Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Higher Order SM Block Control of Nonlinear Systems with Unmodeled Actuators

Alexander G. Loukianov * **Leonid Fridman** ** **Lose M. Cañedo** * **Adolfo Soto-Cota** *** **Edgar N. Sanchez** *

* *CINVESTAV, Unidad Guadalajara, Apartado Postal 31-438, Plaza La Luna, Guadalajara, Jalisco, C.P. 45091, Mexico, e-mail: [louk][canedoj][sanchez]@gdl.cinvestav.mx*
** *Centro Universidad Nacional Autónoma de México, Facultad de Ingeniería, Ciuadad Univercitaria, México. E-mail: lfridman@servidor.unam.mx*
*** *Instituto Tecnológico de Sonora, 5 de Febrero 818 sur, Cd. Obregón, Sonora México. E-mail: adolfosoto@itson.mx*

Nonlinear system, sliding mode control, power systems.

Abstract: In this paper, a control strategy is proposed for robust stabilization of nonlinear perturbed plants in the presence of actuator unmodeled dynamics. The block control technique is applied to design a nonlinear SM manifold for achieving the error tracking, and High Order Sliding Mode (HOSM) algorithm is implemented to ensure finite time convergence of the state vector to the designed SM manifold. A robust exact differentiator is designed to obtain the estimates of the sliding variable and its derivatives. The proposed method is applied to design robust controllers for a power electric system in the presence of the exciter system unmodeled dynamics.

1. INTRODUCTION

The dynamics of the most of the industrial plants (for example electric power system, electromechanical system, electro-hydraulic system and so on) are highly nonlinear and, moreover, include actuator dynamics which increase the relative degree of the complete system. To stabilize the plant dynamics it is naturally to applied some feedback linearization (FL) technique: block control(Loukianov [2002]), backstepping (Krstic et al [1995]) or input-output linearization (Isidori [1992]), since the model of these plants can be presented in the nonlinear block controllable form or (the same) strict-feedback one. All these control techniques require to calculate the time derivatives of the plant dynamics vector fields (Lie derivatives), results in a computationally expensive control algorithm, and the closed-loop system is susceptible to plant parameter variations and disturbances. To simplify the control the fast actuator dynamics is usually skipped, and to overcome the robust problem the sliding mode (SM) control (Utkin [1999]) in combination with FL technique (Loukianov [2002]), can be can be applied. However, the presence of the actuator unmodeled fast dynamics can destroy the desired behavior of the sliding mode control systems causing lost of robustness and accuracy and provoking the chattering effect (?]). Therefore, the problem of control design for the systems with unmodeled actuator dynamics becomes to be a big challenge.

In the present paper we propose a control schemes based on the combination of the block control and sliding mode control techniques. First, the block control technique is used to design a nonlinear sliding manifold for achieving the error tracking. Then the High Order Sliding Mode (HOSM) algorithm (Levant [1993]) is implemented to ensure finite time convergence of the state vector to the designed SM manifold in the presence of the actuator unmodeled dynamics. Finally, a robust exact differentiator (Levant [2003]) is used to obtain the estimates of the sliding variable and its derivatives. The proposed method is applied to design robust controllers for a power electric system in the presence the exciter fast unmodeled dynamics (Fridman et al. [2007]). The simulations results show the reasonable behavior of the designed controllers.

2. THE IDEA OF NONLINEAR BLOCK HIGHER ORDER SLIDING MODE CONTROLLER

The principal advantage of SM control is robustness in the presence of external and internal disturbances. In subsection 2.1, the first order SM controller is presented, and the HOSM control is described in subsection 2.2

2.1 Nonlinear block controllers with a first order sliding mode

Consider a class of nonlinear SISO systems presented (possibly after a nonlinear transformation) in Nonlinear Block Controllable form (NBC-form) consisting of r blocks

⋆ This work was supported by CONACYT Mexico, through Projects 46069Y, 56819 and 57801Y.

(Loukianov [2002]) (or strict feedback form (Krstic et al [1995])) subject to uncertainties

$$\begin{aligned} \dot{x}_1 &= f_1(x_1) + b_1(x_1)x_2 + g_1(x_1,t) \\ \dot{x}_i &= f_i(\bar{\mathbf{x}}_i) + b_i(\bar{\mathbf{x}}_i)x_{i+1} + g_i(\bar{\mathbf{x}}_i,t) \\ \dot{x}_r &= f_r(\bar{\mathbf{x}}_r,\bar{\mathbf{x}}_{r+1}) + b_r(\bar{\mathbf{x}}_r,\bar{\mathbf{x}}_{r+1})u + g_r(\bar{\mathbf{x}}_r,\bar{\mathbf{x}}_{r+1},t) \end{aligned} \quad (1)$$

$$\dot{\bar{\mathbf{x}}}_{r+1} = \bar{\mathbf{f}}_{r+1}(\bar{\mathbf{x}}_r,\bar{\mathbf{x}}_{r+1},t), \quad i = 2,\ldots,r-1 \quad (2)$$

$$y = h(\bar{\mathbf{x}}) = x_1 \quad (3)$$

where the state vector $\bar{\mathbf{x}} \in \mathbf{R}^n$ is decomposed as $\bar{\mathbf{x}} = (x_1,\ldots,x_r,x_1,\ldots,x_n)^T = (\bar{\mathbf{x}}_r,\bar{\mathbf{x}}_{r+1})^T$, $\bar{\mathbf{x}}_i = (x_1,\ldots,x_i)^T$, $i = 1,\ldots,r$; y and $u \in \mathbf{R}$; f_i, b_i and h are known sufficiently smooth functions of their arguments, g_i is a uncertain but bounded function, and $b_i(\bar{\mathbf{x}}_i) \neq 0$, $i = 1,\ldots,r$ over the set $D_1 \times D_2$:

$$\begin{aligned} D_1 &= \{\bar{\mathbf{x}}_r \in \mathbf{R}^r \mid \|\bar{\mathbf{x}}_r\|_2 \le r_1\}, \ r_1 > 0 \\ D_2 &= \left\{\bar{\mathbf{x}}_{r+1} \in \mathbf{R}^{n-r} \mid \|\bar{\mathbf{x}}_{r+1}\|_2 \le r_2\right\}, \ r_2 > 0. \end{aligned}$$

