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Stability Analysis of a Closed-loop Thermoforming Reheat Process Using an Affine Quadratic Stability Test

Hamed Azarnoush, Benoit Boulet

Centre for Intelligent Machines
McGill University, Montreal, Quebec, Canada, H3A 2A7
hamed@cim.mcgill.ca, boulet@cim.mcgill.ca

Abstract: The process of manufacturing plastic parts by heating polymer sheets and forming them on a mold is called thermoforming. The heating stage of the thermoforming process is nonlinear and parameter-varying. The heater temperature set-points are usually determined by trial and error. A control design for this system can improve quality, reduce scrap and allow for temperature zoning. In this paper, the problem of stability analysis for a thermoforming process controlled by a static output feedback controller is addressed. An affine quadratic stability (AQS) test is chosen for this analysis. The AQS test requires a number of linear matrix inequalities (LMIs) to hold in order for the system to be stable. There is only one varying parameter in the thermoforming oven model, and as a result the number of LMIs to be computed is limited to five, which makes the AQS test practical. A parameter-dependent Lyapunov function is developed to prove the stability of the system.

1. INTRODUCTION

Forming operations are used to manufacture parts in several different industries, such as aeronautics, automotive, electronics, etc. Thermoforming is a process that produces hollow tub-shaped parts (Throne, 1996 and Moore *et. al*, 2002a). Currently, this process is operated manually and through trial and error. Many steps need to be taken to achieve accurate control of thermoforming. The model for the reheat stage of the thermoforming process is nonlinear (Moore *et al.*, 2002b and Gauthier *et al.*, 2005) and has time-varying parameters. Therefore, one may need to use stability analysis techniques that are applicable to systems with time-varying parameters. Many techniques have been developed by different authors. Zames (1966) was amongst the first authors to consider the stability of systems with nonlinear time-varying feedback. The development of parameter-dependent Lyapunov functions has also received a lot of attention. A parameter-dependent Lyapunov function in the form of $V(x) := x^T P(\theta_1, \theta_2, ..., \theta_n)x$ was considered by Barmish *et al.* (1986), with P defined as $P(\theta_1, \theta_2, ..., \theta_n) := \sum_{i=1}^{n} \theta_i P_i$ where $\theta_1, \theta_2, ..., \theta_n$ are the varying parameters and the P_i 's correspond to vertices of a polytope of uncertain matrices with vertices $A_1,...,A_n$. A few years later, Leal *et al.* (1990) considered a Lyapunov matrix P in the form of $P(\theta_1, \theta_2, ..., \theta_n) := P_0 + \sum_{i=1}^{n} \theta_i P_i$ where P_0 corresponds to the nominal model of the system and P_i is a first-order perturbation of P_0. The criterion developed by Popov (1962) is also based on a parameter-varying Lyapunov function. Haddad *et al.* developed a framework for parameter-dependent Lyapunov function as a less conservative refinement of fixed Lyapunov function. This framework can be considered as a reinterpretation of classical Popov criterion. In the work presented by Gahinet *et al.* (1996), the authors successfully expressed conditions required for affine quadratic stability (AQS) of a system in terms of linear matrix inequalities (LMIs). In this paper, these conditions form an LMI feasibility problem to check the stability of the thermoforming system.

This paper is organized as follows. In Section 2, the problem is formulated and the model to be used is introduced. Section 3 expresses the conditions for AQS in the form of a finite set of LMIs. In Section 4, the model is transformed into a form that fits the AQS test and a solution for the LMI problem is found, implying the system is AQS. Section 5 discusses the simulation results which confirm the theoretical finding in Section 4. Concluding remarks are given in Section 6.

2. PROBLEM STATEMENT AND MODELING

Thermoforming machines are typically composed of an oven for sheet reheat and a vacuum or pressure forming station for shaping the plastic part. Figure 1 shows the AAA thermoforming machine at the Industrial Materials Institute (IMI) of the National Research Council of Canada whose model was used in this paper. The machine was retrofitted with infrared sensors to measure sheet surface temperatures in real time.

Figure 1: AAA thermoforming machine at NRC-IMI

An appropriate modeling of the thermoforming reheat process is an important step towards stabilization and control. The model applied in this paper is based on the model suggested by Moore (2002b). In this model, as shown in Figure 2, the surface of the plastic sheet is divided into S zones, and the thickness is divided into N layers each of which corresponding to a node for zone k. Lateral heat transfer between adjacent zones is neglected as it is small compared to heat propagating perpendicularly to the sheet. Node *1* is located at the upper surface of the sheet and node N at the lower surface. It is standard to have a node at the center of each layer.

