

## Advanced Modelling and Control using a Laboratory Plant with Hybrid Processes

J. Hlava\* and B. Šulc\*\*

\* *Technical University Liberec, Faculty of Mechatronics  
461 17 Liberec, Hálkova 4, Czech Republic  
jaroslav.hlava@tul.cz*

\*\**Institute of Instrumentation and Control Engineering  
166 07 Praha 6, Technická 4, Czech Republic  
bohumi.sulc@fs.cvut.cz*

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**Abstract:** Advanced control theory is usually associated with the use of abstract mathematical tools. It requires much time and a good theoretical background to understand and explain these tools. In ordinary university courses or in continuing professional education organized by employers, it is not easy to meet these requirements. Widely used system simulation and virtual experiments can be a good aid to increase clarity, but they cannot fully demonstrate the problems that a control designer or user may encounter in practical implementation. A laboratory scale plant has been designed for this purpose in the framework of research activities focused on hybrid systems. It exhibits most of the hybrid phenomena typical of process control applications. The plant is also equipped with industrial control hardware, so that educational (as well as research) experiments can be carried out implementing advanced hybrid control algorithms in conditions close to real world applications. The instrumentation provides a remote web access facility. A mathematical and technical description of the pilot plant is included here, and this will enable readers to consider whether a similar device could be useful for their own educational and/or research purposes. Some examples of experimental results are also given.

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### 1. INTRODUCTION

Despite considerable development of virtual experiments and virtual laboratories, experiments with laboratory scale plants will certainly continue to play an important role in control engineering courses. A long list of physical models designed to demonstrate various control problems is given in (Horáček, 2000), and other experimental plants are described in many other papers. Though the literature suggests that a great variety of control problems can be demonstrated with the use of laboratory models, very few experimental plants have been designed to demonstrate hybrid phenomena. Some papers sketch simple model plants that are used to illustrate certain ideas related to hybrid systems, but it was never the intention to really build them. Other papers on hybrid systems use experimental models that were designed for experiments with continuous control. Since hybrid systems are an important area of control theory, a laboratory scale plant specifically designed for research and educational experiments with hybrid systems could be beneficial in many ways.

With this idea, in 2004 we started a project on hybrid system control which received financial support from the Czech Science Foundation. One of the research tasks was the development of a laboratory scale plant that would exhibit a wide range of hybrid phenomena. The intended purpose of the plant was to test hybrid control algorithms and to demonstrate hybrid control problems in reality for both research and educational purposes. The original educational

experiments with hybrid systems using the developed laboratory scale plant aimed to introduce this theory at a practical level. The lack of such laboratory set-ups was solved by an internet access.

The planned plant structure, together with an outline of the mathematical model, the suggested control experiments and the proposed internet access using Web services were presented by the authors at the 16<sup>th</sup> IFAC congress (Hlava *et al.*, 2005). Since then the set-up structure has been revised into its final form, completed, put into operation and tested. In the course of these activities, we have discovered new opportunities for interesting educational experiments, which we will report on in this paper. These suggestions are not restricted to the topic of hybrid system. They deal with control in general (especially from the viewpoint of controller implementation). At present, we see the following areas for utilizing the laboratory pilot plant:

- Mathematical modelling of the plant in selected configuration and comparison with the real properties obtained by measurement and experimental identification or by simulation in Simulink
- Design and testing of control algorithms at various level of complexity (from PID control to hybrid control) especially focused on problems of implementation and operation in real conditions.
- Training in implementation of advanced controllers using industrial PLCs
- HMI design and testing