Suppose

A1) *A solution of the system*

$$\dot{\bar{\mathbf{x}}}_{r+1} = \bar{\mathbf{f}}_{r+1}(0,\bar{\mathbf{x}}_{r+1},t) \quad (4)$$

described zero dynamics in (1)-(3) with any initial condition from $D_1 \times D_2$ converges exponentially to a compact set

$$\|\bar{\mathbf{x}}_{r+1}\|_2 \le b_1 < r_2, \quad b_1 > 0.$$

The general first order SM design procedure is the following. First, the output tracking error is defined as

$$z_1 = y - y_{ref}(t)$$

where is $y_{ref}(t)$ a reference signal. Then, using a Block Control linearized transformation (Loukianov [2002])

$$z_i = \varphi_i(\bar{\mathbf{x}}_i), \quad i = 2,...,r \quad (5)$$

the system (1)-(2) can be represented as

$$\begin{aligned} \dot{z}_1 &= -k_1 z_1 + z_2 + \tilde{g}_1(z_1,t) \\ \dot{z}_i &= -k_i z_i + z_{i+1} + \tilde{g}_i(\bar{\mathbf{z}}_i,t), \qquad i = 2,\ldots,r-1 \\ \dot{z}_r &= \tilde{f}_r(\bar{\mathbf{z}}_r,\bar{\mathbf{x}}_{r+1}) + \tilde{b}_r(\bar{\mathbf{z}}_r,\bar{\mathbf{x}}_{r+1})u + \tilde{g}_r(\bar{\mathbf{z}}_r,\bar{\mathbf{x}}_{r+1},t) \\ \dot{\bar{\mathbf{x}}}_{r+1} &= \mathbf{f}_{r+1}(\bar{\mathbf{z}}_r,\bar{\mathbf{x}}_{r+1},t) \end{aligned} \quad (6)$$

where $\bar{\mathbf{z}}_i = (z_1,\ldots,z_i)^T$, $1,\ldots,r$, $k_j > 0$, $j = 1,\ldots,r-1$ and $\tilde{b}_r = b_1 b_2 \cdots b_r$.

Taking advantage of the system (6) structure we choose

$$s = z_r = \varphi_r(\bar{\mathbf{x}}_r) \quad (7)$$

as a sliding variable for the discontinuous control law

$$u = -u_0 \tilde{b}_r^{-1} sign(s). \quad (8)$$

Proposition 1 *The control law (7)-(8) under the following condition:*

$$u_0 > \left| \tilde{f}_r(\bar{\mathbf{z}}_r,\bar{\mathbf{x}}_{r+1}) + \tilde{g}_r(\bar{\mathbf{z}}_r,\bar{\mathbf{x}}_{r+1},t) \right|$$

guaranties the convergence of the closed-loop system motion to the manifold $s = 0$ in a finite time defined as

$$t_s < t_0 + \frac{1}{\eta}\|\bar{\mathbf{z}}_r(t_0)\|_2, \ \eta > 0.$$

Now, for the system constrained to the sliding manifold $s = z_r = 0$, the system (6) reduces to

$$\begin{aligned} \dot{z}_1 &= -k_1 z_1 + z_2 + \tilde{g}_1(z_1,t) \\ \dot{z}_i &= -k_i z_i + z_{i+1} + \tilde{g}_i(\bar{\mathbf{z}}_i,t) \\ \dot{z}_{r-1} &= -k_{r-1} z_{r-1} + \tilde{g}_{r-1}(\bar{\mathbf{z}}_{r-1},t) \\ \dot{\bar{\mathbf{x}}}_{r+1} &= \bar{\mathbf{f}}_{r+1}(\bar{\mathbf{z}}_{r-1},0,\bar{\mathbf{x}}_{r+1},t), \quad i = 2,\ldots,r-2 \end{aligned} \quad (9)$$

and thus the original nonlinear problem is reduced to analyze the robustness property of the decomposed reduced-order SM dynamics (9) which can be considered as linear system with nonlinear perturbation, unmatched with respect to the control u in the system (6). It is apparent when $s = 0$, stability of the system (9) linear part is defined by of the coefficients k_i, $1,...,r-1$ values. Assume

A2) There exist positive constants q_{ij} and d_i such that

$$\begin{aligned} |\tilde{g}_1(z_1,t)| &\le q_{11}|z_1| + d_1; \\ |\tilde{g}_2(\bar{\mathbf{z}}_2,t)| &\le k_1 q_{21}|z_1| + q_{22}|z_2| + d_2 \\ |\tilde{g}_i(\bar{\mathbf{z}}_i,t)| &\le \sum_{j=1}^{i} k_j^{(i-j)} q_{i,j}|z_j| + d_i, \\ & i = 3,...,r-1, j = 3,...,i. \end{aligned}$$

