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Delay-dependent Stability Criteria for Markovian Switching Networks with Time-varying Delay ⋆

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Abstract: A problem of feedback stabilization of hybrid systems with time-varying delay and Markovian switching is considered. Delay-dependent sufficient conditions for stability based on linear matrix inequalities (LMI's) for stochastic asymptotic stability is obtained. The stability result depended on the mode of the system and of delay-dependent. The robustness results of such stability concept against all admissible uncertainties are also investigated. This new delay-dependent stability criteria is less conservative than the existing delay-independent stability conditions. An example is given to demonstrate the obtained results.

Keywords: Markov chain, Time-varying delay, Makovian switching systems, Stochastic stability.

1. INTRODUCTION

Due to the fact that most of the real world processes arising in biology, chemistry, economics, mechanics, viscoelasticity, physics, physiology, population dynamics, as well as in engineering sciences include some form of delays, study of time-delay systems has always been an important issue in control systems research (Richard [2003]). In addition, modern complex systems are increasingly implemented as distributed control systems, where distributed sensors and actuators are used, and the control loops are closed over a communication network or a field bus. In this case, there will inevitably be time delays induced by the communication network. As the performance requirements of the control system increase, the delays in the control system become a more important issue (Nilsson et al. [1998]).

On the other hand, a surge of interests in studying the class of Markov jump linear systems (MJLS) has been observed for the past decades. The MJLS are dynamical systems subject to abrupt variations in their structures. Since MJLS is natural to represent dynamical systems that are often inherently vulnerable to component failures, sudden disturbances, change of internal interconnections, and abrupt variations in operating conditions, they are an important class of stochastic dynamical systems (Mariton [1990], Boukas [2006], Dragan et al. [2006], Sathananthan [2001], Sathananthan et al. [2008], Ladde and Siljak [1983], Lawrence [1994]) and the references therein.

The feedback stability and stabilizability of time-delay of this important class of Markov jump linear systems is an interesting problem and has been attracting the attention of many researchers in the area of system science. This important class of systems with fixed time delay has been extensively studied and are classified into two categories such as delay-dependent and delay-independent stability and stabilizability conditions, see (Boukas and Liu [2002], Boukas and Al-Muthairi [2006], Mao [2006], Mahmoud [2007]) and the references therein. Due to the use of the information on the length of delays, the delay-dependent criteria is considered to be less conservative than the delay-independent criteria. On the other hand, similar problems with time-varying delay for systems without Markovian switching are encountered in (Mahmoud [2000]). Some robustness results for fixed-delay is considered in (Yuan and Mao [2004]). Time-varying delay without LMI-conditions for stabilization problems are investigated in (Mao [2002]). Robust stability for time-varying delay systems and a good comparison of existing methods without Markovian switching is considered in (Wu et al. [2004]). Mean-square stability results and optimal-control results for MJLS with time-varying delay are investigated in (Kolomanovsky and Maizenberg [2001a,b]). Exponential stability results are reported in (Yue et al. [2003], Yue and Han [2005]).

In this paper, sufficient conditions for feedback stochastic stability and stabilizability of the class of Markov jump linear systems with time-varying delay is considered. Robust stability and stabilizability of such linear uncertain systems with Markovian jumps and time-varying delay are also investigated. A technique to design a state feedback that achieves stochastic stability for MJLS with a time-varying delay is provided. Our delay-dependent sufficient conditions are written in matrix forms which are determined by solving linear matrix inequalities (LMIs). The LMI approach is proved to have significant computational advantage over any other techniques. This is the main strength and advantage of our method compared to existing methods. A second interesting feature of our method is

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that we considered time-varying delay compared to fixed delay which is apparent in most of the existing work for MJLS. Our new criteria is shown to be effective, overcome some of the conservativeness associated with delay-independent results (for comparison of existing methods, see (Wu et al. [2004]) and contain the results of fixed time-delay of (Boukas and Liu [2002]) as a special case. An example is given for illustration.

