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Fractional Order Control of an Unmanned Aerial Vehicle (UAV) ⋆

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Abstract: This paper deals with the trajectory control problem for a rotary-wing nonlinear vehicle model. The control of this kind of systems is one of the most challenging and attractive research areas. The design scheme presented is based on the use of fractional order controllers, originated from the application of the theory of Fractional Calculus to control system design. One of the interesting features of the control strategy proposed is the use of a fractional order derivative to ensure the robustness of the nonlinear system in spite of using a linearized design model. It is shown that by assuring constant phase margin on a frequency range around the roll-off frequencies the resulting control system is robust to parameter variations and nonlinear effects. With this strategy the control design problem becomes much simpler and gives very straightforward tuning rules. Fundamental operational principles are also considered for establishing the bandwidth of the input-output channels of the system. The performance of the controller is shown through nonlinear simulations.

1. INTRODUCTION

In the past years there has been a steady increase in the development of sophisticated unmanned aerial vehicles (UAVs) for military and civilian applications. UAVs are for local reconnaissance, fire control, and detection of intruders. Law enforcement organizations use UAVs for hostage rescue, border patrol, traffic surveillance and riot control (Davis et al. [1998]). The commercial success of UAVs together with the revolutionary advances in the miniaturization of computers, sensors and mechanical actuators have posed new challenges to control engineers. Nowadays, UAVs are considered challenging benchmarks for the development of new controllers. On the other hand, new UAV configurations are being proposed (Kendoul et al. [2005]). Existing UAVs can be classified mainly in two classes: rotary wing vehicles and fixed wing vehicles. For missions requiring the vehicle to remain stationary (hover) or to maneuver in tightly constrained environments rotary wing vehicles have significant advantages over fixed wing vehicles. For example, traffic surveillance around buildings requires a hovering vehicle with good manoeuvrable characteristics. It is important to point out that hover flight consumes approximately twice the power of a similarly loaded fixed wing vehicle moving forward. However, it is expected that new power technologies will allow to achieve reasonable endurance for rotary wing vehicles.

On the other hand, nowadays the better understanding of the potential of Fractional Calculus (FC) and the increasing number of studies related to the applications of fractional order controllers (FOC) in many areas of science and engineering have led to the importance of studying aspects such as the analysis, design, tuning and implementation of these kind of controllers.

In this paper a rotorcraft powered by four non-tilting rotors known as the X4-flier (Hamel et al. [2002]) or the Dragan-flyer is considered. Besides its practical relevance, this system is an interesting case of study since it is a six degrees of freedom mechanical system whose dynamics is described by an under-actuated twelfth order highly coupled nonlinear model.

The goal of this work is to solve the trajectory tracking problem. It is important to point out that this problem has been addressed in Salazar et al. [2005], using a nested saturation control algorithm, and in Hamel et al. [2002], Arujo et al. [1998], Castillo et al. [2007], using back-stepping techniques. However, in this paper the problem is solved by introducing fractional order derivative controllers that guarantees the robustness of the system to variations in its dynamics and parameters, giving a constant phase margin in the frequency response. Since the approach here presented is specifically tailored to our particular rotorcraft dynamics, the control design problem becomes much simpler and gives very straightforward tuning rules. A similar strategy, though with a different application, has been also used in Feliu et al. [2005].

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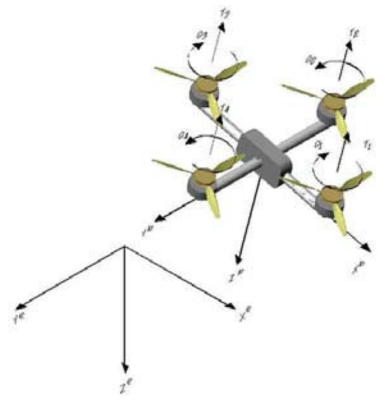

