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A Result on State Estimation of Nonlinear Systems with Application to Fuel Cell Stacks

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Abstract: In this paper,the observation issue of the partial pressure of oxygen and nitrogen and the mass flow rate of dry air in the cathode channel of a fuel cell stack is addressed. The proposed approach considers the mass flow rate of dry air as an unknown input and uses the voltage and the total pressure as measurements. By using the Jacobian of the nonlinear functions and the convexity principe, the observer design problem is turned into a LMI feasibility problem. Simulation results with a detailed model show the good convergence properties of the observer.

Keywords: Fuel cell stacks, nonlinear observer, state and unknown input estimation

1. INTRODUCTION

Fuel cell stacks are known since the XIXth century and have received considerable interest over the years. More recently, with the increase of the oil price, new solutions have to be developed for the energy issues. One solution could be to use hydrogen for stocking energy and to use fuel cell stacks in order to convert it into electricity. Hydrogen could be produced from electric energy produced by any source, including solar cells.

Fuel cells are complex dynamical systems that include several unknown quantities. Over the last decades, tremendous research activities have focused on observer design for these systems. (Arcak et al. (2004); McKay and Stefanopoulou (2004); Görgün et al. (2005); Benallouch et al. (2007b)). These observers can be used for different applications, such as control (Pukrushpan et al. (2002, 2003)), diagnosis (Görgün et al. (2005); Mays et al. (2001)) and communication (Boutayeb et al. (2002); Benallouch et al. (2007a); Liao and Huang (1999)).

The purpose of this paper is to investigate the problem of the observation of the partial pressure of oxygen, nitrogen and the mass flow rate of dry air treated here as an unknown input. When the input is not completely available for measurement, the existence conditions for an unknown input observer are more restrictive than the classical detectability condition (Boutayeb et al. (2002), Ha and Trinh (2004) Corless and Tu (1998)). Unknown input observers find a wide applicability in the design of robust observers, decentralized control, and for fault detection (Chen and Saif (2006)).

This paper is organized as follows. Section 2 presents the models of the cathode flow (Pukrushpan et al. (2004b,a)) and of the stack voltage (Pukrushpan et al. (2004b,a); Larmine and Dicks (2000)). In section 3, a novel algebraic method is introduced for simultaneous estimation of the partial pressure of oxygen, of nitrogen and of the mass flow rate of dry air. It consists in sufficient LMI conditions. Simulation results are presented in Section 4, that illustrate the interest of the proposed method.

List of Symbols

$\bar{R}$: universal gas constant $(\mathrm{J.(mol.K)^{-1}})$
T_{st} : fuel cell temperature (K)
V_{ca} : cathode volume $(\mathrm{m^3})$
I_{st} : current (A)
n_{st} : the number of cells in the stack
F_d : Faraday number (Coulombs)
A_{fc} : fuel cell active area
t_m : membrane thickness
$P_{sat}(T_{st})$: vapor saturation pressure $(\mathrm{P_a})$

2. MODEL

2.1 Cathode Flow Model

The mass continuity and the ideal gas law are used to balance the pressure of the oxygen and nitrogen inside the cathode volume:

$$\begin{cases} \dfrac{dP_{O_2,ca}}{dt} = \dfrac{\bar{R}T_{st}}{M_{O_2}V_{ca}}\left(W_{O_2,ca,in} - W_{O_2,ca,out} - W_{O_2,reacted}\right) \\ \dfrac{dP_{N_2,ca}}{dt} = \dfrac{\bar{R}T_{st}}{M_{N_2}V_{ca}}\left(W_{N_2,ca,in} - W_{N_2,ca,out}\right) \end{cases} \tag{1}$$

where P_{O_2} and P_{N_2} are the oxygen and nitrogen partial pressure, M_{O_2} $(\mathrm{kg.mol^{-1}})$ and M_{N_2} $(\mathrm{kg.mol^{-1}})$ are the molar masses of oxygen and nitrogen, respectively.
$W_{O_2,ca,in}$ and $W_{N_2,ca,in}$ are the oxygen and nitrogen mass flow rate entering the cathode, as shown by:

$$W_{O_2,ca,in} = x_{O_2,ca,in} W_{a,ca,in} \quad (2.a)$$

$$W_{N_2,ca,in} = (1 - x_{O_2,ca,in}) W_{a,ca,in} \quad (2.b)$$

where $x_{O_2,ca,in}$ is the Oxygen mass fraction; and $W_{a,ca,in}$ represent the mass flow rate of dry air considered here in as an unknown input. Furthermore $W_{O_2,ca,out}$ and $W_{N_2,ca,out}$ are the oxygen and nitrogen mass flow rate leaving the cathode:

$$W_{O_2,ca,out} = \frac{P_{O_2} M_{O_2}}{P_{O_2} M_{O_2} + P_{N_2} M_{N_2} + P_{v,ca} M_v} W_{ca,out} \quad (3.a)$$

$$W_{N_2,ca,out} = \frac{P_{N_2} M_{N_2}}{P_{O_2} M_{O_2} + P_{N_2} M_{N_2} + P_{v,ca} M_v} W_{ca,out} \quad (3.b)$$

where $W_{ca,out} = k_{ca,out}(P_{ca} - P_{rm})$ the total flow rate is determined using the simplified orifice equation, $k_{ca,out}$ is the orifice constant, $P_{ca} = P_{O_2} + P_{N_2} + P_{sat}(T_{st})$ is the cathode total pressure, P_{rm} is the return manifold pressure, $P_{v,ca}$ vapor partial pressure and M_v $(kg.mol^{-1})$ is the vapor molar mass.

The rate of oxygen consumed in the reaction is a function of the stack current I_{st} :

$$W_{O_2,reacted} = M_{O_2} \frac{n_{st} I_{st}}{4F_d} \quad (4)$$

In order to simplify the calculation, we assume that $M_{O_2} = M_{N_2} = M_v$, then system (1) can be rewritten in the following form:

$$\begin{bmatrix} \frac{dP_{O_2}}{dt} \\ \frac{dP_{N_2}}{dt} \end{bmatrix} = \frac{\bar{R}T_{st}}{V_{ca}} \Bigg\{ k_{ca,out} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} P_{O_2} \\ P_{N_2} \end{bmatrix} + \begin{bmatrix} \frac{-M_{O_2} n_{st}}{4F_d} \\ 0 \end{bmatrix} I_{st}$$
$$+ k_{ca,out} \begin{bmatrix} \frac{P_{O_2} P_{rm}}{P_{O_2} + P_{N_2} + P_{v,ca}} \\ \frac{P_{N_2} P_{rm}}{P_{O_2} + P_{N_2} + P_{v,ca}} \end{bmatrix} + \begin{bmatrix} x_{O_2,ca,in} \\ (1 - x_{O_2,ca,in}) \end{bmatrix} W_{a,ca,in} \Bigg\} \quad (5)$$

2.2 Stack voltage model

The voltage E produced by one cell is affected to different voltage losses: the loss voltage responsible for the activation polarization is denoted v_{act}; the ohmic voltage loss is denoted v_{ohm}; the concentration polarization is caused by the concentration overpotential, leading to voltage loss v_{conc}, as given by Pukrushpan et al. (2004a). Therefore, the voltage provided by the stack writes:

$$v_{\text{st}} = n_{\text{st}}\left(E - v_{\text{act}} - v_{\text{ohm}} - v_{\text{conc}}\right) \quad (6)$$

The open-circuit voltage is a function of the stack temperature T_{fc}, and oxygen partial pressure P_{O_2}, and hydrogen partial pressure P_{H_2}.

$$E = 4.308 \times 10^{-5} T_{fc} [\ln(P_{H_2}) + \frac{1}{2}\ln(P_{O_2})]$$
$$+1.482 - 8.5 \times 10^{-4} T_{fc}$$

The activation voltage drop is approximated by:

$$v_{\text{act}} = v_0 + v_{\text{a}}\left(1 - e^{-c_1 \frac{I_{\text{st}}}{A_{\text{fc}}}}\right)$$

Where $v_0 = v_0\left(T_{\text{fc}}, P_{\text{ca}}, P_{\text{sat}}\right)$ is the voltage drop at zero current density and $v_{\text{a}} = v_{\text{a}}\left(T_{fc, P_{O_2}, P_{\text{sat}}}\right)$ and c_1 are constants.

