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Implementation of Optimal Decisions in the Presence of Uncertainty

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Abstract: In this work, the implementation of optimal and robust decisions in the presence of various uncertainties comprising the model parameters, external conditions and the closed loop behavior of basic controllers is presented. In order to compute optimal and reliable decisions, a chance constrained optimization problem is formulated. The efficient solution approach is based on the relaxation of the original stochastic problem formulation to a standard NLP problem. By this means, nominal optimal solutions are relocated in order to guarantee both feasibility and process operation as close to the true optimum as possible. The solution implicates the minimization of additional costs which result from conservative strategies so as to compensate for uncertainty. The experimental verification of the developed approach is carried out on a distillation pilot plant for the separation of an azeotropic mixture.

1. INTRODUCTION

Several approaches have been suggested to formulate and solve optimization problems under uncertainty, differing in how uncertainty is handled. Conventional deterministic optimization strategies treat uncertain variables by their expected values and the resulting nominal optimal decisions are not robust. The common procedure in industrial practice is to select an extremely conservative strategy. This implicates that the operation costs will be much higher than required and the strategy applied is far from the optimum. Accordingly it is desirable to include uncertainty explicitly in the problem formulation.

In general, the uncertainty nature and its significance determine the formulations of the objective function and the constraints. To treat the objective function in an optimization or control problem under uncertainty, minimizing the expected value and the variance of the objective function is usually adopted (Sahinidis [2004], Flemming et al. [2007]):

$$\min E\left[f(\mathrm{x},\mathrm{u},\xi)\right] + \omega D\left[f(\mathrm{x},\mathrm{u},\xi)\right] \tag{1}$$

Here, x, u and ξ are state, decision and random vectors, respectively. E and D are the operators of expectation and variance, ω is a weighting factor between the two terms.

Nagy and Braatz [2003] derive an analytical expression that provides an estimate of the distribution of the states and outputs as a function of time. For this purpose, the description of the probability distribution of the uncertain parameters is carried out by a bounded set and the worst case deviation is computed using a series expansion of the model equations. In Govatsmark and Skogestad [2005] the robustness of the operation of a plant is realized choosing the right control structure. By this means, the variation of the objective function value is minimized acounting for uncertainties.

The true process optimum for operation of chemical processes often lies on the boundary of the feasible region defined by active constraints. Thus, the process optimum and the set-points can be infeasible ignoring uncertainty in the parameters, the measurement and the implementation of optimal decicions. Here, the compliance with constraints on process outputs is more challenging than those on process inputs. Moreover, if uncertainty can not be compensated by feedback, the use of conservative margins, called *back-off*, is necessary to ensure feasibility (Srinivasan et al. [2003]). The *back-off* describes the distance between the nominal optimal solution and the correction adding a conservative distance. For constrained process states which are measured online, a direct *back-off* can be implemented and considered in the optimization problem (Barz et al. [2006], Govatsmark and Skogestad [2005], Visser et al. [2000]). Moreover, under the assumption that the set of active constraints is known a priori, an online measurement based optimization strategy can be implemented (Srinivasan et al. [2003]), which tracks the necessary conditions of optimality (*NCO-tracking*). In this way not only infeasibility but also a highly suboptimal operation can be avoided.

However, many variables in the engineering practice can not be measured on-line. These variables often represent the product quality and, thus, their control is desired. The same problem exists with predictive control applications. Here optimal and robust trajectories regarding the compliance of output constraints have to be calculated (Visser et al. [2000]). One approach for the consideration of uncertainties in contraints on output variables is to formulate chance or probabilistic constraints (Arellano-Garcia et al. [2007], Wendt et al. [2002]). Thus, it is required to evaluate the probability of violating these constraints at the decided operation point. Besides, they have to be satisfied with a user defined probability level.

$$\Pr\left\{\mathrm{h}_i\left(\mathrm{x},\mathrm{u},\xi\right) \le 0, \quad i = 1,\cdots,m\right\} \ge \alpha \tag{2}$$

$$\Pr_i\left\{\mathrm{h}_i\left(\mathrm{x},\mathrm{u},\xi\right) \le 0\right\} \ge \alpha_i, \quad i = 1,\cdots,m \tag{3}$$

