

## Wall Juggling of one Ball by Robot Manipulator with Visual Servo

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**Abstract:** This paper propose a method to achieve wall juggling of a ball *without* rebound from a table by a racket attached to a robot manipulator with two visual camera sensors. The proposed method is composed of juggling preservation problem and ball regulation problem. The juggling preservation problem means going on hitting the ball iteratively. The ball regulation problem means regulating the hitting position of the ball. The juggling preservation problem is achieved by the tracking control of the racket position for a symmetry trajectory of the ball with respect to a horizontal plane. The ball regulation problem is achieved by controlling the racket orientation, which is determined based on a discrete transition equation of the hitting position of the ball. The effectiveness of the proposed method is shown by an experimental result.

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### 1. INTRODUCTION

Since Ball juggling is a typical example to represent dexterous rhythmic tasks of humans, the ball juggling by robots has been studied by many researchers. The kinds of the juggling are toss juggling, paddle juggling, bouncing juggling, wall juggling and so on. Especially, the paddle juggling are studied by many researchers because it is simple iterative hitting of a ball by a racket. The paddle juggling by robots is composed of three parts: the iteration of hitting the ball, the regulation of the ball state which are the height and the incident angle to the racket, and the regulation of the hitting position of the ball. M. Buehler et al. (1994) proposed the mirror algorithms for the paddle juggling of one or two balls by a robot having one degree of freedom in two dimensional space, where the robot motion was symmetry of the ball motion with respect to a horizontal plane. This method achieved hitting the ball iteratively. R. Mori et al. (2005) proposed a method for the paddle juggling of a ball in three dimensional space by a racket attached to a mobile robot, where the trajectory of the mobile robot was determined based on the elevation angle of the ball. This method achieved hitting the ball iteratively and regulating the incident angle. A. Nakashima et al. (2006) proposed a method for the paddle juggling of a ball in three dimensional space by a racket attached to a robot manipulator having 5 degrees of freedom. This method achieved hitting the ball iteratively, regulating the incident angle and the hitting point simultaneously. These methods were the feedback control based on the ball state. On the other hand, T. M. H. Dijkstra (2004) proposed an open loop algorithm without the ball state for one-dimensional paddle juggling of one ball. They derived the condition for the racket trajectory to stabilize the periodic trajectory of the ball.

However, the paddle juggling can be achieved easier than the other jugglings because there does not exist any contacts with environments around robots in the paddle

juggling. There are few studies of the other jugglings. M. Takeuchi et al. (2002) considered the wall juggling of a ball, which means hitting a ball against a wall iteratively by a racket attached to a robot having 2 degrees of freedom. They allowed rebound from a table after the rebound from the wall. They analyzed the stability of periodic trajectory around its equilibrium states of the ball and proposed a feedback law to stabilize the trajectory. On the other hand, there is the another wall juggling which means hitting the ball against the wall *without* the rebound from the table. In this juggling, the racket motion is almost in the parallel direction to the wall because it is forced to hit the ball before the rebound while the racket in the former can move in any direction. This constrained racket motion make the wall juggling without the rebound more difficult.

This paper propose a method to achieve wall juggling of a ball without rebound from a table by a racket attached to a robot manipulator with two visual camera sensors. The proposed method is composed of juggling preservation problem and ball regulation problem. The juggling preservation problem means going on hitting the ball iteratively. The ball regulation problem means regulating the hitting position of the ball. The juggling preservation problem is achieved by the tracking control of the racket position for a symmetry trajectory of the ball with respect to a horizontal plane. The ball regulation problem is achieved by controlling the racket orientation, which is determined based on a discrete transition equation of the hitting position of the ball. The effectiveness of the proposed method is shown by an experimental result.

In Section 2, the system configuration and the models of the manipulator and the ball motion are shown. The manipulator dynamics are linearized for preliminary in Section 3. In Section 4, the control designs for the juggling preservation and the ball regulation problems are shown. An experimental result is shown in Section 5 and the conclusion is described in Section 6.

