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Nonlinear implicit on-line observer : application to the estimation in binary distillation columns

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Abstract: The current demand for more complex models has initiated a shift away from state-space models towards models described by both dynamic and algebraic equations, so-called implicit systems. These models arise as the natural product of real processes.
In this paper, we consider design of observers for a class of implicit systems. The proposed observer combines the dynamic Newton-Raphson algorithm as well as a high gain method. It is described by differential equations and it can be initialized outside the algebraic constraints. The main results of the paper include conditions that ensure the exponential convergence of the observer error. Design methodology is presented and the performance of the proposed observer illustrated using an application to a binary distillation column model.
Keywords: Nonlinear system, implicit system, high gain observer, constant gain.

1. INTRODUCTION

State observation for non-linear ODE models has been studied for quite some time and is still an active area of research. However, chemical engineering systems such as separation columns are described by nonlinear mixed differential (e.g., mass and energy balances) and algebraic (e.g., thermodynamic relationships) equations. Existing methods solve such equations by sequentially satisfying first the differential equations and then the algebraic equations. Such methods have been shown to lack robustness when solving large complex problems like the dynamics of crude units, Ballard and Brosillow, [1978]. As mentioned in Mclellan, [1994], implicit systems arise in problems such as distillation in which the vapor-liquid equilibrium conditions impose algebraic constraints on the process.

The focus of this work is to design observers for systems described by nonlinear implicit equations (DAEs with index 1). Nonlinear implicit systems are considered in different works for example, Zimmer and Meier, [1997] gave an extension of the extended Kalman filter observer for an implicit system based on the transformation of the original system to an ODE one on a restricted manifold (see also Becerra, et al., [2001]).

In this paper, we show that a dynamic Newton-Raphson algorithm can be used in order to design a nonlinear observer for our implicit system. This observer is described by differential equations only and no optimization algorithm is required to estimate the unknown state. Moreover, this observer can be initialized outside the constraint algebraic set. This result is then applied to state estimation for a binary distillation column model. The dynamic simulation of distillation columns is made basically of two parts : one part is the development of dynamic models supported by the physical and thermodynamical phenomena and the other part is the numerical methods of solution of model's equations. A typical rigorous model of a single tray of a distillation column, consists of equations describing holdups (concentrations), vapor flow (energy balance), liquid flow, bubble point (tray temperature), pressure drop and vapor-liquid relationships, Gani, et al., [1986], Rouchon, [1990]. Increasing demands of high product quality, minimal waste generation and low energy consumption have led to an increased demand for tighter control of columns. In 1990, Lang and Gilles presented a full-order nonlinear observer for distillation columns. Temperatures are measured at different points of the column and compared to the observer's output temperatures. A new nonlinear profile position observer which uses tray temperatures instead of tray compositions, in order to overcome the problem related to the online composition measurement, is proposed in Joonho, et al., [1990].

The outline of the paper is as follows. In section 2, we recall some fundamental results on observer design and we give a first observer for implicit systems. In section 3, we give some sufficient conditions under which an observer based on differential equations can be used in order to estimate the unknown state of the implicit system. Finally, in section 4, an application to a binary distillation column model is presented.

2. PROBLEM STATEMENT

We consider a class of multi output implicit systems of the following canonical form :

$$\begin{cases} \dot{x}(t) = f(x(t), z(t), u(t)) & x \in \mathbb{R}^n \\ \varphi(x, z) = 0 & z \in \mathbb{R}^n \\ y(t) = Cx & y \in \mathbb{R}^p \end{cases} \quad (1)$$

$$x = \begin{bmatrix} x^1 \\ \vdots \\ x^p \end{bmatrix} ; z = \begin{bmatrix} z^1 \\ \vdots \\ z^p \end{bmatrix} ; x^i = \begin{bmatrix} x_1^i \\ x_2^i \\ \vdots \\ x_{n_i}^i \end{bmatrix} \in \mathbb{R}^{n_i},$$

$z^i = \begin{bmatrix} z_1^i \\ z_2^i \\ \vdots \\ z_{n_i}^i \end{bmatrix} \in \mathbb{R}^{n_i}$ for $i = 1, \dots, p$ $(n = \sum_{i=1} p(n_i)$; $y = [C_1 x^1, \cdots, C_p x^p]^T$ where $C_i = (1, 0 \dots, 0)$ $(C_i x^i = x_1^i)$, u is an known signal such that $\forall t$, $u(t) \in U$ a bounded domain of $\mathbb{R}^m$