Fig. 1. Rotary wing vehicle

2. THE MODEL

The rotary wing vehicle is shown in Figure 1. It is powered by four non-tilting rotors attached to a rigid frame. The dynamical model of the rotary wing vehicle considered can be obtained as follows. Let $0x^e y^e z^e$ denote a right-hand inertial frame (earth frame) such that z^e **points downwards** into the centre of the earth and $0x^b y^b z^b$ a right-hand frame fixed to the center of mass of the aircraft structure (body frame). The vehicle dynamics in the body frame is described by (Roskam [1982])

$$\begin{aligned} m\,\dot{V}^b_{CM} + m\,\Omega \times V^b_{CM} &= F^b_e \\ I\,\dot{\Omega} + \Omega \times I\,\Omega &= M^b_e \end{aligned} \tag{1}$$

where m represents the vehicle mass, $V^b_{CM} = [\,u\ v\ w\,]^\top$ denotes the linear velocity of the vehicle center of mass expressed in the body frame, $\Omega = [\,p\ q\ r\,]^\top$ denotes the angular velocity of the body frame, I is the vehicle inertia matrix [1], F^b_e represents the external applied forces expressed in the body frame, and M^b_e represents the external applied moments expressed in the body frame.

In order to express the motion dynamics (1) referred to earth axis, it is necessary to specify the orientation of the body axis with respect to the earth frame. Considering the classical Euler yaw-pitch-roll rotation sequence, the rotation matrix is given by

$$R = \begin{bmatrix} c_\theta c_\psi & c_\theta s_\psi & -s_\theta \\ c_\psi s_\theta s_\phi - s_\psi c_\phi & s_\psi s_\theta s_\phi + c_\psi c_\phi & c_\theta s_\phi \\ c_\psi s_\theta c_\phi + s_\psi s_\phi & s_\psi s_\theta c_\phi - c_\psi s_\phi & c_\theta c_\phi \end{bmatrix} \tag{2}$$

where $\eta = [\,\phi\ \theta\ \psi\,]^\top$ are the Euler angles with ϕ, θ, and ψ the roll, pitch, and yaw angular displacements, respectively, and $c_x = \cos(x)$ and $s_x = \sin(x)$. On the other hand, Ω is related to the Euler angles velocity as follows (Roskam [1982])

$$\Omega = W\dot{\eta} = \begin{bmatrix} 1 & 0 & -\sin(\theta) \\ 0 & \cos(\theta) & cos(\theta)\sin(\phi) \\ 0 & -\sin(\phi) & \cos(\theta)\cos(\phi) \end{bmatrix} \dot{\eta} \tag{3}$$

From equation (2) the relationship between the velocity components in the earth frame and the velocity components in the body frame is defined as

$$V^b_{CM} = R\,V^e_{CM} \tag{4}$$

[1] As the vehicle has two symmetry axes $I = \mathrm{diag}\{I_{xx}, I_{yy}, I_{zz}\}$.

where $V^e_{CM} = [\,\dot{x}\ \dot{y}\ \dot{z}\,]^\top$ is the linear velocity of the vehicle centre of mass expressed in the earth frame. Thus, the vehicle dynamics expressed in the earth frame can be written as

$$\begin{aligned} m\,\dot{V}^e_{CM} &= R^\top F^b_e \\ I_\eta\,\ddot{\eta} + W^\top I\dot{W}\dot{\eta} + W^\top (W\,\dot{\eta} \times I\,W\,\dot{\eta}) &= W^\top M^b_e \end{aligned} \tag{5}$$

where the following facts have been considered: $R^{-1} = R^\top$, $\dot{R}\,R^\top\,V^b_{CM} = -Sk(\Omega)\,V^b_{CM}$ with $Sk(\Omega)$ a matrix such that $\Omega \times V^b_{CM} = Sk(\Omega)\,V^b_{CM}$, and $I_\eta = W^\top I\,W$.

The external applied forces expressed in the body frame are the vehicle weight and the total thrust produced by the four rotors, that is,

$$F^b_e = \begin{bmatrix} -m\,g\sin(\theta) \\ m\,g\cos(\theta)\sin(\phi) \\ m\,g\cos(\theta)\cos(\phi) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -T_T \end{bmatrix} \tag{6}$$

where $T_T = \sum_{i=1}^4 T_i$ with T_i the thrust of each rotor. It is shown in Gessow and Myers [1978] that the thrust generated by each rotor can be expressed as

$$T_i = C_{T_i}\,\pi\,r_i^4\rho\,\omega_i^2$$

where C_{T_i} is the rotor i thrust coefficient, ρ is the air density, r_i is the radius of rotor i and ω is the angular velocity of rotor i.

