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New Approaches to Control Education ⋆

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Abstract: In this paper, we describe recent developments in theory and computational aids that signal a new approach to a design oriented control education curriculum at both the undergraduate and graduate levels. The main features of this approach are: a) Analytical results developed for low order controllers such as Proportional-Integral-Derivative (PID) and lead/lag controllers which account for 99% of controllers in applications, b) Development of tools to obtain complete sets of controllers achieving stability, performance and their efficient graphical representations and c) An approach to design based on raw measured data rather than identified models. The paper is illustrated with examples of design using both recently developed commercially available software and custom software developed by us.

1. INTRODUCTION

We propose a design oriented undergraduate control curriculum wherein real-world problem scenarios are introduced and the theory is developed to address such issues. This approach is possible due to recent results obtained by us and others where an analytical approach to "design" is feasible. The main features of this theory are an emphasis on the design of controllers of low and fixed order (Hara et al. [2006], Henrion et al. [2004], Haddad et al. [1993]), based on measured data (Park and Ikeda [2004], Yasumasa et al. [2005]) on the plant and required to satisfy multiple performance specifications.

In this paper, we specifically discuss recent results on the synthesis and design of Proportional-Integral-Derivative (PID) controllers which are most important in applications (Silva et al. [2005], Tantaris et al. [2002, 2003]). In the following sections, we discuss the new approach to design applied to these classes of controllers. Next we consider the design of continuous and discrete-time controllers, with and without time delay. We then describe the extension of these results to model free data-based design of control systems. Finally, we present a commercially available software developed by National Instruments using our methods.

2. A NEW APPROACH TO CONTROL EDUCATION

In the new approach, stress is laid on low order controller design which is predominantly used in the industry. This is an attempt to bring academia closer to industrial control. In recent results the entire stabilizing set of controllers for such systems has been obtained in terms of linear inequalities. It can be mentioned that methods such as Routh-Hurwitz criteria can also give inequalities but they get highly nonlinear in nature and cannot be solved analytically (Datta et al. [2000]). Traditional methods of controller design find a single controller which achieves one particular desired performance criterion. On the other hand, with our approach, we have the entire set of stabilizing values, and we can then solve the more realistic problem of computing subsets achieving several various desired performance objectives (Mitra et al. [2007]).

In general any design problem can be formulated as follows: There are a set of n adjustable design variables in a system which are labeled as $x_1,\ x_2, \cdots,\ x_n$ and let

$$\bar{x} = [x_1, x_2, \cdots, x_n] \in R^n$$

Consider m performance criteria $f_1,\ f_2, \cdots,\ f_m$ which are functions of these adjustable variables such that

$$f_1(x_1, x_2, \cdots, x_n) \in R, \cdots f_1(x_1, x_2, \cdots, x_n) \in R$$

There are performance requirements described by $f_1 > F_1$ or $f_1 < F_1$ where F_1 is some specified value. This in turn gives a solution set $S_1 \in R^n$ which is a set of all values of x which satisfies the above criteria. In general

$$S_i = \{\bar{x}\ :\ f_i(\bar{x}) \geq F_i\} \text{ where } i = 1, 2, \cdots, k. \tag{1}$$

In case of a control design problem there will be a system which will have multiple such specifications and the objective is to find a set of these adjustable variables which make the closed loop system stable and also simultaneously satisfy all performance requirements. In mathematical terms, let

$$S_0 := \{\bar{x}\ :\ \text{Closed loop system is stable}\} \tag{2}$$

and S_i as described in (1), Then our objective is to find a set S such that

$$S = \cap_{i=1}^{k} S_0 \cap S_i \tag{3}$$

Generally the functions f_i are highly non-linear functions of the design parameters and it is not possible to solve

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them analytically. With the high speed computational power available today sometimes these can be solved point by point in considerably less time. Nevertheless it is desirable to develop theory that yields a "linear" solution whenever possible. This is shown below.