The nonlinear dynamic f satisfies the following triangular structure:

$$f(x,z,u) = \begin{bmatrix} f^1(x,z,u) \\ f^2(x,z,,u) \\ \vdots \\ f^{p-1}(x,z,u) \\ f^p(x,z,u) \end{bmatrix};$$

$$f^i(x,z,u) = \begin{bmatrix} f_1^i(x_1^i, x_2^i, z_1^i, u) \\ f_2^i(x_1^i, x_2^i, x_3^2, z_2^i, u) \\ \vdots \\ f_{n_{i-1}}^i(x, z_{n_{i-1}}^i, u) \\ f_{n_i}^i(x, z_{n_i}^i, u) \end{bmatrix}$$

and $\varphi = \begin{bmatrix} \varphi^1(x^1, z^1) \\ \vdots \\ \varphi^p(x^p, z^p) \end{bmatrix}$; $\varphi^i = \begin{bmatrix} \varphi_1^i \\ \dots \\ \varphi_{n_i}^i \end{bmatrix}$ are assumed to be sufficiently smooth and that for $i = 1, \dots, p$:

C1)

$$\begin{cases} \varphi_1^i(x^i, z^i) &= \varphi_1^i(x_1^i, x_2^i, z_1^i) \\ \varphi_j^i(x^i, z^i) &= \varphi_j^i(x_{j-1}^i, x_j^i, x_{j+1}^i, z_j^i) \quad \text{for } 1 \le j \le n_i - 1 \\ \varphi_{n_i}^i(x^i, z^i) &= \varphi_{n_i}^i(x, z_{n_i}^i) \end{cases}$$

C2) Set $\mathcal{M} = \{(x,z) \in \mathbb{R}^n \times \mathbb{R}^n, \text{ s.t. } \varphi(x,z) = 0\}$ and $\Omega_\epsilon = \{(x,z), \quad d((x,z), \mathcal{M}) < \epsilon\}$, where $\epsilon > 0$ is a constant and $d((x,z), \mathcal{M})$ denotes the distance between (x,z) and $\mathcal{M}$, then:

$\forall (x,z) \in \Omega_\epsilon$, $\frac{\partial \varphi_j^i}{\partial z^i} |_{(x,z)} \neq 0$

Condition C2) above means that the set $\mathcal{M}$ is a smooth submanifold of $\mathbb{R}^n \times \mathbb{R}^n$.

Using C1), C2) and the implicit function theorem, we can deduce that there are smooth functions γ_j^i, such that for $i = 1, ..., p$, we have:

$$\begin{cases} z_1^i &= \gamma_1^i((x_1^i, x_2^i)) \\ z_j^i &= \gamma_j^i((x_{j-1}^i, x_i^i, x_{j+1}^i)) \quad \text{for } 1 \le j \le n_i - 1 \\ z_{n_i}^i &= \gamma_{n_i}^i(x) \end{cases}$$

at last locally.

In the sequel, we will use the following notations : $\widetilde{f}_j^i(x,u) = f_j^i(x, \gamma_j^i(x), u)$, where γ_j^i is given above, and $\widetilde{f} = [\widetilde{f}^1, \dots, \widetilde{f}^p]^T$, where $\widetilde{f}^i = [\widetilde{f}_1^i, \dots, \widetilde{f}_{n_i}^p]^T$.

Now let us consider the following system:

$$\begin{cases} \dot{x} = \widetilde{f}(x,u) \\ y = Cx \end{cases} \tag{2}$$

From the above constructions, trajectories $x(t)$ of (1) are the same as the ones generated by (2). Consequently, to estimate trajectories $x(t)$ of (1), it suffices to design an observer for system (2). To do so, the following assumptions are required :

H1) $\frac{\partial \widetilde{f}_j^i}{\partial x_{j+1}^i}$ doesn't change sign and $|\frac{\partial \widetilde{f}_j^i}{\partial x_{j+1}^i}(x,u)| \ge \varrho$, for some $\varrho > 0$ and for every (x,u).

H2) $\widetilde{f}$ is a global Lipschitz function, namely:
$\exists c > 0$, $\forall u \in U$, we have $\|\widetilde{f}(x,u) - \widetilde{f}(x',u)\| \le c\|x - x'\|$, $\forall x, x'$.

Using **H1)**, system (2) takes a normal form (or canonical form) characterizing a class of uniform observable systems (see Gauthier, Kupka, [1994], H. Hammouri and M. Farza, [2003])).

Theorem *2.1.* (Gauthier, Kupka, [1994], Hammouri, H., et al [2001])
Under assumptions **H1)** and **H2)**, an observer for system (1) takes the following form:

$$\{ \dot{\hat{x}} = \widetilde{f}(\hat{x}, u) + K(C\hat{x} - y) \tag{3}$$

where K must be judiciously chosen.
Moreover this observer exponentially converges.