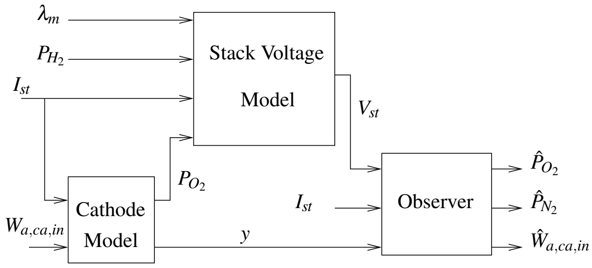

Fig. 1. Bloc diagram of the Fuel cell and the observer

The ohmic overvoltage writes:

$$v_{ohm} = I_{st} \frac{R_{ohm}}{A_{fc}}$$

where $R_{ohm} = \frac{t_m}{\sigma_m\left(\lambda_m, T_{\text{fc}}\right)}$ is the electrical resistance by unit of surface, which has units of $\Omega.m^2$. σ_m is the membrane conductivity, it is a function of fuel cell temperature and membrane water content λ_m.

The concentration loss is given by:

$$v_{\text{conc}} = I_{\text{st}}^{c_3+1} \frac{1}{A_{\text{fc}}} \left(\frac{c_2}{i_{\max} A_{\text{fc}}}\right)^{c_3}$$

where c_2, c_3 and $i_{\max}$ are constants that depend on the temperature and the reactant partial pressure.

2.3 Measurements

We consider that the measurements available the system are: the Stack voltage v_{st} and the total pressure at the cathode defined by:

$$y = P_{ca} - P_{sat}(T_{st}) = P_{O_2} + P_{N_2}$$

3. MAIN RESULTS

3.1 Reformulation of the model

The equations of the system can be reformulated as following:

$$\begin{cases} \dot{x} = Ax + BW_{a,ca,in} + f(x,y) + DI_{st} \\ y = Cx \\ V_{st} = h(x, y, I_{st}) \end{cases} \quad (7)$$

where $A = \frac{\bar{R}T_{st}}{V_{ca}} k_{ca,out} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, $B = \frac{\bar{R}T_{st}}{V_{ca}} \begin{bmatrix} x_{O_2,ca,in} \\ (1 - x_{O_2,ca,in}) \end{bmatrix}$

$$f(x,y) = \frac{\bar{R}T_{st} k_{ca,out}}{V_{ca}} \begin{bmatrix} \frac{P_{O_2} P_{rm}}{P_{O_2} + P_{N_2} + P_{v,ca}} \\ \frac{P_{N_2} P_{rm}}{P_{O_2} + P_{N_2} + P_{v,ca}} \end{bmatrix}, \quad C = [1 \ \ 1]$$

$$D = \frac{\bar{R}T_{st}}{V_{ca}} \begin{bmatrix} \frac{-M_{O_2} n_{st}}{4F_d} \\ 0 \end{bmatrix}$$

Assume that matrix B has full column rank. Then there exists a matrix N such that:

$$T = [N \ \ B] \quad (8)$$

is non singular. Let define T_1 and T_2 such that:

$$T^{-1} = \begin{bmatrix} T_1^T & T_2^T \end{bmatrix}^T \tag{9}$$

The change of state variable $x = Tz$ is introduced, leading to:

$$\begin{cases} \dot{z} = T^{-1}ATz + T^{-1}BW_{a,ca,in} + T^{-1}f(Tz,y) + T^{-1}DI_{st} \\ y = CTz \\ V_{fc} = h(Tz,y,I_{st}) \end{cases} \tag{10}$$

where

$$\bar{A} = T^{-1}AT = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

and $T^{-1}B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

In order to eliminate the component $W_{a,ca,in}$, we multiply the state equation (10) by matrix $E = [1 \;\; 0]$, yielding the following descriptor system:

$$\begin{cases} E\dot{z} = \mathscr{A}z + T_1 f(Tz,y) + T_1 DI_{st} \\ y = Hz \\ V_{fc} = h(Tz,y,I_{st}) \end{cases} \tag{11}$$

with $\mathscr{A} = [a_{11} \;\; a_{12}]$ and $H = CT$.