The external moments in the body frame are defined as follows. The pitching motion is actuated by the moment around y^b produced by increasing the thrust of rotor 2 and reducing the thrust of rotor 4. The roll movement is generated in a similar way, that is, by producing a differential thrust between rotors 1 and 3. Due to the torque applied to the rotor shaft by the motors, a reactive torque of the same magnitude but opposite direction is experienced on the structure of the vehicle. By manipulating these reaction torques it is possible to control the yaw moment. That is,

$$M^b_e = \begin{bmatrix} L \\ M \\ N \end{bmatrix} = \begin{bmatrix} (T_3 - T_1)\ \ell \\ (T_2 - T_4)\ \ell \\ \displaystyle\sum_{i=1}^4 Q_i \end{bmatrix} \tag{7}$$

where ℓ is the distance between the rotor rotation axis and the aircraft center of mass and Q_i is the reactive moment produced by rotor i. This reactive moment is given as (Gessow and Myers [1978])

$$Q_i = C_{Q_i}\,\pi\,r_i^5\rho\,\omega_i^2$$

As shown in Hamel et al. [2002], there exist a globally defined change of coordinates from $[\,-T_T\ L\ M\ N\,]^\top$ to $[\,\omega_1^2\ \omega_2^2\ \omega_3^2\ \omega_4^2\,]^\top$ for $C_{T_i} > 0$ and $C_{Q_i} > 0$.

3. THE CONTROL PROBLEM

Regarding the control strategy, first, the vehicle vertical motion controller is designed and then, through the pitch and roll angles, the vehicle position in the plane is controlled. The motion in the yaw direction is controlled independently.

From the first equation in (5) the translational dynamics is described by

$$\begin{aligned} m\,\ddot{x} &= T_T \sin(\theta) \\ m\,\ddot{y} &= -T_T \cos(\theta)\sin(\phi) \\ m\,\ddot{z} &= -T_T \cos(\theta)\cos(\phi) + m\,g \end{aligned} \tag{8}$$

These equations are typical for the X4-flier (Hamel et al. [2002]).

In order to preserve the altitude of the aircraft, priority on the heave motion control is necessary, fact that any helicopter pilot would certify. Thus, the heave response should be faster than that of the longitudinal and lateral dynamics. These dynamical characteristics can be achieved by selecting the bandwidths of the input-output channels accordingly. Therefore, the control design can be stated in terms of specifications that guarantee altitude control and robustness before executing other manoeuvres. Longitudinal and lateral control systems are specified in terms of the heave closed-loop dynamics and the achievement of appropriate bandwidth separation.

Heave Control. It is assumed that during normal flight pitch and roll angles are bounded. Otherwise the vertical force component of the thrusters will not compensate the weight of the aircraft. Thus, $-35° < \theta < 35°$ and $-35° < \phi < 35°$ so that $0.67 < \cos\theta\cos\phi < 1$ in order to preserve enough lifting vertical thrust for maintaining a prescribed altitude.

Under this restriction and assuming $m = 1kg$, a control design model from the third equation in (8) can be defined

$$\ddot{z} = -K_z T_T + g \tag{9}$$

with $K_z = \frac{\cos\theta_n \cos\phi_n}{m_n}$, where θ_n and ϕ_n are time varying but bounded and m_n is a nominal value of the mass. The rotor thrust T_T should compensate the effect of gravity and add extra thrust for executing manoeuvres. That is,

$$T_T = \frac{1}{\hat{K}_z}(T_v + \hat{g}) \tag{10}$$

where $\hat{g}$ is the nominal value of the acceleration due to gravity and $\hat{K}_z$ is constant. The resulting heave dynamics is

$$\ddot{z} = \frac{-K_z}{\hat{K}_z}(T_v + \hat{g}) + g \tag{11}$$

which can be rewritten as

$$\ddot{z} = -\bar{K}_z T_v + \bar{g_z} \tag{12}$$

where $\bar{K_z} = \frac{K_z}{\hat{K}_z}$ and $\bar{g_z} = -\hat{K}_z\hat{g} + g$. Clearly T_v should be designed as a stabilizing controller for a double integrator with uncertain gain under the influence of a constant disturbance. That is, the design model is

$$\frac{Z(s)}{T_v(s)} = g_z(s) = -\frac{\hat{K}_z}{s^2} \tag{13}$$

and

$$T_v(s) = c_z(s)\; e_z(s) \tag{14}$$

where $c_z(s)$ is the controller expressed as a transfer function and $e_z(s)$ is the heave error.

