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A Totally Stable Adaptive Control for Path Tracking of Time-Varying Autonomous Underwater Vehicles

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Abstract: This paper deals with the problem of adaptive path tracking of autonomous underwater vehicles with time-varying dynamics. The controller design is based on a speed-gradient adaptive law. A high-performance control behavior is aimed, so the full actuator dynamics is considered together with that of the vehicle. To this end, a state/disturbance observer is developed in the state feedback employing inverse dynamics. It is proved that the error paths can converge asymptotically to null when only the nonlinear static characteristic of the thrusters is involved in the design. When the actuator dynamics is considered too, only attractivity of the error paths to a residual set can be stated. The framework for this last proof relies on the concept of total stability. One main characteristic of our approach is that it can cope with a wide variety of bounded time-varying parameters with no limitations at all on their rates or a-priori knowledge.

Keywords: Time-varying nonlinear dynamics; Oceanic vehicles; Nonlinear adaptive control; State/disturbance observer; Asymptotic tracking; Total stability; Speed-gradient methodology

1. INTRODUCTION

In the past decade, new design tools and systematic design procedures has been developed to adaptive control for a set of general classes of nonlinear systems with uncertainties, for instance, integrator backstepping (Krstić *et al.* [1995]), speed-gradient control (Fradkov *et al.* [1999]), among others.

In the absence of modeling uncertainties, adaptive controllers can achieve in general global boundedness, asymptotic tracking, passivity of the adaptation loop irrespectively of the relative degree, and systematic improvement of transient performance (Krstić *et al.* [1993]).

When modeling uncertainties are included in the controller design, the basic adaptive laws can generally be modified so that the system can tolerate, in a global sense, a large diversity of unmodeled dynamics in the form of linearly and nonlinearly parameterized uncertainties enclosing time-varying parameters (Zhang and Ioannou [1996], Zhang and Ioannou [1998], Ikhouane and Krstić [1998], Arcak *et al.* [2000]).

Generally speaking, robust stabilization with respect to unknown time-varying parameters demands the *a-priori* knowledge of bounds of compact parameter sets among other considerations. Nevertheless, only in few cases, asymptotic tracking may be achieved under special conditions of signals in the control loop.

Common applications of submarine vehicles employed as platforms of mechanical tools or scientific instrumental can often be described as time-varying dynamics, with a-priori unknown parameters which may also change suddenly, periodically or erratically (El-Hawary [2001]). These facts support here the aim of control them adaptively. Moreover, in sampling missions, speed and high-performance in path tracking are usually required, even in critical cases by perturbed scenarios with currents and waves, in where robustness properties are also demanded.

The main objective of the paper is to present high-performance adaptive controllers for path tracking of arbitrarily time-varying systems with hydrodynamics. The description of the actuator dynamics as parasitic or dominant in comparison with the vehicle dynamics, plays an important role in the design and analysis of the approach and its convergence of the error tracking paths using the framework of total stability. A case study for an autonomous navigation vehicle with flying paths in 6 degrees of freedom (DOF), aims finally to illustrate features of the presented approach per simulation.

2. TIME-VARYING NONLINEAR DYNAMICS

According to (Fossen [1994]), the vehicle dynamics is

$$\dot{\mathbf{v}}=M^{-1}(t)(-C(t,\mathbf{v})\mathbf{v}-D(t,|\mathbf{v}|)\mathbf{v}+\mathbf{g}(t,\boldsymbol{\eta})+\boldsymbol{\tau}_c(t)+\boldsymbol{\tau}_t(t)) \quad (1)$$

$$\dot{\boldsymbol{\eta}}=J(\boldsymbol{\eta})(\mathbf{v}+\mathbf{v}_c), \quad (2)$$

where $\mathbf{v}=[u,v,w,p,q,r]^T \in \Re^{6\times1}\times\Re_0^+$ describes the body motion modes: surge, sway, heave, roll, pitch and yaw, respectively, and $\boldsymbol{\eta}\in\Re^{6\times1}\times\Re_0^+$ with the form $\boldsymbol{\eta}=[x,y,z,\varphi,\theta,\psi]^T$ for the same modes, respectively, but observed from the earth-fixed frame. The J is the rotation matrix expressing the transformation from the inertial frame to the body-fixed frame. The matrices M (inertia matrix), C (Coriolis matrix) and D (drag matrix) and the

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vectors $\mathbf{g}$ (buoyancy vector), $\mathbf{v}_c$ (current vector), $\boldsymbol{\tau}_c$ (cable force) and $\boldsymbol{\tau}_t$ (thruster force) are explained in (Jordán and Bustamante [2006]). As indicated therein, some of them can be structurally decomposed into combinations of time-varying and state-dependent matrices as in the case of $C = \sum_{i=1}^{6} C_i(t).\times C_{v_i}(v_i)$, $D = D_l(t) + \sum_{i=1}^{6} D_{q_i}(t)\ |v_i|$ and $\mathbf{g} = B_1(t)\mathbf{g}_1(\boldsymbol{\eta}) + B_2(t)\mathbf{g}_2(\boldsymbol{\eta})$, with ".×" being an element-by-element product.

Finally, the generalized thrust force applied on O is

$$\mathbf{f} = B^T \left(BB^T\right)^{-1} \boldsymbol{\tau}_t, \quad (3)$$

where the constant matrix $B \in \Re^{6\times n_\tau}$ contains position coordinates with respect to O of the n_τ thrusters. The thruster dynamics is (*cf.* Fossen [1994])

$$\mathbf{f} = K_1 (|\mathbf{n}|\,.\mathbf{n}) - K_2 (|\mathbf{n}|\,.\mathbf{v}_a) \quad (4)$$

$$\mathbf{n} = \mathbf{n}_1 + \mathbf{n}_2; \quad \mathbf{n}_1 = G_1(s)\ \mathbf{f}; \quad \mathbf{n}_2 = G_2(s)\ \mathbf{u}_a \quad (5)$$

$$\mathbf{u}_a = G_{PID}(s)(\mathbf{n}_r - \mathbf{n}), \quad (6)$$

where $(\mathbf{x}.\mathbf{y})$ represents an element-by-element product of vectors, $|\mathbf{n}|$ is a vector with elements of $\mathbf{n}$ but in absolute value, $\mathbf{n}$ and $\mathbf{n}_r : \Re_+ \rightarrow \Re^{n_\tau\times 1}$, $\mathbf{n}_1$, $\mathbf{n}_2$ are auxiliary vectors, $\mathbf{v}_a : \Re_+ \rightarrow \Re^{n_\tau\times 1}$ is the axial flow velocity of the thruster set, $\mathbf{u}_a$ is the armature voltage vector, K_1, $K_2 \in \Re^{n_\tau\times n_\tau}$ gain diagonal matrices of the thrust static characteristic, G_1 and G_2 represent diagonal matrices with strictly proper Laplace transfer functions, and similarly, G_{PID} a diagonal matrix with Laplace transfer functions on the diagonal representing usually PID controllers for the open-loop thruster DC motors.

