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Robust Delay Block Stabilization via Integral Sliding Mode Control

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Abstract: In this paper, a new discontinuous control strategy is proposed for robust stabilization of a class of uncertain multivariable linear time-delay system with delays in both the state and control variables. The integral sliding mode control technique is applied to compensate the uncertainty term and then a predictor is used to obtain free-delay closed-loop system with desired spectra. For systems presented in Block Controllable (BC) form with delay, a block dead time compensation algorithm which gives a sliding manifold is derived. An example of the application of the proposed control strategy is presented.

1. INTRODUCTION

Robust feedback stabilization of time-delay systems remains is one of the most challenging problems in control theory because many industrial processes are modelled by delay differential equations. It is well-known that the delay can dramatically limit the performance and sometimes destabilizes the closed-loop system. This problem has been extensively studied and several controllers and stability criteria based on optimal control method including H_∞ and LMIs approaches Fridman et al. [2003], used basically Lyapunov- Krasovskii functional have been proposed.

On the other hand, a sliding mode (SM) control has attractive properties such as decomposition of the original control design procedure and robustness to plant parameter variations and external matched disturbances Utkin et al. [1999]. In addition, SM control achieves fast transient response of the closed-loop system. Due to these advantages and simplicity implementation SM approach was used to design robust stabilizing discontinuous controllers for the delayed systems by Gouaisbaut et al. [2002], Li et al. [2004], Shtessel et al. [2003] and Xia et al. [2003].

However, most of these controllers were proposed for systems with only delay in the state, while the direct implementation of discontinuous control in systems with input delay can cause ostillations Fridman et al. [1996]. This problem can be treated via a predictor-based controller proposed by Roh et al. [1999], but the conditions to preserve uncertainties matching condition in the transformed predicted system, are very restrictive Nguang [2001]. Moreover, the problem of designing a predictor-based control for system with both delayed state and input is still open to the best knowledge of the authors.

In this paper, a new sliding mode control strategy is proposed for robust stabilization of a class of uncertain systems with delays in both the state and control variables, firstly in general form and then in Block Controllable form (BC form) Loukianov et al. [2003]. The control vector is divided in three parts. The first part is designed using the integral sliding mode (ISM) technique Utkin et al. [1999] which enables to preserve the matching condition for the unknown perturbation. In addition, the switching function includes an auxiliary variable that allows to compensate the perturbation. The second part of the control cancels the known undesired matched dynamics, including delayed part. And the third, nominal part of the control is selected to stabilize the nominal delay-free dynamics. Based on this nominal system, two predictors are designed for the state and auxiliary variables to compensate for the input delay, and therefore ensure the sliding mode stability and achieve chattering-free sliding mode motion. In the case of BC form with delay, part of the state variables is used as a fictitious control for the other variables. Such presentation and using the block control techniques allows to relax the matching condition with respect to retarded part of the plant dynamics, introduced in general case. Note that the various plants are presented in BC form with delay Loukianov et al. [2006].

2. GENERAL FORM WITH TIME DELAY

Consider a linear time-delay system with uncertainties described by the following state equation:

$$\dot{x}(t) = Ax(t) + Cx(t-\tau_1) + Bu(t-\tau_2) + f(x(t),t) \quad (1)$$
$$x(t) = \varphi_1(t),\ u(t) = \varphi_2(t)\ \forall t \in [t_0 - \tau, t_0],\ t_0 \geq 0$$

where $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$ are the state and control vectors, respectively; the unknown function $f \in \mathbb{R}^m$ represent the system nonlinearity and any model uncertainties in the system including external disturbances; rank $B = m$; τ_1 and τ_2 are known time delays, $\tau_1 \geq \tau_2$. We use the following assumptions:

A1) The pair (A, B) is controllable;

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A2) The perturbation term $f(x(t),t)$ is Lipshitz, continuous and satisfies the matching condition:

$$f(x(t),t) = B\bar{f}(x(t),t) \tag{2}$$

with $\bar{f} \in \mathbb{R}^m$ bounded by

$$\left\|\bar{f}(x(t),t)\right\| \leq \alpha(x(t),t) \tag{3}$$

for some known scalar function $\alpha(\cdot)$.

A3) There is a matrix $\bar{C} \in \mathbb{R}^{m\times n}$ such that $C = B\bar{C}$.