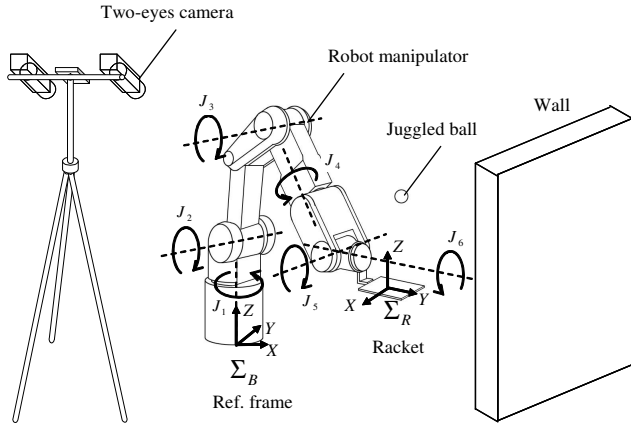


Fig. 1. Wall Juggling of a Ball by Camera-Robot System

## 2. MODELING

### 2.1 System Configuration

In this paper, we consider control of wall juggling of one ball by a racket attached to a robot manipulator with two-eyes camera as shown in Fig. 1. The robot manipulator has 6 joints, which is denoted by  $J_i$  ( $i = 1, \dots, 6$ ) shown in Fig. 1. In this paper, the juggled ball is supposed to be smaller and lighter than the racket. The physical parameters of the manipulator, the racket and the ball are shown in Section 5. The reference frame  $\Sigma_B$  is attached at the base of the manipulator. The position and orientation of the racket are represented by the frame  $\Sigma_R$ , which is attached at the center of the racket. The camera is calibrated with respect to  $\Sigma_B$ . Therefore, the position of the juggled ball can be measured as the center of the ball which the two cameras system detects (B. K. Ghosh et al. (1999)). It is obvious that the orientation of the racket about  $z$ -axis with respect to  $\Sigma_B$  does not effect on the ball motion at the time when the racket hits the ball. Therefore we need only 5 degrees of freedom of the manipulator. In the following discussion, the fourth joint  $J_4$  is assumed to be fixed by an appropriate control.

### 2.2 Dynamical Equation of Manipulator

The dynamical equation of the manipulator is given by

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + \mathbf{N}(\mathbf{q}) = \boldsymbol{\tau}, \quad (1)$$

where  $\mathbf{q} \in \mathbb{R}^5$  and  $\boldsymbol{\tau} \in \mathbb{R}^5$  describe the joint angles and the input torques respectively, and  $\mathbf{M} \in \mathbb{R}^{5 \times 5}$ ,  $\mathbf{C} \in \mathbb{R}^{5 \times 5}$  and  $\mathbf{N} \in \mathbb{R}^5$  are the inertia matrix, the coriolis matrix and the gravity term respectively. For the latter discussion, we introduce the following coordinate transformations:

$${}^B\mathbf{p}_R := \mathbf{h}_1(\mathbf{q}) \in \mathbb{R}^3, \quad {}^B\boldsymbol{\theta}_R := \mathbf{h}_2(\mathbf{q}) \in \mathbb{R}^2, \quad (2)$$

where  ${}^B\mathbf{p}_R = [{}^B p_{Rx} \quad {}^B p_{Ry} \quad {}^B p_{Rz}]^T \in \mathbb{R}^3$  is the position of  $\Sigma_R$  with respect to  $\Sigma_B$ ,  ${}^B\boldsymbol{\theta}_R = [{}^B \theta_{Rx} \quad {}^B \theta_{Ry}]^T \in \mathbb{R}^2$  is the orientation of  $\Sigma_R$  with respect to  $x$ - and  $y$ - axes of  $\Sigma_B$ . The left superscripts  $B$  of vectors denote that the vectors are expressed with respect to the frame  $\Sigma_B$ . This notation is utilized for another frames in the following. The functions  $\mathbf{h}_1(\mathbf{q})$  and  $\mathbf{h}_2(\mathbf{q})$  describe the relationships between the position/orientation of the racket and the joint angles respectively. Differentiating (2) with respect

to time  $t$  and getting together the resultant equations yield the velocity relationship

$$\dot{\mathbf{x}}_R = \mathbf{J}_R(\mathbf{q})\dot{\mathbf{q}}, \quad (3)$$

where

$$\mathbf{J}_R := [(\partial \mathbf{h}_1 / \partial \mathbf{q})^T \quad (\partial \mathbf{h}_2 / \partial \mathbf{q})^T]^T \in \mathbb{R}^{5 \times 5}$$

$$\mathbf{x}_R := [{}^B \mathbf{p}_R^T \quad {}^B \boldsymbol{\theta}_R^T] \in \mathbb{R}^5.$$

Applying the relationship (3) for the dynamical equation (1) yields the dynamical equation with respect to the position and orientation of the racket

$$\mathbf{M}_R(\mathbf{q})\ddot{\mathbf{x}}_R + \mathbf{C}_R(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{x}}_R + \mathbf{N}_R(\mathbf{q}) = \mathbf{J}_R(\mathbf{q})^{-T} \boldsymbol{\tau}, \quad (4)$$

where

$$\mathbf{M}_R := \mathbf{J}_R^{-T} \mathbf{M} \mathbf{J}_R^{-1} \in \mathbb{R}^{5 \times 5}$$

$$\mathbf{C}_R := \mathbf{J}_R^{-T} \mathbf{C} \mathbf{J}_R^{-1} + \mathbf{J}_R^{-T} \mathbf{M} \frac{d}{dt} \mathbf{J}_R^{-1} \in \mathbb{R}^{5 \times 5}$$

$$\mathbf{N}_R := \mathbf{J}_R^{-T} \mathbf{N} \in \mathbb{R}^5.$$

### 2.3 Equation of Motion and Rebound Phenomenon of Ball

For the modeling of the equation of motion and the rebound phenomenon of the ball, we make the following assumptions:

[Assumption 1] There does not exist the air resistance for the ball.