3. ADAPTIVE CONTROLLER

Consider now the asymptotic path tracking target as defined by

$$\lim_{t\to\infty} (\boldsymbol{\eta}(t) - \boldsymbol{\eta}_r(t)) = \mathbf{0}, \quad \lim_{t\to\infty} (\mathbf{v}(t) - \mathbf{v}_r(t)) = \mathbf{0}, \quad (7)$$

for arbitrary finite initial conditions $\boldsymbol{\eta}(t_0) \in \mathcal{S}_\eta$ and $\mathbf{v}(t_0) \in \mathcal{S}_v$ and smooth positioning and kinematic path references $\boldsymbol{\eta}_r(t) \in \mathcal{S}_\eta$ and $\mathbf{v}_r(t) \in \mathcal{S}_v$, respectively. The sets $\mathcal{S}_i$ will indicate a compact set for the respective variable i. Let the desired control performance be established by the energetic cost functional

$$Q(t, \widetilde{\boldsymbol{\eta}}, \widetilde{\mathbf{v}}) = \frac{1}{2}\widetilde{\boldsymbol{\eta}}^T \widetilde{\boldsymbol{\eta}} + \frac{1}{2}\widetilde{\mathbf{v}}^T M(t)\widetilde{\mathbf{v}}, \quad (8)$$

which is a radially unbounded and nonnegative in the error space $\mathcal{S}_\eta \times \mathcal{S}_v$, where

$$\widetilde{\boldsymbol{\eta}} = \boldsymbol{\eta} - \boldsymbol{\eta}_r \quad (9)$$

$$\widetilde{\mathbf{v}} = \mathbf{v} - J^{-1}(\boldsymbol{\eta})\dot{\boldsymbol{\eta}}_r + J^{-1}(\boldsymbol{\eta}) K_p \widetilde{\boldsymbol{\eta}}, \quad (10)$$

and K_p is a design gain matrix with $K_p = K_p^T \geq 0$.

A suitable selection of $\boldsymbol{\tau}_t$ is (Jordán and Bustamante [2006])

$$\boldsymbol{\tau}_t(t) = \sum_{i=1}^{6} U_i.\times C_{v_i}(v_i)\mathbf{v} + U_7\mathbf{v} + \sum_{i=1}^{6} U_{i+7}\ |v_i|\ \mathbf{v} + \quad (11)$$

$$+ U_{14}\mathbf{g}_1 + U_{15}\mathbf{g}_2 + U_{16}\mathbf{d} - U_{17}\widetilde{\mathbf{v}} - K_v\widetilde{\mathbf{v}} - J^T\widetilde{\boldsymbol{\eta}}\ ,$$

where $U_i \in \Re^{6\times 6}$ are controller matrices obtained by (Fradkov *et al.* [1999])

$$\dot{U}_i = -\Gamma_i \frac{\partial \dot{Q}(U_i)}{\partial U_i}, \ \text{for } t \in \Re_+, \quad (12)$$

with $\Gamma_i = \Gamma_i^T \geq 0$ a positive-definite design gain matrix.

Theorem 1. (*Asymptotic convergence*).

Consider the system in (1)-(2) with bounded and continuous path references η_r and v_r, and known continuous disturbances τ_c and v_c. The control system with:

a) bounded, piecewise-continuous, time-varying elements $p_{jk}(t)$ in the physical matrices M, $C_1, \cdots, C_6$, D_l, $D_{q_1}, \cdots, D_{q_6}$, B_1 and B_2, stated in Section 2, with existing bounded $M^{-1}(t)$ for $t \geq t_0$, and eventual jumps ΔM_i in $M(t)$ being finite and isolated

b) the generalized force τ_t calculated as in (11) and

c) the variable controller matrices U_i's generated by integration of the adaptive laws (12),

ensures:

1) the asymptotic path tracking, *i.e.,* (7), for arbitrary initial conditions $\eta(t_0) \in \mathcal{S}_\eta$, $v(t_0) \in \mathcal{S}_v$, and $U_i(t_0) \in \mathcal{S}_U$, and provided that the design matrices in (10), (11) and (12) satisfy $K_p = K_p^T \geq 0$, $K_v = K_v^T \geq 0$ and $\Gamma_i = \Gamma_i^T \geq 0$ for $i = 1, \cdots, 17$, respectively, and

2) that all signals in the adaptive control loop are bounded.

Proof.

Invoking the continuity of $\boldsymbol{\tau}_t$, $\mathbf{v}_c$, $\boldsymbol{\tau}_c$ and the right-hand side of the $\dot{U}_i$'s in (12),it can concluded that there exists a scalar function $L(\beta)$ for any $\beta > 0$, such that it is valid

$$|\boldsymbol{\tau}_t(\boldsymbol{\eta}, \mathbf{v}, U_i)| + |\mathbf{v}_c| + |\boldsymbol{\tau}_c| + \sum_{i=1}^{17} \left\| \Gamma_i \frac{\partial \dot{Q}}{\partial U_i} \right\| \leq L(\beta) \quad (13)$$

for $|\boldsymbol{\eta}| \leq \beta$, $|\mathbf{v}| \leq \beta$, $|\mathbf{v}_c| \leq \beta$, $|\boldsymbol{\tau}_c| \leq \beta$ and $\| U_i \| \leq \beta$ in $\mathcal{S}_\eta$, $\mathcal{S}_v$, $\mathcal{S}_{v_c}$, $\mathcal{S}_{\tau_c}$, and $\mathcal{S}_U$, respectively, where $\|.\|$ is some matrix induced norm. After inclusion of global Lipschitz continuous feedback functions and exogenous perturbations on the right-hand of the $\dot{\boldsymbol{\eta}}$ and $\dot{\mathbf{v}}$, it can argued that there exists a unique, global Lipschitz continuous solution with components $\boldsymbol{\eta}$, $\mathbf{v}$ and U_i on $\mathcal{S}_\eta$, $\mathcal{S}_v$ and $\mathcal{S}_U$, respectively, all of them for $t \in \Re_+$.