3.2 Full-order observer design

We consider the nonlinear observer of the following form:

$$\begin{cases} \dot{\omega} = N\omega + Ly + RT_1 f(T\hat{z},y) + RT_1 DI_{st} + K(\hat{V}_{fc} - V_{fc}) \\ \hat{z} = \omega + Qy \end{cases} \tag{12}$$

where $\hat{z}$ denotes the state estimation vector of z. L, R, K are matrices to be determined such that $\hat{z}$ converges asymptotically to z.

The main contribution of the paper consists in LMI sufficient condition for the observer synthesis problem. This result is formulated in the following theorem.

Theorem 1. The observer error $e(t)$ converge asymptotically towards zero if there exist matrices $P = P^T > 0$, X and Y of appropriate dimensions such that the following LMIs are feasible:

$$\begin{aligned} &\big(R\mathscr{A}\big)^T P + P\big(R\mathscr{A}\big) + \big(RT_1 J_f(\alpha)T\big)^T P + P\big(RT_1 J_f(\alpha)T\big) \\ &\quad - H^T X - X^T H + \big(J_h(\beta)T\big)^T Y + Y^T\big(J_h(\beta)T\big) < 0 \\ &\qquad \forall\, \alpha, \beta \in \mathscr{V}_{\mathscr{H}}. \end{aligned} \tag{13}$$

If these LMIs are feasible, matrices F, K, N and L are given by: $F = P^{-1}X^T$, $K = P^{-1}Y^T$, $N = R\mathscr{A} - FH$ and $L = F + NQ$.

Proof.

Consider the error vector:

$$e = \hat{z} - z, \tag{14}$$

Substituting (11) and (12) into (14), we obtain:

$$e = \omega + (QH - I_2)z \tag{15}$$

where I_2 represents the identity matrix of dimension 2.
Let R be a matrix such that:

$$RE + QH = I_2 \tag{16}$$

Then (15) becomes $e = \omega - REz$ and the error dynamics write:

$$\begin{aligned} \dot{e} = {} & Ne + (LH + NRE - R\mathscr{A})z + RT_1\{f(T\hat{z},y) - f(Tz,y)\} \\ & + K\{h(T\hat{z},y,I_{st}) - h(Tz,y,I_{st})\} \end{aligned}$$

We assume that f is differentiable on $\mathrm{Co}(z(t),\hat{z}(t))$. From the differential mean value theorem (see in Appendix), there exist constant vectors η_1 and $\eta_2 \in \mathrm{Co}(z(t),\hat{z}(t))$ such that:

$$f(T\hat{z},y) - f(Tz,y) = \sum_{i,j=1}^{2} e_2(i)e_2^T(j)\frac{\partial f_i}{\partial z_j}(\eta_i,y)T(\hat{z}-z)$$

with $e_2(1) = [1 \;\; 0]^T$ and $e_2(2) = [0 \;\; 1]^T$. For simplicity, we introduce the notations $J_f(\rho) = \sum_{i,j=1}^{2} e_2(i)e_2^T(j)\rho_{ij}$ and $\rho_{ij} = \frac{\partial f_i}{\partial z_j}(\eta_i,y)$ which is equivalent to

$$f(T\hat{z},y) - f(Tz,y) = J_f(\rho(y))T(\hat{z}-z)$$

A same reasoning yields $J_h(\mu) = \sum_{j=1}^{2} e_2^T(j)\mu_j$ with $\mu_j = \frac{\partial h}{\partial z_j}(\eta_j,y)$ we obtain

$$h(T\hat{z},y,I_{st}) - h(Tz,y,I_{st}) = J_h(\mu(y))T(\hat{z}-z)$$

If we set

$$F = L - NQ$$

and:

$$N = R\mathscr{A} - FH$$

then the error dynamics write:

$$\dot{e} = \big(R\mathscr{A} - FH + RT_1 J_f(\rho)T + KJ_h(\mu)T\big)e \tag{17}$$