[Assumption 2] The surfaces of the ball, the racket and the wall are uniform and smooth. Therefore, the components of the velocity of the ball parallel to the surfaces do not change due to the rebound phenomenon, that is, the restitution coefficients parallel to the surfaces are zero.

[Assumption 3] The mass of the racket is bigger than the mass of the ball such that the racket velocity does not change due to the rebound phenomenon.

Define the ball position as  ${}^B \mathbf{p}_b := [{}^B p_{bx} \quad {}^B p_{by} \quad {}^B p_{bz}]^T \in \mathbb{R}^3$ . From Assumption 1, the equation of motion of the ball is given by

$${}^B \dot{p}_{bx} = v_{bx}, \quad {}^B \dot{p}_{by} = v_{by}, \quad (5)$$

$$m {}^B \ddot{p}_{bz} = -mg, \quad (6)$$

where  $v_{bx}$  and  $v_{by}$  are the velocity components in the  $(x, y)$  plane respectively,  $m[\text{kg}]$  is the mass of the ball and  $g[\text{m/s}^2]$  is the gravitational constant. Note that (5) represents the ball motion in the  $(x, y)$  plane, which is the uniform motion, and  $v_{bx}$  and  $v_{by}$  are changed only at the rebounds from the racket and the table.

From Assumption 2 and 3, the mathematical model of the rebound phenomenon at the rebound from the racket is given by

$${}^B \dot{\mathbf{p}}_b(t_r + dt) = \mathbf{R}_{BR} \mathbf{E} \mathbf{R}_{BR}^T ({}^B \dot{\mathbf{p}}_b(t_r) - {}^B \dot{\mathbf{p}}_R(t_r)) + {}^B \dot{\mathbf{p}}_R(t_r), \quad (7)$$

where  $t_r$  is the start time of the collision between the ball and the racket,  $dt$  is the time interval of the collision,  $\mathbf{R}_{BR} \in \mathbb{R}^{3 \times 3}$  is the rotation matrix from  $\Sigma_R$  to  $\Sigma_B$  and  $\mathbf{E} := \text{diag}(0, 0, -e) \in \mathbb{R}^{3 \times 3}$  represents the restitution coefficients of the  $x$ ,  $y$  and  $z$  directions with respect to  $\Sigma_R$ . Note that the rebound phenomenon from the wall is the same as (7) without  ${}^B \dot{\mathbf{p}}_R(t_r)$ .

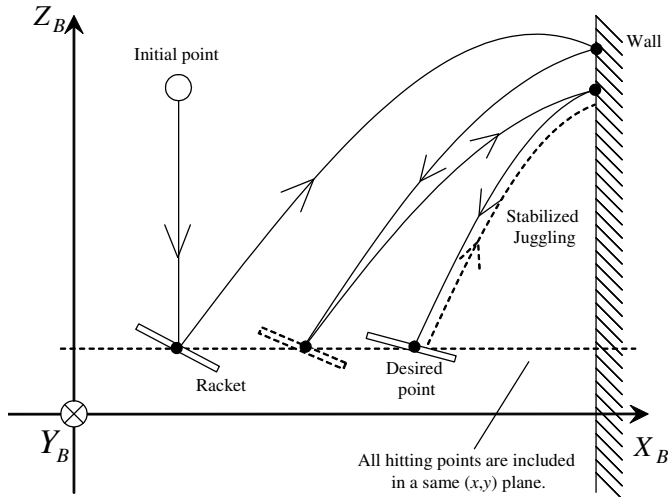


Fig. 2. The concept for Wall Juggling of one Ball  
 3. LINEARIZING COMPENSATOR FOR MANIPULATOR

As for the preliminary to control the wall juggling, we linearize the manipulator dynamics (4) by the following linearizing compensator:

$$\tau = \mathbf{J}_R^T (\mathbf{M}_R \mathbf{u}_R + \mathbf{C}_R \dot{\mathbf{x}}_R + \mathbf{N}_R), \quad (8)$$

where  $\mathbf{u}_R := [\mathbf{u}_{Rp}^T \ \mathbf{u}_{R\theta}^T]^T \in \mathbb{R}^5$  is the new input for  $\ddot{\mathbf{x}}_R$ . Substituting (8) into (4) results in

$$\mathbf{M}_R (\ddot{\mathbf{x}}_R - \mathbf{u}_R) = \mathbf{0}.$$

Since  $\mathbf{M}_R = \mathbf{J}_R^{-T} \mathbf{M} \mathbf{J}_R^{-1}$  is the positive definite matrix because the inertia matrix  $\mathbf{M}$  is positive definite,  $\mathbf{M}_R$  always has the inverse matrix. Therefore, we get the linearized equations given by

$${}^B \ddot{\mathbf{p}}_R = \mathbf{u}_{Rp}, \quad (9)$$

$${}^B \ddot{\boldsymbol{\theta}}_R = \mathbf{u}_{R\theta}. \quad (10)$$

The tracking control of  ${}^B \mathbf{p}_R$  and  ${}^B \boldsymbol{\theta}_R$  can be easily realized by PD controller for  $\mathbf{u}_{Rp}$  and  $\mathbf{u}_{R\theta}$ , for example. In the following discussion, let us consider the desired values of  ${}^B \mathbf{p}_{R_d}$  and  ${}^B \boldsymbol{\theta}_R$  to realize the wall juggling of the ball.