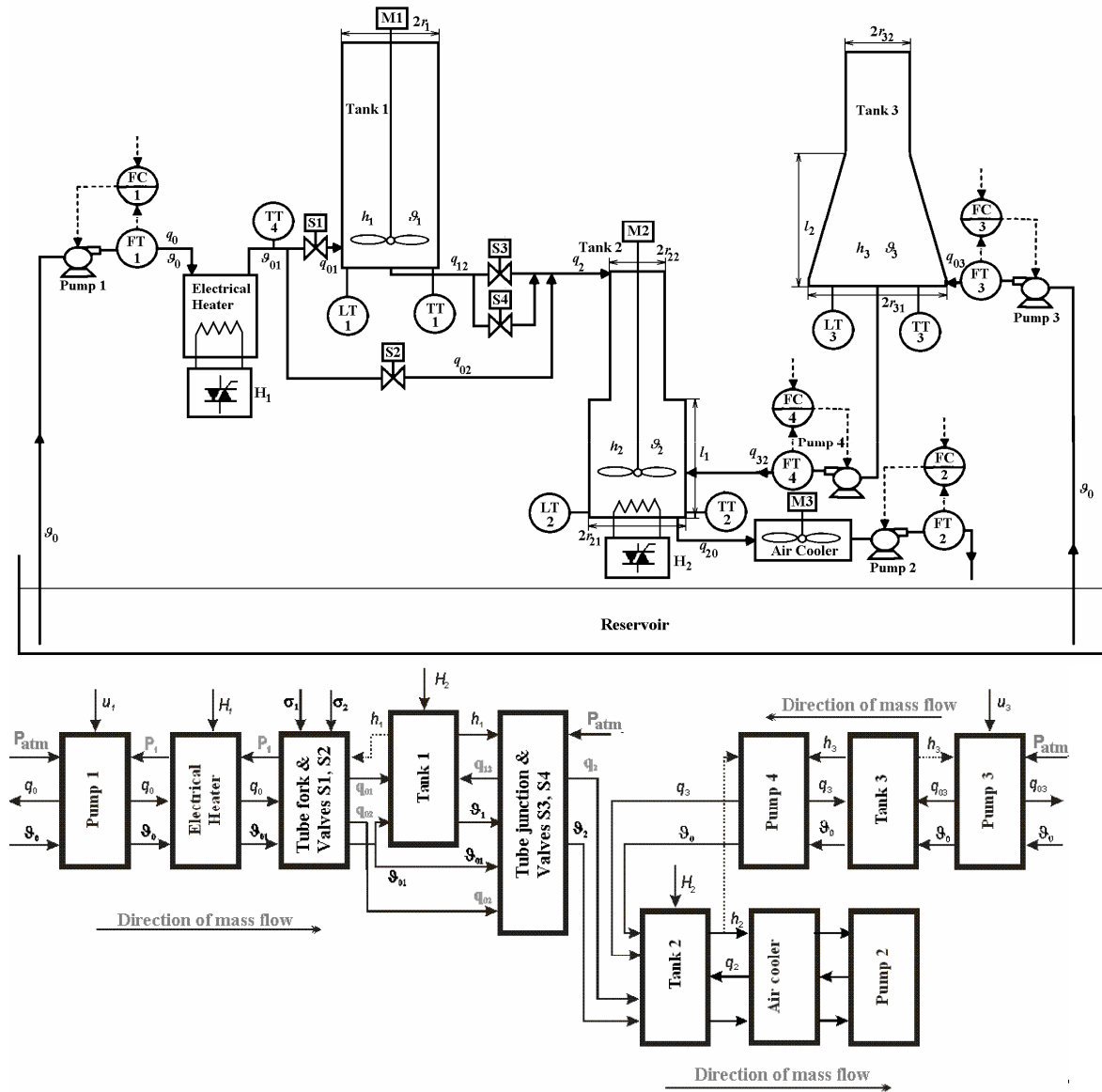


Fig. 1. Functional and block scheme (model) of the laboratory scale plant. Symbols in black and italics represent measured or adjustable variables while in grey are symbols of variables used in modelling. The meaning of the symbols in the upper part of the figure is as follows: FT, LT, TT are flow, level and temperature transmitter respectively, FC – flow controller, S–solenoid valve, M – motor,  $r_1 = 5.64$  cm,  $r_{21} = 5.8$  cm,  $r_{22} = 3$  cm,  $r_{31} = 6$  cm,  $r_{32} = 2.9$  cm), tank height  $l_{max} = 80$  cm,  $l_1 = l_2 = 40$  cm