2. PROBLEM FORMULATION

Consider the continuous-time delay system

$$\dot{x}(t) = A(\eta_t)x(t) + A_1(\eta_t)x(t-\tau(t)) + B(\eta_t)u(t) \quad (1)$$

where $x(t) \in \Re^n$, $u(t) \in \Re^m$ are the state vector and control input, respectively and $A_1(\cdot)$, $A(\cdot)$, $B(\cdot)$ are matrices of appropriate dimensions and $\tau(t)$ is an unknown time-varying continuous delay factor satisfying:

$$0 \leq \tau(t) \leq \tau^0, \;\; \dot{\tau}(t) \leq \tau^+ < 1$$

where τ^0, τ^+ are known bounds. The initial condition of the system is specified as $(\eta_0, \phi(.))$ with $x(s) = \phi(s)$, $s \in [-\tau^0, 0]$. Define

$$x_s(t) := x(s+t), \quad \text{for } -\tau^0 \leq s \leq 0.$$

Note that $\{(x_t, \eta_t), t \geq 0\}$ is a Markov process defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with an associated nondecreasing family of σ-algebras $\mathcal{F}(t) \in \mathcal{F}$. Note that the jump dynamics described by the Markov chain $S = \{1, 2, .., N\}$ are the multiple modes of $u(t, x(t))$. Let us introduce the indicator function of the regime of S by $\xi(t) \in \Re^s$ with components

$$\xi_{\eta_t}(t) = \begin{cases} 1, & \eta_t = i \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

for $i = 1, 2, ..., N$, whose dynamics or transitions is given by a Markov chain

$$d\xi(t) = \Lambda^T \xi(t)dt + dM(t) \quad (3)$$

where $M(t)$ is an $\mathcal{F}(t)$-martingale and Λ is the chain generator an $N \times N$ matrix. The entries λ_{ij} for $i, j = 1, 2, \cdots, N$ of Λ are interpreted as transition rates

$$P\{\eta_{t+\Delta} = j \mid \eta_t = i\} = \begin{cases} \lambda_{ij}\Delta + o(\Delta), & \text{for } i \neq j \\ 1 + \lambda_{ii}\Delta + o(\Delta), & \text{for } i = j \end{cases} \quad (4)$$

The main objective of this paper is to establish delay-dependent sufficient conditions for stochastic stability, and its robustness results of (1) that can be evaluated by LMI techniques. To proceed, we first introduce the following definition of stability criteria.

Definition 1. System (1) with $u(t) \equiv 0$ is said to be *stochastically stable* if there exists a constant $T(\eta_0, \phi(\cdot))$ such that

$$\mathbf{E}\left[\int_0^\infty \|x(t)\|^2 dt \;:\; (\phi(\cdot), \eta_0)\right] \leq T(\eta_0, \phi(\cdot)) \quad (5)$$

3. PRELIMINARIES

Let, V be a vector-functional satisfying the following property:

For any function $\phi(s)$ that is continuous for $-\tau^0 \leq s < 0$ the function

$$\hat{V}(t, x, \eta(t)) = V(t, x, \phi_t(s), \eta(t))$$

is twice continuously differentiable with respect to x and continuously differentiable at least once with respect to t. Here, $x = \phi(t)$, and

$$\phi_t(s) = \left\{\phi(t+s), -\tau^0 \leq s < 0\right\}.$$

The class of functionals V that yields these properties with respect to $\hat{V}$ is denoted by D. We will also write $V(t, x, \eta(t))$ when we drop the argument ϕ_t from $V(t, x, \phi_t(s), \eta(t))$.

To establish the infinitesimal generator, we associate a vector Lyapunov functional $\hat{V}(t, x(t), \eta(t))$ to the system (1) with the *average dini derivative* defined as follows:

$$\mathcal{L}\hat{V}(t,x,k) = \lim_{h \to 0^+} \frac{1}{h}\Big\{\mathbf{E}[\hat{V}(t, x + hf(t, x_t, x, \eta_t)), \eta(t+h)) \mid x(t) = x, \eta(t) = k] - \hat{V}(t, x, k)\Big\}$$

where

$$f(t, x_t, x, \eta_t) = A(\eta_t)x(t) + A_1(\eta_t)x(t - \tau(t)).$$

The following lemma estimates this average derivative (differential generator) by virtue of the system (1) at the point(t, x, k).

