phi_F^{(w,u)} := \frac{\partial\boldsymbol{\phi}_{uf}}{\partial\boldsymbol{w}_F} \in \mathbb{C}^{(n_u\times N)\times(n_{wf})}$, $\Phi_H := \frac{\partial\boldsymbol{\phi}_{uh}}{\partial\boldsymbol{\phi}_e} \in \mathbb{C}^{(n_u\times N)\times(n_e\times N)}$ and $\Phi_H^{(w,u)} := \frac{\partial\boldsymbol{\phi}_{uh}}{\partial\boldsymbol{w}_H} \in \mathbb{C}^{(n_u\times N)\times(n_{wh})}$.

Considering the LTI case in Fig. 1, the plant G can be written with increment format as

$$\Delta\boldsymbol{\phi}_e = \Phi_G(\Delta\boldsymbol{\phi}_{uf} + \Delta\boldsymbol{\phi}_{uh}) \tag{9}$$

With regard to the small increment of parameter $\boldsymbol{w}$, i.e., $\boldsymbol{w}_F \to \boldsymbol{w}_F + \Delta\boldsymbol{w}_F$ and $\boldsymbol{w}_H \to \boldsymbol{w}_H + \Delta\boldsymbol{w}_H$, it is straightforward to write

$$\Delta\boldsymbol{\phi}_e = \Phi_G(\Phi_F^{(w,u)}\Delta\boldsymbol{w}_F + \Phi_H^{(w,u)}\Delta\boldsymbol{w}_H + \Phi_H\Delta\boldsymbol{\phi}_e) \tag{10}$$

Using notations $\Delta\boldsymbol{\phi}_{uf}^w := \Phi_F^{(w,u)}\Delta\boldsymbol{w}_F$, $\Delta\boldsymbol{\phi}_{uh}^w := \Phi_H^{(w,u)}\Delta\boldsymbol{w}_H$ and $\Delta\boldsymbol{\phi}_{uh}^y := \Phi_H\Delta\boldsymbol{\phi}_e$, the incremental relationship (10) can be graphically described as shown in Fig 2.

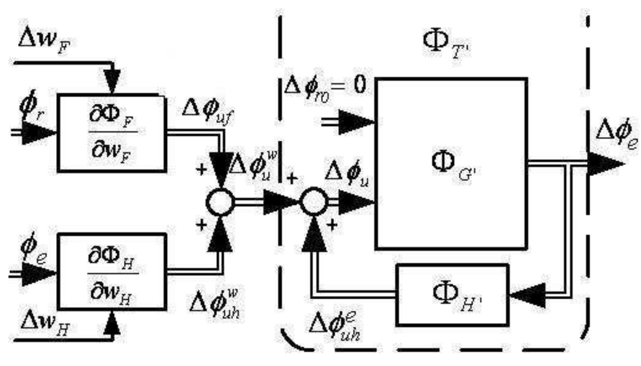

Fig. 2. Block diagram of small increment in frequency domain

According to (10), the input/output increment mapping through the plant dynamics G is $\Delta\boldsymbol{\phi}_u \mapsto \Delta\boldsymbol{\phi}_e$.

Note that the physical increment in the feedback path comprises two parts: $\boldsymbol{\phi}_{uh}^{w}$ caused by the change of controller parameter $\Delta\boldsymbol{w}_H$, and $\boldsymbol{\phi}_{uh}^{y}$ caused by the change of output $\Delta\boldsymbol{\phi}_e$. Denoting $\Delta\boldsymbol{\phi}_u^w := \Delta\boldsymbol{\phi}_{uf}^w + \Delta\boldsymbol{\phi}_{uh}^w$, if $(I-\Phi_G\Phi_H)^{-1}$ exists, the input/output mapping $\Delta\boldsymbol{\phi}_u^w \mapsto \Delta\boldsymbol{\phi}_e$ can be rewritten from (10) as

$$\Delta\boldsymbol{\phi}_e = (I-\Phi_G\Phi_H)^{-1}\Phi_G(\Delta\boldsymbol{\phi}_{uf}^w + \Delta\boldsymbol{\phi}_{uh}^w) \quad (11)$$