Remark *2.1.* In Gauthier, Kupka, [1994], Hammouri, H., et al [2001], it is shown that the gain K depend only on the bounds ϱ and c given by assumptions **H1)** and **H2)**, respectively.

The above theorem allows to state the following corollary (see the proof given in Gauthier, Kupka, [1994], Hammouri, H., et al [2001]):

Corollary *2.1.* There exists a symmetric positive definite quadratic function $W(e) = e^T S e$ and $\alpha > 0$ such that for every (x, e) and for every $u \in U$, we have:

$$\frac{\partial W}{\partial e}(e)[\widetilde{f}(x+e,u) - \widetilde{f}(x,u) + KCe] \le -\alpha W(e) \tag{4}$$

Unfortunately, the design of an observer of the form (3) assumes that the functions γ_j^i are known in closed form. However, in many cases γ cannot be explicitly expressed. In this case an observer for system (1) takes the form:

$$\begin{cases} \dot{\hat{x}} &= f(\hat{x}, \hat{z}, u) + K(C\hat{x} - y) \\ \varphi(\hat{x}, \hat{z}) &= 0 \end{cases} \tag{5}$$

In practice, the computation of the state estimate $\hat{x}$ requires an ODE numerical method combined with an optimization algorithm. The global convergence is guaranteed under the following assumptions:

H3) $f(x,z,u)$ is a global Lipschitz function w.r.t. (x,z): $\exists c > 0$; $\forall u \in U$, $\|f(x,z,u) - f(x',z',u)\| \le c\|(x,z) - (x',z')\|$.

H4) the map $(x,z) \to (x, \varphi(x,z))$ is one to one from Ω_ϵ into $\mathbb{R}^n \times \mathbb{R}^n$, where Ω_ϵ is the tubular neighborhood of $\mathcal{M}$ given in **C2)** above.

Remark *2.2.* Since

$$\frac{\partial \gamma_j^i}{\partial xj+1} = (\frac{\partial \varphi_j^i}{\partial z})^{-1} \frac{\partial \varphi_j^i}{\partial x_{j+1}},$$

it follows that bounds c and ϱ of H1), H2) may be calculated independently of the knowledge of the analytic expression of γ_j^i, i.e. the gain K calculation doesn't depend on the knowledge of γ_j^i.

In the following section, we will give an observer for system (1) having only an ODE structure. Namely, neither an optimization algorithm will be required not and the analytic expressions of γ_j^i needed to be known.

3. HIGH GAIN OBSERVER HAVING AN ODE STRUCTURE FOR IMPLICIT SYSTEMS (1)

From **C2)** and **H4)**, the following map:

$$(x,z) \overset{\Psi}{\to} \begin{pmatrix} x \\ \varphi(x,z) \end{pmatrix} = \begin{pmatrix} x \\ \eta \end{pmatrix} \quad (6)$$

forms a diffeomorphism from Ω_ϵ into an open subset of $\mathbb{R}^n \times \mathbb{R}^d$.
Set $F(u,x,\eta) = f(u,x,\gamma(x,\eta))$, where $\gamma(x,\eta)$ is the implicit function (generally unknown) given by the implicit function theorem applied to $\varphi(x,z) = \eta$.
The following assumption replaces the assumption H2).
H5) F is a global Lipschitz function w.r.t. (x,η), namely: $\exists c > 0;\ \forall u \in U;\ \forall (x,\eta),(x',\eta'),\ \|F(u,x,\eta) - F(u,x',\eta')\| \le c\|(x,\eta)-(x',\eta')\|$

Theorem *3.1.* Let K be the matrix gain given in (3) and assume that H1) holds and F satisfies **H5)**. Then an observer which allows to estimates the unknown state $(x(t),z(t))$ of system (1) takes the following form:

$$\begin{cases} \dot{\hat{x}} = f(\hat{x},\hat{z},u) + K(C\hat{x}-y) \\ \dot{\hat{z}} = -(\frac{\partial\varphi}{\partial z}|_{\hat{x},\hat{z}})^{-1}(\frac{\partial\varphi}{\partial x}|_{\hat{x},\hat{z}})[f(\hat{x},\hat{z},u) \\ \qquad +K(C\hat{x}-y)] - (\frac{\partial\varphi}{\partial z}|_{\hat{x},\hat{z}})^{-1}\Lambda\varphi(\hat{x},\hat{z}) \end{cases} \quad (7)$$

where Λ is any constant symmetric positive definite matrix.