Lemma 1. (Sathananthan et al. [2008]) Suppose that $\hat{V}(t, x(t), \eta(t)) \in \mathcal{C}\left[\mathcal{J} \times \Re^n \times C \times S, \Re^N_+\right]$, $\frac{\partial \hat{V}(t,x,j)}{\partial t}$, $\frac{\partial \hat{V}(t,x,j)}{\partial x}$ exist and are continuous for $(t, x) \in \mathcal{J} \times \Re^n$. Let $\eta(t)$ be a right continuous Markov chain with transition intensity matrix $\Lambda = (\lambda_{ij})_{s \times s}$. Then

$$\mathcal{L}\hat{V}(t,x,k) = \frac{\partial \hat{V}(t,x,k)}{\partial t} + \left[\frac{\partial \hat{V}(t,x,k)}{\partial x}\right]^T \cdot f(t, x, x_t, k) + \sum_{j=1}^s \lambda_{kj}\left[\hat{V}(t,x,j) - \hat{V}(t,x,k)\right]$$

4. STABILITY AND STABILIZATION CRITERIA

Before we state the stability conditions, we consider the following matrix inequality.

Lemma 2. (Mao [2006]) Let $x \in \Re^n$, $y \in \Re^m$, $N \in \Re^{n \times m}$, $\hat{N} \in \Re^{n \times m}$, and $\alpha > 0$. If $N^T N \leq \hat{N}$, then

$$2x^T N y \leq \alpha |x|^2 + \frac{1}{\alpha} y^T \hat{N} y \quad (6)$$

We now establish sufficient conditions for the stochastic stability of (1).

Theorem 1. The system (1) with $u(t) \equiv 0$ is stochastically stable if one of the following two equivalent conditions hold:

(i) If there exists symmetric, positive-definite matrices
$$P = (P(1), P(2), \cdots, P(N)) > 0, \;\; H_2 > 0, \;\; H > 0$$
with $H_\tau = (1 - \tau^+)H$, satisfying the algebraic Riccati inequalities (ARI)

$$[A(\eta_t)+A_1(\eta_t)]^T P(\eta_t)+P(\eta_t)[A(\eta_t)+A_1(\eta_t)] \\ +\tau^0 H_2+\tau^0 A^T(i)H_\tau A(i) \\ +2\tau^0 P(\eta_t)A_1(\eta_t)H_\tau^{-1}A_1^T(\eta_t)P(\eta_t) \\ +\sum_{j=1}^{N}\lambda_{\eta_t j}P(j)\equiv\Omega(\eta_t)<0$$

$$A_1^T(\eta_t)H_\tau A_1(\eta_t)\le H_2$$

(ii) If there exists symmetric, positive-definite matrices

$$P=(P(1),P(2),\cdots,P(N))>0,\ H_2>0,\ H>0$$

with $H_\tau=(1-\tau^+)H$, satisfying the LMI's

$$\begin{bmatrix} J(i) & \tau^0 P(i)A_1(i) \\ \tau^0 A_1^T(i)P(i) & -\frac{\tau^0}{2}H_\tau \end{bmatrix}<0, \tag{7}$$

$$\begin{bmatrix} -H_2 & A_1^T(i)H_\tau \\ H_\tau A_1(i) & -H_\tau \end{bmatrix}<0,\ \forall i\in S \tag{8}$$

where

$$J(i)=[A(i)+A_1(i)]^T P(i)+P(i)[A(i)+A_1(i)] \\ +\sum_{j=1}^{N}\lambda_{ij}P(j)+\tau^0 A^T H_\tau(i)A(i)+\tau^0 H_2$$

Proof. Using the Leibniz-Newton formula, we can write

$$x(t)-x(t-\tau(t))=\int_{t-\tau(t)}^{t}\dot{x}(s)ds$$

Thus, substituting this in (1), the system (1) is equivalent to the system

$$\dot{x}(t)=[A(\eta_t)+A_1(\eta_t)]x(t) \tag{9}$$

$$-\int_{t-\tau(t)}^{t}[A_1(\eta_t)A(\eta_\theta)x(\theta)+A_1(\eta_t)A_1(\eta_\theta)x(\theta-\tau)]d\theta$$

with the initial condition of the system $x(s)=\phi(s)$, $s\in[-2\tau^0,0]$. Let us define the process $\{x(t),t\ge 0\}$ defined on $C[-2\tau^0,0]$ as $x_s(t)=x(t+s),\ s\in[t-2\tau^0,t]$. Note that $(x(t),\eta_t)$ is a Markov process. Consider the Lyapunov functional of the following form:

$$V(x(t),\eta_t)=x^T(t)P(\eta_t)x(t) \\ +\int_{-\tau^0}^{0}\int_{t+\theta}^{t}x^T(s)H_1(\eta_s)x(s)dsd\theta \\ +\int_{-\tau^0}^{0}\int_{t-\tau(t)+\theta}^{t}x^T(s)H_2x(s)dsd\theta \tag{10}$$

and

$$H_1(\eta_t)=A^T(\eta_t)H_\tau A(\eta_t)$$

Let $\mathcal{L}$ be the infinitesimal generator of $\{(x(t),\eta_t),t\ge 0\}$. We use the inequality in Lemma 2 and after some algebraic manipulations, we get

$$\mathcal{L}V(x(t),\eta_t)\le x^T(t)\left[\tau^0 H_1(\eta_t)+\tau^0 H_2(\eta_t)\right]x(t) \\ +x^T(t)\Big[(A(\eta_t)+A_1(\eta_t))^T P(\eta_t)+P(\eta_t)(A(\eta_t)+A_1(\eta_t)) \\ +\sum_{j=1}^{N}\lambda_{\eta_t j}P(j)\Big]x(t)+x^T(t)\Big[\tau^0 P(\eta_t)A_1(\eta_t)H_\tau^{-1}A_1^T(\eta_t) \\ +\tau^0 P(\eta_t)A_1(\eta_t)H_\tau^{-1}A_1^T(\eta_t)\Big]x(t)\le x^T(t)\Omega(\eta_t)x(t).$$

Therefore, we obtain

$$\mathcal{L}V(x(t),\eta_t)\le-\min_{i\in S}\{-\Omega(i)\}x^T(t)x(t). \tag{11}$$

Integrating and taking expected value for both sides of (11), we have

$$\left[\min_{i\in S}\lambda_{\min}(-\Omega(i))\right]\mathbf{E}\left[\int_0^t x^T(s)x(s)ds\ \middle|\ (x(0),\eta_0)\right] \\ \le\mathbf{E}\left[V(x(0),\eta_0)\right].$$

Hence, we get

$$\mathbf{E}\left[\int_0^t x^T(s)x(s)ds\ \middle|\ (x(0),\eta_0)\right]\le\frac{\mathbf{E}\left[V(x(0),\eta_0)\right]}{\min_{i\in S}\lambda_{\min}(-\Omega(i))}$$

holds for any $t>0$. This completes the proof.

Remark 1. If $\tau^+=0$, and τ^0 is fixed this Theorem1 is identical to the delay-dependent stability criterion of (Boukas and Liu [2002]) (Lemma 8.5, page 195) with fixed-delay and Markovian switching. In other words, this theorem is more generalized than (Boukas and Liu [2002]) (Lemma 8.5, page 195) with fixed-delay and Markovian switching.

We now consider the problem of synthesizing a state feedback controller

$$u(t)=K(\eta_t)x(t) \tag{12}$$

that *stochastically* stabilizes the system (1). The following theorem gives a stabilizability condition.

Theorem 2. If there exists symmetric positive definite matrix

$$X=(X_1,X_2,\cdots,X_N)>0, H_2>0,\ H>0$$

with $H_\tau=(1-\tau^+)H$ satisfying

$$\begin{bmatrix} \mathbf{J}_1 & \mathbf{J}_2 & \mathbf{J}_3 & S_i(X) \\ \mathbf{J}_4 & \mathbf{J}_5 & 0 & 0 \\ \tau^0 X_i & 0 & -\tau^0 U_1 & 0 \\ S_i^T(X) & 0 & 0 & -\mathcal{X}_i(X) \end{bmatrix}<0 \tag{13}$$

where

$$\mathbf{J}_1:=J_0(i)+2\tau^0 A_1(i)(1-\tau^+)^{-1}UA_1^T(i) \\ \mathbf{J}_2:=\tau^0[A(i)X_i+B(i)Y_i]^T \\ \mathbf{J}_3:=\tau^0 X_i^T \\ \mathbf{J}_4:=\tau^0[A(i)X_i+B(i)Y_i] \\ \mathbf{J}_5:=-\tau^0(1-\tau^+)^{-1}U$$

and

$$\begin{bmatrix} -U_1 & U_1A_1^T(i) \\ A_1(i)U_1 & -(1-\tau^+)^{-1}U \end{bmatrix}<0,\ \forall i\in S \tag{14}$$