To achieve the robustness property with respect to unknown but bounded uncertainty, the controller gains k_i, $i = 1,...,r-1$ have to be chosen hierarchically high. Thus, since $\tilde{g}_1(\bar{z}_1,t)$ does not depend on k_1, the value of this coefficient can be chosen such that the term $k_1 z_1$ in the first block of (9) will dominate. By block linearization procedure, the term $\tilde{g}_2(\bar{z}_2,t)$ depends on k_1 but not k_2,...,k_{r-1}. Then for fixed k_1, the appropriate choice of k_2 value the term $k_2 z_2$ in the second block of (9) will be also dominating, and so on. Finally, a constructive step-by-step Lyapunov technique approach (Loukianov [2002]) establishes the stability property of the sliding mode motion on the manifold $z_r = 0$, and provides the required values of the controller gains $k_1,...,k_{r-1}$. So

Theorem 1. *Let Assumptions **A1** and **A2** hold. Then there exist positive scalars $k_1,...,k_{r-1}$ and $h_1,...,h_{r-1}$ such that a solution of the system (9) is uniformly ultimately bounded, i.e.*

$$\limsup_{t\to\infty} |z_i(t)| \le h_i, \ i = 1,...,r-1.$$

To derive the linearized transformation (5) it needs to calculate the successive derivatives of $f_1(x_1)$, $f_i(\bar{\mathbf{x}}_i)$ and $b_1(x_1)$, $b_i(\bar{\mathbf{x}}_i)$, $i = 2,\ldots,r-1$ in (1)-(2), that results in a computationally expensive control algorithm. Moreover, to achieve robustness with respect to unmatched perturbation $\tilde{g}_1(z_1,t)$ and $\tilde{g}_i(\bar{\mathbf{z}}_i,t)$, $i = 2,\ldots,r-1$ in (9) the controller gains $k_1,...,k_{r-1}$ must be sufficiently high. To solve these problems a HOSM controller combined with a robust exact differentiator (Levant [2003]) will be applied in the next section.

2.2 Nonlinear block controllers with higher order sliding modes

Assume that the system (1)-(2) models both the plant and its actuator with the relative degrees k and q, respectively, so $k + q = r$. Therefore, choosing s_0 (5)

$$s_0 = z_{k+1} = \varphi_{k+1}(\bar{\mathbf{x}}_{k+1}), \ 1 < k < r-2 \quad (10)$$

as a sliding variable, and then taking its successive derivatives, straightforward calculations give

$$\begin{aligned} \dot{z}_1 &= -k_1 z_1 + z_2 + \tilde{g}_1(z_1,t) \\ \dot{z}_i &= -k_i z_i + z_{i+1} + \tilde{g}_i(\bar{\mathbf{z}}_i,t), \quad i=2,\ldots,k-1 \\ \dot{z}_k &= -k_k z_k + s_0 + \tilde{g}_k(\bar{\mathbf{z}}_k,t), \qquad (11) \\ \dot{s}_j &= s_{j+1}, \qquad j=0,\ldots,q-2 \end{aligned}$$

$$\begin{aligned} \dot{s}_{q-1} &= \tilde{f}_{q-1}(\bar{\mathbf{z}}_k, \bar{\mathbf{s}}_{q-1}, \bar{\mathbf{x}}_{r+1}) + \tilde{b}_{q-1}(\bar{\mathbf{z}}_k, \bar{\mathbf{s}}_{q-1}, \bar{\mathbf{x}}_{r+1})u \\ &\quad + \tilde{g}_{q-1}(\bar{\mathbf{z}}_k, \bar{\mathbf{s}}_{q-1}, \bar{\mathbf{x}}_{r+1}, t) \qquad (12) \\ \dot{\bar{\mathbf{x}}}_{r+1} &= \bar{f}_{r+1}(\bar{\mathbf{z}}_k, \bar{\mathbf{s}}_{q-1}, \bar{\mathbf{x}}, t) \end{aligned}$$

where $\bar{\mathbf{s}}_{q-1} = (s_0, s_1, ..., s_{q-1})^T$, $\tilde{b}_{q-1} = b_1 b_2 \cdots b_r$ and $k+q=r$. Denote

$$\begin{aligned} N_{1,q} &= |s_0|^{\frac{p}{q}}, \qquad i=1,...,q-1 \\ N_{i,q} &= \left(|s_0|^{\frac{p}{q}} + |s_1|^{\frac{p}{q-1}} + \cdots + |s_{i-1}|^{\frac{p}{q-i+1}}\right)^{\frac{q-i}{p}} \\ N_{q-1,q} &= \left(|s_0|^{\frac{p}{q}} + |s_1|^{\frac{p}{q-1}} + \cdots + |s_0|^{\frac{p}{2}}\right)^{\frac{1}{p}}, \qquad (13) \\ \psi_{0,q} &= s_0, \quad \psi_{1,q} = s_1 + \beta_1 N_{1,q} sign(\psi_{0,q}) \\ \psi_{i,q} &= s_i + \beta_i N_{i,q} sign(\psi_{i-1,q}), \quad i=2,...,q-1 \end{aligned}$$

where $\beta_1, ..., \beta_{q-1}$ and p are positive numbers. Then the controller

$$u = -u_0 \tilde{b}_{q-1}^{-1} sign[\psi_{q-1,q}(s_0, s_1, ..., s_{q-1})] \qquad (14)$$