Now, consider Q in (8) and following candidate of Ljapunov function

$$V(t, \widetilde{\boldsymbol{\eta}}, \widetilde{\mathbf{v}}, U_i) = Q\left(t, \widetilde{\boldsymbol{\eta}}, \widetilde{\mathbf{v}}\right) + Q_1(U_i - U_i^*) \quad (14)$$

with

$$Q_1(U_i - U_i^*) = \frac{1}{2}\sum_{i=1}^{17}\sum_{j=1}^{6} (\mathbf{u}_{i_j} - \mathbf{u}_{i_j}^*)^T \Gamma_i^{-1} (\mathbf{u}_{i_j} - \mathbf{u}_{i_j}^*) \quad (15)$$

and $\mathbf{u}_{i_j}$ the column vector j of U_i, and analogously, $\mathbf{u}_{i_j}^*$ the column vector j of every matrix $U_i^* : \Re_+ \rightarrow \Re^{6\times 6}$ defined as

$$U_i^*(t) = C_i(p_{jk}(t)), \text{ with } i = 1, ...6 \quad (16)$$

$$U_7^*(t) = D_l(p_{jk}(t)) \quad (17)$$

$$U_i^*(t) = D_{q_i}(p_{jk}(t)), \text{ with } i = 8, ...13 \quad (18)$$

$$U_{14}^*(t) = B_1(p_{jk}(t)) \quad (19)$$

$$U_{15}^*(t) = B_2(p_{jk}(t)) \quad (20)$$

$$U_{16}^*(t) = M(p_{jk}(t)) \quad (21)$$

$$U_{17}^*(t) = \dot{M}_c(p_{jk}(t))\ , \quad (22)$$

with $p_{jk}(t)$ time-varying elements in every physical matrix U_i^* and $M_c = M(t) - \sum_i \Delta M_i \delta(t - t_i)$ with $t_i \in \mathcal{S}_{t'}$ describing isolated time points for mass jumps.

In the following, consider $\dot{Q}(U_i)$ with (11) for $t \in \Re_+ \cap \mathcal{S}_{t'}$, *i.e.,* excluding the time points of sudden mass changes. Thus, taking the time derivative on V on any open interval (t_i, t_{i+1}) with $t_i, t_{i+1} \in \mathcal{S}_{t'}$, one achieves

$$\dot{V}\left(t,\widetilde{\boldsymbol{\eta}},\widetilde{\mathbf{v}},\widetilde{\mathbf{x}}_j,U_i\right)=\dot{Q}\left(t,\widetilde{\boldsymbol{\eta}},\widetilde{\mathbf{v}},U_i\right)+\sum_{i=1}^{17}\sum_{j=1}^{6}(\mathbf{u}_{i_j}\text{-}\mathbf{u}^*_{i_j})^T\Gamma_i^{-1}\dot{\mathbf{u}}_{i_j}. \quad (23)$$

As $\dot{Q}(U_i)$ is chosen globally convex in any compact convex set in the space of the controller parameter, it is valid that for any pairs of controller matrices (U_i, U_i')

$$\dot{Q}\left(\widetilde{\boldsymbol{\eta}},\widetilde{\mathbf{v}},U_i'\right)-\dot{Q}\left(\widetilde{\boldsymbol{\eta}},\widetilde{\mathbf{v}},U_i\right)\geq\sum_{i=1}^{17}\sum_{j=1}^{6}(\mathbf{u}'_{i_j}-\mathbf{u}_{i_j})^T\frac{\partial\dot{Q}}{\partial\mathbf{u}_{i_j}} \quad (24)$$

for any $U_i \in \mathcal{S}_U$, $U_i' \in \Re^{6\times 6}$, $\boldsymbol{\eta} \in \Re_+ \to \mathcal{S}_\eta$ and $\mathbf{v} \in \Re_+ \to \mathcal{S}_v$. Hence, for the particular choice of pairs (U_i, U_i^*) in (24) with $\mathbf{u}'_{i_j} \equiv \mathbf{u}^*_{i_j}(t)$ in (16)-(22) and $\dot{\mathbf{u}}_{i_j} = -\Gamma_i\frac{\partial\dot{Q}}{\partial\mathbf{u}_{i_j}}$ from (12), one achieves from (23)-(24)

$$\dot{V}\left(t,\widetilde{\boldsymbol{\eta}},\widetilde{\mathbf{v}},U_i\right)\leq\dot{Q}\left(\widetilde{\boldsymbol{\eta}},\widetilde{\mathbf{v}},U_i^*(t)\right). \quad (25)$$

Thus, using U_i^* in (11) one attains

$$\dot{Q}\left(\widetilde{\boldsymbol{\eta}},\widetilde{\mathbf{v}},U_i^*(t)\right)=-\widehat{\boldsymbol{\eta}}^T K_p\widehat{\boldsymbol{\eta}}-\widehat{\mathbf{v}}K_v\widehat{\mathbf{v}} \quad (26)$$

and so together with (8), (25) can be further bounded on every open interval $(t_i, t_{i+1}) \subset (t_0, \infty)$ as

$$\dot{V}\left(t,\widetilde{\boldsymbol{\eta}},\widetilde{\mathbf{v}},U_i\right)\leq -c(t)Q\left(t,\widetilde{\boldsymbol{\eta}},\widetilde{\mathbf{v}}\right)\leq -c_0\, Q\left(t,\widetilde{\boldsymbol{\eta}},\widetilde{\mathbf{v}}\right)\leq 0 \quad (27)$$

where c and c_0 are positive real values that accomplish

$$c(t)=\frac{\max\left\{\lambda_i\left(K_p\right),\lambda_i\left(K_v\right)\right\}}{\max\left\{\lambda_i\left(I/2\right),\lambda_i\left(M(t)/2\right)\right\}} \quad (28)$$

$$c_0=\sup_{t\in\Re_+}c(t), \quad (29)$$

and $\lambda_i(.)$ are the eigenvalues of the matrix indicated in parenthesis. Accordingly, as $\dot{V}$ is decreasing on (t_i, t_{i+1}), it is valid with (27)

$$V\left(t_{i+1}\right)-V\left(t_i\right)\leq -c_0\int_{t_i}^{t_{i+1}}Q(t)dt<\infty. \quad (30)$$