In our case, we consider PID controllers in which there are three adjustable variables k_p, k_i and k_d. The closed loop system has performance criteria, typically gain margin, phase margin, overshoot, rise-time. For this case, firstly the entire set S_0 of k_p, k_i and k_d which stabilizes the given plant is determined. The design objective is to find subsets of the set S_0 attaining given performance specifications gain-margin $> F_{GM}$, phase-margin $> F_{PM}$, over-shoot $< F_{OS}$ etc. Then with the aid of computer the set $S = S_{GM} \cap S_{PM} \cap S_{OS} \cdots$ is obtained which lies within S_0. This set meets all the design objectives.

With a design method like this, and software developed applying the algorithms developed, one can effectively design controllers satisfying several design constraints. Additionally the custom software developed also has the capability of analysis wherein, once a particular controller is chosen from the stable set, its time response, required control signal, error signal are also obtained from built-in functions.

For educational purposes, once the theory is explained, such tools allow the students to explore actual design problems. For undergraduate course work, even the theory may be omitted so that they may have a true experience in control system design.

3. DESIGN OF PID CONTROLLERS

In this section, we discuss PID Controller design for continuous time systems when the model of the system is known. Algorithms have been developed for systems with and without time delays (Silva et al. [2005]). Here, a brief overview is provided along with illustrative examples.

3.1 Continuous-Time PID for Delay-Free Plants

Let us consider a plant P(s) given by

$$P(s) = \frac{N(s)}{D(s)} \tag{4}$$

which is to be stabilized by a PID controller of structure

$$C(s) = K_p + \frac{K_i}{s} + K_d s \tag{5}$$

in a unity feedback loop. The characteristic polynomial of the closed loop system $\delta(s)$ is given by

$$\delta(s) = N(s)(K_p s + K_i + K_d s^2) + sD(s) \tag{6}$$

Now we construct a polynomial $\nu(s) = \delta(s)N(-s)$.

When this polynomial is evaluated at $s = \jmath\omega$, it is seen that the imaginary part contains only K_p while the real part contains K_i and K_d, that is the parameters are "separated". Using the Generalized Hermite-Biehler Theorem, we can then obtain linear inequalities in K_i and K_d for fixed K_p for the stabilizing set. For the detailed algorithm, see (Mitra et al. [2007]).

To illustrate, consider the following example where for a given plant $P(s)$ the entire set of PID controllers is to be determined.

$$P(s) = \frac{s^3 - 2s^2 - s - 1}{s^6 + 2s^5 + 32s^4 + 26s^3 + 65s^2 - 8s + 1} \tag{7}$$

On entering the numerator and denominator in the GUI as shown in Fig. 1, we obtain a range of K_p values for which there may exist a set in $K_i - K_d$ space. For this example, we observe that this range is between -24 and 0. When a particular value of K_p is selected, say $K_p = 15$, a $K_i - K_d$ set is obtained generated from the linear inequalities obtained from the real part of the polynomial evaluated at the real, distinct zeros of odd multiplicities of the imaginary part at the chosen value of K_p. In this case, 2 disjoint regions are obtained as shown in Fig. 1.

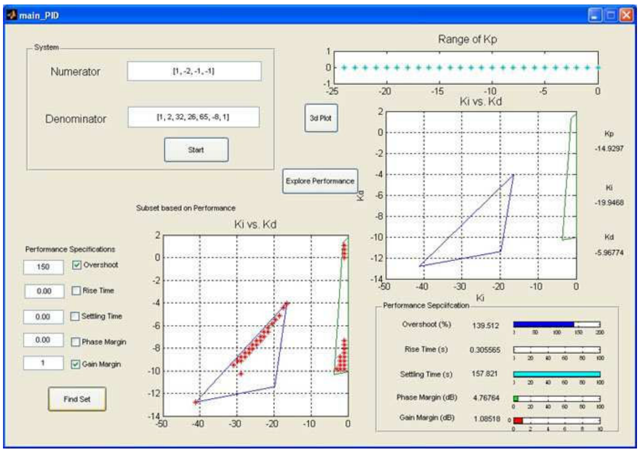

Fig. 1. GUI for PID Controller design for a continuous time system.