## 2. THE LABORATORY SCALE PLANT

To explain the range of experiments that can be performed with the plant, we will first describe the structure of the plant in the form as it was finally built. The structure is sketched in the upper part of Fig. 1. A system representation, in the form of block diagram, is added in the lower part of the figure. The system is based on a block scheme using Pressure (level) – Flow rate – Temperature block representation, by means of which we can show the important problem of the correct separation of process variables that distinguish inputs and outputs. The problem will be commented on later, but in order to demonstrate the relationship between real technical elements and their models it is good to have both reality and model in one figure. Fig. 2 is a photograph of the plant. The plant is not an exact representation of a particular industrial

process, but both the plant itself and its instrumentation are well representative of many plants commonly used in the process industries. The measured and controlled variables (water level, temperature and flow) are common variables often occurring in control tasks. Where possible, the sensors and actuators were chosen from standard industrial ranges.

The basic components of the plant are three water tanks. Tanks 2 and 3 have special shapes that introduce changes in dynamics. The tanks are thermally insulated to make the heat losses negligible (as thermal insulation hides tank shapes, all tanks look the same in the plant photo). Water from the reservoir mounted under the plant is drawn by Pump 1 and Pump 3 (delivery flow rate 0-4.5 l/min) to the respective tanks. The delivery rates can be continuously changed by changing the armature voltage of the DC motors driving the pumps. The flow rates are measured using turbine flow-

meters. Steady state measurement performed with the pumps confirm that the delivered water flow is relatively independent from the hydrostatic pressure connected with the water levels in the tanks. Nevertheless, there is a certain dependence and there are also slight non-linearities in the steady state characteristics of the pumps. To compensate for these effects, it is beneficial to use slave flow rate controllers, which are denoted with FC in Fig. 1. The higher level controllers can then use the flow rates and not the armature voltages as their manipulated variables.

The flow from Pump 3 is fed directly to Tank 3. The flow from Pump 1 goes through a storage water heater (2 kW) and it is further controlled by a solenoid valve S1. The power consumption of the heater can be changed continuously. Another continuously controlled heater is mounted on the bottom of Tank 2 (800 W). The temperatures are measured with Pt1000 sensors at the points shown in Fig. 1. Depending on the selected control scenario, the inflows to Tanks 1 and 3 can be used as manipulated variables or as disturbances.

In addition to the pumps, the delivery flow rates of which can be changed continuously, the plant includes another way of manipulating the flow: solenoid valves. These discrete valued actuators control the flow from Tank 1 to Tank 2 (valves S3, S4,  $k_v=5$  l/min each). This flow is changed in three steps: no valve open, one open, both valves open. The valves close and open instantaneously. Tank 1 can be by-passed by closing S1 and opening S2. The air-water heat exchanger with cooling fan at the output from Tank 2 keeps the water temperature in the reservoir roughly constant during the experiments. The water levels are measured using pressure sensors. These can be configured either as continuous sensors or as level switches (indicating three different water levels).



Fig. 2 Photograph of the laboratory scale plant

### 3. CONTROL HARDWARE AND SOFTWARE ENVIRONMENT

The plant is controlled from a PC using two data acquisition boards (11 analog inputs, 6 analog outputs, 6 digital outputs) and the necessary interface hardware (power amplifiers, solid state relays, signal conditioning devices etc.). The basic software tool for identification and control experiments is the Real-Time Toolbox ([www.humusoft.cz](http://www.humusoft.cz)), which allows an easy connection of the Matlab/Simulink environment with the real world. Any control algorithm modelled in Simulink can be used to control the plant using the input/output blocks of the Real-Time Toolbox. As all of the capabilities of Matlab/Simulink are available, this option provides unbeatable flexibility in testing advanced control algorithms.

However, a PC with data acquisition boards running Matlab/Simulink is certainly not a typical industrial control system. For teaching purposes, it is very advantageous to provide the students with an opportunity to design and operate the control of this plant using real industrial control hardware. For this reason this plant can alternatively be controlled using the WinCon-8000 control system produced by ICP DAS. The changeover from PC to WinCon-8000 and vice versa is very simple: two connectors with analog and digital inputs/outputs have to be reconnected.