One infers from (27) that $Q(t)$ is also a decreasing function on every interval (t_i, t_{i+1}). So, there exists a $\mathcal{KR}$-function $\phi_1(\widehat{\boldsymbol{\eta}}, \widehat{\mathbf{v}})$ (see for instance Vidyasagar [1993]) that fulfills for all $t \in \Re_+$

$$\phi_1(\widehat{\boldsymbol{\eta}},\widehat{\mathbf{v}})=\frac{1}{2}\widehat{\boldsymbol{\eta}}^T\widehat{\boldsymbol{\eta}}+\frac{1}{2}\sup_{t\in\Re_+}\lambda_{\max}\left(M(t)\right)\widehat{\mathbf{v}}^T\widehat{\mathbf{v}} \quad (31)$$

$$0\leq Q(t,\widehat{\boldsymbol{\eta}},\widehat{\mathbf{v}})\leq\phi_1(\widehat{\boldsymbol{\eta}},\widehat{\mathbf{v}}), \quad (32)$$

with $\lambda_{\max}$ being the maximal eigenvalue of $M(t)$. Hence, by (30) one gets along the trajectories on (t_i, t_{i+1})

$$V\left(t_{i+1}\right)-V\left(t_i\right)\leq -\int_{t_i}^{t_{i+1}}\phi_1(t)dt\leq 0. \quad (33)$$

Due to (27), the function termed as $\chi(t_i, t_{i+1}) = \int_{t_i}^{t_{i+1}}\phi_1(t)dt > 0$ characterizes a series in the metric space $\left(\widehat{\boldsymbol{\eta}}, \widehat{\mathbf{v}}\right)$ along the solutions with elements satisfying

$$\chi(t_i,t_{i+1})>\chi(t_{i+1},t_{i+2}). \quad (34)$$

Thus, for every $\varepsilon > 0$, there is an integer N sufficiently large such that for $i, j \geq N$ one has $|\chi(t_j, t_{i+1}) - \chi(t_i, t_{i+1})| < \varepsilon$, *i.e.*, $\chi(t_i, t_{i+1})$ is a Cauchy sequence in the space $\widehat{\boldsymbol{\eta}}$, $\widehat{\mathbf{v}}$ and its limit for infinite number of time points t_i is a constant greater or equal to zero.

Clearly, if $t_i, t_{i+1} \to \infty$ then, by (33) and (34), one accomplishes $(V(t_{i+1}) - V(t_i)) \to 0$ and $\chi(t_i, t_{i+1}) \to 0$ as well. So one deduces that $\lim_{t\to\infty}(\boldsymbol{\eta}(t) - \boldsymbol{\eta}_r(t)) = 0$. In consequence and using (10), $\lim_{t\to\infty}\left(\mathbf{v} - J^{-1}(\boldsymbol{\eta})\dot{\boldsymbol{\eta}}_r\right) = \lim_{t\to\infty}\left(\mathbf{v} - J^{-1}(\boldsymbol{\eta}_r)\dot{\boldsymbol{\eta}}_r\right) = \lim_{t\to\infty}(\mathbf{v} - \mathbf{v}_r) = 0$ as well.

On the contrary, if the last time point for a mass jump is $t_i < \infty$, then at the limit $t \to \infty$, the integral on both sides of (27) on the period (t_i, ∞) leads

$$V(t_i)-V(\infty)\leq c_0\int_{t_i}^{\infty}Q\left(t,\widetilde{\boldsymbol{\eta}},\widetilde{\mathbf{v}}\right)dt<\infty. \quad (35)$$

Since the integral (35) exists and $Q\left(t, \widetilde{\boldsymbol{\eta}}, \widetilde{\mathbf{v}}\right)$ is uniformly continuous and bounded on (t_i, ∞) one can invoke the Lemma of Barbalat to show that $Q(t)$ tends to zero for $t \to \infty$ (see for instance Fradkov et al. [1999]). Consequently, it yields that the error trajectories of the result 1) converge to zero asymptotically.

To demonstrate the result 2) it can be stated from the previous results that

$$\int_{t_0}^{\infty}Q\left(t,\widetilde{\boldsymbol{\eta}},\widetilde{\mathbf{v}}\right)d\tau=\sum_{i=0}^{n}\chi(t_i,t_{i+1})<\infty. \quad (36)$$

with n an integer with $n \in [1, \infty]$. So, with (8) and the boundness of M and $\widetilde{\mathbf{v}}$, it can be stated also $\int_{t_0}^{\infty}\left|\widetilde{\mathbf{v}}\right|d\tau < \infty$ and $\int_{t_0}^{\infty}\left|\widetilde{\mathbf{v}}\right|^2 d\tau < \infty$. Thus from (12), it follows

$$|U_i|\leq c_i\int_{t_0}^{\infty}\left|\widetilde{\mathbf{v}}\right|d\tau<\infty,\ for\ c_i>0. \quad (37)$$

Similarly, for the particular case

$$|U_{17}|\leq c_{17}\int_{t_0}^{\infty}\left|\widetilde{\mathbf{v}}\right|^2 d\tau<\infty,\ for\ c_{17}>0. \quad (38)$$

Hence, it is concluded that the U_i's are also bounded. ■

4. ACTUATORS

The main idea so far has consisted in designing a high-performance control system for the vehicle with a fast global response. The open-loop vehicle dynamics considered is in general much more dominant than the thruster dynamics. Many authors consider thruster dynamics as parasitic and in turn it is neglected in the controller design. This is true for vehicles with large inertia and relatively slow motions. However, as one is interested in a high-performance path tracking, the actuators are taken into account by involving their inverse dynamics. The way to do this will consist in developing a state/disturbance observer.

Let the generalized force $\boldsymbol{\tau}_t$ be calculated in (11) by the adaptive controller and take $\mathbf{f}$ in (3) as an ideal thrust. Furthermore, let this thrust be referred to as $\mathbf{f}_{ideal}$. So, from (4) the ideal thrust fulfills

$$\mathbf{f}_{ideal}=K_1\left(|\mathbf{n}|\,.\mathbf{n}\right)-K_2\left(|\mathbf{n}|\,.\mathbf{v}_a\right), \quad (39)$$

As (39) is quasiconvex for $\mathbf{v}_a \neq \mathbf{0}$, then $\mathbf{n}(t)$ would result finitely discontinuous. As the thruster is a passive system, the consequence of these discontinuities is that the reference input $\mathbf{n}_r$ would be unbounded. To tackle this difficulty and be able to implement $\mathbf{n}_r$, one can relax slightly the condition of perfect tracking of $\mathbf{f}_{ideal}$. To this end, one can define ideal values for the auxiliary vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ in (5) as

$$\mathbf{n}_1=G_1(s)\mathbf{f}_{ideal} \quad (40)$$

$$\bar{\mathbf{n}}_2=G_3(s)\mathbf{n}-\mathbf{n}_1, \quad (41)$$

with $G_3(s)$ a diagonal matrix with Laplace functions of a stable low-pass filter for $\mathbf{n}$. The filter matrix G_3 is selected *ad-hoc* with a wide band of frequency so as to enable the filtered vector to be continuous and with only a little distortion with respect to $\mathbf{n}$.