Considering LTI closed loop dynamics $T := \{G,H\}$, the FRF of T is defined as

$$\Phi_T := (I-\Phi_G\Phi_H)^{-1}\Phi_G \in \mathbb{C}^{(n_e\times N)\times(n_u\times N)}, \quad (12)$$

From (11), the derivative of $\boldsymbol{\phi}_e$ with respect to controller parameters $\boldsymbol{w}_H$ and $\boldsymbol{w}_F$ can be written as

$$\frac{\partial\boldsymbol{\phi}_e}{\partial\boldsymbol{w}_H} = \Phi_T\Phi_H^{(w,u)} \quad (13)$$

and

$$\frac{\partial\boldsymbol{\phi}_e}{\partial\boldsymbol{w}_F} = \Phi_T\Phi_F^{(w,u)} \quad (14)$$

According to Parseval's theorem (Oppenheim and Willsky, 1996), it is straightforward to write (5) in the frequency domain format as

$$J = \frac{1}{N^2}\sum_{i=1}^{n_e}\sum_{j=0}^{N-1}\phi_e^{i*}(\omega_j)q_i\phi_e^i(\omega_j) = \frac{1}{N^2}\boldsymbol{\phi}_e^* Q_F\boldsymbol{\phi}_e \quad (15)$$

where $Q_F \in \mathbb{R}^{(n_e\times N)\times(n_e\times N)}$ is the performance weighting matrix Q in the frequency domain.

The derivative of performance J with respect to controller parameters can be written as

$$\frac{\partial J(\boldsymbol{w})}{\partial w_i} = \frac{2}{N^2}\boldsymbol{\phi}_e^* Q_F\Phi_T\Phi_C^{(w_i,u)} \quad (16)$$

where $\Phi_C^{(w_i,u)} := \frac{\partial\Phi_C(\boldsymbol{w},\boldsymbol{\phi}_e,\boldsymbol{\phi}_r)}{\partial\boldsymbol{w}_i}$.

To summarize, since the output $\boldsymbol{y}$, the controller H and the reference $\boldsymbol{r}$, F are either detectable or a priori known by the design engineer, $\boldsymbol{\phi}_e$ and $\Phi_C^{(w_i,u)}$ are both available in (16). If Φ_T can be either a priori known or estimated through experiments, $\nabla J(\boldsymbol{w})$ can be computed using (16).

3.2 Tuning of MIMO systems in the frequency domain

In (16) the key to estimate $\frac{\partial J(\boldsymbol{w})}{\partial w_i}$ is to compute Φ_T, which has $n_e\times N$ rows and $n_u\times N$ columns.

In this subsection, the above conclusion about gradient estimate in the frequency domain, i.e., (13),(14) and (16), is studied for the case of LTI systems with periodic signals.

First note that some signals in engineering can often be considered to have finite discrete spectrum, especially in manufacturing problems, harmonic signal recovery and compensation. For a periodic output $\boldsymbol{e}$ with common period N, only a finite set of frequencies, $\boldsymbol{\Omega} = \{\omega_1,\ldots,\omega_{n_\Omega}\}$, are included in $\boldsymbol{\phi}_e$, the other elements in $\boldsymbol{\phi}_e$ are 0. Therefore, in order to find $\frac{\partial J(\boldsymbol{w})}{\partial w_i}$ in (16), only the rows in Φ_T with respect to $\boldsymbol{\Omega}$ is required to be computed, which will be denoted by $\Phi_T|_\Omega$ in the following discussion. There will be similar notations used such as $\Phi_G|_\Omega$, $\Phi_H|_\Omega$ and $\Phi_F|_\Omega$.