On selecting a particular point in the $K_i - K_d$ set, the performance specifications for the closed loop system for the particular controller values are evaluated and are displayed. On exploring different points in the stabilizing region, we can observe the trends in different performance specifications across the region. Further, when some performance criteria like gain margin $< 1db$ and overshoot $< 110\%$ are specified, the subset satisfying the required criteria is also displayed. The entire 3-D stabilizing set of $K_p - K_i - K_d$ is also obtained as shown in Fig. 2.

3.2 Continuous-Time PID for systems with delay

The plant and controller are as in equations (4) and (5). Consider a delay of L_0 in the loop which can be described as

$$P(s) = \left[\frac{N(s)}{D(s)}\right] e^{-L_0 s} \tag{8}$$

The aim is to find the stabilizing values of K_p,K_i and K_d which will stabilize the plant with delay L_0. For the detailed algorithm, see (Silva et al. [2005]). In this case,for a fixed K_p, first we evaluate the stabilizing set without time delay and call this set S_N. Next we find the set of all $K_i - K_d$ lines as ω goes from $-\infty$ to ∞ for which the system will be unstable for the given delay and denote

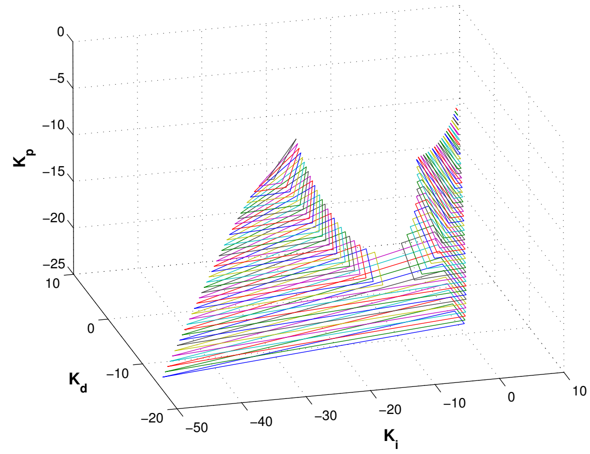

Fig. 2. 3D stabilizing set of PID Controller parameters for continuous time system.

this by S_{L,K_p}. Subtracting S_{L,K_p} from S_N, we obtain the stabilizing set.

As an example, consider a plant with delay of 1 unit described as:

$$P(s) = \left[\frac{s^3 - 4s^2 + s + 2}{s^5 + 8s^4 + 32s^3 + 46s^2 + 46s + 17}\right] e^{-1s} \qquad (9)$$

First, we obtain the stabilizing set for this system without any delay. The stabilizing set for $K_p = 1$ is as shown in Fig. 3. Next, we draw all the $K_i - K_d$ lines which form the set S_{L,K_p}. The region for which the plant is stable with a delay of 1 unit is given by $S_N/S_{L,K_p}$ as shown by the shaded region in Fig. 3.

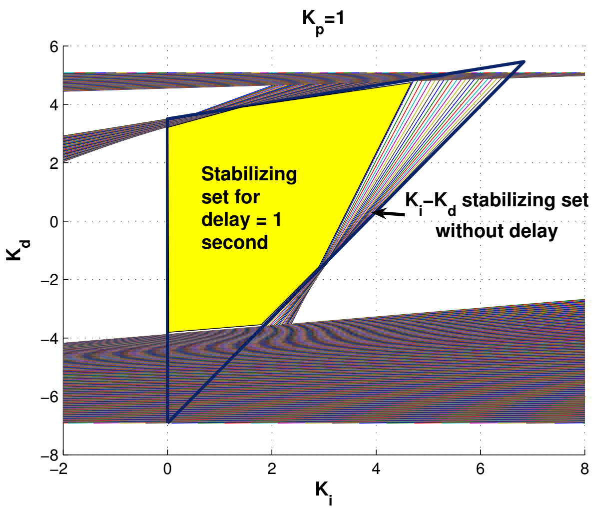

Fig. 3. Stabilizing set for system with time delay

4. DATA BASED DESIGN OF PID CONTROLLERS

In most practical design problems, we do not have the model of the system and instead have some input output measurement data of the system. In that case, identification of the plant becomes necessary if one is constrained to use a model based theory. In this section, we illustrate design of PID controllers directly from data without identifying the system.