The setpoint vector $\mathbf{n}_r$ will be now estimated by means of a state/disturbance observer. For simplicity in the notation, let the estimation be described for one generic thruster alone with scalar variables $n, n_1, n_2, \bar{n}_2$ and n_r as elements in $\mathbf{n}$, $\mathbf{n}_1$, $\mathbf{n}_2$, $\bar{\mathbf{n}}_2$ and $\mathbf{n}_r$, respectively. Furthermore, the Laplace transfer function in $G_2 G_{PID}(s)$ in (5)-(6) can be characterized in state space as

$$\dot{\mathbf{x}} = A\mathbf{x} + \mathbf{b}\,(n_r - n) \tag{42}$$

$$n_2 = n - n_1 = \mathbf{c}^T\mathbf{x} \tag{43}$$

$$\bar{n}_2 = g_3(s)n - n_1 = -\left(1 - g_3(s)\right)n + \mathbf{c}^T\mathbf{x}, \tag{44}$$

with $(A, \mathbf{b}, \mathbf{c})$ a minimal matrix set of the system representation in the observer form, $\mathbf{x}$ the state vector and $g_3(s)$ is an element of $G_3(s)$ in (41). Then let

$$\dot{\hat{\mathbf{x}}} = A\hat{\mathbf{x}} + \mathbf{b}\,\hat{e}_c + \mathbf{k}_{n_2}(\bar{n}_2 - \hat{n}_2), \tag{45}$$

be an estimation equation for $\mathbf{x}$, with $\mathbf{k}_{n_2}$ a gain vector for the rate error, and

$$\hat{n}_2 = \mathbf{c}^T\hat{\mathbf{x}} \tag{46}$$

$$\hat{e}_c = k_n\bar{n}_2 + k_{\dot{n}}\dot{\bar{n}}_2 + \mathbf{k}_{\hat{x}}^T\hat{\mathbf{x}}, \tag{47}$$

where $\hat{n}_2$ and $\hat{e}_c$ are estimations of n_2 and the input error $(n_r - n)$, respectively, and k_n, $k_{\dot{n}}$ and $\mathbf{k}_{\hat{x}}$ are gains for the appropriate components of $\hat{e}_c$.

On the other hand, with (42), (45), (46) and (44), the state error $\breve{\mathbf{x}} = \mathbf{x} - \hat{\mathbf{x}}$ accomplishes

$$\dot{\breve{\mathbf{x}}} = \left(A - \mathbf{k}_{n_2}\mathbf{c}^T\right)\breve{\mathbf{x}} + \mathbf{b}\,\tilde{e}_c + \mathbf{k}_{n_2}\left(1 - g_3(s)\right)n. \tag{48}$$

with $\tilde{e}_c = (n_r - n) - \hat{e}_c$.

Using (42)-(43) one gets

$$\dot{n}_2 = \mathbf{c}^T A\mathbf{x} + \mathbf{c}^T\mathbf{b}\,(n_r - n), \tag{49}$$

which combined with (47) and (44) gives the input estimation error

$$\tilde{e}_c = \left(1 - k_{\dot{n}}\mathbf{c}^T\mathbf{b}\right)(n_r - n) - \left(k_n\mathbf{c}^T + k_{\dot{n}}\mathbf{c}^T A\right)\mathbf{x} - \mathbf{k}_{\hat{x}}^T\hat{\mathbf{x}} + \left(1 - g_3(s)\right)\left(k_n n + k_{\dot{n}}\dot{n}\right). \tag{50}$$

Moreover, there exist particular values for $\mathbf{k}_{\hat{x}}$, k_n and $k_{\dot{n}}$ in (50) accomplishing

$$\begin{aligned} &\left(1 - k_{\dot{n}}\mathbf{c}^T\ \mathbf{b}\right) = 0 \\ &\mathbf{k}_{\hat{x}}^T = -\left(k_n\mathbf{c}^T + k_{\dot{n}}\mathbf{c}^T A\right), \end{aligned} \tag{51}$$

so that

$$k_{\dot{n}} = \frac{1}{b_{m-1}} \tag{52}$$

$$\mathbf{k}_{\hat{x}}^T = -\left[\frac{-a_{m-1}}{b_{m-1}} + k_n, \frac{1}{b_{m-1}}, 0, ..., 0\right], \tag{53}$$

with m the order of the system $G_2 G_{PID}(s)$. So the perturbation error turns into

$$\tilde{e}_c = -\left(k_n\mathbf{c}^T + k_{\dot{n}}\mathbf{c}^T A\right)\breve{\mathbf{x}} + \left(1\text{-}g_3(s)\right)\left(k_n n + k_{\dot{n}}\dot{n}\right). \tag{54}$$

In turn, the dynamics of the state error in (48) with (54) and the choice

$$\mathbf{k}_{n_2} = -k_n\mathbf{b}, \tag{55}$$

turns into

$$\dot{\breve{\mathbf{x}}} = (I - k_{\dot{n}}\mathbf{b}\mathbf{c}^T)A\breve{\mathbf{x}} + k_{\dot{n}}\mathbf{b}\left(1 - g_3(s)\right)\dot{n}, \tag{56}$$

where $(1 - g_3(s))$ is a high-pass filter that magnifies the errors $\breve{\mathbf{x}}$ and $\tilde{e}_c$ when rapid changes of n occur. It is seen in (56), that only the high-frequency components of $\dot{n}$ will excite the error system. In this sense, it is meaningful for the stability analysis to consider two cases, namely, when $\dot{n} = \dot{n}_s$ and when $\dot{n} = \dot{n}_s + \Delta n_i\delta(t - t_i)$, where $\dot{n}_s$ is the continuous part of $\dot{n}$ and Δn_i are the jumps of n.

Finally, the setpoint $\mathbf{n}_r$ to input the thrusters is calculated with help of (47) as

$$\hat{\mathbf{n}}_r(t) = \hat{\mathbf{e}}_c(t) + G_3(s)\mathbf{n}(t). \tag{57}$$

The proof of convergence of the adaptive control system together with observer is given in Section 6.

5. EVALUATION OF THRUST ERROR

Let the thrust error be defined as the difference between the real thrust $\mathbf{f}$ in the system and that one calculated in (11) by the control algorithm, it is $\mathbf{f}_{ideal}$. Let this thrust difference be referred to as $\Delta\mathbf{f}$. The appearance of $\Delta\mathbf{f}$ is the consequence of the inability of the energy system of reproduce arbitrary $\mathbf{f}_{ideal}$ faithfully when it is required the generation of a train of impulses for $\hat{n}_r$ each time that n need to be discontinuous in time. Instead of that, the system uses the filter matrix G_3 that provides a filtered version of n and $\dot{n}$ which can be implemented according to an energetic viewpoint.