under the condition

$$u_0 >> \left| \tilde{f}_{q-1}(\bar{z}_k, \bar{s}_{q-1}, \bar{x}_{r+1}) + \tilde{g}_{q-1}(\bar{z}_k, \bar{s}_{q-1}, \bar{x}_{r+1}, t) \right|$$

provides for appearance sliding mode on the q-sliding point set

$$s_i = 0, \; i=0,...,q-1 \qquad (15)$$

in finite time (Levant [1993]). The dynamics on the q-sliding set (15) are described by the reduced $(n-q)$-order system

$$\begin{aligned} \dot{z}_1 &= -k_1 z_1 + z_2 + \tilde{g}_1(z_1,t) \\ \dot{z}_i &= -k_i z_i + z_{i+1} + \tilde{g}_i(\bar{\mathbf{z}}_i,t), \quad i=2,\ldots,k-1 \\ \dot{z}_k &= -k_k z_k + \tilde{g}_k(\bar{\mathbf{z}}_k,t) \qquad (16) \\ \dot{\bar{\mathbf{x}}}_{r+1} &= \bar{f}_{r+1}(\bar{\mathbf{z}}_k,,0,\bar{\mathbf{x}}_{r+1},t). \end{aligned}$$

The HOSM controller (13)-(14) in the combination with the q-order exact robust differentiator (Levant [2003]) achieves the closed-loop system (11)-(12) and (13)-(14) robustness with respect to the matched $g_r(\bar{\mathbf{x}}_r,t)$ as well unmatched $g_i(\bar{\mathbf{x}}_i,t)$, $i=k+1,\ldots,r-1$ perturbation terms in (1)-(3).

In the following we present an example of application of the proposed method to control an electric power system.

3. HOSM CONTROLLER FOR A SYNCHRONOUS GENERATOR WITH EXCITER SYSTEM

3.1 Problem statement

The excitation control system functionally consists of the exciter and Automatic Voltage Regulator (AVR) (see Fig.1).

The aim of this regulator is to keep the terminal voltage equal to the prescribed value, V_{ref}. To provide sufficient

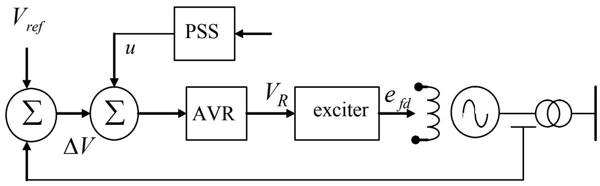

Fig. 1. Excitation control system

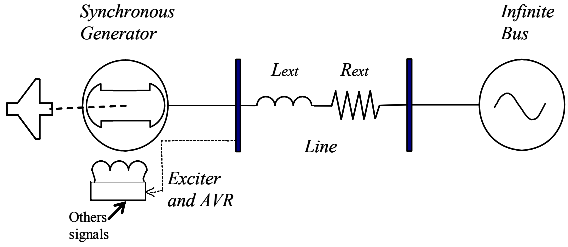

Fig. 2. Single machine - infinite bus

damping multi-modal oscillations at all credible operating conditions a supplementary control loop, known as the Power System Stabilizer (PSS) is often added. Traditionally, the PSS design is based on linearized dynamics equations (see for example, Sauer et al. [1998]), and consequently only local stability for a specific operation point is achieved.

To provide low sensitivity of the closed loop system with respect to large perturbations, a SM controller was proposed in (Loukianov [2004]). The proposed first order SM controller, however, can loose robustness and produce oscillations (chattering) in the presence of the exciter system unmodeled dynamics with the relative degree two, (which were not counted in (Loukianov [2004])). So, to design a bounded controller for synchronous generator keeping of insensitivity with respect to perturbation and the reduction of chattering despite the presence of the unmodeled exciter dynamics, the proposed above control approach is applied.

3.2 State space plant model

The complete mathematical model of the single machine infinite-bus system (see Fig. 2) consists of electrical and mechanical dynamics and load constraints, and after Park's transformation (Sauer et al. [1998]), it can be expressed in the state-space form as follows

$$\begin{bmatrix} \dot{\mathbf{x}}_1 \\ \dot{\mathbf{x}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{f}_1(\mathbf{x}_1,\mathbf{x}_2) \\ \mathbf{f}_2(\mathbf{x}_1,\mathbf{x}_2) \end{bmatrix} + \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix} e_{fd} + \begin{bmatrix} \mathbf{d}_1 \\ 0 \end{bmatrix} T_m \qquad (17)$$

where $\mathbf{x}_1 = (x_1,x_2,x_3)^T = (\delta,\omega,\lambda_f)^T$, $\mathbf{x}_2 = (x_4,,...,x_8)^T = (\lambda_g,\lambda_{kd},\lambda_{kq},i_d,i_q)^T$; δ is the power angle; ω is the angular velocity; ω_s is the rated synchronous speed; λ_f is the field flux; λ_g, λ_{kd}, λ_{kq}, i_d and i_q are the direct-axis and quadrature-axis stator fluxes and currents, respectively; e_{fd} is the excitation voltage, and the mechanical torque T_m is assumed to be a slowly varying function of time. Thus: $\dot{T}_m = 0$,

$$\mathbf{f}_1 = \begin{bmatrix} (x_2 - \omega_s) \\ (-a_{23}x_8x_3 + a_{24}x_7x_4 \\ -a_{25}x_8x_5 + a_{26}x_7x_6 + a_{28}x_7x_8) \\ (-a_{33}x_3 + a_{35}x_5 - a_{37}x_7) \end{bmatrix}$$