Proof *1.* One can see that the diffeomorphism Ψ defined by (6), transforms system (7) into the following form:

$$\begin{cases} \dot{\hat{x}} = F(\hat{x},\eta,u) + K(C\hat{x}-y) \\ \dot{\eta} = -\Lambda\eta \end{cases} \quad (8)$$

By construction, we have $F(x,0,u) = \widetilde{f}(x,u)$ (where $\widetilde{f}$ is given in (2)). Thus, to prove the theorem, it suffices to show that $\widehat{x}(t)$ exponentially estimate the unknown trajectory $x(t)$ of system (3).
Let S be the symmetric positive definite matrix given in corollary 2.1 and let $e(t) = \widehat{x}(t) - x(t)$ be the error between trajectories of systems (8) and (2). Then we obtain:

$$\begin{cases} \dot{e} = F(u,x,\eta) - \widetilde{f}(u,x) + KCe \\ \quad = F(u,\widehat{x},0) - \widetilde{f}(u,x) + KCe + F(u,x,\eta) - F(u,\widehat{x},0) \end{cases} \quad (9)$$

Setting $W = e^T S e$ and using the fact that $F(u,\widehat{x},0) = \widetilde{f}(u,\widehat{x})$, we get:

$$\begin{aligned} \frac{dW}{dt} &= \frac{\partial W}{\partial e}(e)[\widetilde{f}(x+e,u) - \widetilde{f}(x,u) + KCe] \\ &+ \frac{\partial W}{\partial e}(e)(F(u,\widehat{x},\eta) - F(u,\widehat{x},0)) \end{aligned} \quad (10)$$

From remark 2.1, we deduce that:

$$\frac{dW}{dt} = -\alpha W(e) + \frac{\partial W}{\partial e}(e)(F(u,\widehat{x},\eta) - F(u,\widehat{x},0)) \quad (11)$$

where $\alpha > 0$ is a constant.
Combining the fact that F is a global Lipschitz function (hypothesis **H5)**) and that $\dot{\eta} = -\Lambda\eta$, where Λ is a S.P.D. matrix, we deduce that :
$\|F(u,\widehat{x},\eta) - F(u,\widehat{x},0)\| \le \beta e^{-\lambda t}$, for some constant $\lambda > 0$.

Combining this last fact with (11), we deduce that $e(t)$ exponentially converges to 0.

4. APPLICATION TO A BINARY DISTILLATION COLUMN

4.1 Mathematical model

The dynamical model of a binary distillation column consists of a set of ordinary differential equations derived from mass and energy balances around each tray of the column and a set of algebraic equations introduced by the thermodynamic equilibrium between the liquid and vapor composition (see Gani, et al., [1986], Distefano, [1968], Becerra, et al. [2001], Lang and Gilles, [1990]).
This case study considers a binary distillation column separating methanol and water. The column consists of ten trays, a boiler and a total condenser. The steady-state column operating point is listed in Table 1. The simulations are based on the following assumptions (theoretical trays, feed is at liquid state, thermal equilibrium, constant pressure, total condensation of the overheat vapor, molar vapor hold up is negligible, etc); see the work of Targui, [2000] for more details.
The continuous binary column is made of n trays numbered from the condenser $i = 1$ to the boiler $i = n$. The column is divided into three sections : the rectifying section ($i = 2,\dots,f-1$), the feed tray $i = f$ and the stripping section ($f+1,\dots,n-1$).
According to the above mentioned assumptions, the set of equations can be derived for the light component as follows : Total material balances

$$\begin{cases} \dot{N}_1 = V_2 - L_1 - D \qquad \text{condenser} \\ \dot{N}_i = V_{i+1} + L_{i-1} - V_i - L_i + \delta(i)F \quad (i = 2,\dots,n-1) \\ \dot{N}_i = L_{n-1} - V_n - W \qquad \text{boiler} \end{cases} \quad (12)$$

Light component material balances

$$\begin{cases} \dot{x}_{11} = \frac{1}{N_1}\cdot(V_2 y_{12} - L_1 x_{11} - D x_{11}) \qquad \text{condenser} \\ \dot{x}_{1i} = \frac{1}{N_i}\cdot(V_{i+1}y_{1i+1} + L_{i-1}x_{1i-1} - V_i y_{1i} - L_i x_{1i} \\ \qquad + \delta(i)F z_F) \qquad (i = 2,\dots,n-1) \\ \dot{x}_n = \frac{1}{N_n}\cdot(L_{n-1}x_{1n-1} - V_n y_{1n} - W x_{1n}) \qquad \text{boiler} \end{cases} \quad (13)$$