2.1 Controller design

Under assumptions A2 and A3, system (1) is represented as

$$\dot{x}(t) = Ax(t) + B[u(t-\tau_2) + \bar{C}x(t-\tau_1) + \bar{f}(x(t),t)]. \tag{4}$$

To cancel the perturbation term $\bar{f}(\cdot)$ in (4) the method of integral SM will be used. For, let us first redefine the control to be

$$u(t) = u_0(t) + u_1(t) + u_2(t) \tag{5}$$

where $u_1 \in \mathbb{R}^m$ will be designed to reject the perturbation term, $\bar{f}(\cdot)$; $u_2 \in \mathbb{R}^m$ is chosen to cancel the delay term, $\bar{C}x(t-\tau_1)$, i.e.

$$u_2(t) = -\bar{C}x(t-\Delta),\ \Delta = \tau_1 - \tau_2 \tag{6}$$

and $u_0 \in \mathbb{R}^m$ is the nominal part of the control. Substituting (5) and (6) into (4) yields

$$\dot{x}(t) = Ax(t) + B[u_0(t-\tau_2) + u_1(t-\tau_2) + \bar{f}(x(t),t)]. \tag{7}$$

A predictor is designed as

$$\xi(t) = e^{A\tau_2}x(t) + \int_{-\tau_2}^{0} e^{-A\theta}Bu_0(t+\theta)d\theta \tag{8}$$

with a predictive state $\xi(t) \in \mathbb{R}^n$. Now define a sliding variable $s(t)$ of the form

$$s(t) = G\xi(t) + w(t) \tag{9}$$

where $s \in \mathbb{R}^m$, $G \in \mathbb{R}^{m\times n}$ is a design matrix, and

$$\dot{w}(t) = -G[A\xi(t) + Bu_0(t)],\quad w(0) = -G\xi(0). \tag{10}$$

Taking the time derivative of (9) results in

$$\dot{s}(t) = G[e^{A\tau_2}\dot{x}(t) + A\int_{-\tau_2}^{0} e^{-A\theta}Bu_0(t+\theta)d\theta \\ +Bu_0(t) - e^{A\tau_2}Bu_0(t-\tau_2)] + \dot{w}(t). \tag{11}$$

Using (7) in (11) and then (8) yields

$$\dot{s}(t) = G[A\xi(t) + Bu_0(t)] \\ +Ge^{A\tau_2}B[u_1(t-\tau_2) + \bar{f}(x(t),t)] + \dot{w}(t).$$

Choosing $G = B^Te^{-A\tau_2}$ and using (10) we have

$$\dot{s}(t) = M[u_1(t-\tau_2) + \bar{f}(x(t),t)] \tag{12}$$

where $M = B^TB > 0$. To eliminate the input delay in (12) the following predictor is used:

$$\sigma(t) = s(t) + M\int_{-\tau_2}^{0} u_1(t+\theta)d\theta.$$

The straightforward calculations via (12) gives

$$\dot{\sigma}(t) = \dot{s}(t) + M[u_1(t) - u_1(t-\tau_2)] \\ = M[u_1(t) + \bar{f}(x(t),t)]. \tag{13}$$

To induce a sliding motion on the sliding manifold $\sigma(t) = 0$ the control component u_1 is selected as

$$u_1(t) = -\rho_1(x(t), x(t-\tau(t)), t)\frac{\sigma(t)}{\|\sigma(t)\|} \tag{14}$$

where $\rho_1(x(t), x(t-\tau(t)), t)$ is a positive scalar function for control gain.

Taking the time derivative of a Lyapunov functional candidate $V_\sigma(t) = \frac{1}{2}\sigma^T(t)M^{-1}\sigma(t)$ along the trajectories of (13) with control (14) and using (3), yields

$$\dot{V}_\sigma(t) \leq -\|\sigma(t)\|\,[\rho_1(x(t), x(t-\tau_1,t) - \alpha(x(t),t)$$

If we choose the control gain as

$$\rho_1(x(t), x(t-\tau(t),t) - \alpha(x(t),t) \geq \rho_0 > 0 \tag{15}$$

(ρ_0 is a constant) then $\sigma(t)$ vanishes and SM motion occurs on manifold $\sigma(t) = 0$ in finite time.

2.2 Sliding mode dynamics

The delayed system (7) is transformed by (8) into a system which is delay free in the nominal part u_0 of the control:

$$\dot{\xi}(t) = A\xi(t) + Bu_0(t) + e^{A\tau_2}B[u_1(t-\tau_2) + \bar{f}(x(t),t)]. \tag{16}$$

Now, formally setting $\dot{\sigma}(t) = 0$ (13) we have

$$\dot{\sigma}(t) = M[u_1(t) + \bar{f}(x(t),t)] = 0. \tag{17}$$

Solving (17) for u_1 shows that

$$u_{1eq}(t) = -\bar{f}_1(x(t),t) \tag{18}$$

where u_{1eq} is the equivalent control value for u_1. Substituting (18) into (16), the SM motion for $\xi(t)$ on $\sigma(t) = 0$ is described by following perturbed system:

$$\dot{\xi}(t) = A\xi(t) + Bu_0(t) + e^{A\tau_2}B\Delta f_{\tau,eq}(t) \tag{19}$$

where

$$\Delta f_{\tau,eq}(t) = \bar{f}_1(x(t),t) - [\bar{f}_1(x(t-\tau_2),t) \\ = -u_{1eq}(t) + u_{1eq}(t-\tau_2) \tag{20}$$

It implies that that:

1. The predictor (8) enables to conserve the matching condition (2) with respect to the part of the control, namely u_1.

2. With the equivalent control $u_{1eq}(t)$ we can cancel the perturbation term in the system (19) at time t but not at time $t+\tau_2$. However, due to this cancellation at time t the system (19) contents the perturbation of $O(\tau_2)$ order only.