#### 4. CONTROL DESIGN FOR WALL JUGGLING

##### 4.1 Control Objectives

Figure 2 illustrates the concept of the wall juggling. Without loss of generality, the wall is located parallel to the  $(Y_B, Z_B)$  plane. The initial point of the ball is above the racket and the ball is freely released. The control purpose is to achieve the wall juggling of the ball at the desired hitting point  ${}^B \mathbf{p}_{R_d}$  by hitting the ball with the racket iteratively. This is described by the following specified control problems:

- (1) [Juggling Preservation Problem] The juggling Preservation problem means going on hitting the ball iteratively. The control of the ball position is not included in this problem.
- (2) [Ball Regulation Problem] The ball regulation problem means regulating the hitting position of the hit

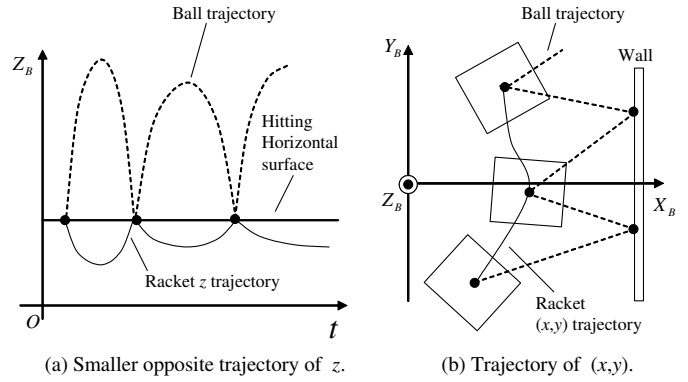


Fig. 3. Control Scheme for the Racket Position Control  
 ball. The regulation of the ball height at the rebound from the wall is not included in this problem.

In the following sections, the control designs for these control problems are shown.

##### 4.2 Control Design for Juggling Preservation Problem

The control design for the juggling preservation problem with controlling the racket position  ${}^B \mathbf{p}_R$  is shown. We firstly consider that the ball is always hit by the racket in a same  $(x, y)$  plane as shown in Fig. 2. More specific determination of the racket position is shown in Fig. 3. The trajectories of the  $z$  coordinates of the ball and the racket are shown in the right figure (a), where the vertical axis is the  $Z_B$  coordinate and the horizontal axis is time  $t$  respectively. The  $z$  coordinate of the racket is forced to follow the symmetric trajectory of the  $z$  coordinate of the ball with respect to a horizontal plane. This plane is called the hitting horizontal plane in this paper. Due to this determination, the racket can hit the ball at the same height without the prediction of the  $z$  coordinate of the ball.

On the other hand, let us consider the  $(x, y)$  trajectories of the racket in order to guarantee that the racket always hits the ball. For the paddle juggling, A. Nakashima et al. (2006) considered a method to force the  $(x, y)$  coordinates of the racket to follow the  $(x, y)$  coordinates of the ball. We modify this method for the wall juggling because the racket clashes the wall with the method. The trajectories of the  $(x, y)$  coordinates of the ball and the racket are shown in the left figure (b), where the vertical and horizontal axes are the  $Y_B$  and  $X_B$  coordinates respectively. The  $y$  coordinate of the racket is forced to follow the  $y$  coordinate of the ball while the  $x$  coordinate of the racket moves to the point of fall of the ball before the  $z$  coordinate of the ball is equals to the height of the hitting horizontal plane. The prediction of the point of fall is shown in Section 4.4.

To sum up, the desired value of the racket position  ${}^B \mathbf{p}_{R_d}$  to satisfy the mentioned in the above is given by

$${}^B \mathbf{p}_{R_d} := \begin{bmatrix} {}^B \hat{p}_{b_x} \\ {}^B p_{b_y} \\ {}^B p_h - k_h ({}^B p_{b_z} - {}^B p_h) \end{bmatrix}, \quad (11)$$

where  ${}^B \hat{p}_{b_x}$  is the predicted value of the  $x$  coordinate of the ball,  ${}^B p_h$  is the height of the hitting horizontal plane and  $0 < k_h < 1$  is the constant to make the  $z$  coordinate

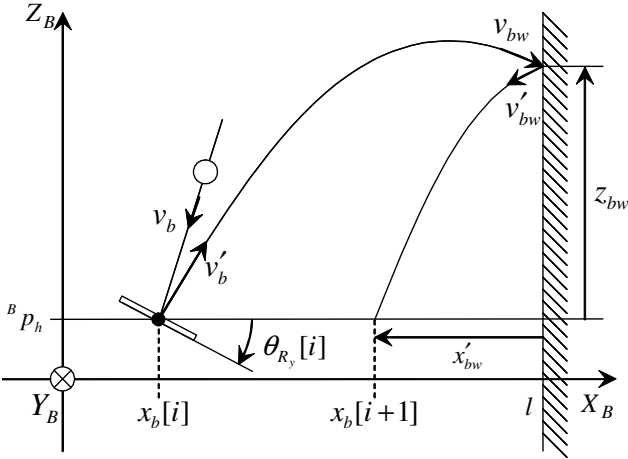


Fig. 4. Ball transition due to Rebound from Wall

of the racket small appropriately. Note that  $k_h$  effects on the height of the hitting ball.