Hence (12) can be rewritten in the finite frequency format as

$$\Phi_T|_\Omega = (\boldsymbol{I}-\Phi_G|_\Omega\Phi_H^i|_\Omega)^{-1}\Phi_G|_\Omega, \quad (17)$$

and similarly (16) can be rewritten as

$$\nabla J(\boldsymbol{w}^i) = \frac{2}{N^2}\boldsymbol{\phi}_e^*|_\Omega\Phi_T^i|_\Omega\Phi_C^{(w,u)}|_\Omega. \quad (18)$$

Remark 1. It should be noted that there was no limitation about the linearity of the system and the spectrum of $\boldsymbol{y}$, $\boldsymbol{r}$ and $\boldsymbol{u}$. Theoretically, the gradient based tuning described by (13), (14) and (16) is applicable for most control problems.

However, in LTI systems, the matrix Φ_T^i is a diagonal matrix that represents independent frequency responses in the frequency domain. Eqn. (18) can be solved as n_Ω sub-problems for every single frequency ω_i. The computation of gradient estimate in the frequency domain is simple for LTI systems.

In case of periodic signals, the signals' spectra have only n_Ω non-zero values to proceed. When the frequency number n_Ω is much less than the common period N, the gradient estimate in the frequency domain can be greatly simplified relative to that in the time domain, which has N data to proceed.

Secondly, an indirect estimate of Φ_T is more convenient for online tuning. According to (12), if Φ_G can be estimated, Φ_T can be solved since H is known by the designer.

Note that if Φ_G is assumed to be an LTI system, then the FRF is independent in the different frequencies. To ease the notation, for a single frequency FRF of Φ_G, the $\Phi_G(\omega)$ is used in the following discussion, and the extension to complete Φ_G is straightforward.

Note that $\Phi_G(\omega) \in \mathbb{C}^{n_e\times n_u}$ has $n_e\times n_u$ unknown variables, which can be solved through a full-rank $n_e\times n_u$ equation matrix.

Considering plant G, it is straightforward to get an equation system

$$\Delta\phi_e(\omega) = \Phi_G(\omega)\Delta\phi_u(\omega), \quad (19)$$

which gives n_e equations.

Therefore, considering the case of the full rank equations, given n_u such equation groups as in (19), $\Phi_G(\omega)$ can be obtained by solving an equation system with $n_u\times n_e$ equations.

To summarize, under the assumption of a finite frequency set $\boldsymbol{\omega}$ for the disturbance and assuming an LTI system, we have the following tuning strategy in the frequency domain:

At the i-th iteration,

(1) Estimate $\Phi_G|_\Omega$ by solving the equation set from (19);
(2) Calculate $\Phi_T^i|_\Omega$ with (17);
(3) Obtain the derivative of J using (18);
(4) Update the controller parameter $\boldsymbol{w}$ with

$$\boldsymbol{w}^{i+1} = \boldsymbol{w}^i - \mu\nabla J(\boldsymbol{w}^i) \quad (20)$$

where μ is a proper step size to update the controller.

Remark 2. As above stated, at least n_u different equation groups as (17) are required to determine $\Phi_G(\omega)$, which means n_u pairs of difference data $\{\Delta\boldsymbol{u},\Delta\boldsymbol{y}\}$ are required. In the implementation, in order to get the estimate of $\Phi_G|_\Omega$, $1+n_u$ experiments are required to yield n_u pairs of $\{\Delta\boldsymbol{u},\Delta\boldsymbol{y}\}$.

For LTI systems, $\Phi_G|_\Omega$ is considered unchanged and can be estimated offline. $\Phi_T|_\Omega(\Phi_G|_\Omega,\Phi_H^i|_\Omega)$ can be updated with the change of H^i. Therefore, as shown in Fig. 3, to make N_T times gradient based tuning, plus the n_u additional experiments, N_T+n_u time iterations are necessary to perform.