The WinCon-8000 is an embedded platform that has the capabilities of both traditional PLCs and industrial PCs. It is based on Intel Strong ARM CPU, and it runs a Windows CE .NET operating system that is suitable for real time control (hard real-time capability, small core size, fast boot speed, achievable deterministic control). WinCon includes PLC-style and expandable I/O Direct backplane access to I/O with a wide range of available expansion I/O modules including analog inputs and outputs with a sampling period up to 2 ms.

The control algorithms can be programmed in several ways. Microsoft Embedded Visual C++, Visual Basic .NET or Visual C# can be used. Another option is the new control software environment REX (description available in Balda *et al.*, 2005). REX software supports some real time operating systems including Windows CE .NET used by WinCon while the programming can be done with a standard PC in a Simulink-compatible environment. The authors of REX have defined the main features of this software as follows:

- compatibility with Matlab-Simulink
- development of industrial control block library (blockset)
- openness of the system for easy creation of new algorithms
- suitability for control engineering teaching
- support of industrial standards and internet technologies.

REX also offers OPC (OLE for Process Control) functionality, which can be used for communication of process stations with SCADA/HMI levels. REX and WinCon in connection with this plant provide students with an opportunity to obtain hands-on experience of applying an industrial control system and advanced control algorithms to a relatively complex plant.

#### 4. MATHEMATICAL MODEL OF THE PLANT

##### 3.1. Formulae for classical nonlinear modelling of the plant

Assuming that the liquid in the tanks is incompressible, the mass balance equation of a single tank is expressed as

$$\frac{dV(t)}{dt} = \frac{d}{dt} \left( \int_0^{h(t)} A(x) dx \right) = A(h) \frac{dh(t)}{dt} = q_{in}(t) - q_{out}(t) \quad (1)$$

where  $h$  is water level,  $A(h)$  cross sectional area at a level  $h$ , and  $q_{in}$ ,  $q_{out}$  are inlet and outlet volume flow rates. Assuming constant liquid heat capacity  $c$ , negligible heat losses and ideal mixing, the energy balance equation of a single tank is

$$\frac{d}{dt} (\rho c V(t) (\mathcal{G}(t) - \mathcal{G}_{ref})) = q_{in}(t) \rho c (\mathcal{G}_{in}(t) - \mathcal{G}_{ref}) - q_{out}(t) \rho c (\mathcal{G}(t) - \mathcal{G}_{ref}) + H(t) \quad (2)$$

where  $H$  is heater power output,  $\mathcal{G}_{in}$  inlet water temperature,  $\mathcal{G}$  water temperature in the tank. Due to the assumption of perfect mixing, the outlet water temperature can be equated with  $\mathcal{G}$ . Eq. (2) is then simplified to

$$V(h) \dot{\mathcal{G}}(t) = q_{in}(t) (\mathcal{G}_{in}(t) - \mathcal{G}(t)) + H(t) / \rho c \quad (3)$$

The volume flow rate through a valve characterized by flow coefficient  $k_v$  from a tank with water level  $h$  can be expressed using the basic flow equation, where hydrostatic pressure  $\rho gh$  is substituted for pressure drop across the valve

$$q = 0.1 k_v \sqrt{gh} \quad (4)$$

Flow in (4) is given in the same units as flow coefficient  $k_v$ .

##### 3.2 Formulae for hybrid modelling of the plant

Model of the whole plant is built by putting together the equations for water levels and temperatures. As the plant includes on/off valves and tank shapes induce abrupt changes in dynamics, the plant model has a hybrid character.