Due to determining the desired value of the racket position  $^B \mathbf{p}_R$  by (11), there always exists the racket under the ball at the hit and the ball is hit in the same horizontal plane automatically.

#### 4.3 Control Design for Ball Regulation Problem

The control design for the ball regulation problem with controlling the racket orientation  $^B \theta_{R_y}$  is shown. Suppose that the juggling preservation problem is achieved.

*Control of the x coordinate of the ball* The transition of the  $x$  coordinate of the ball due to the rebound from the racket and the wall in  $(x, z)$  plane is illustrated in Fig. 4.  $x_b[i]$  is the  $x$  coordinate of the ball at the  $i$ th hit,  $\theta_{R_y}[i]$  is the racket orientation about  $Y_B$ -axis at the  $i$ th hit,  $\mathbf{v}_b \in \mathbb{R}^2$  and  $\mathbf{v}'_b \in \mathbb{R}^2$  are the ball velocity just before and after the hit, and  $\mathbf{V}_R \in \mathbb{R}^2$  is the racket velocity. The constant  $i$  ( $i = 0, 1, 2, \dots$ ) represents the number of the hitting. We derive the discrete transition equation of the  $x$  coordinate of the ball in order to design the controller.

From the model of the rebound phenomenon (7), the relationship between  $\mathbf{v}_b$ ,  $\mathbf{v}'_b$  and  $\mathbf{V}_R$  is given by

$$\mathbf{v}'_b = \begin{bmatrix} 1 - (1+e) \sin^2 \theta_{R_y} & \frac{1+e}{2} \sin 2\theta_{R_y} \\ \frac{1+e}{2} \sin 2\theta_{R_y} & 1 - (1+e) \cos^2 \theta_{R_y} \end{bmatrix} \mathbf{v}_b + \begin{bmatrix} (1+e) \sin^2 \theta_{R_y} & -\frac{1+e}{2} \sin 2\theta_{R_y} \\ -\frac{1+e}{2} \sin 2\theta_{R_y} & (1+e) \cos^2 \theta_{R_y} \end{bmatrix} \mathbf{V}_R, \quad (12)$$

where  $e$  is the restitution coefficient between the ball and the racket in the normal to the racket. Since the racket position  $^B \mathbf{p}_R$  follows the target position (11), the racket velocity  $\mathbf{V}_R$  is expressed as the following by differentiating (11):

$$\mathbf{V}_R = \begin{bmatrix} 0 \\ -k_h v_{bz} \end{bmatrix}. \quad (13)$$

Substituting (13) into (12) yields

$$\mathbf{v}'_b = \begin{bmatrix} \{1 - (1+e) \sin^2 \theta_{R_y}\} v_{bx} + \frac{(1+e)(1+k_h)}{2} \sin 2\theta_{R_y} v_{bz} \\ \frac{1+e}{2} \sin 2\theta_{R_y} v_{bx} + \{1 - (1+e)(1+k_h) \cos^2 \theta_{R_y}\} v_{bz} \end{bmatrix}. \quad (14)$$

On the other hand, from (5)–(7), the relationship between  $x_b[i]$  and  $x_b[i+1]$  is given by

$$x_b[i+1] = -e_w x_b[i] + (1+e_w)l - \frac{2e_w}{g} v'_{bx} v'_{bz}, \quad (15)$$

where  $e_w$  is the restitution coefficient between the ball and the wall in the normal to the wall and  $l$  is the distance of the wall from the origin of the reference frame.

Substituting (14) into (15), we can get the discrete transition equation of  $x_b[i]$ . However, since the resultant equation is nonlinear, we linearize it around  $\theta_{R_y} = \bar{\theta}_{R_y}$  and  $k_h = \bar{k}_h$  for the control design:

$$x_b[i+1] = -e_w x_b[i] + (1+e_w)l - \frac{2e_w}{g} \mathbf{B} \begin{bmatrix} \theta_{R_y} - \bar{\theta}_{R_y} \\ k_h - \bar{k}_h \end{bmatrix}, \quad (16)$$

where

$$\mathbf{B} := \begin{bmatrix} \frac{\partial(v'_{bx} v'_{bz})}{\partial \theta_{R_y}} & \frac{\partial(v'_{bx} v'_{bz})}{\partial k_h} \end{bmatrix} \Big|_{\theta_{R_y} = \bar{\theta}_{R_y}, k_h = \bar{k}_h}. \quad (17)$$