Longitudinal Control. The longitudinal dynamics

$$\ddot{x} = \frac{1}{m}\sin\theta\; T_T \tag{15}$$

with the heave control system becomes

$$\ddot{x} = \frac{1}{m}\sin\theta\;\frac{1}{\hat{K}_z}(c_z(s)\; e_z(s) + \hat{g})\tan\theta \tag{16}$$

$$\ddot{\theta} = \tau_\theta \tag{17}$$

This abuse of notation allows to write the longitudinal dynamics in closed form, where τ_θ is the pitch moment and acts as the input. The system has relative degree $r_d = 4$ and the input-output relationship is

$$\begin{aligned} x^{(IV)} = {} & K_x(c_z(s)\; e_z(s) + \hat{g})\sin\theta\;\ddot{\theta}^2 + \\ & K_x(c_z(s)\; e_z(s) + \hat{g})\cos\theta\tau_\theta \end{aligned} \tag{18}$$

with $K_z = \frac{\hat{m}}{\cos\phi_n\cos\theta_n}$, where ϕ_n, θ_n and $\hat{m}$ are the nominal fixed values. Note that the term $c_z(s)\; e_z(s)$ will vanish if the heave control has high performance. On the other hand, $\sin\theta\;\ddot{\theta}^2$ is in general negligible.

A design model can be obtained by a simple linearization around straight level flight, resulting

$$\frac{X(s)}{\tau_\theta(s)} = \frac{g}{s^4} \tag{19}$$

Lateral Control. The lateral control can be analyzed similarly. That is

$$\ddot{y} = \frac{1}{m}\; sin\phi\;\cos\theta\;\; T_T \tag{20}$$

By substituting the heave controller, the above equation is transformed into

$$\ddot{y} = \hat{K}_y\; sin\phi\;\cos\theta\;(c_z(s)\; e_z(s) + \hat{g}) \tag{21}$$

with

$$\ddot{\phi} = \tau_\phi \tag{22}$$

where τ_ϕ is the roll control torque and $\hat{K}_y = \frac{\hat{m}}{m\cos\phi_n\cos\theta_n}$. The system input-output relationship is

$$\begin{aligned} y^{(IV)} = {} & \hat{K}_y\cos\theta(c_z(s)\; e_z(s) + \hat{g})\sin\phi\;\phi^2 - \\ & \hat{K}_y\cos\theta(c_z(s)\; e_z(s) + \hat{g})\cos\phi\;\;\tau_\phi \end{aligned} \tag{23}$$

Linearizing (24) considering straight level flight, the following design model is obtained

$$\frac{Y(s)}{\tau_\phi(s)} = -\frac{g}{s^4} \tag{24}$$

Further considerations. It is clear that the design models defined here may lack aspects such as nonlinearities and coupling. Also uncertainty due to parameter variations should be considered. Nevertheless, it has been proven that the vertical system is dominated by the double integral action and the longitudinal and lateral dynamics by two double integrators (Kendoul et al. [2005], Hamel et al. [2002], Salazar et al. [2005]). In recent years the control of such systems has called the attention of the control community. As follows, a new strategy is proposed by applying fractional order controllers.