where

$$J_0(i) = X_i[A(i) + A_1(i)]^T + [A(i) + A_1(i)]X_i + B(i)Y_i + Y_i^T B(i)\lambda_{ii}X_i$$

$$S_i(X) = \left[\sqrt{\lambda_{i1}}X_i, \cdots, \sqrt{\lambda_{ii-1}}X_i, \cdots, \sqrt{\lambda_{ii+1}}X_i, \cdots, \sqrt{\lambda_{iN}}X_i\right],$$

$$\mathcal{X}_i(X) = \mathrm{Diag}\left[X_1, \cdots, X_{i-1}, X_{i+1}, \cdots, X_N\right],$$

$$U = H^{-1}, \qquad U_1 = H_2^{-1}$$

then controller

$$u(t) = K(\eta_t)x(t) \tag{15}$$

with

$$K(i) = Y_i X_i^{-1} \tag{16}$$

stochastically stabilizes the system (1).

Proof. Substituting (15) into (1) yields the dynamics of the closed-loop system described by

$$\dot{x}(t) = \underbrace{[A(\eta_t) + B(\eta_t)K(\eta_t)]}_{\bar{A}(\eta_t)} x(t) + A_1(\eta_t)x(t-\tau)$$
$$= \bar{A}(\eta_t)x(t) + A_1(\eta_t)x(t-\tau). \tag{17}$$

Then from Theorem 1, it suffices to show that there exists symmetric, positive definite matrix,

$$P = (P(1), \cdots, P(N)) > 0, \quad H_2 > 0, \quad H > 0,$$

with $H_\tau = (1-\tau^+)H$ such that

$$\left[\bar{A}(\eta_t) + A_1(\eta_t)\right]^T P(\eta_t) + P(\eta_t)\left[\bar{A}(\eta_t) + A_1(\eta_t)\right] + \tau^0 H_2 + \tau^0 \bar{A}^T(i)H_\tau \bar{A}(i) \tag{18}$$
$$+2\tau^0 P(\eta_t)A_1^T(\eta_t)H_\tau^{-1}A_1(\eta_t)P(\eta_t) + \sum_{j=1}^{N}\lambda_{\eta_t j}P(j) < 0,$$

where

$$\bar{A}(\eta_t) = A(\eta_t) + B(\eta_t)K(\eta_t). \tag{19}$$

Suppose that

$$X_i = P^{-1}(i) \tag{20}$$

and

$$Y_i = K(i)X_i, \quad U = H^{-1}, \quad U_1 = H_2^{-1} \tag{21}$$

Using Schur complement, some algebraic manipulations and the above expressions for $\mathcal{X}_i(x)$, $S_i(x)$ and Y_i, we obtain the inequality (13).

5. ROBUST STABILITY AND STABILIZATION CRITERIA

In the previous section, we investigated the stability and stabilizability of the time-delay system with Markovian switching given by (1). The conditions given are under the assumption that no uncertainties are presented in the system or system parameters. In this section, we consider the case when the plant parameters are subject to perturbations. Under this consideration, we study the conditions for robust stability and robust stabilization of the time-delay systems with Markovian switching.

Consider the system (1) with

$$\begin{aligned} A_\triangle(\eta_t) &= A(\eta_t) + \Delta A(\eta_t)\\ B_\triangle(\eta_t) &= B(\eta_t) + \Delta B(\eta_t)\\ A_{1\triangle}(\eta_t) &= A_1(\eta_t) + \Delta A_1(\eta_t)\end{aligned} \tag{22}$$

where

$$\begin{aligned} \Delta A(\eta_t) &= D(\eta_t)\Delta(\eta_t)E_a(\eta_t)\\ \Delta B(\eta_t) &= D(\eta_t)\Delta(\eta_t)E_b(\eta_t)\\ \Delta A_1(\eta_t) &= D(\eta_t)\Delta(\eta_t)E_1(\eta_t).\end{aligned} \tag{23}$$

Note that $A(\eta_t)$, $B(\eta_t)$, $A_1(\eta_t)$, D_a, E_a, E_b, E_1 are known matrices of appropriate dimensions. $\Delta(\eta_t)$ is an unknown time-varying matrix of appropriate dimension that represent the parameter uncertainties in the system. We say that the uncertainty $\Delta(\eta_t)$ is *admissible* if it satisfy the following condition:

$$\Delta^T(\eta_t)\Delta(\eta_t) \le I \tag{24}$$

Before we state the condition for robust stability, we consider the following lemma which will be used to prove the result.