For simplicity, suppose that the control gain  $k_h$  is a constant  $\bar{k}_h$ . Therefore, the linearized discrete equation of the  $x$  coordinate of the ball for the control design is given by

$$x_b[i+1] = -e_w x_b[i] + (1+e_w)l - \frac{2e_w}{g} b_1 \Delta \theta_{R_y}, \quad (18)$$

where  $b_1$  is the first component of  $\mathbf{B}$  and  $\Delta \theta_{R_y} := \theta_{R_y} - \bar{\theta}_{R_y}$ . For the system (18), we propose the following state feedback law:

$$\Delta \theta_{R_y} = -k_x (x_b[i] - x_{bd}), \quad (19)$$

where  $k_x$  is the control gain and  $x_{bd}$  is a target value of the  $x$  coordinate of the hitting position. Substituting (19) into (18) yields the closed loop system

$$x_b[i+1] = (-e_w + \frac{2e_w}{g} b_1 k_x) x_b[i] + (1+e_w)l - \frac{2e_w}{g} b_1 x_{bd} k_x. \quad (20)$$

From (20), the conditions to stabilize  $x_b$  to  $x_{bd}$  are given by

$$\left| -e_w + \frac{2e_w}{g} b_1 k_x \right| < 1 \quad (21)$$

$$(1+e_w)l - \frac{2e_w}{g} b_1 x_{bd} k_x = x_{bd}. \quad (22)$$

The condition (22) leads to the control gain  $k_x$ :

$$k_x = \frac{g}{2e_w b_1} \left\{ \frac{(1+e_w)l}{x_{bd}} - 1 \right\}. \quad (23)$$

Substituting (23) into (21) yields the range of the target value  $x_{bd}$ :

$$\frac{1+e_w}{2+e_w} l < x_{bd} < \frac{1+e_w}{e_w} l. \quad (24)$$

*Control of the y coordinate of the ball* From Assumption 2, the transition of the  $y$  coordinate of the ball is the same as the one of the ball without the rebound from the wall. Therefore, it is enough to consider the rebound between the ball and the racket as shown in Fig. 5, where  $y_b[i]$  is the  $y$  coordinate of the ball at the  $i$ th hit,  $\theta_b[i]$  is the incident angle at the  $i$ th hit and  $\theta_{R_x}[i]$  is the racket orientation about  $X_B$ -axis at the  $i$ th hit. The linearized discrete transition equation of  $y_b[i]$  and  $\theta_b[i]$  around  $\theta_b[i] = 0$  and  $\theta_{R_x}[i] = 0$  is given by A. Nakashima et al. (2006):

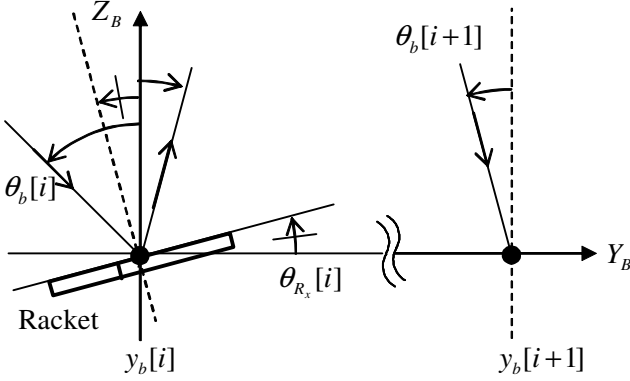


Fig. 5. Ball transition due to Rebound at Racket

$$\begin{bmatrix} y_b \\ \theta_b \end{bmatrix} [i+1] = \begin{bmatrix} 1 & \frac{2v_{b0}^2}{g} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_b \\ \theta_b \end{bmatrix} [i] + \begin{bmatrix} -\frac{4v_{b0}^2}{g} \\ -2 \end{bmatrix} \theta_{Rx} [i], \quad (25)$$

where  $v_{b0} := \sqrt{(v_{by}[i])^2 + (v_{bz}[i])^2}$ . From the transition equation (25), the controller for  $\theta_{Rx}[i]$  to stabilize  $(y_b[i], \theta_b[i])$  is easily obtained as

$$\theta_{Rx} [i] = -[k_{yp} \quad k_{y\theta}] \begin{bmatrix} y_b[i] - y_{bd} \\ \theta_b[i] \end{bmatrix}, \quad (26)$$

where  $y_{bd}$  is the target value and  $k_{yp} > 0$  and  $k_{y\theta} > 0$  are the control gains for  $y_b[i]$  and  $\theta_b[i]$  to be determined such that the eigenvalues of the closed system is smaller than 1.

#### 4.4 Prediction of Ball State

As mentioned in Section 4.2, we need the point of fall of the  $y$  coordinate of the ball  $\hat{p}_{bx}$  at the  $i$ th hit. Furthermore, from (14), (17), (23) and (25), we need the ball velocity just before the  $i$ th hit  $v_{bx}[i]$ ,  $v_{by}[i]$  and  $v_{bz}[i]$ . From (26), we also need the ball position  $y_b[i]$  and the incident angle  $\theta_b[i]$ . In this subsection, a simple prediction method of these variables is proposed. Note here that  $\hat{p}_{bx}$  is described by  $x_b[i]$ .