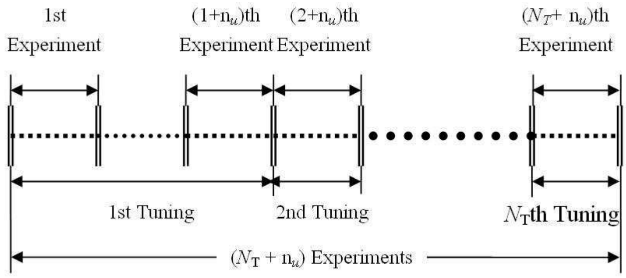

Fig. 3. Experiment and tuning iteration in FD-IT

Compared to the multi-variable time-domain IFT method (Hjalmarsson, 2002), the obvious advantage of FD-IT is that FD-IT requires much less iterations to tune than IFT. As stated in (Hjalmarsson, 1999; Hjalmarsson, 2002; Jansson and Hjalmarsson, 2004), $n_u \times n_e$ gradient experiments for feedback controller H and $n_u \times n_r$ gradient experiments for feed forward controller F are necessary, which gives $(1+n_u \times n_e + n_u \times n_r)$ experiments in order to compute all gradients for one tuning. In order to perform N_T iterations, FD-IT can finish within $N_T + n_u$ experiments, while IFT requires as many as $N_T \times (1+n_u \times n_e + n_u \times n_r)$ experiments.

4. DIRECT DERIVATION OF CLOSED LOOP DYNAMICS FOR LTI SYSTEMS

In the above derivations the infinitesimal increment equations such as (10) are used because nonlinearities can be locally linearized with infinitesimal incremental equations. In the linear time invariant (LTI) case, the conclusion of the proposed gradient estimate theory can be directly deduced through derivation of the closed loop dynamics.

A typical linear feedback and feedforward control system can be expressed with closed loop dynamics T in the time domain as

$$\boldsymbol{z} = \frac{GF(\boldsymbol{w}_F)}{1-GH(\boldsymbol{w}_H)}\boldsymbol{u}, \tag{21}$$

where the plant G is an LTI system, LTI feedback controller $H(\boldsymbol{w}_H)$ and feedforward controller $F(\boldsymbol{w}_F)$ have tunable parameters $\boldsymbol{w}_H$ and $\boldsymbol{w}_F$, respectively.

It can be derived in the frequency domain (Dorsey, 2002) that

$$\boldsymbol{\phi}_z = [I - \Phi_G \Phi_H(\boldsymbol{w}_H)]^{-1}\Phi_G \Phi_F(\boldsymbol{w}_F)\boldsymbol{\phi}_u \tag{22}$$

as illustrated in Fig. 1, $\boldsymbol{\phi}_e = \boldsymbol{\phi}_z - \boldsymbol{\phi}_r$. While $\boldsymbol{r}$ is not correlated to $\boldsymbol{w}_H$ and $\boldsymbol{w}_F$, it is straightforward to get

$$\frac{\partial \boldsymbol{\phi}_e}{\partial \boldsymbol{w}_H} = \frac{\partial \boldsymbol{\phi}_z}{\partial \boldsymbol{w}_H}, \frac{\partial \boldsymbol{\phi}_e}{\partial \boldsymbol{w}_F} = \frac{\partial \boldsymbol{\phi}_z}{\partial \boldsymbol{w}_F} \tag{23}$$

According to (22), it is straightforward to obtain

$$\frac{\partial \boldsymbol{\phi}_z}{\partial \boldsymbol{w}_F} = [I - \Phi_G \Phi_H]^{-1}\Phi_G \frac{\partial \Phi_F(\boldsymbol{w}_F)}{\partial \boldsymbol{w}_F}\boldsymbol{\phi}_u = \Phi_T \frac{\partial \Phi_F(\boldsymbol{w}_F)}{\partial \boldsymbol{w}_F}\boldsymbol{\phi}_u \tag{24}$$

which is the same as (14).