The value $\alpha \in [0,1]$ represents the probability level. Since α can be defined by the user, it is possible to select different levels and make a compromise between optimality of costs and the risk of constraint violation. Equation (2) denotes the simultaneous formulation (*joint chance constraints*) with α being a defined probability of holding all constraints at once, while (3) corresponds to the formulation of single probabilities ensuring each inequality separately. In this form, different confidence levels α_i can be assigned to different outputs based on their requirements.

For linear systems with uncertain parameters characterized by Gaussian PDF, different solutions have been proposed. Visser et al. [2000] presents a cascade scheme for the computation of a robust set-point trajectory where margins from constraints are introduced. Loeblein and Perkins [1999] and Schwarm and Nikolaou [1999] use the information of the uncertainty directly in a model predictive control algorithm in order to guarantee feasibility concerning output constraints. In engineering praxis the process model gM and thus the relations between uncertain parameters and output variables are highly nonlinear.

$$\mathrm{gM}\left(\mathrm{x}, \mathrm{u}, \xi\right) = 0 \tag{4}$$

While the inputs have known stochastic properties, the output probability distribution is unknown. This issue is emphasized in Figure 1 for the vapour-liquid-equilibrium of a binary azeotropic mixture (acetonitrile-water). It is demonstrated that for variations in temperature T in form of a Gaussian distribution, the output variable concentration x_{Ac} shows a strongly non-symmetric behaviour which is not Gaussian. One possible way is to evaluate

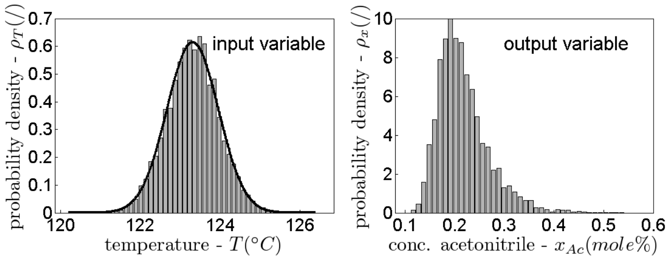

Fig. 1. Effect of controller deviations in Temperature (known input) on the concentration (unknown process state): Prediction based on a simple VLE-Model for acetonitrile/water.

the integral over the uncertain output variable space by efficient sampling methods and an approximated integration Wendt et al. [2002]. An alternativ efficient approach is discussed in this paper.

1.1 Implementation of Robust Optimal Decisions in an hierarchical automation scheme

The development and implementation of optimal control strategies is normally achieved through a hierarchical system of layers (Govatsmark and Skogestad [2005]). In the optimization upper layer, decisions about the optimal process states with respect to various objectives are taken. The results are then sent in form of set-points to the regulatory control layer where the optimal strategies are implemented keeping the system state at the optimal operating point (Fig. 2). Overstepping existing constraints

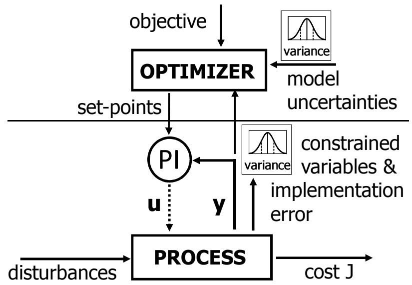

Fig. 2. Hierarchical process control scheme with basic regulatory control and optimization layer.

makes the operation infeasible, which not only means a loss of quality but also a safety risk. However, the inherent plant disturbance spectrum leads inevitably to controller performance deviations from their optimal set-points. To guarantee robust implementation of optimal decisions for all process states, the controller deviations and model uncertainties are required to be part of the control system i.e. to integrate implementation errors and stochastic parameters. By this means, effects of disturbances and model uncertainties can be compensated and robust set-points for the regulatory control layer are then obtained. Accordingly, the challenge of plant operation optimization lies in reducing the conservative distance to the constraints and pushing the plant to its limits without exceeding critical limitations in the presence of uncertainty.