As shown in Fig. 6,  $(x_b(t), y_b(t), z_b(t))$  are the ball position and  $(v_{bx}(t), v_{by}(t), v_{bz}(t))$  are the ball velocity at time  $t$  between the  $i-1$ th and  $i$ th hits.  $y_b(t)$  and  $v_{by}(t)$  are

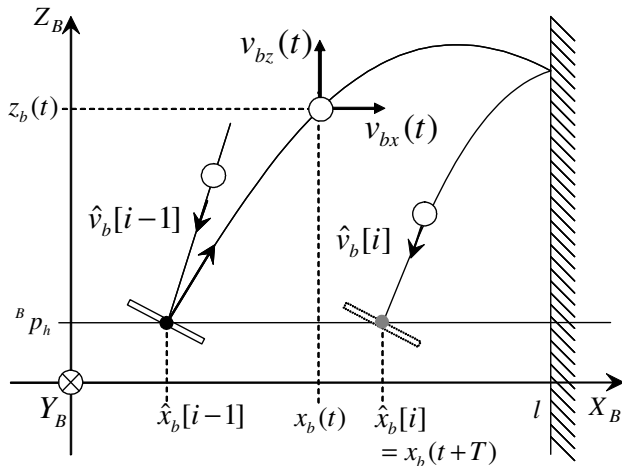


Fig. 6. Prediction of Ball State

omitted for simplicity. Suppose the position and velocity of the ball can be detected by the visual sensor. From Assumption 1, the velocity  $(v_{bx}[i], v_{by}[i])$  at the  $i$ th hit are the same as  $(v_{bx}(t), v_{by}(t))$ . For the velocity  $v_{bz}[i]$ , the predicted  $z$  velocity  $\hat{v}_{bz}[i]$  can be obtained from  $v_{bz}(t)$  and  $z_b(t)$  before the  $i$ th hit based on the energy conservation law:

$$\hat{v}_{bz}[i] = \sqrt{g(z_b(t) - p_h) + v_{bz}^2(t)} \quad (27)$$

Furthermore, the predicted incident angle  $\hat{\theta}_b[i]$  can be obtained from  $v_{by}[i]$  and  $\hat{v}_{bz}[i]$  by  $\hat{\theta}_b[i] = \tan^{-1}(v_{by}[i]/\hat{v}_{bz}[i])$ .

On the other hand, the predicted point of fall  $\hat{p}_{bx} := \hat{x}_b[i]$  can be obtained as

$$\hat{x}_b[i] = \begin{cases} l - e_w v_{bx}(t)(T - T_w) & (v_{bx} > 0) \\ x(t) + v_x(t)T & (v_{bx} < 0) \end{cases}, \quad (28)$$

where

$$T := \frac{v_{bz}(t) + \sqrt{v_{bz}^2(t) + 2g(z_b(t) - p_h)}}{g}, \quad T_w := \frac{l - x_b(t)}{v_{bx}(t)}. \quad (29)$$

$T$  is the time interval from time  $t$  to the time of the  $i$ th hit and  $T_w$  is the time interval from time  $t$  to the time of the  $i$ th rebound from the wall. Note that  $y_b[i]$  can be easily predicted by the same method as (28) without the rebound from the wall.

## 5. EXPERIMENTAL RESULT

The effectiveness of the proposed method is shown by an experimental result. The physical parameters of the robot are shown in Fig 7.  $J_i$  and  $W_i$  ( $i = 1, \dots, 6$ ) denote the  $i$ th joint and the center of mass of the  $i$ th link respectively. The values of  $W_1, W_2, W_3, W_4, W_{56}$  are 6.61, 6.60, 3.80, 2.00 and 2.18 [kg] respectively. the restitution coefficients  $e$  and  $e_w$  are 0.84 and 0.91. The distance of the wall  $l$  is set to 0.9[m]. The sampling period for the manipulator control is 120[Hz]. The racket is a square plate made from aluminum. The sides, the thickness and the mass are 150, 3[mm] and 0.20[kg] respectively. The ball is a ping-pong ball with the radius and the mass being 35[mm] and  $2.50 \times 10^{-3}$ [kg]. The vision system is composed of two CCD cameras and the image processing system. The pixel and the focal length of the each camera are 768(H)  $\times$  498(W) and 4.8[mm] respectively. The image processing system can measure the