Lemma 3. (Wang [1992]) Let A, D, Δ, E be real matrices of appropriate dimensions with $\|\Delta\| \le 1$. Then, we have

(i) for any matrix $P > 0$ and scalar $\epsilon > 0$ satisfying $\epsilon I - EPE^T > 0$,

$$(A + D\Delta E)P(A + D\Delta E)^T \le APA^T + APE^T(\epsilon I - EPE^T)^{-1}EPA^T + \epsilon DD^T \tag{25}$$

(ii) for any matrix $P > 0$ and scalar $\epsilon > 0$ satisfying $P - \epsilon DD^T > 0$,

$$(A + D\Delta E)^T P^{-1}(A + D\Delta E) \le A^T\left(P - \epsilon DD^T\right)^{-1}A + \frac{1}{\epsilon}E^T E \tag{26}$$

We now state the LMI based sufficient condition for the system (1) to be robust stochastically stable when $u(t) \equiv 0$.

Theorem 3. If there exists symmetric, positive definite matrices $X = (X_1, X_2, \cdots, X_N)$, and scalars $\epsilon_i > 0$ $i = 1, 2, 3, 4$ with $U = H^{-1} > 0, U_1 = H_2^{-1} > 0$, satisfying the LMI

$$\begin{bmatrix} J_0(i) & J_{12} & J_{13} & S_i(X)\\ J_{12}^T & -J_{22} & 0 & 0\\ J_{13}^T & 0 & -J_{33} & 0\\ S_i^T(X) & 0 & 0 & -\chi_i\end{bmatrix} < 0, \tag{27}$$

$$\begin{bmatrix} -U_1 & U_1A_1^T(i) & U_1E_1^T(i)\\ A_1(i)U_1 & -(1-\tau^+)^{-1}U + \epsilon_3 D(i)D^T(i) & 0\\ E_1(i)U_1 & 0 & -\epsilon_3 I\end{bmatrix} < 0, \tag{28}$$

for every $i \in S$ where

$$J_0(i) = X_i[A(i) + A_1(i)]^T + [A(i) + A_1(i)]X_i + 2\tau^0 A_1(i)(1-\tau^+)^{-1}UA_1^T(i) + 2\tau^0\epsilon_2 D(i)D^T(i) + \epsilon_4 D(i)D^T(i) + \lambda_{ii}X_i$$

$$J_{12} = \left[X_i A^T(i), \quad A_1(i)(1-\tau^+)^{-1} U E_1^T(i)\right]$$

$$J_{22} = \text{Diag}\begin{bmatrix} (\tau^0)^{-1}[(1-\tau^+)^{-1}U - \epsilon_1 D(i)D^T(i)] \\ (2\tau^0)^{-1}[\epsilon_2 I - E_1(i)(1-\tau^+)^{-1}UE_1^T(i)] \end{bmatrix}$$

$$J_{13} = \begin{bmatrix} X_i \\ E_a(i)X_i \\ (E_a(i)+E_1(i))X_i \end{bmatrix}^T$$

$$J_{33} = \text{Diag}\left[(\tau^0)^{-1}U_1, \quad \frac{\epsilon_1}{\tau^0}I, \quad \epsilon_4 I\right],$$

then system (1) is robust stochastically stable when $u(t) \equiv 0$.

The following theorem provides an LMI-based sufficient condition for the system (1) to be robust stochastically stable with the feedback

$$u(t) = K(\eta_t)x(t). \tag{29}$$

Theorem 4. If there exists symmetric, positive definite matrices $X = (X_1, X_2, \cdots, X_N)$, matrices $Y = (Y_1, Y_2, .., Y_N)$ and scalars, $\epsilon_i > 0, i = 1,2,3,4$, $X = (X_1, X_2, \cdots, X_N)$, and scalars $\epsilon_i > 0$ $i = 1,2,3,4$ with $U = H^{-1} > 0, U_1 = H_2^{-1} > 0$ satisfying the LMI

$$\begin{bmatrix} \hat{J}_0(i) & \hat{J}_{12} & \hat{J}_{13} & S_i(X) \\ \hat{J}_{12}^T & -\hat{J}_{22} & 0 & 0 \\ \hat{J}_{13}^T & 0 & -\hat{J}_{33} & 0 \\ S_i^T(X) & 0 & 0 & -\chi_i \end{bmatrix} < 0, \tag{30}$$