2. CHANCE CONSTRAINED SOLUTION APPROACH

For this purpose, in this work, chance constrained optimization is proposed, i.e. the objective function (e.g. costs) is improved and the constraints are then to be satisfied with a predefined confidence level (*chance constraints*). Thus, the resulting robust decisions ensure the probability of satisfying constraints, i.e. the reliability of being feasible. The essential challenge lies then in the computation of the probabilities of complying with the constraints, and their gradients, which means a multivariate integration over the unknown PDF of the constrained output variables. In order to relax the stochastic optimization problem to a standard NLP problem two steps are required Wendt et al. [2002]. First, the constraints of the problem formulation are reduced to the inequality constraints only. This is realized following the sequential solution approach and solving the equality constraints (mostly model equations gM) in an extra simulation layer. Second, the chance constrained region of the outputs is mapped to a bounded region of the uncertain input variables with known PDF. The method relies upon the sufficient condition of a monotonic relationship between the constrained output variables $\mathrm{y}_i \in \mathrm{Y}_i$ with $\mathrm{y}_i = \mathrm{h}_i(\mathrm{x}, \mathrm{u}, \xi)$ and at least one uncertain input variable, here $\xi_s \in \Xi_s$ where Ξ_s is a subspace of Ξ. So, for instance if $\xi_s \uparrow \Rightarrow \mathrm{y}_i \uparrow$, then:

$$\Pr\left\{\mathrm{y}_i \leq \mathrm{y}_i^{max}\right\} = \int_{-\infty}^{\xi_i^{max}} \tilde{\rho}\left(\mathrm{y}_i\right) d\mathrm{y}_i \tag{5}$$

$$= \Pr\{\xi_s \leq \xi_s^{max}\} = \int_{-\infty}^{\xi_s^{max}} \rho(\xi_s)\, d\xi_s$$

And for the multivariate case:

$$\Pr\{\mathrm{y}_i \leq \mathrm{y}_i^{max}\} = \Pr\{\xi_s \leq \xi_s^{max}\} \quad (6)$$

$$= \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} \int_{-\infty}^{\xi_s^{max}} \rho(\xi_1, \cdots, \xi_{s-1}, \xi_s)\, d\xi_s\, d\xi_{s-1} \cdots d\xi_1$$

where $\rho(\xi)$ is now the joint distribution function of ξ. The input boundary ξ_s^{max} is computed through reverse projection based on the output value of y_i^{max} in the simulation layer.

$$\xi_s^{max} = \mathrm{gM}^{-1}(\xi_1, \cdots, \xi_{s-1}, \mathrm{y}_i^{max}, \mathrm{u}) \quad (7)$$

The boundaries of the infinite integrals in (6) are chosen as a function of the standard deviation σ, to $[-3\sigma, +3\sigma]$. In principle, the solution approach can be used to solve problems under uncertainties with any kind of joint correlated multivariate distribution function, provided that the density function is available or can be approximated. Using equations (6) and (7) a single probability (3) can be computed. The extension for the joint probabilities (2) is discussed in (Arellano-Garcia et al. [2006], Wendt et al. [2002]).

2.1 Illustrating example

A simple example with one uncertain parameter ξ is considered in problem (8). Whereas the objective function J is only a function of the decision variables u_1, u_2, the constraint dependent variables y_1 and y_2 are affected also by the uncertain parameter ξ. The uncertainty ξ possesses a strong nonlinear influence on the constraints Py_1, Py_2. Thus, a chance constrained optimization problem is formulated with the additional parameters α_1, α_2 defining the probability level of satisfying each constraint Py_1 and Py_2 (3).

$$\begin{aligned} \min_{u_1,u_2}\; & J(u_1, u_2) & (8)\\ \text{s.t.}\; & 0.75 \leq u_1 \leq 1.75, \quad 1 \leq u_2 \leq 3.5 \\ & Py_1 := \Pr\{y_1(u_1, u_2, \xi) \leq 0\} \geq \alpha_1 \\ & Py_2 := \Pr\{y_2(u_1, u_2, \xi) \leq 0\} \geq \alpha_2 \\ \text{and}\; & \xi \sim \mathcal{N}(\mu = 0,\ \sigma = 2/7) \end{aligned}$$