Choose a quadratic Lyapunov function as $V = e^T Pe$. Its time-derivative writes:

$$\begin{aligned} \dot{V} = {} & \big(R\mathscr{A} - FH + RT_1 J_f(\rho)T + KJ_h(\mu)T\big)^T P \\ & + P\big(R\mathscr{A} - FH + RT_1 J_f(\rho)T + KJ_h(\mu)T\big) \end{aligned}$$

Based on the Lyapunov stability theory, if $\dot{V}$ is negative-definite then the convergence of the estimation error is guaranteed, which is equivalent to:

$$\begin{aligned} \Gamma = {} & \big(R\mathscr{A}\big)^T P + P\big(R\mathscr{A}\big) - H^T X - X^T H + \big(RT_1 J_f(\rho)T\big)^T P \\ & + P\big(RT_1 J_f(\rho)T\big) + \big(J_h(\mu)T\big)^T Y + Y^T\big(J_h(\mu)T\big) < 0 \end{aligned} \tag{18}$$

with $X = F^T P$ and $Y = K^T P$.

Since Γ is affine in ρ and μ, the relationship holds for any ρ and μ in $\mathscr{H}$ as soon as it is verified at the vertices, that is to say for $\alpha, \beta \in \mathscr{V}_{\mathscr{H}}$ (Appendix). By using the notations $X = F^T P$ and $Y = K^T P$, condition (18) is equivalent to (13), which completes the proof.

3.3 Estimation of the mass flow rate of dry air

The aim of this section is to estimate the mass flow rate of dry air $W_{a,ca,in}$. For this, we need to estimate firstly the state x. Since $z \to \hat{z}$ when $t \to \infty$, then the estimate of x is given by $\hat{x} = T\hat{z}$.

Now, let use the state estimates to reconstruct the unknown input. From (7), we have:

$$\hat{W}_{a,ca,in} = (CB)^{-1}(\dot{y} - CA\hat{x} - Cf(\hat{x},y) - CDI_{st})$$

4. SIMULATION RESULTS FOR A FUEL CELL MODEL

We apply this approach to the fuel cell model. Refereing to Subsection 3.1, we choose:

$$N = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{19}$$

We get:

$$T = \begin{bmatrix} 1 & 2.441 \\ 0 & 8.035 \end{bmatrix} \quad (20)$$

and

$$\bar{A} = \begin{bmatrix} -22.81 & 7.451 \times 10^{-9} \\ 0 & -22.81 \end{bmatrix} \quad (21)$$

Matrices R et Q are chosen in order to satisfy Eq. (16):

$$R = \begin{bmatrix} 1 \\ -0.5100 \times 10^{-8} \end{bmatrix} \quad (22)$$

$$Q = 10^{-7} \begin{bmatrix} 0 \\ 0.9544 \end{bmatrix} \quad (23)$$

The initial conditions considered in the results presented in the sequel are:

$$\left[P^0_{O_2} \; P^0_{N_2}\right]^T = \left[1.0951 \times 10^4 \;\; 8.2070 \times 10^4\right]^T (Pa)$$

In the current case, a simplification is possible, allowing to obtain a formulation with 3 uncertain parameters instead of 4. The Jacobian then write $J_f(\rho) = \sum_{i=1}^3 J_{f_i}\rho_i$ are:

$$J_{f_1} = \begin{bmatrix} 0 & -1 \\ 0 & 1 \end{bmatrix}, \quad (24)$$

$$J_{f_2} = \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix} \quad (25)$$

and

$$J_{f_3} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad (26)$$

with $\underline{\rho}_1 = 1.597$, $\bar{\rho}_1 = 2.187$, $\underline{\rho}_2 = 11.70$, $\bar{\rho}_2 = 14.28$, $\underline{\rho}_3 = 3.315$ and $\bar{\rho}_3 = 6.867$.