4. CONTROL DESIGN

4.1 Introduction to FOC

Fractional calculus is a generalization of the integration and differentiation to the non-integer (fractional) order fundamental operator ${}_a\mathrm{D}_t^\alpha$, where a and t are the limits and α ($\alpha \in \Re$) is the order of the operation. Among many different definitions, two commonly used for the general fractional integro-differential operation are the Grünwald-Letnikov (GL) definition and the Riemann-Liouville (RL) definition (Podlubny [1999]). The GL definition is

$$ {}_a\mathrm{D}_t^\alpha f(t) = \lim_{h\to 0} h^{-\alpha} \sum_{j=0}^{[\frac{t-a}{h}]} (-1)^j \binom{\alpha}{j} f(t-jh) \tag{25}$$

where $[\cdot]$ means the integer part, while the RL definition is

$$ {}_a\mathrm{D}_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{\mathrm{d}^n}{\mathrm{d}t^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} \mathrm{d}\tau \tag{26}$$

for $(n-1<\alpha<n)$ and where $\Gamma(\cdot)$ is the *Euler's gamma* function.

For convenience, the Laplace domain notion is commonly used to describe the fractional integro-differential operation. The Laplace transform of the RL fractional derivative/integral (26) under zero initial conditions for order α $(0<\alpha<1)$ is given by

$$\pounds\{{}_a\mathrm{D}_t^{\pm\alpha} f(t)\} = s^{\pm\alpha}F(s). \tag{27}$$

As follows a trajectory controller for the rotor craft following the fractional operation approach is designed.

4.2 The fractional order integrator

Fractional integro-differential operators were introduced in a feedback structure by Bode in Bode [1940] and Bode [1945]. Originally the aim was to design feedback amplifiers with uniform performance in spite of changes of the amplifiers gain. Bode presented an elegant solution to this robust design problem, which he referred to as the *ideal cutoff characteristic* or the *ideal loop transfer function*. In a Nyquist plot, such a characteristic is reflected as a straight line through the origin, giving a phase margin invariant to gain changes. This ideal system is a fractional integrator with transfer function $G(s) = \left(\frac{\omega_{cg}}{s}\right)^\alpha$, known as *Bode's ideal transfer function*, where ω_{cg} is the gain crossover frequency and the constant phase margin is $\varphi_m = \pi - \alpha\frac{\pi}{2}$. This frequency characteristic is very interesting in terms of robustness to parameters changes or uncertainties.

4.3 Controller design

Firstly the controller for the z-axis movement is designed. As shown previously in (13), the plant to control is a double integrator with transfer function $G_z(s) = \hat{K}_z/s^2$, with $\hat{K}_z = 0.82m/sec^2$. The condition for a constant phase margin can be expressed as

$$\arg\left[C(j\omega)\frac{\hat{K}_z}{(j\omega)^2}\right] = constant, \forall\omega \tag{28}$$

where $C(s)$ is the controller. The resulting phase margin φ_m is

$$\varphi_m = \arg\left[C(j\omega)\right] \tag{29}$$

For a constant phase margin $0<\varphi_m<\pi/2$ the controller that achieves this must be of the form

$$C(s) = k_z s^\alpha, arg(C(s)) = \alpha\frac{\pi}{2} = \varphi_m, \alpha = \varphi_m\frac{2}{\pi} \tag{30}$$

with $0<\alpha<1$. This $C(s)$ is a fractional derivative controller of order α.

The design of the controller thus involves the selection of two parameters: (1) α, the order of the derivative, which determines: (a) the overshoot of the step response, (b) the phase margin, or (c) the damping; (2) k_z, the controller gain, which determines for a given α: (a) the speed of the step response, or (b) the crossover frequency.

In the frequency domain, the selection of these parameters can be regarded as choosing a fixed phase margin by selecting α, and choosing a crossover frequency ω_{cg} by selecting k_z for a given α. That is

$$\alpha = \frac{2}{\pi}\varphi_m, k_z = \frac{(\omega_{cg})^{2-\alpha}}{\hat{K}_z} \tag{31}$$