The required monotonic relationship between both stochastic outputs and the uncertain parameter is fulfilled within the feasible region. In this example, a positive monotony between ξ and y_1, y_2 is given. Since all relations are given in an explicit form, the effect of ξ on the original constraints y_1 and y_2 can be plotted (Fig. 3). They are marked as straight and dashed lines for different values $\xi = [-3\sigma : \sigma : +3\sigma]$. From Figure 3 can be seen that while the uncertain parameter ξ changes the exponential trait of the constraint y_1, for the quadratic constraint y_2 a term of third degree becomes active and the feasible area can become non convex for certain values of ξ. The objective function is displayed by (dotted) isolines of $J(u_1, u_2)$ and can be interpreted as heights with respect to the u_1-u_2-plane. Before solving the chance constrained optimization problem, the stochastic variable ξ is replaced by its expected value, here $\mu = 0$. Doing so, the nominal optimal decisions can be obtained solving an deterministic optimization problem with the constraints:

$$\begin{aligned} y_1(u_1, u_2, \xi = 0) &\leq 0 & (9)\\ y_2(u_1, u_2, \xi = 0) &\leq 0 \end{aligned}$$

For the nominal solution both output constraints are active and the minimum of J at $u = [0.7, 2]$ can not be achieved (see Fig. 3). Moreover, if the stochastic nature of ξ is considered, a feasible operation can not be guaranteed.

The solution of the chance constrained problem formulation is given for different probability level $\alpha_i = [0.1,\ 99.9]\%$. It should be noted that α_1 und α_2 are varied here simultaneously. However, it is also possible to define different values α in order to consider one constraint as more relevant than the other (e.g. for safety reasons). It should though noted that like in the nominal case both constraints are active for a wide range of $\alpha_i = [2,\ 98]\%$. For $\alpha_i \geq 98\%$ only Py_2, for $\alpha_i \leq 2\%$ only Py_1 remains active. In addition to the nonlinear relation between u, ξ and y, the probability integration of the unknown PDF causes also a strong nonlinear effect on Py_1 and Py_2. Thus, the solution exhibits discontinuity around $\alpha_i \approx 98\%$. Following this, the realized *back-off* is the distance between the nominal solution and the solution obtained by problem (8) and a function of the probability level α_i. For high values of α_i additional costs (in the sense of the objective J) are necessary in order to hold the constraints in the presence of the considered uncertainty on ξ. The worst result is obtained for the highest probability of holding the constraints $\alpha_i \approx 99.9\%$ (see Fig. 3). For lower values of α_i, the objective function value J increases. Predictably for $\alpha_i \leq 50\%$ the solution is less conservative as for the nominal result.

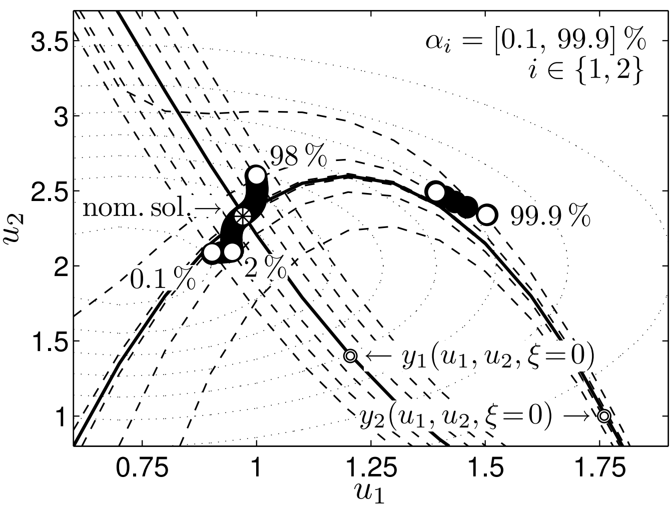

Fig. 3. Objective function $J(u_1, u_2)$ (dotted isolines) and constraint states $y_1(u_1, u_2, \xi)$, $y_2(u_1, u_2, \xi)$ (straight/dashed lines for $\xi = [-3\sigma : \sigma : +3\sigma]$) and nominal and robust solutions for different probability levels α_i.