According to (22), the derivative matrix $\frac{\partial \boldsymbol{\phi}_z}{\partial \boldsymbol{w}_H}$ first gives

$$\frac{\partial \boldsymbol{\phi}_z}{\partial \boldsymbol{w}_H} = [I - \Phi_G \Phi_H]^{-2}\Phi_G \frac{\partial \Phi_H(\boldsymbol{w}_H)}{\partial \boldsymbol{w}_H}\Phi_G \Phi_F \boldsymbol{\phi}_u \tag{25}$$

In LTI systems the $[I - \Phi_G \Phi_H]^{-1}$, Φ_G and $\frac{\partial \Phi_H(\boldsymbol{w}_H)}{\partial \boldsymbol{w}_H}$ are all diagonal matrixes, and their positions are exchangeable in (25), which gives

$$\begin{aligned}
&[I - \Phi_G \Phi_H]^{-2}\Phi_G \frac{\partial \Phi_H(\boldsymbol{w}_H)}{\partial \boldsymbol{w}_H}\Phi_G \Phi_F \boldsymbol{\phi}_x \\
&= \{[I - \Phi_G \Phi_H]^{-1}\Phi_G\}\{\frac{\partial \Phi_H(\boldsymbol{w}_H)}{\partial \boldsymbol{w}_H}\}\{[I - \Phi_G \Phi_H]^{-1}\Phi_G \Phi_F \boldsymbol{\phi}_x\} \\
&= \Phi_T \frac{\partial \Phi_H(\boldsymbol{w}_H)}{\partial \boldsymbol{w}_H}\boldsymbol{\phi}_z
\end{aligned} \tag{26}$$

Considering the three braced items in (26), using the notation T of the closed loop dynamics and system output spectrum in (22), it is straightforward to get that

$$\frac{\partial \boldsymbol{\phi}_e}{\partial \boldsymbol{w}_H} = \Phi_T \frac{\partial \Phi_H(\boldsymbol{w}_H)}{\partial \boldsymbol{w}_H}\boldsymbol{\phi}_z \tag{27}$$

which is the same as (13).

Therefore the new proposed gradient theory can be intuitively explained from the LTI case derivatives by the deduction through the derivation of system dynamics.

5. SIMULATION

This section illustrates the usefulness of FD-IT for tracking control through simulation in MATLAB. Frequency-selective filter (FSF) based FD-IT is used in this simulation, for the details of the filtering algorithm we refer to (Luo and Veres, 2007*b*).

5.1 Elliptic track simulation

The block diagram of the SIMULINK-based MIMO system to be controlled is given in Fig. 4. It is a 2-input and 2-output LTI system. $y1$, $y2$, $r1$ and $r2$ denote the data acquired for output and reference signals. The output $\boldsymbol{y}1$ and $\boldsymbol{y}2$ represent positions of x-axis and y-axis in a 2-dimension space, respectively.

The sampling frequency is 4kHz. The signal $\boldsymbol{r}_0$ is a 50Hz harmonic signal, leading to:

$$r_0(t) = \sin(50\pi t) \tag{28}$$

Modules $Uf1$ and $Uf2$ denote the sensor noise in the feedforward paths. They are assumed as white noise with standard deviation 0.001.

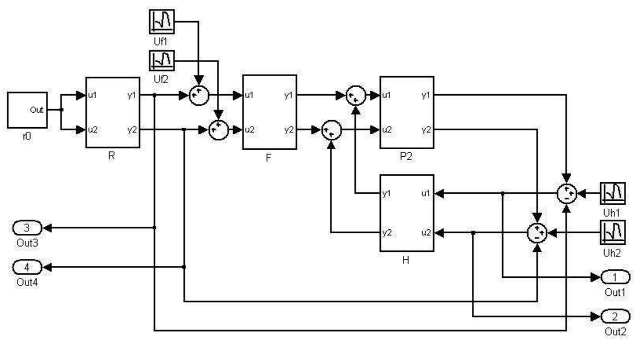

Fig. 4. Block diagram for simulation

In Fig. 4 the unknown plant P is given by

$$\begin{bmatrix} \dfrac{0.15q^{-7}-0.3q^{-8}}{1+0.2q^{-1}-0.2q^{-2}} & \dfrac{0.02q^{-6}-0.03q^{-7}}{1+0.02q^{-1}+0.01q^{-2}} \\ \dfrac{-0.02q^{-6}-0.01q^{-7}}{1+0.02q^{-1}-0.01q^{-2}} & \dfrac{-0.2q^{-8}-0.3q^{-9}}{1+0.1q^{-1}-0.2q^{-2}} \end{bmatrix}, \quad (29)$$