The nominal component of the control, $u_0(t)$ in (19) now is selected as

$$u_0(t) = K_0\xi(t) \tag{21}$$

where $K_0 \in \mathbb{R}^{n\times n}$ is chosen under the assumption A1 such that the matrix $(A + BK_0)$ be Hurwitz.

However, instead of the system (19) we have to analyze the behavior of the original predicted variable $\zeta(t) = x(t+\tau_2)$. Using (8) this variable can be defined on $\sigma(t) = 0$ as

$$\zeta(t) = \xi(t) + \int_{-\tau_2}^{0} e^{-A\theta}B\Delta f_{\tau,eq}(t+\theta)d\theta. \tag{22}$$

The dynamics for $\zeta(t)$ on $\sigma(t) = 0$ are given by

$$\dot{\zeta}(t) = A\zeta(t) + Bu_0(t) + B\Delta f_{\tau,eq}(t+\tau_2) \tag{23}$$

where $\Delta f_{\tau,eq}(t+\tau_2)$ is the predicted value for $\Delta f_{\tau,eq}(t)$ defined in (20), and it can be expressed as

$$\Delta f_{\tau,eq}(t+\tau_2) = \bar{f}_1(x(t+\tau_2),t) - \bar{f}_1(x(t),t) = \bar{f}_1(\zeta(t),t) - \bar{f}_1(\zeta(t-\tau_2),t)$$

Expressing

$$\xi(t) = \zeta(t) - \int_{-\tau_2}^{0} e^{-A\theta} B \Delta f_{\tau,eq}(t+\theta) d\theta \zeta(t)$$

from (22) and substituting it then in the nominal control (21), i.e. $u_0(t) = K_0\xi(t) = K_0\zeta(t) - K_0 \int_{-\tau_2}^{0} e^{-A\theta} B\Delta f_{\tau,eq}(t+\theta)d\theta$ and then into (23) yields

$$\dot{\zeta}(t) = \bar{A}\zeta(t) + B[\Delta f_{\tau,eq}(t+\tau_2) - K_0 \int_{-\tau_2}^{0} e^{-A\theta} \Delta f_{\tau,eq}(t+\theta)d\theta] \quad (24)$$

with $\bar{A} = (A + BK_0)$. To study behavior of $\zeta(t)$ we use a Lyapunov function $V_\zeta(t) = \frac{1}{2}\zeta^T(t)P\zeta(t)$ with P positive definite solution of the Lyapunov equation

$$\bar{A}^T P + P\bar{A} = -I_n$$

where I_n is the identity matrix. Differentiating V_ζ along the trajectories of (24) yields

$$\dot{V}_\zeta(t) \le -\zeta^T(t)\zeta(t) + 2\zeta^T(t)PB[\Delta f_{\tau,eq}(t) - K_0 \int_{-\tau_2}^{0} e^{-A\theta} \Delta f_{\tau,eq}(t+\theta)d\theta]$$

By assumption A2 there is a constant $q_1 > 0$ such that $\|\Delta f_{\tau,eq}(t)\| \le q_1\tau_2$. Moreover, $\|e^{-At}\| \le q_2 e^{\lambda_{\max}(A)t}$, $q_2 > 0$ and therefore $\left\| K_0 \int_{-\tau_2}^{0} e^{-A\theta} \Delta f_{\tau,eq}(t+\theta)]d\theta \right\| \le q_3\tau_2$ with $q_3 = \frac{q_1 q_2}{\lambda_{\max}(A)} \|K_0\| (1 + e^{-\lambda_{\max}(A)\tau_2})$. Thus

$$\dot{V}_\zeta(t) \le -(1-\beta)\|\zeta(t)\|^2 - \beta\|\zeta(t)\|^2 + 2\|\zeta(t)\| \|PB\| (q_1+q_3)\tau_2 \le -(1-\beta)\|\zeta(t)\|^2, \quad \forall\ \|\zeta(t)\| \ge \mu$$

with

$$\mu = \frac{2q_0\tau_2 \|PB\|}{\beta},\ 0 < \beta < 1 \quad (25)$$

where $q_0 = q_1 + q_3$. Therefore, a solution of the perturbed system (24) is ultimately bounded by

$$\|\zeta(t)\| \le \sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}}\ \mu. \quad (26)$$

The results obtained are stated as follows.

Theorem 1. *Let conditions A1, A2, A3 and (15) be satisfied. Then a solution of the system (1) closed by the following control:*

$$u(t) = K_0\xi(t) - \bar{C}x(t-\Delta) - \rho_1(x(t), x(t-\tau(t)), t)\frac{\sigma(t)}{\|\sigma(t)\|}$$

with $(A + BK_0)$ Hurwitz matrix, is ultimately bounded by (26).