The frequency specifications required for the z-system are: phase margin $\varphi_m = 70°$, approximately, and crossover frequency around $\omega_{cg} = 10$ *rad/sec*. Therefore, accordingly to the previous equation, the parameters of the fractional derivative controller are $\alpha = 0.77$ and $k_z = 20.71$, resulting

$$C_z(s) = 20.71s^{0.77} \tag{32}$$

For the control of the motion in the x and y axes the same design method is followed. However, the dynamics in these two axes is given by two double integrators with transfer function $G_{x,y}(s) = g/s^4$ (from (19) and (24)), with $g = 9.8m/sec^2$. For that reason, the resulting controller will be of the form $C_{x,y}(s) = k_{x,y}s^{2+\alpha}$, that is, a double integer derivative (s^2) to reduce the dynamics to a double integrator, and also a fractional order derivative (s^α) to fulfill the design specifications as explained in the previous case. These specifications are, for both x and y systems: phase margin $\varphi_m = 70°$, and crossover frequency around $\omega_{cg} = 5$ *rad/sec*. Therefore, the parameters of the fractional order derivative controller for both motions are $\alpha = 0.77$ and $k_{x,y} = 0.7388$, resulting

$$C_{x,y}(s) = 0.7388s^{2.77} \tag{33}$$

4.4 Implementation of controllers $C_x(s)$, $C_y(s)$, and $C_z(s)$

When fractional order controllers have to be implemented, fractional transfer functions are replaced by integer transfer functions which approximate the frequency response of the fractional controller on a frequency range around the cross over frequency. There are many different ways of finding such approximations.

One of the methods commonly used for the implementation of fractional order operators is the Oustaloup continuous approximation (see Oustaloup [1995]). With this method a rational transfer function whose frequency response fits the original function within a frequency range

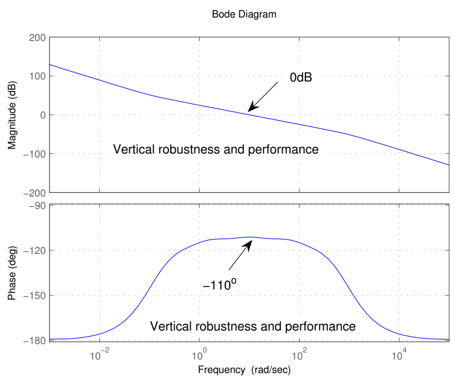

Fig. 2. Bode plots of the system with controller $C_z^5(s)$

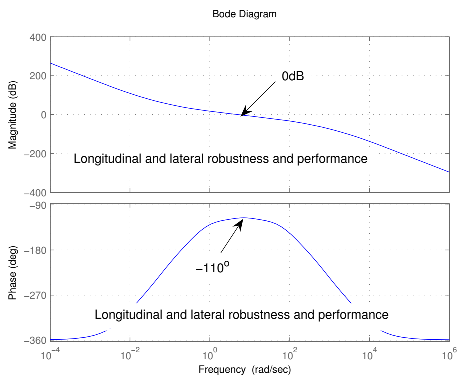

Fig. 3. Bode plots of the system with controller $C_{x,y}^5(s)$

is obtained. Controller $C_z(s)$ will be approximated by a fifth-order transfer function $C_z^5(s)$ with $\omega_h = 1000rad/\sec$ and $\omega_b = 0.1rad/\sec$. Controllers $C_{x,y}(s)$ will be approximated by a fifth-order transfer function $C_{x,y}^5(s)$ with $\omega_h = 1000rad/\sec$ and $\omega_b = 0.05rad/\sec$. The resulting controllers are

$$C_z^5(s) = 4225\frac{(s+196)(s+31)(s+5)(s+0.78)(s+0.12)}{(s+809)(s+128)(s+20.32)(s+3.22)(s+0.51)} \quad (34)$$

$$C_{x,y}^5(s) = 1.5\cdot 10^8\frac{(s+24)(s+3.3)(s+0.45)(s+0.062)(s+0.008)}{(s+5771)(s+796)(s+110)(s+15.16)(s+2.1)} \quad (35)$$

Approximations of greater order can be considered in order to improve the adjustment of the frequency response of the rational controllers to those of the irrational controller. It was observed that a fifth-order transfer function rendered an acceptable approximation. However, a transfer function of lower order could also be effective. The frequency and time responses of the system with these controllers are discussed next, together with stability requirements.