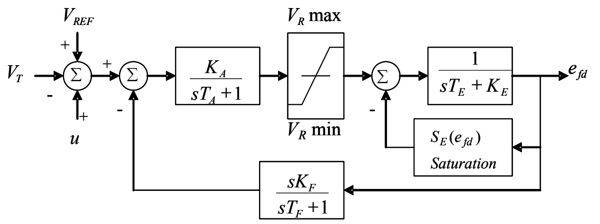

Fig. 3. Type 1 exciter control system

$$\mathbf{f}_2 = \begin{bmatrix} (-a_{44}x_4 + a_{46}x_6 - a_{48}x_8) \\ (a_{53}x_3 - a_{55}x_5 - a_{57}x_7) \\ (a_{64}x_4 - a_{66}x_6 - a_{68}x_8) \\ (-a_{71}\cos x_1 + a_{73}x_3 + a_{75}x_5 \\ -a_{77}x_7 + x_2(-a_{74}x_4 - a_{76}x_6 + a_{78}x_8)) \\ (-a_{81}\sin x_1 + a_{84}x_4 + a_{86}x_6 \\ -a_{88}x_8 + x_2(a_{83}x_3 + a_{85}x_5 - a_{87}x_7)) \end{bmatrix}$$

$$\mathbf{b}_1 = [0, 0, b_3]^T, \mathbf{b}_2 = [0, 0, 0, b_7, 0]^T, \mathbf{d}_1 = [0, d_m, 0]^T$$

a_{ij} , $(i, j = 2, ..., 8)$, b_3 , b_7 and d_m are positive constant parameters.

3.3 Exciter control system (actuator)

In this paper we consider the typical exciter system of IEEE Type 1 which includes the continuously acting AVR and exciter. The block diagram for this system is shown in Fig. 3.

From the block diagram we write the following equations:

$$T_E\frac{de_{fd}}{dt} = -(K_E + S_E)e_{fd} + \bar{V}_R \quad (18)$$

$$T_A\frac{dV_R}{dt} = -V_R + K_A R_f + K_A(V_{ref} - V_t) + u \quad (19)$$

$$T_F\frac{dR_f}{dt} = -R_f - \frac{K_F(K_E + S_E)}{T_E}e_{fd} + \frac{K_F}{T_E}V_R \quad (20)$$

where V_t is the generator terminal voltage, V_{ref} is the regulator reference voltage setting, V_R is the exciter input, R_f is the rated feedback stabilizing transformer, T_E and K_E are the exciter time constant and gain, respectively; T_F and K_F are the regulator stabilizing circuit time constant and gain, respectively; T_A and K_A are the regulator amplifier time constant and gain, respectively. The saturation $\bar{V}_R$ function is approximated in this case by the smooth function $\bar{V}_R(V_R) = \frac{2V_R}{\pi}\tan^{-1}\frac{\lambda\pi V_R}{2V_{R\min}}$ where $\lambda = 1$ is the slope of $\bar{V}_R(V_R)$; $S_E(e_{fd}) = Ae^{Be_{fd}}$, $A = 0.031$ and $B = 6.93$, and the control u to be bounded by

$$|u| \le u_0 \quad (21)$$

with $u_0 > 0$.

It is important to note that the actuator (18) - (20) has already a voltage control regulator. In this case only a rotor speed stabilizing controller design is considered. The advantages of the proposed controller is that, this controller can be implemented to a existing exciter control system with AVR, changing only the Power System Stabilizer (PSS).

3.4 High order sliding mode block controller for synchronous generator

To satisfy the control objective, rotor angle stability, we define the control error as

$$z_2 = x_2 - \omega_s \equiv \varphi_2(x_2) \quad (22)$$

Then using the first subsystem in (17) and then (18)-(19) and (22), straightforward calculations result in

$$\dot{z}_1 = z_2 \quad (23)$$
$$\dot{z}_2 = f_2(\mathbf{x}_2, T_m) + b_2(\mathbf{x}_2)x_3 \quad (24)$$
$$\dot{x}_3 = f_3(\mathbf{x}_2) + b_3 e_{fd} \quad (25)$$
$$\dot{e}_{fd} = f_f(e_{fd}) + b_f V_R \quad (26)$$
$$\dot{V}_R = f_R(e_{fd}, V_R, V_{ref}, V_t, R_f) + b_R u \quad (27)$$

where $z_1 = x_1 \equiv \varphi_1(x_1)$, $f_2(\mathbf{x}_2, T_m) = a_{24}x_7x_4 - a_{25}x_8x_5 + a_{26}x_7x_6 + a_{28}x_7x_8 + d_m T_m$, $b_2(\mathbf{x}_2) = -a_{23}x_8$, $f_3(\mathbf{x}_2) = -a_{33}x_3 + a_{35}x_5 - a_{37}x_7$, $f_f(e_{fd}) = -\frac{K_E + S_E}{T_E}e_{fd}$, $b_f = \frac{1}{T_E}$, $f_R(\cdot) = \frac{1}{T_A}[-V_R + K_A R_f + K_A(V_{ref} - V_t)]$, $b_R = \frac{1}{T_A}$. The second subsystem in (17) and equation (20) describe the internal power system dynamics.

It can be noted that the excitation voltage e_{fd} was taken in (Loukianov [2004]), as usually, as the control input for the power system, and implementation of the derived there discontinuous control algorithm in real life conditions, that is, in the presence of the additional exciter system dynamics (18) - (20), yields chattering.