$$\dot{h}_1(t) = (1/A_1) (q_0(t) \sigma_0(t) - 0.1 k_v \sigma_1(t) \sqrt{gh_1(t)}) \quad (5)$$

$$\dot{\mathcal{G}}_1(t) = q_0(t) \sigma_0(t) (\mathcal{G}_{01}(t) - \mathcal{G}_1(t)) / A_1 h_1(t) \quad (6)$$

$$\dot{h}_2(t) = \begin{cases} (1/A_{21}) \left( \begin{matrix} 0.1 k_v \sigma_1(t) \sqrt{gh_1(t)} \\ + q_{32}(t) - q_{20}(t) \end{matrix} \right) & \text{if } m_1 = 0 \\ (1/A_{22}) \left( \begin{matrix} 0.1 k_v \sigma_1(t) \sqrt{gh_1(t)} \\ + q_{32}(t) - q_{20}(t) \end{matrix} \right) & \text{if } m_1 = 1 \end{cases} \quad (7)$$

$$\dot{\mathcal{G}}_2(t) = \begin{cases} \left( \begin{matrix} 0.1 k_v \sigma_1(t) \sqrt{gh_1(t)} (\mathcal{G}_1(t) - \mathcal{G}_2(t)) \\ + q_{32}(t) (\mathcal{G}_3(t) - \mathcal{G}_2(t)) + \frac{H(t)}{\rho c} \end{matrix} \right) / A_{21} h_2(t) & \text{if } m_1 = 0 \\ \left( \begin{matrix} 0.1 k_v \sigma_1(t) \sqrt{gh_1(t)} (\mathcal{G}_1(t) - \mathcal{G}_2(t)) \\ + q_{32}(t) (\mathcal{G}_3(t) - \mathcal{G}_2(t)) + \frac{H(t)}{\rho c} \end{matrix} \right) / (A_{21} l_1 + A_{22} (h_2(t) - l_1)) & \text{if } m_1 = 1 \end{cases} \quad (8)$$

$$\dot{h}_3(t) = \begin{cases} \frac{q_3(t) - q_{32}(t)}{\pi (r_{31} - h_3(t) \Delta r / l_2)^2} & \text{if } m_2 = 0 \\ \frac{q_3(t) - q_{32}(t)}{A_{32}} & \text{if } m_2 = 1 \end{cases} \quad (9)$$

$$m_1(t^+) = \begin{cases} 0 & \text{if } h_2(t) \leq l_1 \\ 1 & \text{if } h_2(t) > l_1 \end{cases} \quad (10)$$

$$m_2(t^+) = \begin{cases} 0 & \text{if } h_3(t) \leq l_2 \\ 1 & \text{if } h_3(t) > l_2 \end{cases} \quad (11)$$

where  $A_i = \pi r_i^2$ ,  $\Delta r = r_{31} - r_{32}$ ,  $m_1$  and  $m_2$  are discrete state variables, discrete valued input  $\sigma_0$  assumes values 0,1 (S1 closed, S1 open),  $\sigma_1$  assumes values 0,1,2 (no valve open, S3 open, S3 and S4 open),  $H$  is power output of heater H<sub>2</sub>,  $k_v$  is flow coefficient of S3 and S4. As Tank 3 is not heated,  $\mathcal{G}_3$  roughly equals  $\mathcal{G}_0$  and the equation for  $\mathcal{G}_3$  is not important. Model (5) to (11) assumes that S2 is closed. Opening S2 while closing S1 is used if a simpler plant with just Tank 2 and 3 is required. Model (5) to (11) can be compared with general state equations of hybrid system as given, e.g., by Branicky *et al.* (1998). A hybrid system is described by

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{m}(t), \mathbf{u}(t)) \quad (12)$$

$$\mathbf{y}(t) = \mathbf{g}(\mathbf{x}(t), \mathbf{m}(t), \mathbf{u}(t)) \quad (13)$$

$$\mathbf{m}(t^+) = \boldsymbol{\phi}(\mathbf{x}(t), \mathbf{m}(t), \mathbf{u}(t), \boldsymbol{\sigma}(t)) \quad (14)$$

$$\boldsymbol{\sigma}(t^+) = \boldsymbol{\psi}(\mathbf{x}(t), \mathbf{m}(t), \mathbf{u}(t), \boldsymbol{\sigma}(t)) \quad (14)$$

where  $\mathbf{x}(t)$ ,  $\mathbf{u}(t)$  and  $\mathbf{y}(t)$  are continuous state, input and output respectively. Besides continuous state, these equations include discrete state  $\mathbf{m}(t)$ . The development of the discrete state is described by (13), where  $\boldsymbol{\sigma}(t)$  is discrete input and  $\boldsymbol{\sigma}(t)$  is discrete output. Equation (14) models the state jumps. It is evident that model (5) to (11) includes all features of a general hybrid system except for state jumps. However, state jumps are only present in certain mechanical systems (systems with collisions). State variables in process control applications (temperature, liquid level etc.) cannot be abruptly changed in steps.