5. SIMULATION RESULTS

The results from a nonlinear simulation of equation (8) with controllers (34) and (35) are shown next. First of all, the responses obtained with controller $C_z^5 s$ for the z-motion and controllers $C_{x,y}^5(s)$ for the x-motion and y-motion are presented. The Bode plots of the open loop systems for the different axis motions are shown in Figures 2 and 3, respectively. As can be observed, the gain crossover frequencies for the z and x-y motions are $\omega_{cg} = 10\ rad/sec$ and $\omega_{cg} = 5\ rad/sec$, respectively. The phase margin is $\varphi_m = 70°$ for the three systems. Besides, an almost constant phase margin is ensured in each case within a frequency range of two decades around the gain crossover frequencies to guarantee a system response exhibiting robustness performance, independently of parameters uncertainties.

The time responses of the z-system, x-system, and y-system are presented in Figure 4, showing the motion animation in Figure 5. The trajectory tracking problem is solved with a minor error, as can be observed. The initial (x,y,z) position of the rotorcraft in the space is (0,12,20) (in meters). The input reference for the z-motion is given by $r_z = 20e^{-0.1t}$, and the input references for the x-motion and y-motion are $r_x = 12sin(0.12t)$ and $r_y = 12cos(0.12t)$, respectively, describing the parametric

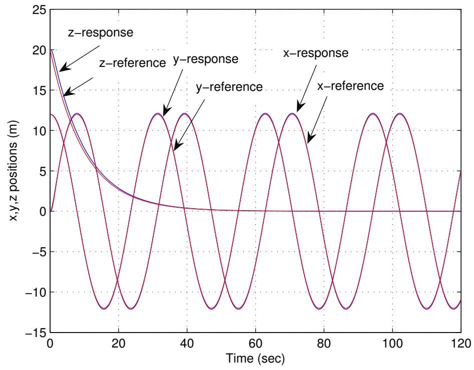

Fig. 4. Time responses of the x,y,z systems

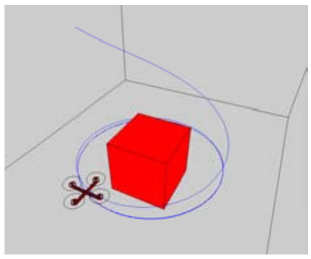

Fig. 5. Motion animation

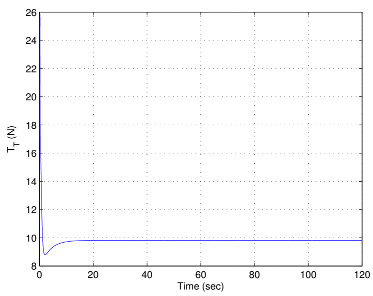

Fig. 6. Rotor thrust T_T

equation of a circumference. Parameters T_T, θ, and ϕ are shown in Figures 6, 7, and 8, respectively, preserving the restriction $0.67 < \cos\theta\cos\phi < 1$ stated before.

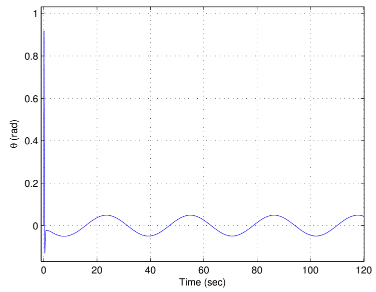

Fig. 7. Pitch angular displacement θ

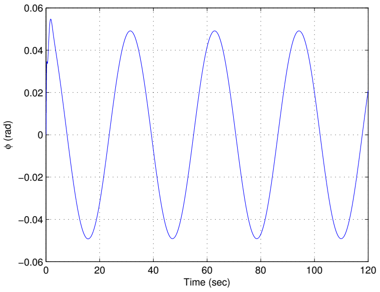

Fig. 8. Roll angular displacement ϕ

6. CONCLUSIONS

The trajectory control problem of a rotary wing vehicle, known as the X4-flier, has been solved by applying fractional order derivative controllers. The design specifications were defined by considering basic operation requirements of rotary wing aircraft. Such specifications are fundamental for achieving decoupling and stability. It is shown that fractional controllers eliminate the nonlinear effects and parameter uncertainty by maintaining appropriate stability margins over a specified range of the frequency response. An advantage of the control system here proposed is that the controllers are expressed and designed as transfer functions. Moreover, the controllers are implemented as integer order transfer functions, so that no need of state estimation is required. The effectiveness of the controller is shown by simulation results.

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