$$\begin{bmatrix} -U_1 & U_1 A_1^T(i) & U_1 E_1^T \\ A_1(i)U_1 & -(1-\tau^+)^{-1}U + \epsilon_3 D(i)D^T(i) & 0 \\ E_1(i)U_1 & 0 & -\epsilon_3 I \end{bmatrix} < 0, \tag{31}$$

for every $i \in S$.
where

$$\begin{aligned} \hat{J}_0(i) &= X_i[A(i)+A_1(i)]^T + [A(i)+A_1(i)]X_i + B(i)Y_i \\ &+ Y_i^T B^T(i) + 2\tau^0 A_1(i)(1-\tau^+)^{-1}UA_1^T(i) \\ &+ 2\tau^0\epsilon_2 D(i)D^T(i) + \epsilon_4 D(i)D^T(i) + \lambda_{ii}X_i \end{aligned}$$

$$J_{12} = \begin{bmatrix} A(i)X_i + B(i)Y_i \\ E_1(i)(1-\tau^+)^{-1}UA_1^T(i) \end{bmatrix}^T$$

$$J_{22} = \text{Diag}\begin{bmatrix} (\tau^0)^{-1}\left[(1-\tau^+)^{-1}U - \epsilon_1 D(i)D^T(i)\right] \\ (2\tau^0)^{-1}\left[\epsilon_2 I - E_1(i)(1-\tau^+)^{-1}UE_1^T(i)\right] \end{bmatrix}$$

$$J_{13} = \begin{bmatrix} X_i \\ E_a(i)X_i + E_b(i)Y_i \\ \left[E_a(i)+E_1(i)\right]X_i + E_b(i)Y_i \end{bmatrix}^T$$

$$J_{33} = \text{Diag}\left[(\tau^0)^{-1}U_1, \frac{\epsilon_1}{\tau^0}I, \quad \epsilon_4 I\right],$$

with

$$\bar{A}(\eta_t) = A(\eta_t) + B(\eta_t)K(\eta_t),$$
$$\bar{E}_a(i) = E_a(i) + E_b(i)K(i).$$

then system (1) is robust stochastically stable when

$$u(t) = K(\eta_t)x(t) \tag{32}$$

with

$$K(i) = Y_i X_i^{-1}, i = 1,2,...S.$$

Example 1. Consider the delay-system

$$\dot{x}(t) = A(\eta_t)x(t) + A_1(\eta_t)x(t-\tau) + B(\eta_t)u(t)$$

Suppose η_t is a three-state Markov chain $S = \{1,2,3\}$, with

$$\Lambda = \begin{bmatrix} -5 & 3 & 2 \\ 1 & -3 & 2 \\ 4 & 3 & -7 \end{bmatrix},$$

$$A(1) = \begin{bmatrix} -4 & 0.25 & 3 \\ -1 & -1.50 & 1 \\ 0 & 2.00 & -1 \end{bmatrix}, \ A_1(1) = \begin{bmatrix} -1 & 1.75 & 1 \\ -2 & -1.50 & 5 \\ 1 & 1.00 & -4 \end{bmatrix}$$

$$A(2) = \begin{bmatrix} -0.20 & 1 & -0.1 \\ 0.05 & -4 & 2.0 \\ 0.10 & 0 & 0.1 \end{bmatrix}, \ A_1(2) = \begin{bmatrix} -0.20 & 1 & 0.0 \\ 0.05 & -3 & -2.0 \\ 1.00 & 1 & -0.9 \end{bmatrix}$$

$$A(3) = \begin{bmatrix} 1 & 0 & 0.5 \\ 5 & -4 & 1.0 \\ 2 & 0 & -4.0 \end{bmatrix}, \ A_1(3) = \begin{bmatrix} -6 & 0 & 0.5 \\ -2 & 2 & 3.0 \\ 2 & 1 & 0.0 \end{bmatrix}$$

$$B(1) = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \quad B(2) = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \quad B(3) = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}.$$

The random time-varying delay is shown in Fig. 1. We now solve the LMIs above to obtain

$$\begin{aligned} K(1) &= [-0.1299, \quad 4.5837, \quad 4.3058] \\ K(2) &= [3.5787, \quad 1.2674, \quad -1.4421] \\ K(3) &= [13.0724, \quad 7.5845, \quad 11.5128]. \end{aligned}$$