##### 3.3 Comparison of inductive and deductive approaches to system identification

This laboratory scale plant as a real physical model opens many opportunities to study how various types of mathematical models can be obtained and used in control system design, and to what extent we can rely on conclusions from simulation or computational design methods.

Another important educational advantage of the plant is the fact that quite accurate first principles modelling is possible, because the described processes adhere to basic and simple physical principles such as mass and energy balances. Thus, the derivation of the plant model can easily be understood by all students who have completed at least an introductory course in physics. Students can concentrate on control and identification problems without spending too much time learning complicated physical laws.

The mathematical model of the plant is a hybrid system. Some parts (Tank 2) are exactly described with a piecewise affine (PWA) system, while other parts can be approximated with a PWA system. First principles model can be compared with models obtained by experimental identification methods developed for PWA systems. These methods are described in (Ferrari-Trecate *et al.*, 2003) and are easily available to students in the form of a free Hybrid Identification Toolbox. This toolbox is now distributed as a part of the free Multi-Parametric (MPT) Toolbox for designing model predictive controllers for PWA systems (control.ee.ethz.ch/~mpt/).

### 3.4. Simulation and real controller performance

One of the reasons why industry is very cautious about adopting advanced control algorithms is that the algorithms are quite successful in simulation but their contribution may become questionable under real conditions. This is usually because some important real world phenomena have been neglected during simulation (e.g. noise, non-linearities, saturation with wind-up effect, sudden changes in parameters). It is very useful to demonstrate this by testing algorithms on the set-up and confronting results with those obtained via simulation.

## 5. SOLVING CONTROL PROBLEMS

The laboratory scale plant described in section 2 allows us to define many control scenarios of varying complexity. For the sake of brevity, a moderately complex scenario will be outlined for the reader to get some glimpse of the nature of possible control experiments. This scenario is intended for advanced control courses and its purpose is to provide students with a hands-on experience of model predictive control (MPC) of hybrid PWA systems.

Model predictive control of PWA systems is an evolving field and the students come in contact with an important research topic. The main references are (Borrelli, 2003) and more recently (Christophersen, 2007). The experiments are made considerably easier by the existence of the MPT Toolbox, which has many routines for analysis and design of MPC for PWA systems. An earlier version of this toolbox was described in (Kvasnica *et al.*, 2004), and current 2.6.2 version was released in December 2006. Since MPT toolbox includes a Simulink MPT controller block, students can evaluate the performance of the controllers with a real plant simply by connecting this block with the input/output blocks of the Real-Time Toolbox.

This method of evaluation is very fast and simple, but it has an obvious drawback in being too academic. However using the methods included in MPT toolbox, we can obtain a MPC control law in an explicit form. Most of the computations are performed off-line in advance and the explicit control law has the form of a discrete time PWA system. This control law can then be implemented relatively easily using WinCon industrial control hardware.

The control task considered here is water level control in tanks 2 and 3. The controlled variable is  $h_2$  and the

manipulated variable is  $q_{03}$ . The standard procedure for avoiding tank overflow as described in (Corripio, 2001) is applied to controlling the flow through Pumps 4 and 2. The flow from Tank 3 to Tank 2 is made directly proportional to water level  $h_3$  and the outflow from Tank 2 is proportional to water level  $h_2$ .

$$q_{32}(t) = k_3 h_3(t); \quad q_{20}(t) = k_2 h_2(t) \quad (15)$$

The values of  $k_2$  and  $k_3$  are determined in such a way that the outflows from the tanks are equal to the maximum flow rate achievable by Pump 3 if the respective water levels are close to their maximum values.