The Jacobian of $J_h(\mu) = \sum_{i=1}^2 H_i\mu_i$ is obtained with:

$$H_1 = [1 \;\; 0] \quad (27)$$

and

$$H_2 = [0 \;\; 1] \quad (28)$$

with $\underline{\mu}_1 = -6.639 \times 10^7$, $\bar{\mu}_1 = -6.945 \times 10^6$, $\underline{\mu}_2 = -9.342 \times 10^{-5}$ and $\bar{\mu}_2 = -3.717 \times 10^{-5}$.

Resolution of the LMIs of (13) provides the following solutions:

$$P = \begin{bmatrix} 9.010 \times 10^{-8} & 2.940 \times 10^{-6} \\ 2.940 \times 10^{-6} & 9.987 \times 10^{-1} \end{bmatrix}, \quad F = \begin{bmatrix} -7.685 \times 10^{-1} \\ 2.600 \times 10^{-6} \end{bmatrix},$$

$$N = \begin{bmatrix} -22.04 & 8.051 \times 10^6 \\ -4.700 \times 10^{-7} & -27.71 \end{bmatrix}, \quad K = \begin{bmatrix} 0.245 \\ -3 \times 10^{-7} \end{bmatrix}$$

and

$$L = \begin{bmatrix} 0.666 \\ 0 \end{bmatrix}$$

Simulations were done, considering step variations of the load. The results are presented in Fig. 2 to 5.

5. CONCLUSION

We have proposed an efficient method for designing a nonlinear observer for fuel cells systems with unknown inputs. The proposed theorem makes use of two elementary principles: first the notion of Jacobian that allows to simplify the equations, second the convexity principle for deriving a LMIs feasibility problem.

Based on this design method, a nonlinear observer was designed for the estimation of the partial pressure of oxygen and

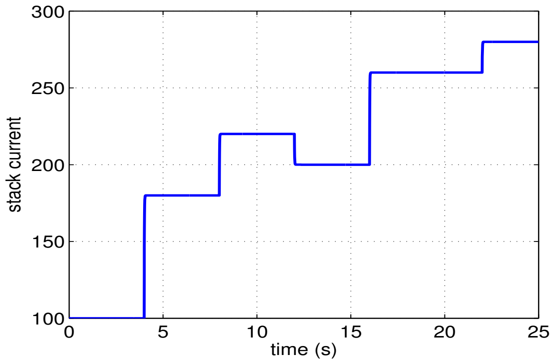

Fig. 2. stack current (A)

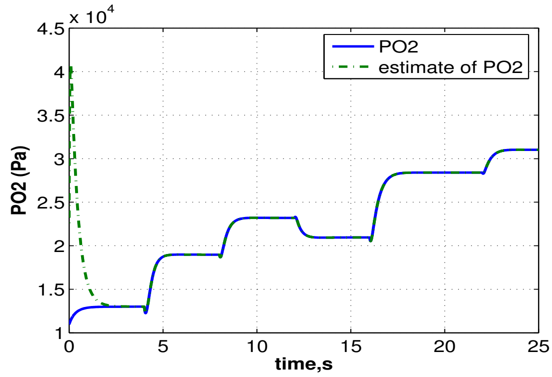

Fig. 3. Response of P_{0_2} and its estimate

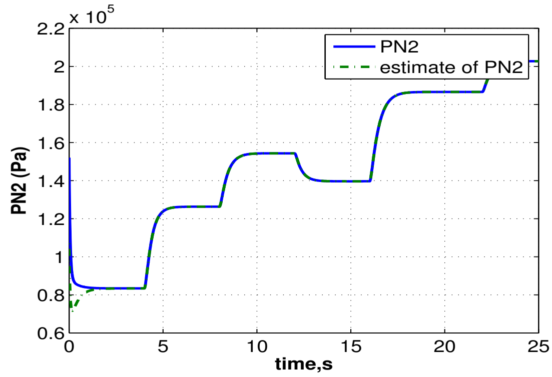

Fig. 4. Response of P_{N_2} and its estimate

nitrogen in addition to the mass flow rate of dry air. Simulation results showed the fast convergence of the obtained estimator.