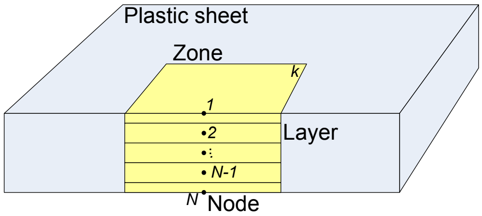

Figure 2: Depiction of zones, nodes and layers

The radiant energy absorption is modeled and added to Moore's model to obtain the model discussed by Gauthier *et al.* (2005). For a plastic sheet with five layers, the model for zone i is given by:

$$\begin{bmatrix} \dot{x}_1^i \\ \dot{x}_2^i \\ \dot{x}_3^i \\ \dot{x}_4^i \\ \dot{x}_5^i \end{bmatrix} = \begin{bmatrix} -2a(h+b) & 2ab & 0 & 0 & 0 \\ ab & -2ab & ab & 0 & 0 \\ 0 & ab & -2ab & ab & 0 \\ 0 & 0 & ab & -2ab & ab \\ 0 & 0 & 0 & 2ab & -2a(h+b) \end{bmatrix} \begin{bmatrix} x_1^i \\ x_2^i \\ x_3^i \\ x_4^i \\ x_5^i \end{bmatrix}$$
$$+ \begin{bmatrix} 2ah & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 2ah \end{bmatrix} \begin{bmatrix} T_{\infty_{top}} \\ T_{\infty_{bottom}} \end{bmatrix} - a \begin{bmatrix} 2c_1F_{T_i} & 2c_5F_{B_i} \\ c_2F_{T_i} & c_4F_{B_i} \\ c_3F_{T_i} & c_3F_{B_i} \\ c_4F_{T_i} & c_2F_{B_i} \\ 2c_5F_{T_i} & 2c_1F_{B_i} \end{bmatrix} \begin{bmatrix} (x_1^i)^4 \\ (x_5^i)^4 \end{bmatrix} \tag{1}$$
$$+a \begin{bmatrix} 2c_1 & 2c_5 \\ c_2 & c_4 \\ c_3 & c_3 \\ c_4 & c_2 \\ 2c_5 & 2c_1 \end{bmatrix} \begin{bmatrix} \sum_{j=1}^{P_T} F_{T_{ij}} T_{S_{T_j}}^{\;4} \\ \sum_{j=1}^{P_B} F_{B_{ij}} T_{S_{B_j}}^{\;4} \end{bmatrix}$$

In this model, x_j^i is the temperature of the jth node on the ith zone in degrees Celsius, h is the convection factor, $T_{\infty_{top}}$ and $T_{\infty_{bottom}}$ are, respectively, the ambient air temperatures of the top and bottom of the plastic sheet. F_{T_i} and F_{B_i} are parameters dependent on the view factors between the top heaters and the zones, and the bottom heaters and the zones, respectively. $T_{S_{T_j}}$ and $T_{S_{B_j}}$ are the temperatures of the jth top and bottom heaters, respectively. Parameters a, b and c_j depend on the characteristics of the plastic sheet, including physical dimensions, conductivity and emissivity.

The objective here is to control the temperatures of the heaters in a way that the node temperatures converge to a set of desired trajectories. Among the parameters defined in the model, the parameter a is a varying parameter which will attract most of our attention. This parameter is defined as:

$$a = \frac{1}{\rho C_p \Delta x} \tag{2}$$

in which ρ , C_p and Δx are the density, specific heat capacity and the distance between corresponding nodes on two adjacent layers, respectively. The values of all of the parameters are expressed in SI units in Table 1. The specific heat capacity, C_p, varies with temperature of the plastic sheet. Therefore, the control design has to consider the parameter varying aspect of the system as well. An effective control cannot be designed without having some guarantee about the stability of the system. An affine quadratic stability test is used to analyze the stability of the closed-loop process. This test is discussed in the next section.