4.1 Continuous-Time Systems

In this case, the only information needed is the frequency response of the system. For a detailed theory, (Keel and Bhattacharyya [2005]). Here we describe one example and show how the PID stabilizing set is obtained using the GUI developed in MATLAB. Let the frequency response of a plant be as shown in Fig. 4.

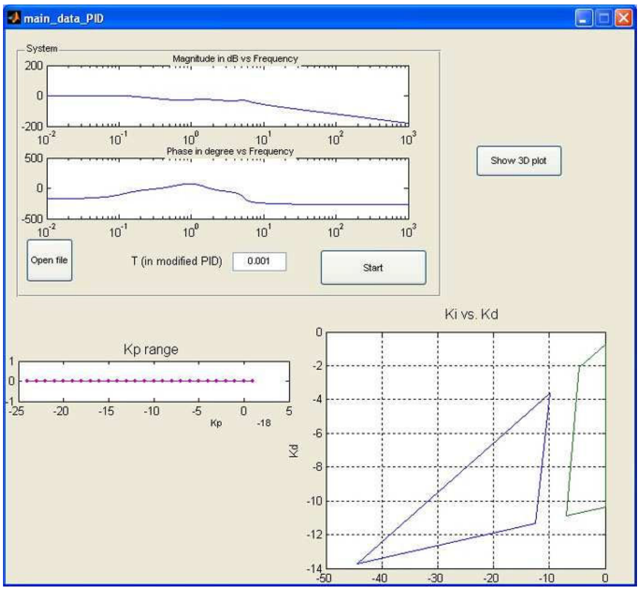

Fig. 4. GUI for model free design of continuous time PID controller

With this data, K_p Vs ω is plotted as shown in Fig. 5. It can be seen that for a fixed value of K_p, the graph has many values of ω. For a given plant, there are a minimum number of cuts required to have stabilizing set in $K_i - K_d$ space. This determines the range of K_p. On selecting a particular value of K_p in this range, say $K_p = -18$ inequalities are obtained in $K_i - K_d$ space based on the ω values and the plant response at those values where the K_p line cuts the graph. The stable set is shown in Fig. 4.

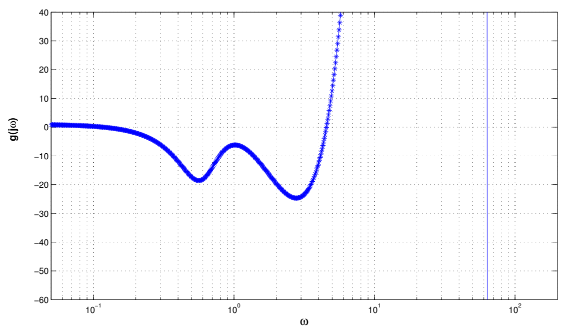

Fig. 5. K_p Vs ω plot

Data robust design of PID controllers has also been developed which can robustly stabilize a given plant when there is uncertainty with respect to the measured data.

4.2 Discrete-Time Systems

In many discrete time plants, it is not possible to obtain the frequency response of the plant but the step response of the plant may be available. In that case, we obtain the Markov parameters from the step response data and then obtain the stabilizing set of PID controllers. For details, see (Mitra et al. [2007]).

Here, we illustrate an example with the GUI developed. Consider the step response data of the plant as shown in Fig. 6.