where there are white noise signals $Uh1$ and $Uh2$ added with standard deviations 0.01 in $y1$ and $y2$ output paths.

The reference signal $\boldsymbol{r}$ to be tracked is filtered by dynamics R as

$$\begin{bmatrix} \dfrac{0.1q^{-21}}{1-0.2q^{-2}} & 0 \\ 0 & \dfrac{0.8q^{-4}}{1+0.4q^{-1}} \end{bmatrix}, \quad (30)$$

which leads to the desired track as an elliptic track shown in Fig. 5: An FSF-based controller structure is used. 1st-order

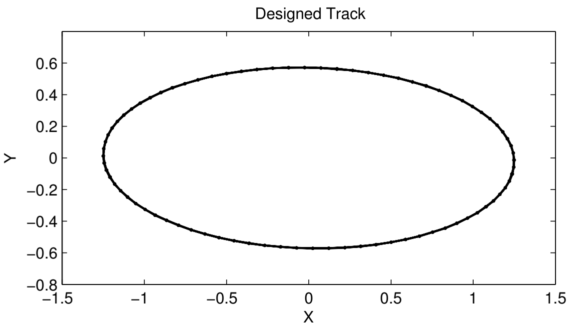

Fig. 5. Desired track

Butterworth bandpass filters are online designed according to the spectrum of $\boldsymbol{y}$ that is 50Hz, and the bandwidths of the FSF are given by the disturbance frequency ± 10 percent which also eliminates the unwanted white noise in the tuning. The 2-th order FIR controllers are used as tunable module in feedback and feedforward paths, which are cascaded with those Butterworth bandpass filters.

In order to describe the the tracking error, the cost function is defined as in (5), where the common period is set to $N = 320$, and the weighting matrix is $Q = \mathrm{diag}([1.0\,1.0])$. The step size (adaptation gain) for feedforward controller tuning is $\mu_f = 8.0$ and step size for feedback controller tuning is $\mu_h = 0.8$.

At the beginning all the initial feedback controller parameters are set to zero, and the initial feed-forward path is set to 0.1. The initial output track is shown in Fig. 6. The first performance criterion without control is 0.9588. In order to perform an initial estimate of G, only the sub-block from $\boldsymbol{r}_1$ to $\boldsymbol{u}_{f1}$ in H is changed to 0.2 in the 2nd iteration, and only the sub-block from $\boldsymbol{r}_2$ to $\boldsymbol{u}_{f2}$ in H is changed to 0.2 in the 3rd iteration.

Fig. 7 shows the updating performance in the simulation: the 2nd and 3rd iterations are manual updates, which give $J(2) = 0.9758$ and $J(2) = 0.9406$. After 40 experiments the final performance is $J = 0.0012$. The final track after tuning is shown in Fig.8.

Compared with Fig. 6 and Fig. 8, it is obvious that the matching performance of the final track greatly improved. It is worthwhile to note that in this simulation example FD-IT only dealt with one gradient computation with respect to frequency 50Hz while the time domain gradient estimation methods often have to process at least 80 data.