3. EXPERIMENTAL CASE STUDY: OPERATION OF A HIGH PRESSURE DISTILLATION COLUMN

3.1 The pilot plant and its basic regulatory control layer

The distillation pilot plant is composed of 28 bubble cup trays and is embedded in a coupled two-pressure column system for the complete separation of a binary homogeneous azeotropic mixture (acetonitrile-water). The column is operated at a pressure of $p^{top} = 2bar$ and the feed is supplied into the column with an acetonitrile concentration below the azeotropic point. The high boiling point component (water) is removed from the bottom whereas at the column top maximal separation is defined by a distillate concentration equal to the concentration of the azeotropic point, which is a function of the column top pressure $x^{az}_{Ac} = f(p^{top})$. Feasible operation of the column is defined by the bottom and top product specifications which can only be measured off-line by a gas-chromatograph. Thus, no direct feed-back control can be applied. Figure 4 shows the individual high-pressure column and the control loops corresponding to the regulatory control layer. Whereas the top specification is controlled in a pure feed-forward mode, setting the reflux ratio, the bottom product concentration is indirectly controlled by the stabilization of the temperature profile in the stripping section. This is done by a temperature control loop controlling the sensitive temperature on a selected tray (set-point: $T^{SP} = 110°C$).

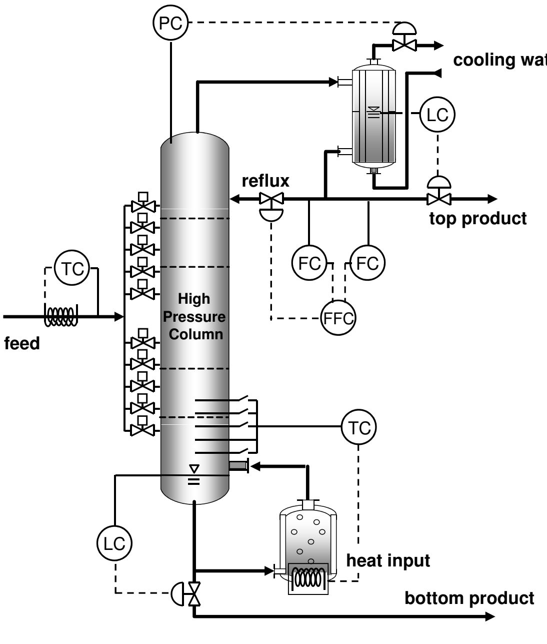

Fig. 4. Basic regulatory control of the column.

3.2 The optimization problem

High top and bottom product purities can only be achieved at a high expense of reflux and reboiler heat being their maximal value $x^{top}_{Ac} \rightarrow x^{Az}_{Ac}(2bar)$ and $x^{bottom}_{Ac} \rightarrow 0mole\%$. The optimal operation is characterized by a minimal heat input satisfying the product specifications. Considering only continuous decisions, the nominal optimal operating point is defined by the active constraints of bottom and top product specifications. The formulated optimization problem is given in (10). For this specific case the minimization of the reboiler duty $\dot{Q}$ can be replaced by the reflux ratio r. Doing so, the objective J is independent from the uncertainties ξ, which are considered later. As a result, the objective function can be treated as in a standard NLP problem (see also Barz et al. [2006], Flemming et al. [2007]). No expected value or variance have to be calculated.