The subsystem (23) - (27) has the NBC-form (or strict feedback form) where the relative degree with respect to the control error z_2 is four. Therefore, to simplify the control algorithm, we first, following the Block Control technique, choose the virtual control in the second block (24) of the form

$$x_3 = -b_2^{-1}(\mathbf{x}_2)[f_2(\mathbf{x}_2, T_m) + k_2 z_2 - z_3] \quad (28)$$

where the term $-k_2 z_2$ presents the desired dynamics for the control error z_2, $k_2 > 0$, and z_3 is a new variable. Now, the sliding variable $s_0 = z_3$ can be calculated from (28) as

$$s_0 = b_2(\mathbf{x}_2)x_3 + f_2(\mathbf{x}_2, T_m) + k_2(x_2 - \omega_s) \equiv \varphi_3(\mathbf{x}_1, \mathbf{x}_2) \quad (29)$$

Using (29) and (23) - (25), the equation of the projection motion of the system (23) - (27) on the subspace s_0 can be derived as

$$\dot{s}_0 = f_0(\mathbf{x}_1, \mathbf{x}_2, T_m) + b_0(\mathbf{x}_2)e_{fd}$$

where $f_0(\cdot)$, $f_0(\cdot) = \frac{\partial\varphi_3}{\partial\mathbf{x}_1}\mathbf{f}_1(\mathbf{x}_1, \mathbf{x}_2) + \frac{\partial\varphi_3}{\partial\mathbf{x}_2}\mathbf{f}_2(\mathbf{x}_1, \mathbf{x}_2)$ is a continuous function, $b_0(\cdot) = b_3 b_2(\cdot)$. Now, using s_0 (29) and its derivatives

$$s_1 = \dot{s}_0 = f_0(\mathbf{x}_1, \mathbf{x}_2, T_m) + b_0(\mathbf{x}_2)e_{fd} \quad (30)$$
$$s_2 = \ddot{s}_0 \quad (31)$$

as new variables, the system (23) - (27) can be represented of the form

$$\dot{z}_1 = z_2 \quad (32)$$
$$\dot{z}_2 = -k_2 z_2 + s_0 \quad (33)$$
$$\dot{s}_0 = s_1, \quad (34)$$
$$\dot{s}_1 = s_2 \quad (35)$$
$$\dot{s}_2 = f_s(\mathbf{x}_1, \mathbf{x}_2, T_m, e_{fd}, V_R, V_{ref}, V_t, R_f) + b_s(\mathbf{x}_2)u \quad (36)$$

where $f_s(\cdot)$ is a smooth function, $b_s(\cdot) = b_0(\cdot)b_R$. Taking in the account that the subsystem (34) - (36) is of the third order and using the constraints (21), we select the following third order SM algorithm (13)-(14) :

$$u = -u_0 sign[\xi_2 + 2(|s_0|^2 + |\xi_1|^3)^{\frac{1}{6}} sign(\xi_1 + |s_0|^{\frac{2}{3}}) sign(s_0)] \quad (37)$$

combined with the 2nd-order exact robust differentiator

$$\dot{\xi}_0 = v_0, \quad v_0 = -\lambda_0 |\xi_0 - s_0|^{\frac{2}{3}} \, sign\, |\xi_0 - s_0| + \xi_1$$
$$\dot{\xi}_1 = v_1, \; v_1 = -\lambda_1 |\xi_1 - v_0|^{\frac{2}{3}} \, sign\, |\xi_1 - v_0| + \xi_2$$
$$\dot{\xi}_2 = -\lambda_2 sign\, |\xi_2 - v_1|$$

where ξ_0, ξ_1 and ξ_2 are the estimates of the sliding variable s_0 and its derivatives s_1 and s_2 , respectively. In (Levant [2003]), it was shown that there exist $\lambda_0 > 0$, $\lambda_1 > 0$ and $\lambda_2 > 0$, such that the estimates ξ_0, ξ_1 and ξ_2 converge to the real variables s_0, s_1 and s_2, respectively, in finite time. Under the following condition:

$$u_0 >> |u_{eq}(\mathbf{x}_1, \mathbf{x}_2, T_m, e_{fd}, V_R, V_{ref}, V_t, R_f)| \qquad (38)$$

where $u_{eq}(\cdot) = b_s^{-1}(\cdot) f_s(\cdot)$, the state vector of the closed-loop system (32) - (37) converges to the set $s_0 = 0, \quad \xi_1 = 0, \quad \xi_2 = 0$ or

$$s_0 = 0, \quad s_1 = 0, \quad s_2 = 0 \qquad (39)$$

in finite time, and sliding mode starts on (39) from this time (Levant [1993]). The condition (38) defines the closed-loop system stability region and obviously holds for all the possible values of V_{ref} and T_m. The sliding motion on (39) is described by the reduced order SME

$$\dot{z}_1 = z_2, \; \dot{z}_2 = -k_2 z_2 \qquad (40)$$
$$\dot{\mathbf{x}}_2 = \mathbf{f}_2(\mathbf{x}_1, \mathbf{x}_2) \qquad (41)$$
$$+ \mathbf{b}_2 u_{eq}(\mathbf{x}_1, \mathbf{x}_2, T_m, e_{fdss}, V_{Rss}, V_{ref}, V_t, \eta)$$
$$\dot{\eta} = -a_1 \eta - a_2 e_{fdss} + a_3 V_{Rss} \qquad (42)$$

where $\eta = R_f$, $a_1 = \frac{1}{T_F}$, $a_2 = \frac{K_F(K_E+S_E)}{T_F T_E}$, $a_3 = \frac{K_F}{T_F T_E}$ and the values $e_{fd_{ss}}$ and $V_{R_{ss}}$ are calculated as solutions for $s_1 = 0$ (29) and $s_2 = 0$ (30), respectively.