It should be noted that the condition A3 can be relaxed if the system (1) can be presented in BC-form with delay.

3. DELAY BLOCK COMPENSATION

In this section, a state feedback control law is developed for a class of system (1) which can be presented in BC-form with delay consisting of r blocks:

$$\begin{aligned}
\dot{x}_1(t) &= A_{11}x_1(t) + C_{11}x_1(t-\tau_1) + B_1x_2(t-\tau_1)\\
\dot{x}_2(t) &= A_{21}x_1(t) + A_{22}x_2(t) + C_{21}x_1(t-\tau_1)\\
&\quad + C_{22}x_2(t-\tau_1) + B_2x_3(t-\tau_1)\\
\dot{x}_i(t) &= \sum_{j=1}^{i} A_{ij}x_j(t) + \sum_{j=1}^{i} C_{ij}x_j(t-\tau_1) \qquad (27)\\
&\quad + B_ix_{i+1}(t-\tau_1),\ i = 3,...,r-1\\
\dot{x}_r(t) &= \sum_{j=1}^{r} A_{ij}x_j(t) + \sum_{j=1}^{r} C_{ij}x_j(t-\tau_1)\\
&\quad + B_r[u(t-\tau_1) + \bar{f}(x(t),t)]
\end{aligned}$$

where $\bar{x}(t) = [x_1(t), ..., x_r(t)]^T$, $\text{rank}B_i = \dim(x_i) = n_i$, $i = 1,...,r$ and $\sum_{i=1}^{r} n_i = n$. The integers $n_1, n_2, ..., n_r$ set the structure of the system and satisfy the following condition $m \ge n_1 \ge n_2 \ge \cdots \ge n_r$. In this paper we consider the case $n_i = m,\ i = 1,...,r$ or $n = r \times m$.

A control strategy for (27) can be designed considering x_{i+1} as a fictitious control vector in the i^{th} block and designing an appropriate predictor for each block. This procedure is outlined in the following.

Step 1. For notational convience let

$$C_{11}x_1(t-\tau_1) := f_1(t-\tau_1).$$

Rewriting the first block of (27) as

$$\dot{x}_1(t) = A_{11}x_1(t) + f_1(t-\tau_1) + B_1x_2(t-\tau_1) \quad (28)$$

and choosing fictitious control $x_2(t)$ in (28) as

$$x_2(t) = B_1^{-1}[-f_1(t) + \varphi_1(t)] \quad (29)$$

results in

$$\dot{x}_1(t) = A_{11}x_1(t) + \varphi_1(t-\tau_1) \quad (30)$$

where $\varphi_1 \in \mathbb{R}^m$ is a new variables vector. Now, defining $z_1(t) = x_1(t+\tau_1)$, a predictor for the block (30) with new fictitious control input φ_1 is designed of the form

$$z_1(t) = e^{A_{11}\tau_1}x_1(t) + \int_{-\tau_1}^{0} e^{-A_{11}\theta}\varphi_1(t+\theta)d\theta. \quad (31)$$

Thus

$$\dot{z}_1(t) = A_{11}z_1(t) + \varphi_1(t). \quad (32)$$

Now, choose the fictitious control $\varphi_1(t)$ in (32) of the form

$$\varphi_1(t) = (K_1 - A_{11})z_1(t) + z_2(t) \quad (33)$$

where $z_2 \in \mathbb{R}^m$ is a new variables vector, and K_1 is a design matrix. Thus

$$\dot{z}_1(t) = K_1z_1(t) + z_2(t). \quad (34)$$

Remark 1. Note that in the case matrix A_{11} is Hurwitz, fictitious control $\varphi_1(t)$ can be chosen as $\varphi_1(t) = K_1 z_1(t) + z_2(t)$ to assign a desired spectrum of matrix $(A_{11} + K_1)$.

The algorithm (29) and (33) defines the transformation for $z_2(t)$ as

$$z_2(t) = B_1 x_2(t) + f_1(t) - (K_1 - A_{11}) z_1(t) \quad (35)$$
$$= B_1 x_2(t) + C_{11} x_1(t) - (K_1 - A_{11}) z_1(t).$$

Step 2. Taking the derivative of (35) gives

$$\dot{z}_2(t) = \bar{A}_{22} z_2(t) + f_2(t - \tau_1) + \bar{B}_2 x_3(t - \tau_1) \quad (36)$$

where , $\bar{A}_{22} = [A_{21} + B_1 A_{22} B_1^{-1}]$ and $\bar{B}_2 = B_1 B_2$. The fictitious control $x_3(t)$ in (36) is chosen as

$$x_3(t) = \bar{B}_2^{-1}[-f_2(t) + \varphi_2(t)] \quad (37)$$

with $\varphi_2(t) \in \mathbb{R}^m$ a new variables vector. Then the block (36) with (37) becomes

$$\dot{z}_2(t) = \bar{A}_{22} z_2(t) + \varphi_2(t - \tau_1). \quad (38)$$

As on the first step, a predictor for $\bar{z}_2(t) = z_2(t + \tau)$ can be designed similar to (31)

$$\bar{z}_2(t) = e^{\bar{A}_{22}\tau_1} z_2(t) + \int_{-\tau_1}^{0} e^{-\bar{A}_{22}\theta} \varphi_2(t + \theta) d\theta.$$