So a component of $\Delta\mathbf{f}$ is defined as (See Fig. 1)

$$\Delta f = k_1\left(|n_t|\,n_t - |n|\,n\right) - k_2\left(|n_t|\,v_a - |n|\,\bar{v}_a\right), \tag{58}$$

where n_t is the true shaft rate homologous to n used in the observer. From the thruster dynamics and (58) one obtains

$$\frac{g_{PID}\,g_2}{1 - g_{PID}\,g_2}\left(\hat{n}_r - n_r\right) = (n_t - n) - \frac{g_1}{1 - g_{PID}\,g_2}\Delta f, \tag{59}$$

which leads to the norm

$$|\Delta f|_\infty \le \frac{\left|\frac{g_{PID}\,g_2}{1 - g_{PID}\,g_2}\right|_1}{\left|\frac{g_1}{1 - g_{PID}\,g_2}\right|_1}\left|\hat{n}_r - n_r\right|_\infty + \frac{1}{\left|\frac{g_1}{1 - g_{PID}\,g_2}\right|_1}\left|n_t - n\right|_\infty. \tag{60}$$

6. THE PERTURBED TRACKING PROBLEM

Let the perturbed path error system of the adaptive control system be obtained by combining (1)-(2) and (9)-(10) as (see Fig. 1)

$$\dot{\breve{\boldsymbol{\eta}}} = -K_p\breve{\boldsymbol{\eta}} + J\breve{\mathbf{v}} + J\ \mathbf{v}_c \tag{61}$$

$$\begin{aligned} \dot{\breve{\mathbf{v}}} = &-M^{-1}(C + D)\left(\breve{\mathbf{v}} + J^{-1}\left(\dot{\boldsymbol{\eta}}_r - K_p\breve{\boldsymbol{\eta}}\right)\right) + \\ &+M^{-1}\,\mathbf{g} - \frac{d\left(J^{-1}\dot{\boldsymbol{\eta}}_r\right)}{dt} + \dot{J}^{-1}K_p\breve{\boldsymbol{\eta}} + \\ &+J^{-1}K_p\left(J\breve{\mathbf{v}} - K_p\breve{\boldsymbol{\eta}} + J\ \mathbf{v}_c\right) + \\ &+M^{-1}\boldsymbol{\tau}_c + M^{-1}B\left(\mathbf{f}_{ideal} + \Delta\mathbf{f}\right), \end{aligned} \tag{62}$$

with $B\left(\mathbf{f}_{ideal} + \Delta\mathbf{f}\right) = \boldsymbol{\tau}_t$ resulting from (3). On the other side, the unperturbed system is obtained by taking $\Delta\mathbf{f} = \mathbf{0}$ in (62).

Definition 1. (Total stability, Hahn, 1959)

The equilibrium point described by $\breve{\boldsymbol{\eta}} = \breve{\mathbf{v}} = 0$ is said totally stable, if for each $\varepsilon > 0$ there exist two positive real values $\delta_1(\varepsilon)$ and $\delta_2(\varepsilon)$ such that the solution $\breve{\boldsymbol{\eta}}(t)$ and $\breve{\mathbf{v}}(t)$ of the error tracking system fulfills

$$\left|\breve{\boldsymbol{\eta}}(t, \breve{\boldsymbol{\eta}}(t_0), \breve{\mathbf{v}}(t_0))\right| < \varepsilon \text{ for } t > t_0 \tag{63}$$

$$\left|\breve{\mathbf{v}}(t, \breve{\boldsymbol{\eta}}(t_0), \breve{\mathbf{v}}(t_0))\right| < \varepsilon \text{ for } t > t_0 \tag{64}$$

provided that $\left|\tilde{\boldsymbol{\eta}}(t_0)\right| < \delta_1$ and $\left|\tilde{\mathbf{v}}(t_0)\right| < \delta_1$ in the state space region delimited by (63)-(64) and $|\Delta\mathbf{f}| < \delta_2$. ■

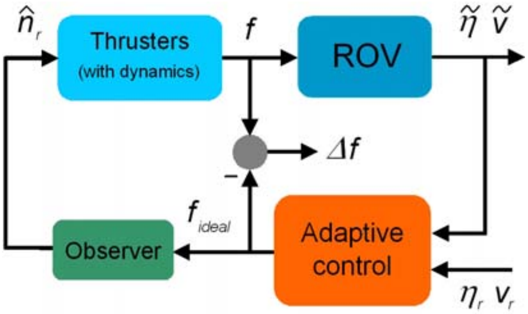

Figure 1 - Perturbed adaptive control system

Theorem 2. (*Stability of the perturbed system*).

Consider the system in (1)-(2) with thruster dynamics (3)-(6), bounded and uniformly continuous path references $\boldsymbol{\eta}_r$ and $\mathbf{v}_r$, a bounded and uniformly continuous $\dot{\mathbf{v}}_r$, and known disturbances $\boldsymbol{\tau}_c$ and $\mathbf{v}_c$. The control system with:

a), b) and c) like *a), b) and c)* in *Theorem I*

d) the reference shaft speed setpoint vector $\hat{n}_r$ attained as in (57),

guarantees that the equilibrium point of the perfect tracking, *i.e.*, $\tilde{\boldsymbol{\eta}} = \tilde{\mathbf{v}} = 0$, is totally stable.

Proof.

Take now the Ljapunov function in (14) into account. From now on one can consider the analysis only for $t \in \mathcal{S}_Q = [t_0, \infty) \cap \mathcal{S}_{t'}$ with $t = t_i \in \mathcal{S}_{t'} \subseteq (t_0, \infty)$ the time points at where sudden mass changes ΔM_i occur, which is a nowhere dense set. Combining (14) and (25) one obtains

$$\dot{V}\left(t, \tilde{\boldsymbol{\eta}}, \tilde{\mathbf{v}}, U_i\right) = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial \tilde{\boldsymbol{\eta}}}\dot{\tilde{\boldsymbol{\eta}}} + \frac{\partial V}{\partial \tilde{\mathbf{v}}}\dot{\tilde{\mathbf{v}}} \leq \tag{65}$$

$$\leq \dot{Q}\left(t, \tilde{\boldsymbol{\eta}}, \tilde{\mathbf{v}}, U_i^*\right) = \frac{\partial Q\left(t, \tilde{\boldsymbol{\eta}}, \tilde{\mathbf{v}}, U_i^*\right)}{\partial t} + $$

$$+ \frac{\partial Q\left(t, \tilde{\boldsymbol{\eta}}, \tilde{\mathbf{v}}, U_i^*\right)}{\partial \tilde{\boldsymbol{\eta}}}\dot{\tilde{\boldsymbol{\eta}}} + \frac{\partial Q\left(t, \tilde{\boldsymbol{\eta}}, \tilde{\mathbf{v}}, U_i^*\right)}{\partial \tilde{\mathbf{v}}}\dot{\tilde{\mathbf{v}}}. \tag{66}$$