$$\min_{r, \mathrm{Y}^{Feed}_l, \mathrm{Y}^{sens}_k} \quad J = \dot{Q} \quad \Rightarrow \quad J = r \tag{10}$$

$$\begin{aligned}
\text{s.t.} \quad & \text{model equations} \quad \text{gM} \\
& x^{top}_{Ac} \geq 63.2\, mol\% \\
& x^{bottom}_{Ac} \leq 0.5\, mol\% \\
& \bigvee_{l \in \mathrm{D}^{Feed}} \begin{bmatrix} \mathrm{Y}^{feed}_l \\ F^{feed}_l = 15\, ltr/h \\ F^{feed}_i = 0, \quad i \in \mathrm{D}^{feed} \backslash \{l\} \\ \mathrm{D}^{feed} = [2:2:10,\ 16:2:24] \end{bmatrix} \\
& \mathrm{Y}^{feed}_l \in \{\text{True, False}\},\ l \in \mathrm{D}^{feed} \\
& \bigvee_{k \in \mathrm{D}^{sens}} \begin{bmatrix} \mathrm{Y}^{sens}_k \\ T_k = 110\,°C \\ T_j = \text{gM}(r, \mathrm{Y}^{feed}_l, \mathrm{Y}^{sens}_k) \\ j \in \mathrm{D}^{sens} \backslash \{k\},\ \mathrm{D}^{sens} = [1:1:5] \end{bmatrix} \\
& \mathrm{Y}^{sens}_k \in \{\text{True, False}\},\ k \in \mathrm{D}^{sens}
\end{aligned}$$

Taking into account the regulatory control layer implemented at the pilot plant, the optimal set-point for the reflux ratio r is calculated. Furthermore the optimal tray $k \in D^{sens}$ for the temperature control loop in the stripping section has to be determined. Fixing the temperature on the respective tray, the reboiler duty $\dot{Q}$ is set by the basic regulatory controller. Additionally, it can be chosen between 10 different feed trays $l \in D^{feed}$, in order to supply the feed at the optimal feed plate into the column. Following this, the optimization problem (10) involves the continuous decision variable reflux ratio r as well as discrete decisions for the optimal feed location Y^{feed}_l and the tray for the temperature control Y^{sens}_k. In order to prevent a simultaneous use of more than one feed tray or temperature tray, respectively, disjunctive conditions are formulated in (10). Stein et al. [2004] presented an approach for the elimination of the discrete decision variables by adding a set of continuous variables and constraints that represent the discrete decision space of the optimization problem. By this means, a standard SQP algorithm can be applied solving a sequence of NLP problems, where the additional constraints are steadily tightened in order to force the relaxed variables to their discrete solution. In this work, the circle conditions applied in Stein et al. [2004] are used for the reformulation of the disjunctive feed conditions. However, it should be noted that it is not possible to relax Y^{sens}_k as well. This is because the control loop for the temperature control is either active or not. A superposition of partially activated control loops is neither in practise nor in a simulation feasible.

For the solution of (10) a steady state rigorous tray by tray column model based on mass and energy balances with a detailed description for the pressure loss is used (Barz et al. [2006]). The physical properties of the mixture are calculated using equations from CHEMCAD whereas for the phase equilibrium the Wilson model is used. This results in a large-scale DAE system with 237 states.

3.3 Nominal optimal solution and impact of uncertainties

The solution of the nominal optimization problem (10) gives a minimal reflux $r \approx 0.4$. The location for the temperature control loop is set to the first tray $Y_k^{sens} = \{\text{True}\}$ for $k = 1$ and the feed is introduced in the stripping section at $Y_l^{feed} = \{\text{True}\}$ for $l = 8$. This shows that the column is rather over-dimensioned and reflux and reboiler duty, respectively, can be minimized using as much trays in the rectifying section as possible. While the top product concentration lays directly on the specification limit $x_{Ac}^{top} = 63.2\, mole\%$, the bottom product concentration is higher than required, with $x_{Ac}^{bottom} \approx 0.3\, mole\%$. This is due to the discrete decision of the tray number for the temperature control loop used for the indirect bottom concentration control.