Note that the linear subsystem (40) described the linearized mechanical dynamics, has the desired eigenvalue $-k_2$, while the subsystem (41)-(42) represents the rotor flux and exciter system internal dynamics. The second equation in (40) with $k_2 > 0$ is asymptotically stable, hence $\lim_{t\to\infty} z_2(t) = 0$, and the angle $z_1(t) = z_1(0) + \int_0^\infty z_2(\gamma) d\gamma$ tends to a steady state value δ_{ss} as the control error $z_2(t)$ tends to zero. On the invariant subspace $z_1 = \delta_{ss}, \; z_2 = 0, s_0 = 0, \, s_1 = 0, \, s_2 = 0, \mathbf{x}_2 \in \mathbf{R}^5, \eta \in \mathbf{R}$ in the state space of closed-loop system (32) - (37) and (41)-(42) the dynamics of $\mathbf{x}_2$ and η are *zero dynamics*. To derive these dynamics, first, using (29) and (30) we calculate the excitation flux x_3 and voltage e_{fd} values on the invariant set (39) as

$$x_{3ss} = b_2^{-1}(\mathbf{x}_2)[f_2(\mathbf{x}_2, T_m)], \; e_{fdss} = b_0^{-1}(\mathbf{x}_2)[f_0(\mathbf{x}_1, \mathbf{x}_2, T_m) \qquad (43)$$

Now, substituting the angle and speed steady state values $x_1 = \delta_{ref}$ and $x_2 = \omega_s$ in (43) and then in subsystem (41)-(42) results in the following linear system with non-vanishing perturbation:

$$\dot{\mathbf{x}}_2 = \mathbf{A}_2 \mathbf{x}_2 + \mathbf{g}_2(\delta_{ss}, \omega_s, \mathbf{x}_2, T_m) \qquad (44)$$
$$\dot{\eta} = -a_1 \eta + g_\eta(\delta_{ss}, \omega_s, \mathbf{x}_2, T_m) \qquad (45)$$

Note that sliding mode dynamics (40)-(42) can be considered as particular case of the SME (16) while the zero dynamics (44)-(45) are particular case of (4). Since the mappings $\mathbf{g}_2$ and g_η in (44)-(45) are smooth and bounded, the matrix $\mathbf{A}_2$ is Hurwitz and $a_1 > 0$, therefore, the assumptions **A1** and **A2** (see Section II) in this case are met. Hence, a solution of (40)-(42) by Theorem 1 is ultimately bounded and, moreover, the control error $z_2(t)$ (22) converges exponentially to zero.

3.5 Simulation results.

The performance of the proposed controller were tested on the complete 8^{th} order model of the generator connected to an infinite bus through a transmission line, Fig.2.

The steady state is computed as $x_1(\infty)$ =1.33140, $x_2(\infty)$ =376.9900, $x_3(\infty)$ =0.82038, $x_4(\infty)$ =-0.79228, $x_5(\infty)$ =0.62594, $x_6(\infty)$ =-0.79247, $x_7(\infty)$ =0.80354, and $x_8(\infty)$ =0.493190.

The controller gains were adjusted to $u_0 = 0.2$, $k_2 = 10$ and the constants for the sliding differentiator were selected as $\lambda_0 = 125$, $\lambda_1 = 115$ and $\lambda_2 = 100$.

The first set of simulations, starting in steady state condition, the mechanical torque experienced at $t = 0.5s$ a pulse0.15$p.u.$ from 0.35 to 0.5 $p.u.$ for 1.0 s, and at $t = 1.5$ s a pulse 0.15 $p.u.$ from 0.5 to 0.35$p.u.$ for 1.0s, (external perturbations). Then at $t = 5$ s, a three-phase short circuit for a period of 150 ms is simulated at the transformer terminals. Figure 4 and 5 depict the angle and velocity responses. The solid line is the response of the proposed HOSM controller applied to the plant while the dotted line is the case the response of the classical PSS+AVR. As evidenced by comparing the dynamic response curves, the proposed HOSM controller provides better damping enhancement than the classical one. The Figure 6 shows that in spite of the strong disturbance the terminal voltage and field flux linkage reach a steady state condition, exhibiting the stability of the closed loop system (HOSM controller).

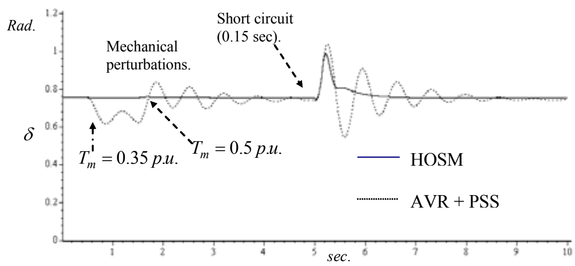

Fig. 4. Power angle during mechanical perturbation and short circuit. Performance comparison Sliding Mode v.s. AVR+PSS controller

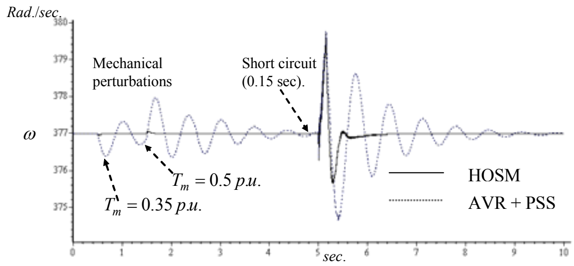

Fig. 5. Rotor velocity during mechanical perturbation and short circuit. Performance comparison Sliding Mode v.s. AVR+PSS controller.