Thus,

$$\dot{\bar{z}}_2(t) = \bar{A}_{22} \bar{z}_2(t) + \varphi_2(t). \quad (39)$$

Now, if we choose $\varphi_2(t)$ in (39) similar to (33) as

$$\varphi_2(t) = (K_2 - \bar{A}_{22}) \bar{z}_2(t) + z_3(t) \quad (40)$$

where $z_3 \in \mathbb{R}^m$ is a new variables vector, and K_2 is a design matrix, then, equation (39) with (40) takes the same form of equation (34), namely

$$\dot{\bar{z}}_2(t) = K_2 \bar{z}_2(t) + z_3(t).$$

Using (37) and (40) yields

$$z_3(t) = \bar{B}_2 x_3(t) + f_2(t) - (K_2 - \bar{A}_{22}) \bar{z}_2(t)$$

where $f_2(t) = \bar{A}_{21} x_1(t + \tau_1) + \bar{C}_{21} x_1(t) + \bar{C}_{22} z_2(t) + \bar{A}_{21}^1 z_1(t + \tau_1)$.

This procedure may be performed iteratively obtaining on the i^{th} step, $i = 3, ..., r - 1$ equation for variable $z_i(t)$ as

$$\dot{z}_i(t) = \bar{A}_{ii} z_i(t) + f_i(t - \tau_1) + \bar{B}_i x_{i+1}(t - \tau_1)$$

Choosing

$$x_{i+1}(t) = \bar{B}_i^{-1}[-f_i(t) + \varphi_i(t)] \quad (41)$$

with new fictitious input variable $\varphi_i(t)$ results in

$$\dot{z}_i(t) = \bar{A}_{ii} z_i(t) + \varphi_i(t - \tau_1).$$

Design of the predictor

$$\bar{z}_i(t) = e^{\bar{A}_{ii}\tau_1} z_i(t) + \int_{-\tau_1}^{0} e^{-\bar{A}_{ii}\theta} \varphi_i(t + \theta) d\theta$$

gives the following equation for predicted variable $\bar{z}_i(t) = z_i(t + \tau_1)$:

$$\dot{\bar{z}}_i(t) = \bar{A}_{ii} \bar{z}_i(t) + \varphi_i(t)$$

Next choose

$$\varphi_i(t) = (K_i - \bar{A}_{ii}) z_i(t) + z_{i+1}(t) \quad (42)$$

we have the desired dynamics for $\bar{z}_i(t)$:

$$\dot{\bar{z}}_i(t) = K_i \bar{z}_i(t) + z_{i+1}(t)$$

where $z_{i+1}(t)$ is a new variables vector and K_i is a design matrix.

To this end, an expression for the new variable $z_{i+1}(t)$ can be now obtained from (41) and (42) as

$$z_{i+1}(t) = \bar{B}_i x_{i+1}(t) + f_i(t) - (K_i - \bar{A}_{ii}) z_i(t)$$

Thus the new variables, obtained from this procedure form a transformation given by

$$\begin{aligned} z_1(t) &= x_1(t + \tau_1) \\ z_2(t) &= B_1 x_2(t) + f_1(t) - (K_1 - A_{11}) z_1(t) \\ z_i(t) &= \bar{B}_{i-1} x_i(t) + f_{i-1}(t) \\ &\quad - (K_{i-1} - \bar{A}_{i-1,i-1}) \bar{z}_{i-1}(t) \\ i &= 3, ..., r. \end{aligned} \quad (43)$$

where $\bar{z}_i(t) = z_i(t + \tau), \quad i = 2, ..., r.$

On the last step, the system (27) can be described in the new variables $z_i(t)$, $i = 1, ..., r$ (43) of the form

$$\begin{aligned} \dot{z}_1(t) &= K_1 z_1(t) + z_2(t) \\ \dot{\bar{z}}_i(t) &= K_i \bar{z}_i(t) + z_{i+1}(t), \ i = 2, ..., r - 1 \\ \dot{z}_r(t) &= \bar{A}_{r,r} z_r(t) + f_r(t) + \bar{B}_r[u(t - \tau_2) + \bar{f}(x(t), t] \end{aligned} \quad (44)$$

where K_i is a design matrix. Now redesigning the control $u(t)$ similar to (5), i.e., $u(t) = u_0(t) - u_1(t) - u_2(t)$, we select $u_2(t) = -\bar{B}_r^{-1} f_r(t)$ that yields

$$\begin{aligned} \dot{z}_r(t) = \bar{A}_{r,r} z_r(t) + \bar{B}_r[&u_0(t - \tau_2) \\ &+ u_1(t - \tau_2) + \bar{f}(x(t), t)]. \end{aligned} \quad (45)$$