However, in the presence of uncertainties deviations from the optimal process states are caused. For the quantitative description, all uncertain variables are replaced by normal-distributed model parameter $\xi \sim \mathcal{N}(\mu, \sigma)$. In practice, besides uncertainties in model parameters, also controller deviations and external uncertainties have to be considered. Controller deviations around the set-point, also called implementation errors (Govatsmark and Skogestad [2005]), were obtained measuring directly the closed loop variance for disturbances. Analysing industrial data (Wozny and Jeromin [1991]) fluctuations of the feed conditions were defined. It should be noted that both feed concentration and flow can vary over a broad range during few hours. This causes the continual adjustment of the manipulated variables in the basic regulatory control layer and disturbs other related control loops. Uncertainties in model parameters were obtained using Maximum Likelihood Parameter Estimation (MLE) for different steady operating points. We found out that the Murphy tray efficiency, which is defined for stripping η_S and rectifying η_R section separately, assumes different values depending on the liquid load of the column. Moreover, the pressure resistance coefficient for the dry pressure loss c_w had to be adjusted in order to fit the experimental data properly. This highlights structural uncertainties and means a limited scope of application for the model. Thus, we identify the corresponding parameters as uncertain too. Table 1 shows all considered uncertainties characterized by their mean σ and the confidence interval of 6σ. As it can be seen the reflux temperature ΔT^{reflux}, the performance of the pressure and temperature control loop p^{top}, T^{sens} and the controlled reflux ratio r have been considered also as stochastic parameters.

Figure 5 shows the theoretical effects of each individual parameter on the top product specification. Based on the nominal optimal decisions the distribution of the uncertain output is plotted by Monte-Carlo sampling for 1000 simulation runs (steady state). Operating at the nominal optimum all uncertain parameters ξ can cause the violation of the top specification. The largest influence arises from the tray efficiency in the stripping section η^S, which is the result of the selected two-point concentration control scheme. Furthermore, the characteristic nonlinear behaviour of the product concentration is apparent. Based on the analysis of the results for bottom and top concentration, four key uncertainties were identified showing a maximal effect on the constrained states, namely: $\xi = [\eta^S, \eta^R, x_{Ac}^{feed}, T^{sens}]^T$.

Table 1. Model uncertainties

uncertain parameters ξ		μ	$\pm 3\sigma$
pressure resistance coef.	$c_w\ (/)$	1.5	1.0
subcooling condensat	$\Delta T^{reflux}\ (^\circ C)$	70.0	9.0
controlled reflux ratio	$r\ (/)$	r^{opt}	0.02
controlled temperature	$T^{sens}\ (^\circ C)$	110.0	7.0
controlled top pressure	$p^{top}\ (bar)$	2.000	0.030
Murphy tray efficiency	η^S, $\eta^R\ (/)$	0.45	0.24
feed flow	$F^{feed}\ (ltr/h)$	15.0	4.5
feed concentration	$x_{Ac}^{feed}\ (mole\%)$	45.0	12.0

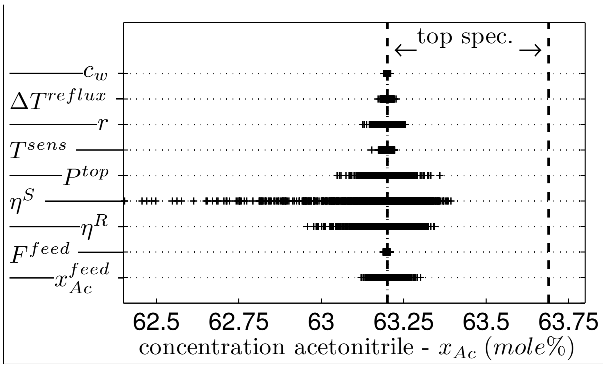

Fig. 5. Effects of individual uncertain parameters on the top product concentration.