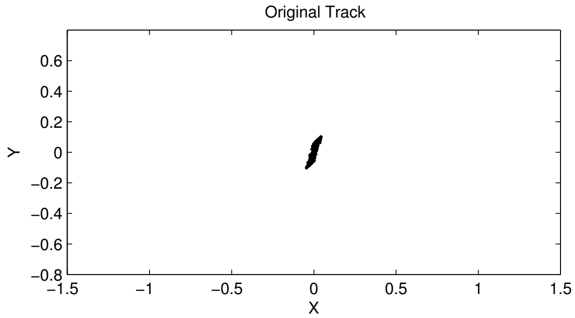

Fig. 6. Initial track

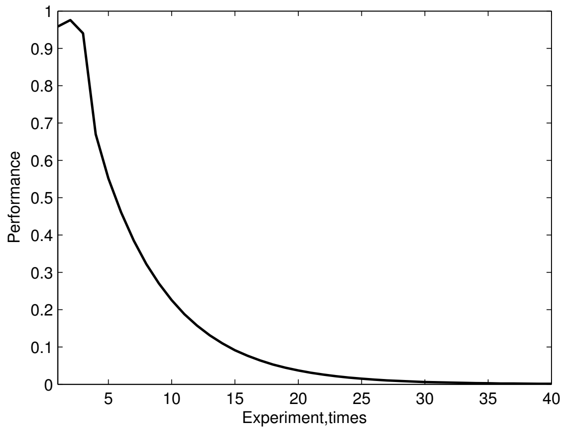

Fig. 7. Performance updating

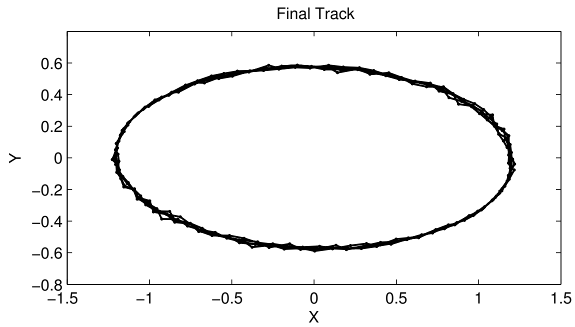

Fig. 8. Final track after 40 experiments

As shown in Fig 4, there is no additional signal path to inject some extra signals for gradient estimation as in the time domain IFT, which simplifies the structure of the control system. As shown in Fig 7, only the first three experiments are manually set, and the subsequent tunings are based on their preceding experiments without extra experiments to carry out, which makes it implementable as a realtime adaptive controller.

6. AVAILABILITY OF ALGORITHMS

Using the basic concepts of 'experiments', 'signals', their 'transforms', etc. most algorithms of this paper are available in a natural language programming format in sEnglish at sysbrain.org in the 'articles/analytic dynamical control' section under 'Frequency domain IFT'. There the algorithms are presented in sentences such as 'Estimate Φ_G over frequency set $\boldsymbol{\Omega}$ using $\boldsymbol{\phi}_e$ and $\boldsymbol{\phi}_u$. Compute Φ_T from Φ_G and Φ_H using feedback formula. Obtain the derivative of J with respect to parameters $\boldsymbol{w}_C$ from $\boldsymbol{\phi}_e$, Φ_T and Φ_C.' etc. All sentences compile into MATLAB code unambiguously for human implementation. The associated sEnglish paper can be read by engineers as well as intelligent physical agents equipped with an sEnglish interpreter that can then use these algorithms. sEnglish papers can be equally read by humans as well as by agents whom can hence develop a shared understanding.

7. CONCLUSIONS

An iterative feedback/feedforward tuning approach has been presented that uses an innovative way of computing gradient estimates of the controller cost function in the frequency domain. Compared to IFT in time domain, this method simplifies both control structure and control operation.

The method is suitable for industrial applications with feedback and feedforward controllers where periodic signals have to be tracked. First a general framework of the IFT approach was provided in the frequency domain and then some detailed techniques were discussed for applications. Secondly, comparisons were made and relationship with the time domain IFT was discussed. The effectiveness, flexibility and robustness of FD-IT was shown by simulation examples.

As the basic scheme was outlined and tested in simulation, the robustness of the obtained controllers is still questionable. Future work on robustification will be possible to perform directly in the frequency domain. Extension of the general framework to other control application such as disturbance rejection, vibration and noise control also require further research.

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