Table 1: The values of the thermoforming process parameters

Parameter	Value
H	6
B	30
c_1	0.1871×10^{-8}
c_2	0.3498×10^{-8}
c_3	0.3197×10^{-8}
c_4	0.2922×10^{-8}
c_5	0.2671×10^{-8}
ρ	950
Δx	0.003

3. AFFINE QUADRATIC STABILITY (AQS)

Following the work of Gahinet *et al.* (1996), let us consider the following parameter-varying system:

$$\dot{x}(t) = A(\theta)x(t), \quad x(0) = x_0 , \tag{3}$$

where t is the time, x is the state vector, x_0 is the initial value of the state vector, A is the state matrix and θ is the varying parameter vector defined as follows:

$$\theta = [\theta_1, \theta_2, ..., \theta_K] \in \mathbb{R}^K , \tag{4}$$

where K is the number of varying parameters in the system. The state matrix $A(\theta)$ is affinely dependent on the parameters θ_i:

$$A(\theta) = \theta_1 A_1 + \theta_2 A_2 + ... + \theta_K A_K \tag{5}$$

where $A_1, A_2, ..., A_K$ are known fixed matrices. Each parameter θ_i is assumed to be bounded in the interval defined below:

$$\theta_i \in [\underline{\theta}_i, \ \overline{\theta}_i] \tag{6}$$

where $\underline{\theta}_i$ and $\overline{\theta}_i$ are known lower and upper bounds for θ_i . The rate of variation for θ_i is well defined and satisfies:

$$\dot{\theta}_i \in [\underline{\upsilon}_i, \ \overline{\upsilon}_i], \tag{7}$$

where $\underline{\upsilon}_i \le 0 < \bar{\upsilon}_i$ are known lower and upper bounds for $\dot{\theta}$. The upper and lower bounds on the parameters define a hyper-rectangle (or parameter box) whose vertices are defined by the following set:

$$V := \left\{ \left(\omega_1, \omega_2, \dots, \omega_K\right) : \omega_i \in \left\{ \underline{\theta}_i,\ \bar{\theta}_i \right\} \right\} \tag{8}$$

Similarly, the vertices of the parameter box for $\dot{\theta}_i$ are given in the set:

$$R := \left\{ \left(\tau_1, \tau_2, \dots, \tau_K\right) : \tau_i \in \left\{ \underline{\upsilon}_i,\ \bar{\upsilon}_i \right\} \right\} \tag{9}$$

Next, affine quadratic stability (AQS) for this system can be defined.

Definition (AQS)- The linear system

$$\dot{x}(t) = A(\theta(t))x(t), \quad x(0) = x_0 \tag{10}$$

is affinely quadratically stable if there exist K+1 symmetric matrices $P_0, P_1, \dots, P_K$ such that:

$$\begin{aligned} &P(\theta) := \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_K P_K > 0 \\ &L(\theta,\dot{\theta}) := A(\theta)^T P(\theta) + P(\theta)A(\theta) + P(\dot{\theta}) - P_0 < 0 \end{aligned} \tag{11}$$

hold for all admissible trajectories of the parameter vector $\theta = [\theta_1, \theta_2, \dots, \theta_K]$ that satisfy (6) and (7).

According to Theorem 3.2 in Gahinet *et al.* (1996), if we let

$$\theta_{\text{mean}} := \left(\frac{\underline{\theta}_1 + \bar{\theta}_1}{2}, \dots, \frac{\underline{\theta}_K + \bar{\theta}_K}{2} \right), \tag{12}$$

then the system (10) is affinely quadratically stable if $A(\theta_{\text{mean}})$ is stable and there exist K+1 symmetric matrices $P_0, P_1, \dots, P_K$ such that

$$P(\theta) := \theta_1 P_1 + \theta_2 P_2 + \dots + \theta_K P_K \tag{13}$$

satisfies

$$A_i^T P_i + P_i A_i \ge 0 \text{ for } i=1,2,\dots,K, \tag{14}$$

and

$$L(\omega,\tau) := A(\omega)^T P(\omega) + P(\omega)A(\omega) + P(\tau) - P_0 < 0 \tag{15}$$

for all $(\omega,\tau) \in V \times R$.

Inequalities expressed by (14) ensure the multi-convexity which reduces the problem of finding an affine parameter-dependent Lyapunov function to an LMI problem. Therefore, the derivative of the Lyapunov function with respect to time is only tested on a finite number of points, i.e., the vertices of the parameter boxes, indicated by (15). When (14) and (15) are feasible, a Lyapunov function for the system is given by:

$$V(x,\theta) := x^T P(\theta)x. \tag{16}$$

In the next section, it is described how the AQS test can be applied to the thermoforming process.