where

$$\delta(i) = \begin{cases} 0 & i \ne f \\ 1 & i = f \end{cases}$$

Volumetric hold up

$$V^{liq,1} = N_i v_i = constant \quad (14)$$

See nomenclature table for notations.
On each tray the liquid and vapor compositions, x_{1i} and y_{1i}, are linked by the liquid-vapor equilibrium law (VLE). In order to complete our implicit system, we need to couple the differential equations (12),(13) with proper equations representing the vapor-liquid equilibria in the trays of the column, i.e. $2 \le i \le n-1$, and the boiler, n. The vapor phase composition in equilibrium with the liquid phase is given by:

$$\begin{cases} y^*_{1i} = K_{1i}(x_{1i},T_i,P)x_{1i} \\ (1-y^*_{1i}) = K_{2i}(x_{1i},T_i,P)(1-x_{1i}) \end{cases} \quad (15)$$

where the equilibrium ratios have the general form

$$K_{1i} = \left[\frac{\gamma_i P^s_i(T_i)}{P}\right],\ K_{2i} = \left[\frac{\gamma_i P^s_i(T_i)}{P}\right] \quad (16)$$

where γ_i indicates the activity coefficient of the ith component in the liquid phase, $P_i^s(T_i)$ is the corresponding vapor pressure and P is the total pressure. The activity coefficient of the component c in the liquid mixture, makes it possible to represent the non ideality in the mixture
The temperature on each tray has been obtained from the stoichiometric equation relative to the equilibrium vapor-phase composition, y_{1i}^*, derived by summing up (15).
The determination of the equilibrium ratios K_{ci}, requires that the temperature on each tray be calculated by solving the non-linear equation :

$$\begin{cases} \sum_{c=1}^{2} y_{ci} - 1 = \sum_{c=1}^{2} K_{ci}(x_{ci}^*, T_i, P)x_{ci}^* - 1 \\ \quad = \varphi_i(x_{ci}^*, T_i, P) = 0; (i = 2, \ldots, f-1) \\ \sum_{c=1}^{2} y_{ci}^* - 1 = \sum_{c=1}^{2} K_{ci}(x_{ci}, T_i, P)x_{ci} - 1 \\ \quad = \overline{\varphi}_i(x_{ci}, T_i, P) = 0; (i = f, \ldots, n) \end{cases} \tag{17}$$

where,

$$\begin{aligned} \varphi_i(x_{ci}^*, T_i) &= x_{ci}^* exp(a1 + \frac{b1}{T_i + c1}) \\ &+ exp(a2 + \frac{b2}{T_i + c2})(1 - x_{ci}^*) - P \\ &= 0 \end{aligned} \tag{18}$$

$$\begin{aligned} \overline{\varphi}_i(x_{ci}, T_i) &= x_{ci} exp(a1 + \frac{b1}{T_i + c1}) \\ &+ exp(a2 + \frac{b2}{T_i + c2})(1 - x_{ci}) - P \\ &= 0 \end{aligned} \tag{19}$$

The composition of the vapor phase leaving the ith tray, y_{1i} is calculated from the corresponding equilibrium value, y_{1i}^*, in order to correct the effect of the mixture on liquid-vapor balance. The definition of these efficiencies are different according to the considered section :

- in the rectifying section the efficiency e_i is defined with respect to the liquid phase :

$$e_i = \frac{x_{1i} - x_{1i-1}}{x_{1i}^* - x_{1i-1}} \tag{20}$$

- in the stripping section including the feed tray the efficiency E_i is defined with respect to the vapor phase :

$$E_i = \frac{y_{1i} - y_{1i+1}}{y_{1i}^* - y_{1i+1}} \tag{21}$$

In all the simulations the value of e_i and E_i has been kept constant.
Looking at expressions (18) and (19), it can be seen that the temperatures which will be compared to the estimated outputs are a nonlinear function of the concentrations. Moreover, the dependency between temperature and concentrations is an implicit one. Expressions (12-13)-(17) for each tray, define the nonlinear implicit model of our binary column.
The state of the model is composed of the liquid compositions profile of the more volatile component on each tray x_{1i}. The top and bottom product compositions $y_1(t) = x_{11}$ and $y_2(t) = x_{1n}$ are the two output variables. The algebraic variable is the temperature T in all the column trays given by the algebraic constraints $\varphi(x_{ci}, T_i) = 0$ from the vapor-liquid equilibrium condition (17). The input vector is $u(t) = (L, V)^T$. The feed F and feed composition z_f are supposed to enter at liquid state.