Designing a predictor

$$\bar{z}_r(t) = e^{\bar{A}_{r,r}\tau_2} z_r(t) + \int_{-\tau_2}^{0} e^{-\bar{A}_{r,r}\theta} \bar{B}_r u_0(t + \theta) d\theta$$

and choosing a switching function $s_r(t) = G_r \bar{z}_r(t) + w_r(t)$, $s_r \in \mathbb{R}^m$ with

$$\dot{w}_r(t) = -G_r[\bar{A}_{r,r} z_r(t) + \bar{B}_r u_0(t - \tau_2)] \quad (46)$$

and $w_r(0) = G_r z_r(0)$. we have $\dot{s}_r(t) = -M_r[u_1(t - \tau_2) - \bar{f}(x(t), t))]$ where $M_r = \bar{B}_r^T \bar{B}_r > 0$, $G_r = \bar{B}_r^T$. The control u_1 is designed as

$$u_1(t) = -\rho_1(x(t), x(t - \tau(t)), t) \frac{s_r(t + \tau_2)}{\|s_r(t + \tau_2)\|}$$

with $s_r(t + \tau_2) = G_r \bar{z}_r(t) + \upsilon_r(t)$, and $\upsilon_r(t) = w_r(t + \tau_2)$ satisfies the following equation:

$$\upsilon_r(t) = w_r(t) - G_r \int_{-\tau_2}^{0} \left[\bar{A}_{rr} \bar{z}_r(t + \theta) + \bar{B}_r u_0(t + \theta)\right] d\theta.$$

To force sliding mode on $s_r = 0$, the condition (15) should be satisfied. Once sliding mode occurs and the system is confined to the manifold $s_r = 0$, a SM equation with the equivalent control $u_{1eq}(t - \tau_2) = \bar{f}(x(t), t)$ and nominal control

$$u_0(t) = \bar{B}_r^{-1}(K_r - \bar{A}_{r,r}) \bar{z}_r(t)$$

with $K_r \in \mathbb{R}^{n \times n}$ is governed by

$$\begin{aligned} \dot{z}_1(t) &= K_1 z_1(t) + z_2(t) \\ \dot{z}_i(t) &= K_i z_i(t) + z_{i+1}(t - \tau_1), i = 2, ..., r - 1 \\ \dot{z}_r(t) &= K_r z_r(t) + e^{A_{r,r}\tau_2} \bar{B}_r \Delta f_{\tau,eq}(t). \end{aligned} \quad (47)$$

Theorem 2. *Let conditions A4, A2 and (15) be satisfied. Then the control*

$$u(t) = \bar{B}_r^{-1}(K_r - \bar{A}_{r,r})\bar{z}_r(t) - \bar{B}_r^{-1}f_r(t) \\ -\rho_1(x(t), x(t-\tau(t)), t)\frac{s_r(t+\tau_2)}{\|s_r(t+\tau_2)\|}$$

with K_i, $i = 1, ..., r$ Hurwitz matrices stabilizes the system (27), and a solution of the closed-loop system (27) and (26) is ultimately bounded.

4. AN APPLICATION EXAMPLE

In this section, the proposed control method is applied to control a high-speed closed air wind tunnel. The main objective of the control is to provide a fast response so to reduce the cost of liquid nitrogen losses during the transient regimes. A linearized model of the wind tunnel is given by Manitius [1984]

$$\begin{aligned} \dot{x}_1(t) &= a_{11}x_1(t) + b_1x_2(t-\tau_1) \\ \dot{x}_2(t) &= x_3(t) \\ \dot{x}_3(t) &= a_{31}x_2(t) + a_{32}x_3(t) + b_3u(t-\tau_2) + f(t) \end{aligned} \tag{48}$$

where the state variables x_1, x_2 and x_3 present the Mach number, actuator position guide vane angle in a driving fan and actuator rate, respectively, $f(t) = 2 + 2cos(0.5t)$, $a_{11} = -0.509$, $b_1 = 0.059$, $a_{31} = -36$, $a_{32} = -96$, $b_3 = 36$ and $\tau_2 = 0.11s$. The nominal delay $\tau_1 = 0.33s$ represents the time of the transport between the fan and the test section.