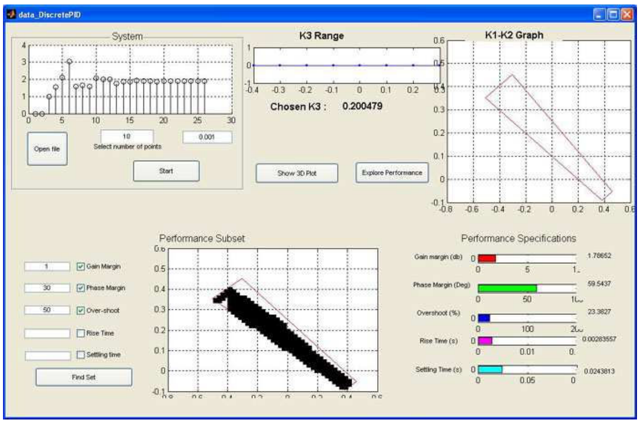

Fig. 6. GUI for discrete time data based PID controller design

On entering the number of points as 10 and sampling time of 0.001s, the feasible range of K_3 is displayed. On choosing $K_3 = 0.2$, the corresponding $K_1 - K_2$ stabilizing set is obtained. On selecting $K_1 = 0.0$ and $K_2 = 0.2$, various time and frequency performance parameters like gain margin, rise time, overshoot etc. are displayed for this chosen controller.

Further on specifying some performance objectives, say Gain Margin $> 1db$, Phase Margin$> 30°$ and Overshoot$<$ 50%, the subset achieving these performances are as shown in Fig. 6. The 3-D stabilizing set is shown in Fig. 7.

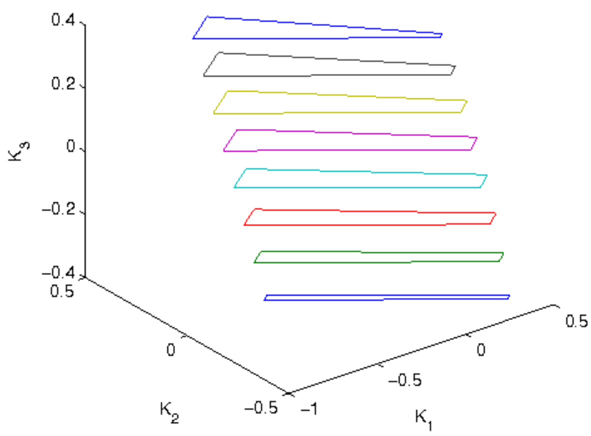

Fig. 7. 3D $K_1 - K_2 - K_3$ stabilizing set of PID Controller parameters where step response data is given

5. COMPUTER AIDED DESIGN USING LABVIEW

The algorithm for the design of a discrete time PID Controller from the frequency response data of a stable system has been programmed in LabVIEW due to its user-friendly graphical environment. The Virtual Instrument (VI) has a front panel that is displayed to the user and a block diagram, where the computations are performed. The inputs to the LabVIEW program are the frequency response data of the stable system, the sampling time and the number of samples to be considered to design the controller. Given these inputs, the entire range of K_3 that can stabilize the system is displayed, and as the user scrolls through the stabilizing range of K_3, entire stabilizing ranges of K_1 and K_2 are displayed. When some value of controller parameters are chosen, the performance parameters like gain margin, phase margin of the open loop and rise time, overshoot, peak time and pole zero placement of the closed loop system can be displayed. Additionally the entire 3-D stabilizing set can also be displayed. Furthermore, when some performance constraints are specified, the subset achieving the desired performance criteria can also be displayed. Once a particular value of controller parameters are chosen based on performance criteria, it can be converted to K_p, K_i and K_d values through a simple linear transformation. The two examples described below illustrate the above capabilities.

Example 1. The file containing the frequency response data of a stable system is fed into the program through the file path box located at the top left hand side of the VI as shown in Fig. 8. When the number of samples and sampling time are selected, the stabilizing range of K_3 is displayed. On selecting a particular value of K_3, (=0.5 in the example), the corresponding stabilizing region in K_1-K_2 space is obtained.