4.2 Observer synthesis

In order to apply the implicit observer given by (7) ($p = 2$) for the trays temperature and compositions of a binary distillation column, the corresponding model (12-13) and (17) must be of the suited triangular form (1). To do so, we consider the followings notations :

$$\begin{cases} x_i = x_i^1; & 1 \le i \le f-1 \\ x_{n-i+1} = x_i^2; & 1 \le i \le n-f \\ z_i = T_i^1; & 1 \le i \le f-1 \\ T_{n-i+1} = T_i^2; & 1 \le i \le n-f \end{cases} \tag{22}$$

With $n_1 = f - 1$, $n_2 = n - f + 1$ and $p = 2$.
With these notations, we obtain:

$$\begin{cases} \dot{x}^1(t) = f^1(x(t), z(t), u(t)) \\ \dot{x}^2(t) = f^2(x(t), z(t), u(t)) \\ \varphi^1(x(t), z(t), u(t)) \\ \varphi^2(x(t), z(t), u(t)) \\ y(t) = \begin{bmatrix} C_1 x^1 \\ C_2 x^2 \end{bmatrix} = \begin{bmatrix} x_1^1 \\ x_1^2 \end{bmatrix} \end{cases} \tag{23}$$

Finally, one has to check whether hypotheses: (H1) and (H5) are verified (see Hammouri, H., et al [2001]). (H3) is the classical assumption always verified in physical processes.
Now applying our main theorem 3.1, the observer for binary distillation columns is then:

$$\begin{cases} \dot{\hat{x}}^1 = f^1(\hat{x}^1, \hat{z}^1, u) + K_1(\hat{x}_1^1 - x_1^1) \\ \dot{\hat{x}}^2 = f^2(\hat{x}^2, \hat{z}^2, u) + K_2(\hat{x}_2^1 - x_2^1) \\ \dot{\hat{z}}^1 = -(\frac{\partial \varphi}{\partial z} |_{\hat{x},\hat{z}})^{-1} (\frac{\partial \varphi}{\partial x} |_{\hat{x},\hat{z}})(f^1(\hat{x}^1, \hat{z}^1, u) \\ \quad + K_1(\hat{x}_1^1 - x_1^1) - (\frac{\partial \varphi}{\partial z} |_{\hat{x},\hat{z}})^{-1} \Lambda \varphi(\hat{x}, \hat{z}) \\ \dot{\hat{z}}^2 = -(\frac{\partial \varphi}{\partial z} |_{\hat{x},\hat{z}})^{-1} (\frac{\partial \varphi}{\partial x} |_{\hat{x},\hat{z}})(f^2(\hat{x}^2, \hat{z}^2, u) \\ \quad + K_2(\hat{x}_2^1 - x_2^1) - (\frac{\partial \varphi}{\partial z} |_{\hat{x},\hat{z}})^{-1} \Lambda \varphi(\hat{x}, \hat{z}) \end{cases} \tag{24}$$

We, still need to calculate the appropriate gain $K = [K_1 \ K_2]$ given in (3):
The authors in Hammouri, H., et al [2001] gave a constant high gain observer based on the triangular form (Assumption H2) satisfied) verified by the binary distillation column model such that :

$$K_i = -\rho_i \Delta_{\theta^{\delta_i}} S_{n_i}^{-1} C_i^T$$

for $i = 1, 2$; $n_1 = f - 1$; $n_2 = n - f + 1$, where ρ_i is a positive constant chosen to have the best convergence, $C_{n_i} = [1, \ldots, 0]$ is a vector of dimension n_i; $\delta_1 > 0$, $\delta_2 > 0$ satisfying: $\frac{2f-3}{2n-2f+3}\delta_1 < \delta_2 < \frac{2f-1}{2n-2f+3}\delta_1$, $\theta > 0$ and $\Delta_{\theta^{\delta_i}}$ is the $(n_i \times n_i)$ diagonal matrix: $\Delta_{\theta^{\delta_i}} = diag(\theta^{\delta_i}, \theta^{2\delta_i}, \ldots, \theta^{n_i\delta_i})$, and finally the S_{n_i}'s are $n_i \times n_i$ symmetric positive definite matrices (S.P.D.) of the form

$$S_{n_i} = \begin{bmatrix} s_{11} & s_{12} & 0 & 0 \\ s_{12} & s_{22} & \ddots & \vdots \\ 0 & \ddots & & 0 \\ \vdots & & \ddots & s_{n_i-1,n_i} \\ 0 & \dots & s_{n_i-1,n_i} & s_{n_i,n_i} \end{bmatrix} \quad i = 1, 2$$