Using (8), it is valid

$$\left|\frac{\partial Q\left(t, \tilde{\boldsymbol{\eta}}, \tilde{\mathbf{v}}, U_i^*\right)}{\partial t}\right| \leq \frac{1}{2}\left\|\frac{\partial M(t)}{\partial t}\right\|\left|\tilde{\mathbf{v}}\right|^2, \tag{67}$$

$$\left|\frac{\partial Q\left(t, \tilde{\boldsymbol{\eta}}, \tilde{\mathbf{v}}, U_i^*\right)}{\partial \tilde{\boldsymbol{\eta}}}\right| \leq \left|\tilde{\boldsymbol{\eta}}\right|, \left|\frac{\partial Q\left(t, \tilde{\boldsymbol{\eta}}, \tilde{\mathbf{v}}, U_i^*\right)}{\partial \tilde{\mathbf{v}}}\right| \leq \|M(t)\|\left|\tilde{\mathbf{v}}\right|. \tag{68}$$

Moreover, as $\boldsymbol{\eta}_r(t)$ and $\mathbf{v}_r(t)$ are uniformly continuous, and the solutions $\boldsymbol{\eta}(t)$ and $\mathbf{v}(t)$ are Lipschitz continuous, and M is uniformly bounded, then there exist positive constants c_0, c_1 and c_2 such that

$$\left|\frac{\partial V}{\partial t}\right| \leq c_0, \left|\frac{\partial V}{\partial \tilde{\boldsymbol{\eta}}}\right| \leq c_1 \text{ and } \left|\frac{\partial V}{\partial \tilde{\mathbf{v}}}\right| \leq c_2, \tag{69}$$

are valid for some domain $\mathcal{D} = \left\{\left|\tilde{\boldsymbol{\eta}}\right| \leq \varepsilon_0, \left|\tilde{\mathbf{v}}\right| \leq \varepsilon_0\right\}$, where ε_0 is a positive constant, and $t \geq t_0$.

As V is a decreasing function for the equilibrium point $\tilde{\boldsymbol{\eta}} = \tilde{\mathbf{v}} = \mathbf{0}$ in the unperturbed system (*cf.* Theorem 1), there exist three functions of class $\mathcal{K}$, referred to as $\alpha(|\tilde{\boldsymbol{\eta}}|, |\tilde{\mathbf{v}}|)$, $\beta(|\tilde{\boldsymbol{\eta}}|, |\tilde{\mathbf{v}}|)$ and $\gamma(|\tilde{\boldsymbol{\eta}}|, |\tilde{\mathbf{v}}|)$ such that $\alpha + Q_1 \leq V \leq \beta + Q_1$ and $\dot{V} \leq -\gamma$ in $\mathcal{D}$, where Q_1 is defined in (15). Considering (61)-(62), one has

$$\dot{V}_{PS}(t, \tilde{\boldsymbol{\eta}}, \tilde{\mathbf{v}}) = \dot{V}_S(t, \tilde{\boldsymbol{\eta}}, \tilde{\mathbf{v}}) + \frac{\partial V_{PS}}{\partial t} + \left(\frac{\partial V_{PS}}{\partial \tilde{\mathbf{v}}}\right)^T M^{-1}B\Delta\mathbf{f}, \tag{70}$$

where $\dot{V}_{PS}$ and $\dot{V}_S$ are the time derivatives of V evaluated along the solutions of the perturbed and unperturbed systems, respectively. Given any $\varepsilon < \varepsilon_0$ there exist a δ_1 and a δ_2 such that $\alpha(\varepsilon) > \beta(\delta_1)$, and a perturbation (60) such that

$$\left|\Delta\mathbf{f}\left(t, \tilde{\boldsymbol{\eta}}, \tilde{\mathbf{v}}\right)\right| \leq \delta_2 \tag{71}$$

for $\tilde{\boldsymbol{\eta}}, \tilde{\mathbf{v}} \in \mathcal{D}$ and $t \geq t_0$. Let

$$\gamma_0 = \min_{\delta_1 \leq |\tilde{\boldsymbol{\eta}}| \leq \varepsilon, \delta_1 \leq |\tilde{\mathbf{v}}| \leq \varepsilon \text{ and } t \geq t_0} \gamma(t, |\tilde{\boldsymbol{\eta}}|, |\tilde{\mathbf{v}}|). \tag{72}$$

For all solutions $\tilde{\boldsymbol{\eta}}\left(t, \tilde{\boldsymbol{\eta}}(t_0), \tilde{\mathbf{v}}(t_0)\right)$, $\tilde{\mathbf{v}}\left(t, \tilde{\boldsymbol{\eta}}(t_0), \tilde{\mathbf{v}}(t_0)\right)$ with $\left|\tilde{\boldsymbol{\eta}}(t_0)\right| \leq \delta_1$, $\left|\tilde{\mathbf{v}}(t_0)\right| \leq \delta_1$ entering the region $\delta_1 \leq \left|\tilde{\boldsymbol{\eta}}(t)\right| \leq \varepsilon$, $\delta_1 \leq \left|\tilde{\mathbf{v}}(t)\right| \leq \varepsilon$ for some time $t = t_1$, one has

$$\dot{V}_{PS}(t, \tilde{\boldsymbol{\eta}}, \tilde{\mathbf{v}}) \leq -\gamma_0 + c_0 + c_2\left\|M^{-1}B\right\|\delta_2. \tag{73}$$

Choosing $\delta_2 = \kappa(\gamma_0 - c_0)/\left(c_2\left\|M^{-1}B\right\|\right)$, where $0 < \kappa < 1$, $\dot{V}_{PS}(t, \tilde{\boldsymbol{\eta}}, \tilde{\mathbf{v}}) < 0$ for all time $t \geq t_1 > t_0$, so that $\left|\tilde{\boldsymbol{\eta}}\left(t, \tilde{\boldsymbol{\eta}}(t_0), \tilde{\mathbf{v}}(t_0)\right)\right| < \varepsilon$, $\left|\tilde{\mathbf{v}}\left(t, \tilde{\boldsymbol{\eta}}(t_0), \tilde{\mathbf{v}}(t_0)\right)\right| < \varepsilon$. Hence, the equilibrium point described by $\tilde{\boldsymbol{\eta}} = \tilde{\mathbf{v}} = \mathbf{0}$ is totally stable. ■