Appendix A. DIFFERENTIAL MEAN VALUE THEOREM (DMVT) FOR VECTOR VALUED FUNCTION (Zemouche et al. (2005))

Let $\varphi : \mathbb{R}^m \mapsto \mathbb{R}^n$. Let $a, b \in \mathbb{R}^m$. We assume that φ is differentiable in $Co(a,b)$. There exist $\eta_i(t) \in Co(a,b)$ for all $i = 1,...,n$, such that:

$$\varphi(a) - \varphi(b) = \left(\sum_{i,j=1}^{n,m} H_{ij} \frac{\partial \varphi_i}{\partial x_j}(\eta_i) \right)(a-b), \quad (A.1)$$

where

$$Co(a,b) = \{\lambda a + (1-\lambda)b(t), 0 \leq \lambda \leq 1\}$$

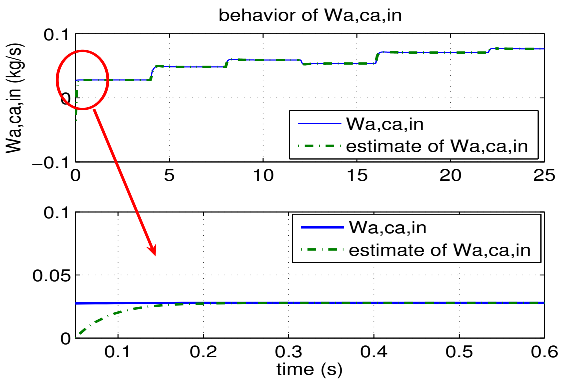

Fig. 5. Response of $W_{a,ca,in}$ and its estimate

and

$$H_{ij} = e_n(i)e_m^T(j),\ i = 1,\dots,n,\ j = 1,\dots,m,$$

where

$$e_n(i) = \big(\underbrace{0,\dots,0,\overbrace{1}^{i\text{ th}},0,\dots,0}_{n\text{ components}}\big)^T$$

is a vector of the canonical basis of $\mathbb{R}^n$. Then, using the notations:

$$\rho_{ij}(t) = \frac{\partial \varphi_i}{\partial x_j}(\eta_i(t)) \qquad (A.2)$$

we deduce that

$$\varphi(a) - \varphi(b) = \left(\sum_{i,j=1}^{n,m} H_{ij}\rho_{ij}(t)\right)(a-b),$$

Proof. we can write $\varphi(x) = \sum_{i=1}^{n} e_n(i)\varphi_i(x)$, where $\varphi_i : \mathbb{R}^n \mapsto \mathbb{R}$ is the ith component of φ. We know that for all scalar function φ_i thats differentiable on $Co(a,b)$. There exists $\eta_i \in Co(a,b)$ such that $\varphi_i(a) - \varphi_i(b) = \frac{\partial \varphi_i}{\partial x}(\eta_i)(a-b)$. For $i = 1\dots.n$ we have $\frac{\partial \varphi_i}{\partial x}(c_i) = \sum_{j=1}^{m} e_m^T(j)\frac{\partial \varphi_i}{\partial x_j}(c_i)$, we deduce that

$$\varphi(a) - \varphi(b) = \left(\sum_{i,j=1}^{n,m} e_n(i)e_m^T(j)\frac{\partial \varphi_i}{\partial x_j}(\eta_i)\right)(a-b),$$

end of prof.

Let assume that parameters $\rho_{ij}(t)$ evolve in a bounded domain $\mathscr{H}_n$ of which 2^{n^2} vertices are defined by:

$$\mathscr{V}_{\mathscr{H}_n} = \left\{\alpha = (\alpha_1,\dots,\alpha_n) \mid \alpha_i \in \{\underline{\rho}_{ij}, \bar{\rho}_{ij}\}\right\} \qquad (A.3)$$

where

$$\bar{\rho}_{ij} = \max_t\left(\rho_{ij}(t)\right) \text{ and } \underline{\rho}_{ij} = \min_t\left(\rho_{ij}(t)\right).$$

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