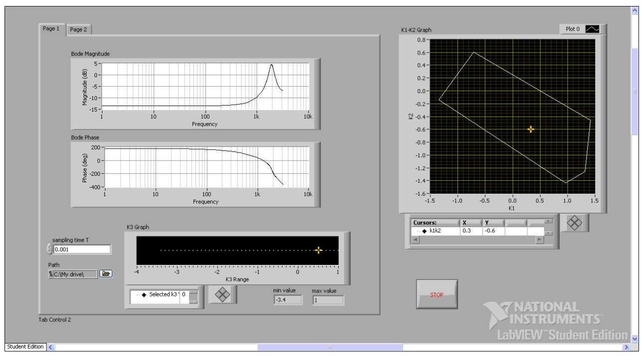

Fig. 8. Front Panel of data based PID controller design

On choosing particular values, $K_1 = 0.35$ and $K_2 = -0.6$ corresponding to the chosen value of K_3, the performance parameters are displayed as shown in Fig. 9.

Note that all the performance indicators are so arranged that higher values correspond to better performance. This helps in better understanding of how the system behaves when browsing through K_1-K_2 values. The pole-zero position corresponding to the above controller parameters is also shown in Fig. 10. The entire 3-D stabilizing region is shown in Fig. 11. On specifying some performance criteria like Gain Margin $> 4db$, Phase Margin $> 25°$ and overshoot $< 50\%$, the subset achieving the required criteria is shown in Fig 12.