Although this scenario is relatively simple it gives many opportunities for experimenting. For example, MPC designs based on various performance indices, prediction horizon lengths etc may be used and compared. It is also possible to experiment with varying complexity of the models. To design a MPC controller for the scenario considered here, we need a model in PWA form. The simplest model is obtained if pumps with slave flow controllers are regarded as linear static elements. Tank 2 is then naturally modelled by a PWA system (two switched linear models). Connecting (15) and (7), the model takes the form

$$\begin{aligned} \tau_{21} \dot{h}_2(t) + h_2(t) &= (k_3/k_2) h_3(t) \text{ if } h_2(t) \leq l_1 \\ \tau_{22} \dot{h}_2(t) + h_2(t) &= (k_3/k_2) h_3(t) \text{ if } h_2(t) > l_1 \end{aligned} \quad (16)$$

where  $\tau_{21} = A_{21}/k_2 = 105.7 \text{ s}$  and  $\tau_{22} = A_{22}/k_2 = 28.3 \text{ s}$ . Model of Tank 3 is linear for  $h_3 > l_2$ . For  $h_3 < l_2$  it can be approximated with a PWA system by local linearizations around working points determined by the linear steady state characteristics  $h_{3S} = q_{3S}/k_3$ . Connecting (9) with (15) and linearizing, the following model is obtained

$$\left( \pi(r_{31} - h_{3S} \Delta r / l_2)^2 / k_3 \right) \Delta \dot{h}_3(t) + \Delta h_3(t) = (1/k_3) \Delta q_3(t) \quad (17)$$

with time constant  $\tau = \pi(r_{31} - h_{3S} \Delta r / l_2)^2 / k_3$ .

The time constant changes from 113.1 s at  $h_3=0$  to 26.8 s at  $h_3=l_2$ . To replace (17) with switched models, the range of  $h_3$  up to  $l_2$  must be divided into parts. Dividing the range from 0 to  $l_2$  into two parts, the following models result: model 1:  $\tau_{31}=78.9 \text{ s}$  valid for  $0 \leq h_3 \leq d_1$ ; model 2:  $\tau_{32}=38.4 \text{ s}$  valid for  $d_1 < h_3 \leq l_2$ ; model 3:  $\tau_{33}=26.8 \text{ s}$  valid for  $l_2 < h_3 \leq l_{\max}$ . The dividing line between the lower sub-ranges is  $d_1=0.235 \text{ m}$ . Defining state vector  $\mathbf{x}(t)=[h_2(t) \ h_3(t)]$ ,  $h_2$  as an output  $y$  and  $q_3$  as an input  $u$  the whole system is described by

$$\dot{\mathbf{x}}(t) = \mathbf{A}_{ij} \mathbf{x}(t) + \mathbf{b}_{ij} u(t); \quad y(t) = \mathbf{c} \mathbf{x}(t) \quad (18)$$

$$\mathbf{A}_{ij} = \begin{bmatrix} -1/\tau_{2i} & k_3/k_2 \tau_{2i} \\ 0 & -1/\tau_{3j} \end{bmatrix}; \quad \mathbf{b}_{ij} = \begin{bmatrix} 0 \\ 1/k_3 \tau_{3j} \end{bmatrix} \quad (19)$$

$$\mathbf{c} = [1 \ 0]; \quad i=1..2, j=1..3$$

This model includes six different dynamics. To design a MPC controller, these equations are discretized (using c2d command) and augmented with equations defining the polyhedral partition of the state input space over which different dynamics are active. The partition is defined by

$$\mathbf{G}_{ij}^x \mathbf{x}(t) \leq \mathbf{G}_{ij}^c; \mathbf{G}_{ij}^x = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & -1 \end{bmatrix}; \mathbf{G}_{11}^c = \begin{bmatrix} l_1 \\ d_1 \\ 0 \\ 0 \end{bmatrix}; \mathbf{G}_{12}^c = \begin{bmatrix} l_1 \\ l_2 \\ 0 \\ -d_1 \end{bmatrix} \quad (20)$$