To assure the exponential convergence of the observer, the matrix S_{n_i} was chosen satisfying the following condition :

$$A_{n_i}^T(t)S_{n_i} + S_{n_i}A_{n_i}(t) - C_{n_i}^T C \leq -\kappa I_{n_i} \qquad \forall\, t \geq 0 \quad (25)$$

where

$$A_{n_i} = \begin{bmatrix} 0 & a_1(t) & 0 & \dots & 0 \\ \vdots & & a_2(t) & & \vdots \\ 0 & & & \ddots & a_{n-1}(t) \\ 0 & \dots & & 0 & 0 \end{bmatrix},$$

I_{n_i} is the $n_i \times n_i$ identity matrix and κ is a positive constant, and

$$\begin{aligned} a_j^1(t) &= \frac{\partial f_j^1}{\partial x_{j+1}^1}(\hat{x}_1^1(t), \dots, \hat{x}_j^1(t), \sigma_j^1(t)); \\ &\quad \text{for } j = 1, \dots, n_1 - 1 \\ a_j^2(t) &= \frac{\partial f_j^2}{\partial x_{j+1}^2}(\hat{x}_1^2(t), \dots, \hat{x}_j^2(t), \sigma_j^2(t)); \\ &\quad \text{for } j = 1, \dots, n_2 - 2 \\ a_{n_2-1}^2(t) &= \frac{\partial f_j^2}{\partial x_{n_2}^2}(\hat{x}^{2-1}, \hat{x}_1^2, \dots, \hat{x}_j^2, \sigma_{n_2}^2(t), u(t)) \end{aligned}$$

Combining this last result and (7) we obtain our final observer structure.

5. SIMULATION RESULTS

To simulate the implicit model, we use a Runge-Kutta method combined with the Newton-Raphson algorithm.

The column is assumed to be stabilized around its steady state. We apply a profile of the feed composition as shown in Fig.1. The output measurements $y_1(t)$ and $y_2(t)$ are obtained from the simulated variables $x_1(t)$ and $x_n(t)$ by adding Gaussian noise of 5%. by fixing $\rho_1 = 1.3; \rho_2 = 2.5, \theta = 3.0$. Performances of the estimator are illustrated

Table 1. Operating point

number of trays and of feed trays	$n = 12,\ f = 8$
liquid volume in condenser	$V^{liq,1} = 10^{-4}m^3$
liquid volume in each tray	$V^{liq,i} = 3 \times 10^{-5}m^3$
liquid volume in boiler	$V^{liq,n} = 3 \times 10^{-4}m^3$
liquid and vapor Murphree efficiency	$e = 0.80,\ e = 0.85$
atmospheric pressure	$P = 760mmHg$

in the Figs. 3-4. This simulation shows that the state estimates converge to their corresponding state variables within at least 5 minutes and the important reduction of the sensibility of noisy measurements in the estimation of algebraic variables.

Table 2. Initial steady state

feed rate	$F^* = 0.3816\frac{\text{mol}}{\text{min}}$
feed composition	$z_F^* = 0.3675\text{mol}$
feed temperature	$T_F^* = 352.17\text{K}$
steady state quality of top product	$x_{11}^* = 0.93$
steady state quality of bottom product	$x_{11}^* = 0.1$
boiler heat flux	$Q_b^* = 9128.27\frac{\text{J}}{\text{min}}$

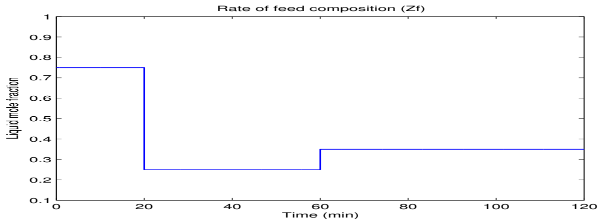

Fig. 1. Time evolution of feed composition.