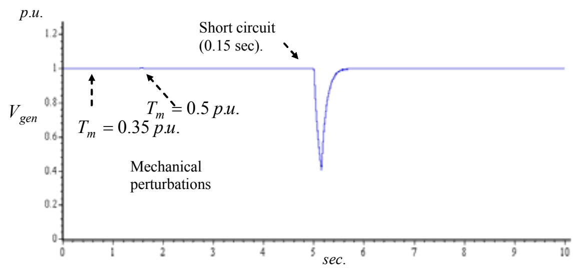

Fig. 6. Generator voltage affected by a 0.15 sec. short circuit (HOSM)

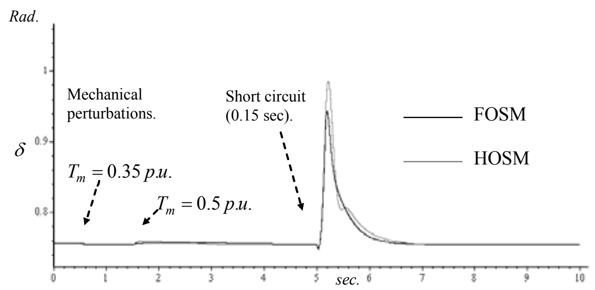

Fig. 7. Power angle, HOSM v.s. FOSM

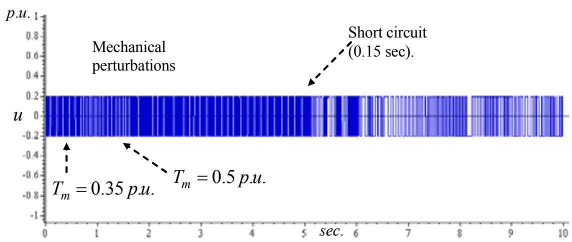

Fig. 8. Control input u under mechanical perturbation and short circuit, (FOSM).

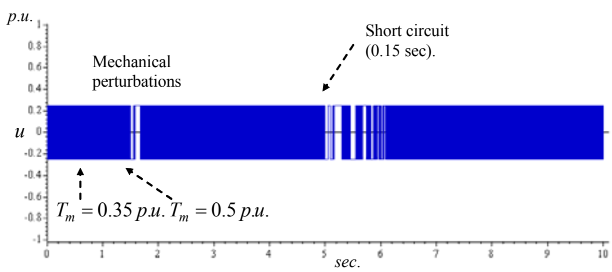

Fig. 9. Control input u under mechanical perturbation and short circuit, (HOSM).

The second set of simulation, Figures 7-9 show the performance of the HOSM controller (solid line) compared with FOSM one (gray line). The events are the same as in the first set simulations. The performance comparison of two controllers exhibits the clear advantage of the proposed HOSM controller since the implementation of the HOSM control in the discrete time gives the precision $(O(h^3))$ while the first order SM will gives us just precision $(O(h))$.

4. CONCLUSIONS

A new control scheme, based on the combination of Block Control and HOSM control techniques is proposed for a class of nonlinear minimum face SISO perturbed systems. This approach enables to ensures a) robustness of the closed-loop system with respect to matched and as well some kind of unmatched perturbations; b) chattering-reduced stability in the presence of an actuator additional dynamics, and reduce complicity of the control algorithm for the plants with high relative degree. The effectiveness of the proposed control scheme was checked by an application to control an electric power system in the presence of exciter unmodeled dynamics.

This method can be easy extended for a class of MIMO nonlinear perturbed systems.

REFERENCES

L. Fridman. An averaging approach to chattering. *IEEE Trans. on Automat. Cont.*, vol. 46, no. 8, pp. 1260–1265, 2001.

L. Fridman, A.G. Loukianov, José M. Cañedo, A. Soto-Cota.High order sliding mode controller for synchronous generator with exciter system. *To appear in IEEE Trans. On Industrial Electronics*, 2008.

A. Isidori. *Nonlinear control systems.* Springer-Verlag, London, 1992.

B. Drazenovic. The invariance conditions for variable structure systems.*Automatica*, vol. 5, pp. 287–295, 2001.

M. Krstic, I. Kanellakopoulos and P. Kokotovic.*Nonlinear and Adaptive Control Design.*John Wiley and Sons, New York, USA, 1995.

A. Levant.Higher-order sliding modes, differentiation and output-feedback control. *Inter. J. Control*, vol. 76, no. 9/10, pp. 924–941, 2003

W. Lin and C. I. Byrnes.Sliding order and sliding accuracy in sliding mode control.*Inter. J. Control*, vol. 58, no. 6, pp. 1247–1263, 1993.

A.G. Loukianov, J.M. Cañedo, V.I. Utkin and J. Cabrera-Vázquez. Discontinuous controller for power system: Sliding-mode block control approach. *IEEE Trans. on Industrial Electronics*, vol. 51, no. 2, pp. 340-353, 2002.

A.G. Loukianov. Robust block decomposition sliding mode control design. *International Journal Mathematical Problems in Engineering: Theory, Methods and Applications*, vol. 8, no. 4-5, pp. 349–365, 2002.

A. Sauer, and M.A. Pai. *Power system dynamics and stability.*Prentice Hall, New Jersey.USA, 1998.

V.I. Utkin, J. Guldner and J.Shi. *Sliding mode control in electromechanical systems.*Taylor and Francis, London, 1999.