A predictor is designed for the first block of (48) similar to (31) as

$$z_1(t) = e^{a_{11}\tau}x_1(t) + \int_{-\tau_1}^{0} e^{-a_{11}\theta}b_1x_2(t+\theta)d\theta \tag{49}$$

Then

$$\dot{z}_1(t) = e^{a_{11}\tau}\dot{x}_1(t) + a_{11}\int_{-\tau_1}^{0} e^{-a_{11}\theta}b_1x_2(t+\theta)d\theta + \\ b_1x_2(t) - e^{a_{11}\tau}b_1x_2(t-\tau)$$

or

$$\dot{z}_1(t) = a_{11}z_1(t) + b_1x_2(t)$$

Introducing the desired dynamics as $(a_{11} - k_1)z_1$, $k_1 > 0$, the transformation (35) is now defined by

$$z_2(t) = k_1z_1(t) + b_1x_2(t)$$

Then the first block of (48) is represented in the new variables $z_1(t)$ and $z_2(t)$ as

$$\dot{z}_1(t) = (a_{11} - k_1)z_1(t) + z_2(t)$$

At the second step, taking the derivative of $z_2(t)$

$$\dot{z}_2(t) = k_1(a_{11} - k_1)z_1(t) + k_1z_2(t) + b_1x_3(t)$$

Choosing the fictitious control $x_3(t)$ as

$$x_3(t) = -b_1^{-1}[k_1(a_{11} - k_1)z_1(t) - (k_1 + k_2)z_2(t) + z_3(t)] \tag{50}$$

yields

$$\dot{z}_2(t) = -k_2z_2(t) + z_3(t), k_2 > 0.$$

From (50) the new variable $z_3(t)$ can be obtained as $z_3(t) = \tilde{a}_{31}z_1(t) + \tilde{a}_{32}z_2(t) + b_1x_3(t)$, then

$$\dot{z}_3(t) = \bar{a}_{31}z_1(t) + \bar{a}_{32}z_2(t) + \bar{a}_{33}z_3(t) \\ +\bar{b}_3[u_0(t-\tau_2) - u_1(t-\tau_2) + u_2(t-\tau_2)] + f(t)$$

where where $\tilde{a}_{31} = k_1(a_{11} - k_1)$ and $\tilde{a}_{32} = (k_1 + k_2)$, $\bar{a}_{31} = \tilde{a}_{31}(a_{11} - k_1) - k_1a_{31} - a_{32}k_1(a_{11} - k_1)$, $\bar{a}_{32} = (\tilde{a}_{31} - \tilde{a}_{32}k_2 + a_{31} + a_{32}(k_1 + k_2))$, $\bar{a}_{33} = \tilde{a}_{32} + a_{32}$ and $\bar{b}_3 = b_1b_3$.

At the last step, redesigning the control $u(t)$ as $u(t) = u_0(t) - u_1(t) - u_2(t)$, we choose first

$$u_2(t) = -\bar{b}_3^{-1}[\bar{a}_{31}z_1(t+\tau_2) + \bar{a}_{32}z_2(t+\tau_2)] \tag{51}$$

that yields

$$\dot{z}_3(t) = \bar{a}_{33}z_3(t) + \bar{b}_3[u_0(t-\tau_2) - u_1(t-\tau_2)] + f(t).$$

The predicted variables $z_1(t+\tau_2) = x_1(t+\tau_1+\tau_2)$ and $z_2(t+\tau_2) = k_1z_1(t+\tau_2) + b_1x_2(t+\tau_2)$ used in (51) can be obtained by using the predictor (49) on the $[0, -(\tau_1+\tau_2)]$ integration interval, and the predictor $x_2(t+\tau_2) = x_2(t) + \int_{-\tau_2}^{0} x_3(t+\theta)d\theta$, respectively. A sliding variable is selected as

$$\begin{aligned} s_3(t) &= \xi_3(t) + w_3(t) \\ \xi_3(t) &= e^{\bar{a}_{33}\tau_2}z_3(t) + \int_{-\tau_2}^{0} e^{-\bar{a}_{33}\theta}\bar{b}_3u_0(t+\theta)d\theta \\ \dot{w}_3(t) &= -[\bar{a}_{33}z_3(t) + \bar{b}_3u_0(t-\tau_2)], \quad w_3(0) = \xi_3(0). \end{aligned}$$

Then

$$\dot{s}_3(t) = e^{\bar{a}_{33}\tau_2}[\bar{b}_3u_1(t-\tau_2) + f(t)]. \tag{52}$$

Using the predictive variable

$$\sigma_3(t) = s_3(t) + e^{\bar{a}_{33}\tau_2}\bar{b}_3\int_{-\tau_2}^{0} u_1(t+\theta)d\theta,$$

the system (52) is transformed into the following delay-free system

$$\dot{\sigma}_3(t) = e^{\bar{a}_{33}\tau_2}[\bar{b}_3u_1(t) + f(t)].$$

The unit control

$$u_1(t) = -\rho_3\frac{\sigma_3(t)}{|\sigma_3(t) + \delta|}, \quad \rho_3 > 0$$

with $\rho_3 > |f(t)|$ ensures the convergence of $\sigma_3(t)$ in a finite time. Then it follows that $\bar{b}_3u_{1eq}(t) = -f(t)$. Selecting