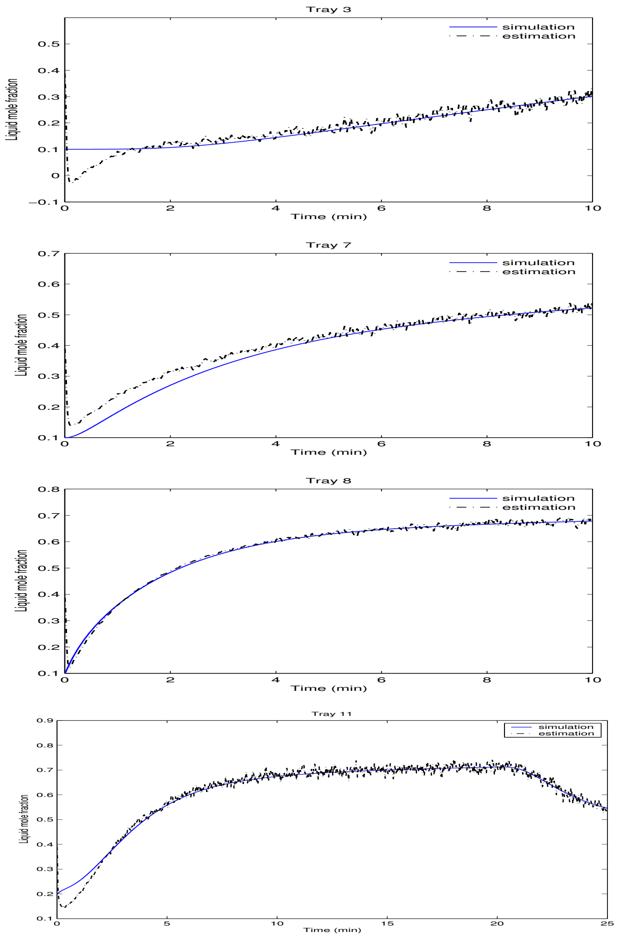

Fig. 2. observer performances for state variables (compositions) from noisy outputs (5% output additive noise, 50% initial error)

6. CONCLUSION

In this work we gave an exponential observer for a class of nonlinear implicit systems which do not require to be initialized at the algebraic constraint manifold $\mathcal{M}$. The calculation of the observer gain is based on the canonical form of uniform observability. This gain is constant and depends on two tuning parameters. The nonlinear constant high gain implicit observer was evaluated by means of digital simulations using the implicit model of the binary distillation column. Simulation results have been given which demonstrate the good performances of the given estimator in coping with model nonlinearities and output noise.

The proposed observer can be generated to a large class of nonlinear implicit system.

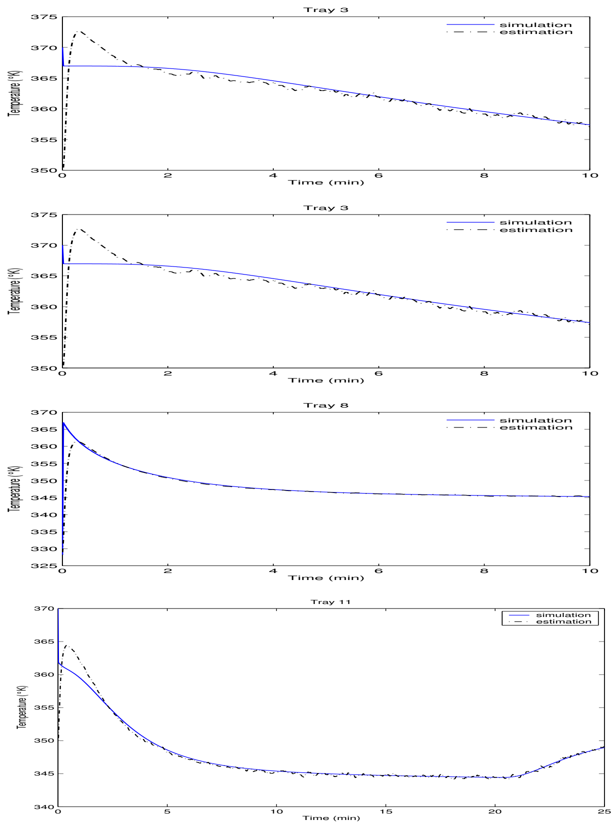

Fig. 3. Observer performances implicit (Temperatures) variables from noisy outputs (5% output additive noise, 50% initial error)

Notations *1.*

f,F	feed tray number, molar feed rate
N_i	molar hold up on the ith tray
K_ci	liquid-vapor equilibrium coefficient
L_i, V_i	molar liquid and vapor rates from ith tray
T_i	temperature in ith tray
h_i	molar liquid enthalpy in ith tray
H_i	molar vapor enthalpy in ith tray
h_F	feed molar enthalpy
D	molar distillate flow rate
W	molar waste flow rate
z_F,T_F	feed composition,feed temperature
x_{ci}, y_{ci}	liquid and vapor composition of component c in the ith tray
x_ci^{eq}	liquid composition of component c in plate i at equilibrium with the vapor phase at y_ci
e_i	Murphree liquid efficiency of ith tray
E_i	Murphree vapor efficiency of ith tray
P	total pressure
v_i	molar volume of the liquid mixture in ith tray
$V^{liq,i}$	liquid volume in ith tray
Q_b, Q_{cond}	heat flux at the reboiler and at the condenser
L_v	reflux volumetric liquid flow rate

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