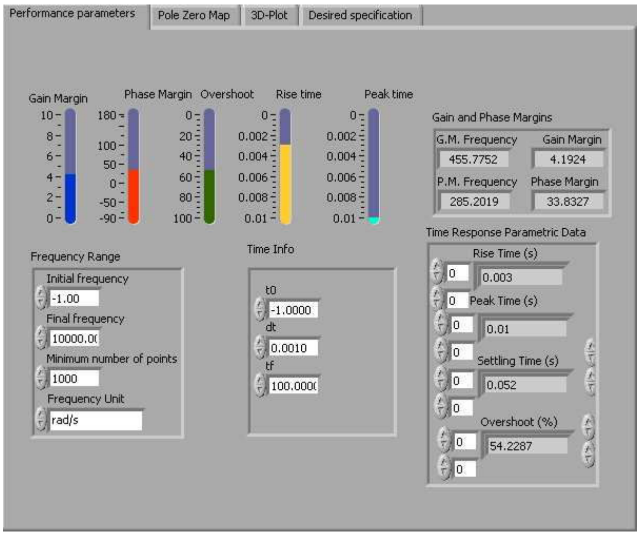

Fig. 9. Performance Parameters

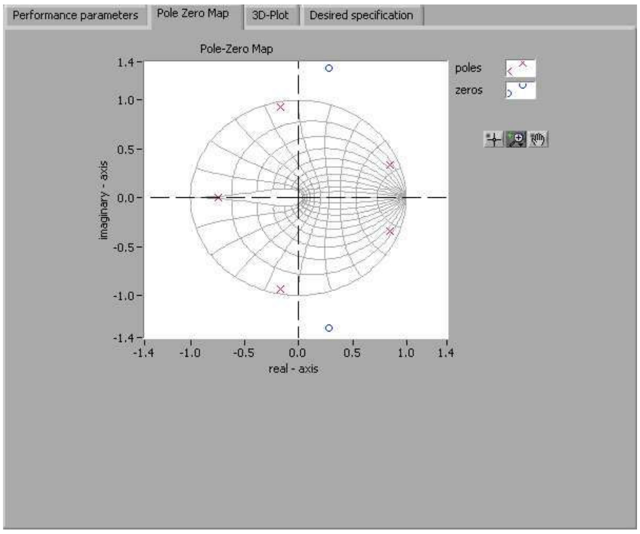

Fig. 10. Pole/zero map

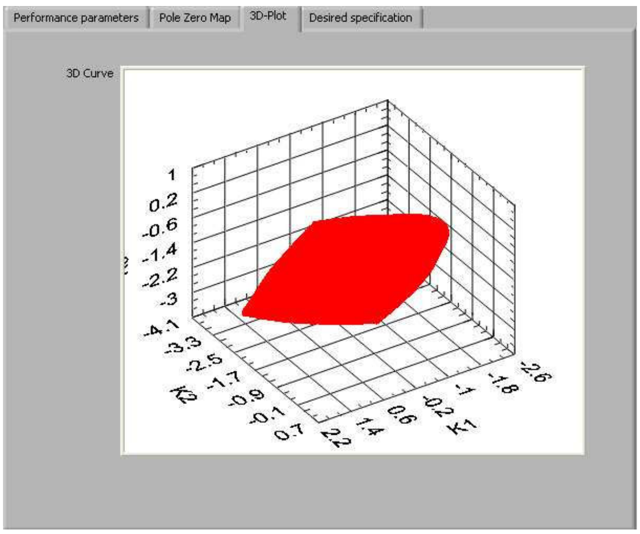

Fig. 11. 3D stabilizing set

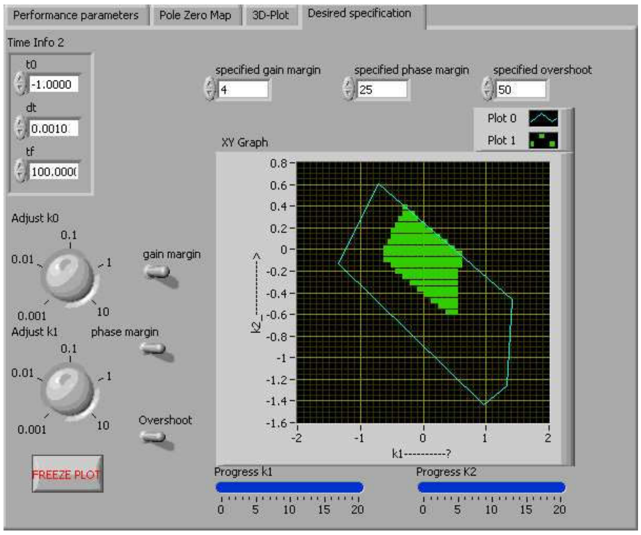

Fig. 12. Performance Subset

Example 2. The frequency response of another stable system as shown in Fig. 13 is obtained from a file. Carrying out similar steps as in the previous example, the $K_1 - K_2$ stabilizing set is obtained for $K_3 = 1.5$.

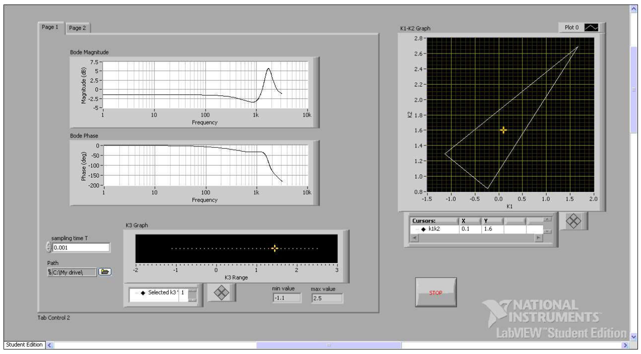

Fig. 13. Front Panel of data based PID controller design

The subset satisfying the criteria Gain Margin $> 2db$ is as shown in Fig. 14. Further when the constraint Overshoot $< 50\%$ is imposed, the set shrinks as shown in Fig. 15. When a third condition Phase Margin $> 14°$ is imposed, the set further shrinks as shown in Fig. 16. This illustrates the fact that as more and more conditions are imposed, the resultant set achieving all specifications is the subset of the previous set.

6. CONCLUDING REMARKS

In this paper, we have outlined the features of a new approach to teaching control systems with an emphasis on the computer aided techniques developed by us. Specifically we have shown how PID controller design for discrete and continuous time, with and without time delay and based on input output data can be explored with this new theory and software. It is expected that this new approach of design will aid control system education constructively and by the use of computer aided techniques, its capabilities can be nicely demonstrated in graduate and undergraduate curricula.