$$\mathbf{G}_{13}^c = \begin{bmatrix} l_1 \\ l_{\max} \\ 0 \\ -l_2 \end{bmatrix}; \mathbf{G}_{21}^c = \begin{bmatrix} l_{\max} \\ d_1 \\ -l_1 \\ 0 \end{bmatrix}; \mathbf{G}_{11}^c = \begin{bmatrix} l_{\max} \\ l_2 \\ -l_1 \\ -d_1 \end{bmatrix}; \mathbf{G}_{11}^c = \begin{bmatrix} l_{\max} \\ l_{\max} \\ -l_1 \\ -l_2 \end{bmatrix}$$

where matrix  $\mathbf{G}_{ij}^x$  is the same for all partial models.

Model predictive controller based on this model can then be designed using the `mpt_control` command of MPT Toolbox. A comparison of the simulated and real responses is shown in Fig. 3 (linear objective, horizon length 3, sampling period 2 s, reference step change to from zero to 0.5 at  $t=0$  s).

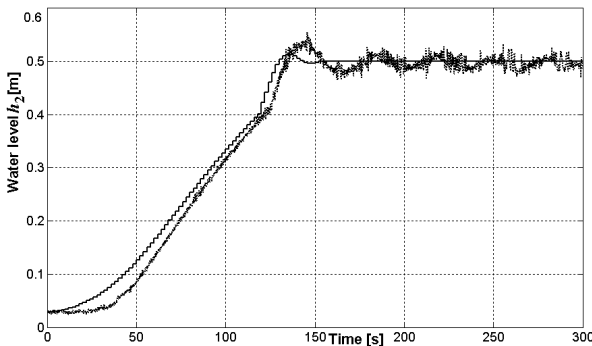


Fig. 3 Simulated and measured MPC control response  $h_2$ —simpler model (solid line simulated, dotted measured)

The difference between simulation and reality is evident. The model can be made more precise if it includes the fact that small flows below a certain minimum cannot be realised by the pumps. This significantly increases the number of different dynamics, and the model becomes too complicated to be presented here. However, it considerably improves the accordance of simulated and real responses as Fig. 4 shows.

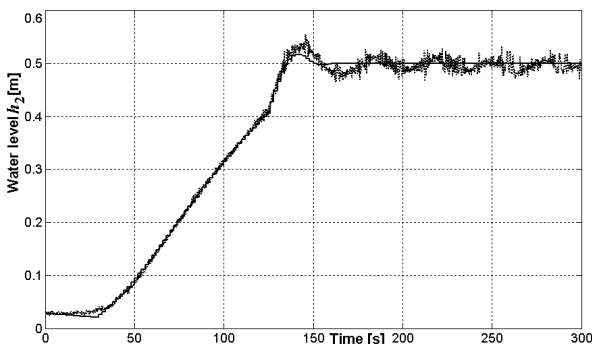


Fig. 4 Simulated and measured MPC control response  $h_2$ —more precise model (solid line simulated, dotted measured)

However, the control performance is not much changed and certain steady state ripple is still observable in the controlled variable. Thus, students may experiment further. It is possible to improve the precision of the model (e.g. by including the pump plus slave controller dynamics) and to tune the design criteria (performance indices and weights, horizon lengths), in order to achieve further control performance improvement.

## 6. CONCLUSION

The laboratory scale plant described here can be used in advanced control courses to provide students with an opportunity to evaluate the performance of current approaches to control of hybrid and PWA systems with a real physical plant and not only in simulation. The control example given in the last section of the paper uses model predictive control, but students can also experiment equally well with other approaches (e.g., with switching control). As plant control can be performed not only by a PC but also by an industrial control system WinCon, students can gain experience with implementation of advanced control algorithms to industrial hardware.

An important feature of the plant is the connection of a real set-up marked by considerably complex dynamic behaviour with industrial control hardware. This feature can also make the plant attractive for use in industrial training courses, where the objective is to inform specialists from industry about the perspectives offered by advanced control methods.

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