The random switching sequence used is given in Fig. 2. Fig. 3 is obtained with the following initial conditions:

$$x(0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \quad x(t) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ for all } t < 0.$$

6. CONCLUDING REMARKS

It is an established fact (Boukas and Liu [2002], Wu et al. [2004]) that delay-dependent stability criteria is less conservative than the delay independent criteria for stability in network control systems with time delays. In this context, delay-dependent sufficient conditions for stability, stabilizability, and robust stability of the class of Markov jump linear systems with time-varying delays have been developed. All conditions are delay-dependent and depended on the mode of the MJLS. The results are extended to deal with systems subjected to admissible perturbation. For a given Markov jump linear system with a time-varying delay, a set of LMI conditions has been given to compute a respective state feedback controller for each case of stability considered. It is shown that delay-dependent stability criterion (Theorem 8.6, Theorem 8.8, Theorem 9.7 and Theorem 9.8) in (Boukas and Liu [2002]) is a special case of this new criteria, and that the new method is less conservative than existing methods.

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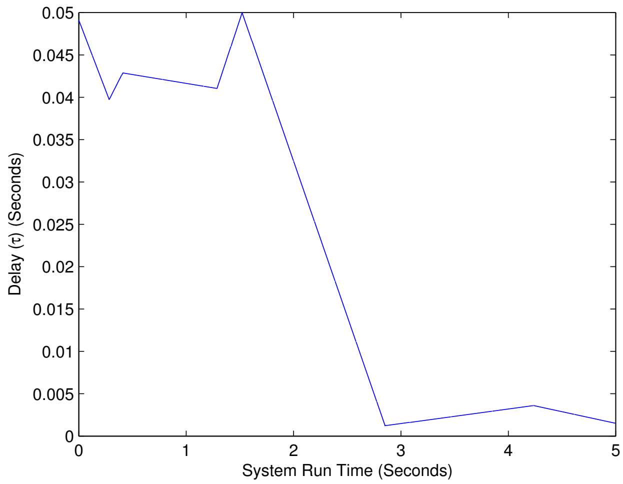

Fig. 1. The time-varying delay used.

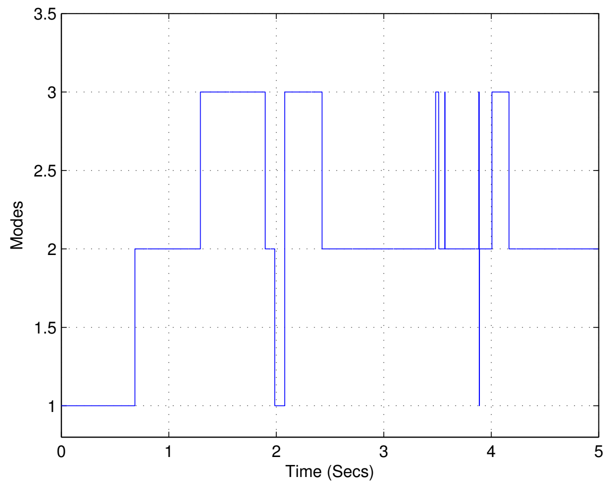

Fig. 2. The random switching sequence used.

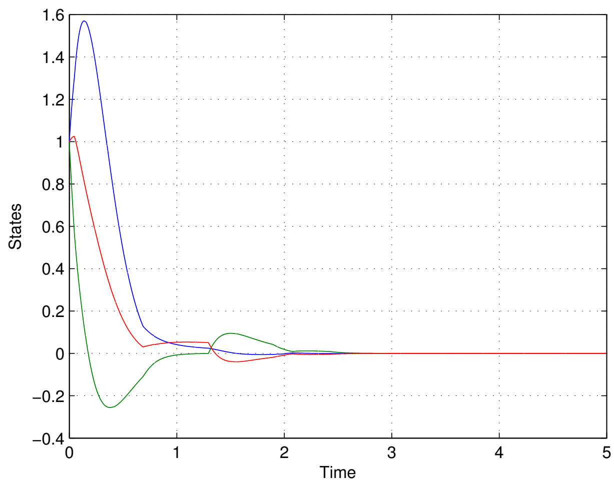

Fig. 3. Response of the feedback system.

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