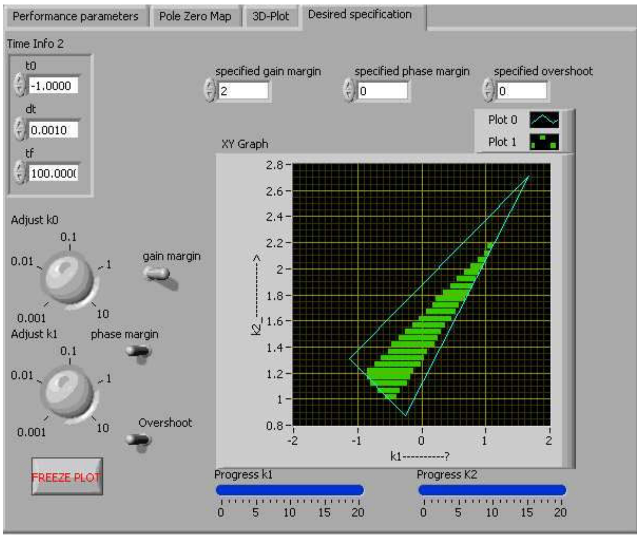

Fig. 14. Subset satisfying gain margin $> 2db$ for the plant

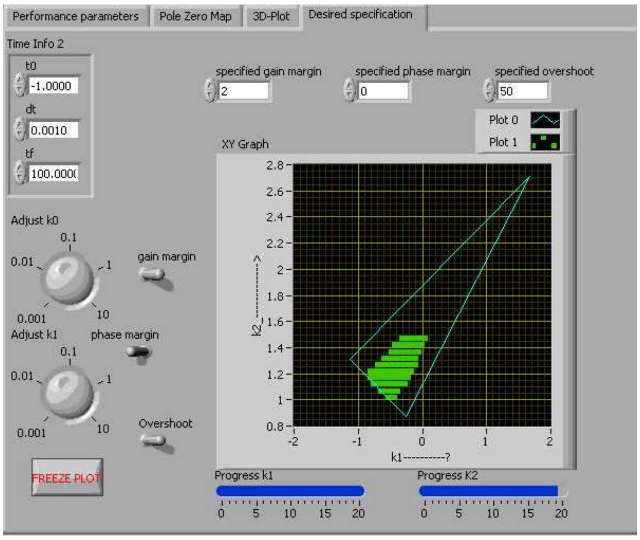

Fig. 15. Subset satisfying gain margin $> 2db$ and overshoot$< 50\%$ for the plant

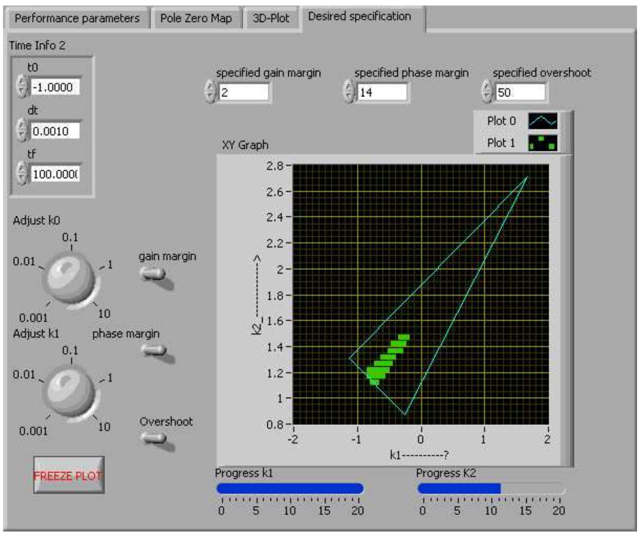

Fig. 16. Subset satisfying gain margin $> 2db$, phase margin$> 14°$ and overshoot$< 50\%$ for the plant

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