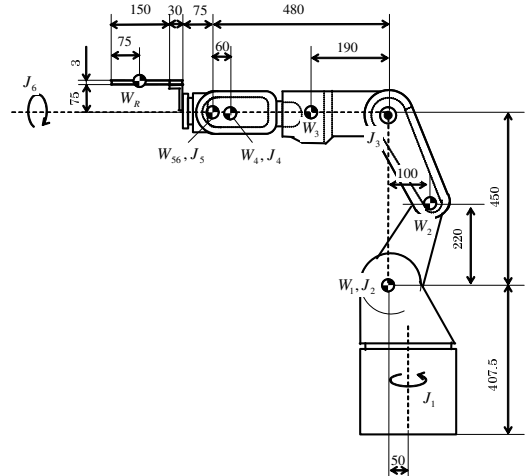


Fig. 7. The physical parameters of the manipulator

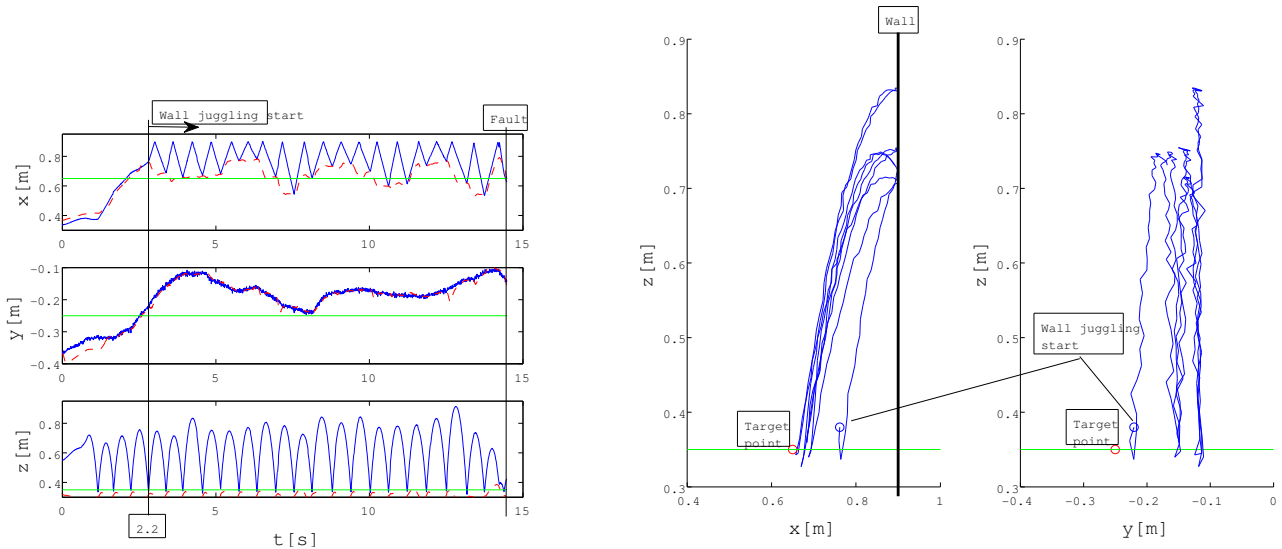


Fig. 8. Experimental result

center of the ball based on its brightness. The sampling period is 120[Hz]. The target values are set to  $x_{bd} = 0.65$ ,  $y_{bd} = -0.25$  and  ${}^B p_h = 0.35$ [m]. The control gains are set to  $k_{yp} = \frac{15}{180}\pi$ ,  $k_{y\theta} = 0.625$  and  $k_h = 0.2$ . The equilibrium value for the linearization  $\theta_{R_y}$  is  $-15$ [deg]. Since enough height of the ball is necessary for the wall juggling, the wall juggling is started when the apex of the ball is higher than 0.75[m].

The experimental result is shown in Fig. 8. The left figure shows the trajectories of the ball and racket positions in the time interval 0–15[s]. The solid and the dashed trajectories represent the ball and the racket respectively. The solid horizontal lines represent the desired hitting values. The wall juggling was started at time  $t = 2.2$ [s] and stopped a time  $t = 14.25$ [s] because the robot couldn't hit the ball against the wall. We can confirm that the wall juggling was achieved 19 times. We can also confirm that the  $x$  and  $y$  coordinates of the racket follow those of the ball and the  $z$  coordinate of the racket is symmetric to the ball with respect to the horizontal surface  ${}^B p_h = 0.35$ [m]. This result shows the effectiveness of the juggling preservation by controlling the racket position.

On the other hand, the right figure shows the ball trajectory from the 1st to 5th hits (2.2–5.5[s]) in the  $(x, z)$  and  $(y, z)$  planes. The blue circle is the initial position and the red circle is the desired hitting point. We can confirm that the  $x$  and  $y$  coordinates of the hitting point stay near the target point. However, the average  $x$  and  $y$  positions have the errors 7.9[cm] and 5.4[cm] from the desired hitting position. As for this reason, the air resistance, the calibration errors and the uncertainty of the robot model can effect on the errors of the hitting position.

## 6. CONCLUSION

This paper proposed a method to achieve wall juggling of a ball without rebound from a table by a racket attached to a robot manipulator with two visual camera sensors. The proposed method was composed of juggling preservation problem and ball regulation problem. The juggling preservation problem was achieved by the tracking control of the

racket position for a symmetry trajectory of the ball with respect to a horizontal plane. The ball regulation problem was achieved by controlling the racket orientation, which was determined based on a discrete transition equation of the hitting position of the ball. The effectiveness of the proposed method was shown by an experimental result.

The proposed method can adjust the ball height but not control it completely. It is our further work because the control of the height improve the stability of the wall juggling.

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