7. CASE STUDY: VEHICLE IN 6 DOF

In order to show the features of the presented adaptive approach with full thruster dynamics, let numerical simulations on a vehicle be considered. A scheduled 3D reference path for a sampling mission is described in Fig. 2. All the system parameters are supposed unknown at the start position. The design parameters for the adaptive laws are tuned as follows

$K_p = \text{diag}(1, 1, 1, 3, 3, 1)$

$K_v = \text{diag}(5{\times}10^4, 5{\times}10^4, 7.5{\times}10^4, 5{\times}10^4, 10^5, 2.5{\times}10^4)$

$\Gamma_i = \text{diag}(4) \quad i = 1, ..., 13$

$\Gamma_{14} = \Gamma_{15} = \text{diag}(200)$, $\Gamma_{16} = \text{diag}(40)$, $\Gamma_{17} = \text{diag}(0.16)$,

and the design parameters for the observer are

$$k_n = 5 \times 10^{-3}$$

$$G_3(s) = \text{diag}\left(\frac{5 \times 10^6}{(s + 1000)(s + 5000)}\right).$$

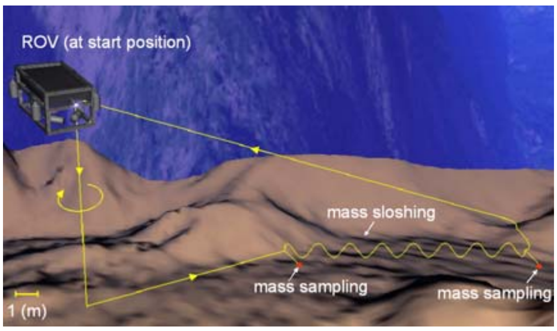

Figure 2 - Reference path of a sampling mission

In Figs. 3-5 the evolutions of vehicle variables in a period containing the events of mass sampling and sloshing are depicted. In Fig. 3 this evolution corresponds to x and θ modus. One notices that no appreciable tracking errors are found by the sudden mass changes and sloshing. Similarly, the true shaft rate n and the respective filtered ideal rate $g_3 n_{ideal}$ displayed in Fig. 4 for one vertical and one horizontal thrusters, show good coincidence. On the contrary, the thrust errors Δf evidence significant rapid changes at the places when sudden mass sampling occur, but also each time that thrusts cross about zero. Nevertheless, the magnitudes of these errors in percentage of the saturation value are less than 0.5%. So the achieved all-round performance of the adaptive control system is significantly high.

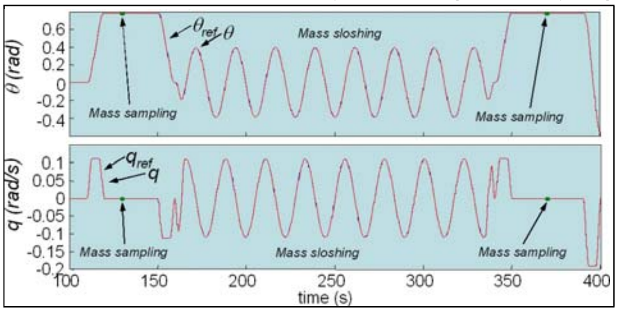

Fig. 3 - Evolution of the of position and kinematics trackings in the modus pitch

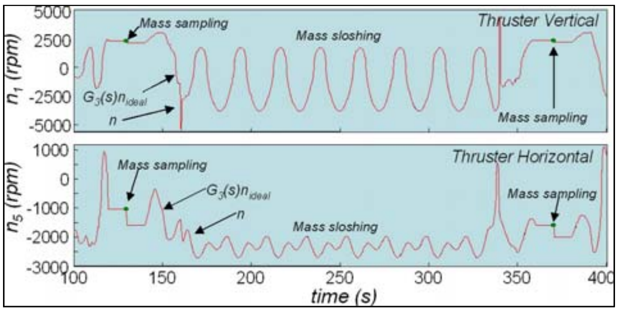

Fig. 4 - Evolution of the true and filtered ideal shaft rates

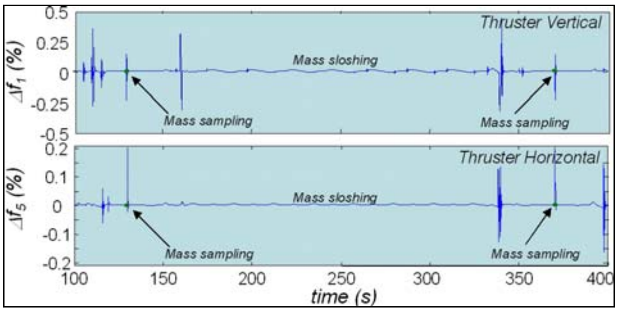

Fig 5 - Evolution of one element of the force error $\Delta\mathbf{f}$ in percentage of the saturation value

8. CONCLUSIONS

In this paper, a speed-gradient adaptive algorithm for asymptotic path tracking in 6 DOF for a large class of autonomous systems with hydrodynamics and in the presence of time-varying parameters has been presented.

The approach embraces arbitrary bounded and smooth exogenous reference paths $\boldsymbol{\eta}_r(t)$ and $\mathbf{v}_r(t)$, and unknown bounded piecewise-discontinuous parameters, which are generally distributed as hundreds of parameters in matrices describing the inertia, buoyancy, hydrodynamics, Coriolis and centripetal forces. These time variations are in compliance with real perturbations and operations in oceanic applications.

The development of the general adaptive control system is carried out in the frame of total stability, in which a state-dependent perturbation is introduced in the analysis and controller design. This concerns the existence of parasitics in the actuators in comparison with the dominant vehicle dynamics, which could commonly be neglected. If, on the contrary, a high-performance controller is aimed (as the approach developed in this paper) the actuator dynamics has to be considered in the controller design to achieve first-rate properties in the tracking behavior. The price to be paid for that is the unavoidable presence of a bounded state-dependent perturbation, that can be minimized conveniently by using ad-hoc design parameters.

In the case that only the static nonlinearities of the thrusters are taken in account, it was proved the adaptive control is asymptotic stable for every arbitrary piecewise-continuous change of parameters, regardless whether the changes are smooth or sudden, periodic or stochastic, increasing or decreasing, or combinations of them. It is pointed out that this case is realistic and therefore important, because often the parasitics can be neglected in vehicles with relative significant inertia and drag resistance. In the case that the full dynamics of the thruster is considered, total stability is proved instead.

Finally, the features of the proposed approach were illustrated in a case study for a remotely operated vehicle with hundreds of unknown parameters and a complex 3D reference path scheduled for a sampling mission. The obtained all-round performance was significantly high.

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