3.4 Robust optimal decisions

As discussed above, operation with the nominal optimal decisions can cause infeasible operation. Thus, some conservatism has to be introduced in order to guarantee feasibility of the constraints depending on ξ. Thus, the original formulation for the constraints corresponding to the bottom and top concentrations in problem (10) are replaced by:

$$
\begin{aligned}
\Pr\left\{x_{Ac}^{top} \geq 63.2\, mole\%\right\} &\geq \alpha_1 = 95\% \qquad (11)\\
\Pr\left\{x_{Ac}^{bottom} \leq 0.5\, mole\%\right\} &\geq \alpha_2 = 95\%
\end{aligned}
$$

The selection of the probability level α means to directly decide on how conservative or aggressive the solution should be (see also Fig. 3). Here, the required probability of holding the constraints is set to $\alpha = 95\%$. Furthermore, the tray efficiencies η^S, η^R are treated as one uncertain variable η. By this means, the dimension of $\xi = [\eta, x_{Ac}^{feed}, T^{sens}]$ and, thus, the computational effort for solving the chance constrained optimization problem is reduced (Wendt et al. [2002]). The solution of the robust optimization problem gives an higher reflux $r \approx 0.74$ in order to compensate for the uncertainties. The robust and optimal location for the temperature control loop is shifted to the second tray $Y_k^{sens} = \{True\}$, $k = 2$ whilst the optimal feed tray remains the same $Y_l^{feed} = \{True\}$, $l = 8$. As expected both product concentrations are generally above the required specification.

3.5 Implementation and Validation

In order to demonstrate the application and to validate the computed results both nominal and robust decisions were implemented at the pilot plant. Here, additionally to the considered uncertainties, other process parameters did not match their assumed values. The biggest bias was observed for the feed concentration $x_{Ac}^{feed} \approx 53\,mole\%$ instead of $45\,mole\%$ (a variation of $15\,\%$), for the feed flow F^{feed} and the reflux ratio r with 7 and $2.5\,\%$, respectively. Both the computed optimal and robust concentration profiles and the analysed samples are shown in Figure 6. The robust optimal operation shows for both the analytical and the experimental values a wider concentration profile, which results from the more conservative operation. Hence, an additional heat amount of $\dot{Q} \approx 6.3\,KW$ (instead of $\dot{Q} \approx 5.6\,KW$) was measured.

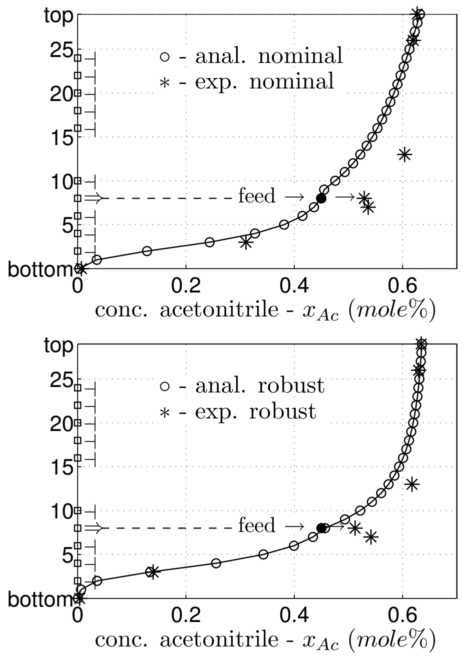

Fig. 6. Analytically vs. experimentally determined concentrations for nominal and robust optimal decisions.

Table 2 shows the results for the simulated and measured product concentrations. The experimental data confirms the analytical results and shows clearly the relevance of the proposed approach.

Table 2. Results for the top and bottom product $x_{Ac}(mole\%)$

specification	nominal operation		robust operation	
	simul.	experim.	simul.	experim.
$x_{Ac}^{top} \geq 63.2$	63.2	62.7 ± 0.5	63.6	63.4 ± 0.5
$x_{Ac}^{bottom} \geq 0.5$	0.3	0.69 ± 0.5	0.05	0.38 ± 0.5

4. CONCLUSIONS

A chance constrained optimization approach has been discussed and experimental verified, which can be applied to constrained process operation under uncertainty allowing the user to make a direct decision about the strategy to be applied w.r.t. reliability or costs. Optimal decisions are then obtained by computing the probabilities of satisfying the constraints. The resulting operating points are as close as possible to the constrained output variables so that the economical benefit can be maximized.

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