$$u_0(t) = -k_3\xi_3(t)$$

and substituting $u_{1eq}(t) = -\bar{b}_3^{-1}\ f(t)$, in the SM equation

$$\dot{\xi}_3(t) = \bar{a}_{33}\xi_3(t) + \bar{b}_3u_0(t) + e^{\bar{a}_{33}\tau_2}[\bar{b}_3u_{1eq}(t-\tau_2) + f(t)]$$

yields

$$\dot{\xi}_3(t) = (\bar{a}_{33} - \bar{b}_3k_3)\xi_3(t) + e^{\bar{a}_{33}\tau_2}\Delta f_{\tau,eq}(t)$$

where $\Delta f_{\tau,eq}(t) = f(t) - f(t-\tau_2)$ and $\bar{a}_{33} - \bar{b}_3k_3 < 0$. The predicted variable $\zeta_3(t) = z_3(t+\tau_2)$ can be defined on $\sigma_3(t) = 0$ as

$$\zeta_3(t) = \xi_3(t) + \bar{b}_3\int_{-\tau_2}^{0} e^{-\bar{a}_{33}\theta}\Delta f_{\tau,eq}(t+\theta)d\theta.$$

Then a SM motion on $\sigma_3(t) = 0$ is described by

$$\begin{aligned} \dot{z}_1(t) &= (a_{11} - k_1)_1z_1(t) + z_2(t) \\ \dot{z}_2(t) &= -k_2z_2(t) + \zeta_3(t-\tau_2) \\ \dot{\zeta}_3(t) &= (\bar{a}_{33} - \bar{b}_3k_3)\zeta_3(t) + \Delta f_{\tau,eq}(t+\tau_2) \\ &\quad -\bar{b}_3k_3\int_{-\tau_2}^{0} e^{-\bar{a}_{33}\theta}\Delta f_{\tau,eq}(t+\theta)]d\theta \end{aligned} \tag{53}$$

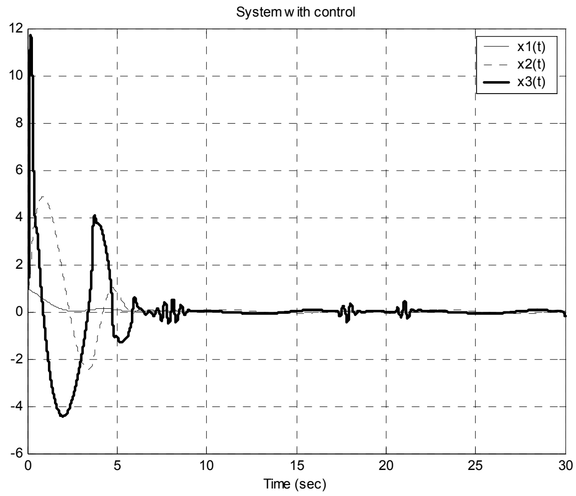

Fig. 1. Response of states the system with control .

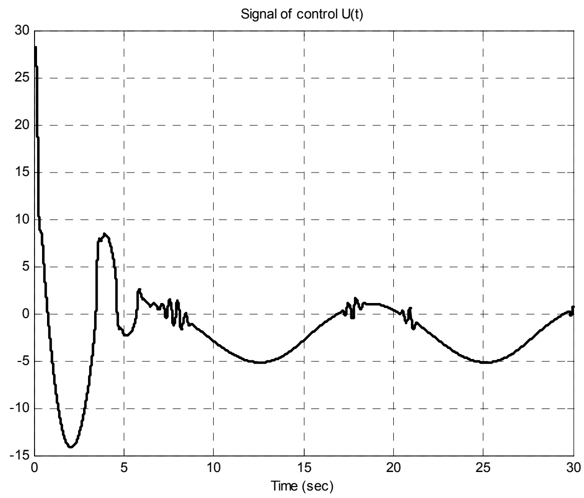

Fig. 2. Response of signal of control.

and a solution of this system is ultimately bounded. Thus, the following resulting control law

$$u(t) = u_0(t) + u_1(t) + u_2(t) = -k_0\xi_3(t)$$
$$-k_r\,|\sigma_3(t)|^{\frac{1}{2}}\,sign(\sigma_3(t))$$
$$-\bar{b}_3^{-1}[\bar{a}_{31}z_1(t+\tau_2) + \bar{a}_{32}z_2(t+\tau_2)].$$

stabilizes the system (48). For the simulation, the values of the control parameters k_0, k_1, k_2 and k_r are adjusted to 0.5, 1, 2 and .5, respectively.The responses of the states with control and input control are shown in Figures 1 and 2.

5. CONCLUSIONS

The decomposition deadtime compensation method based on the integral SM control, has been formulated for linear time-delay systems with uncertainty. For systems which can be presented in the BC form, the proposed predictor-based control design procedure has step-by step character that simplifies the solution of the problem. This method enables to solve one of the classical problem design of pole placement state feedback for linear systems with delayed state and control input.

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