4. AQS FOR THE THERMOFORMING PROCESS

In the previous section the AQS test was explained. However, the thermoforming oven model described in (1) does not have the form of the system given in (3). Therefore, we need to do some adjustments and consider some assumptions to be able to use the AQS test for sheet reheat. If we assume that the effect of the ambient air temperature is negligible and that the radiant energy does not pass through layers, the following model is obtained:

$$\begin{bmatrix} \dot{x}_1^i \\ \dot{x}_2^i \\ \dot{x}_3^i \\ \dot{x}_4^i \\ \dot{x}_5^i \end{bmatrix} = \begin{bmatrix} -2a(h+b) & 2ab & 0 & 0 & 0 \\ ab & -2ab & ab & 0 & 0 \\ 0 & ab & -2ab & ab & 0 \\ 0 & 0 & ab & -2ab & ab \\ 0 & 0 & 0 & 2ab & -2(h+b) \end{bmatrix} \begin{bmatrix} x_1^i \\ x_2^i \\ x_3^i \\ x_4^i \\ x_5^i \end{bmatrix} - a \begin{bmatrix} 2c_1 F_{T_i} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 2c_1 F_{B_i} \end{bmatrix} \begin{bmatrix} (x_1^i)^4 \\ (x_5^i)^4 \end{bmatrix} + a \begin{bmatrix} 2c_1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 2c_1 \end{bmatrix} \begin{bmatrix} \sum_{j=1}^{P_T} F_{T_{ij}} T_{S_{T_j}}^{\ 4} \\ \sum_{j=1}^{P_B} F_{B_{ij}} T_{S_{B_j}}^{\ 4} \end{bmatrix} \tag{17}$$

Fortunately, the nonlinear part of the model is not significant and linearizing the nonlinear part around some equilibrium point can express the behavior of the model for a rather large class of equilibrium points. Linearizing this model results in the following linear state-space system:

$$\begin{bmatrix} \Delta\dot{x}_1^i \\ \Delta\dot{x}_2^i \\ \Delta\dot{x}_3^i \\ \Delta\dot{x}_4^i \\ \Delta\dot{x}_5^i \end{bmatrix} = a \begin{bmatrix} \begin{pmatrix} -2(h+b) \\ -8c_1 F_{T_i} x_{1eq}^{i\ 3} \end{pmatrix} & 2b & 0 & 0 & 0 \\ b & -2b & b & 0 & 0 \\ 0 & b & -2b & b & 0 \\ 0 & 0 & b & -2b & b \\ 0 & 0 & 0 & 2b & \begin{pmatrix} -2(h+b) \\ -8c_1 F_{B_i} x_{5eq}^{i\ 3} \end{pmatrix} \end{bmatrix} \begin{bmatrix} \Delta x_1^i \\ \Delta x_2^i \\ \Delta x_3^i \\ \Delta x_4^i \\ \Delta x_5^i \end{bmatrix} - a \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{18}$$

where x^i_{jeq} is the equilibrium point for the temperature of the jth node of the ith zone, and

$$\Delta x^i_j = x^i_j - x^i_{jeq}\,, \tag{19}$$

$$\begin{aligned} u_1 &= 2c_1 \sum_{j=1}^{P_T} F_{T_{ij}} T_{S_{T_j}}{}^4, \\ u_2 &= 2c_1 \sum_{j=1}^{P_B} F_{B_{ij}} T_{S_{B_j}}{}^4. \end{aligned} \tag{20}$$

Now, let us assume that the first and last states are available for static output feedback control. These states correspond to the top and bottom layer temperatures of zone i, respectively, which are measurable using infrared sensors on the AAA machine. An output feedback gain matrix K has been designed and tested experimentally as:

$$u = \text{-}K \begin{bmatrix} \Delta x^i_1 \\ \Delta x^i_2 \\ \Delta x^i_3 \\ \Delta x^i_4 \\ \Delta x^i_5 \end{bmatrix}, \tag{21}$$

where

$$K = \begin{bmatrix} k_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & k_2 \end{bmatrix}. \tag{22}$$

where $k_1 = k_2 = 1000$. Thus, the closed-loop state equations can be expressed as follows:

$$\begin{bmatrix} \Delta \dot{x}^i_1 \\ \Delta \dot{x}^i_2 \\ \Delta \dot{x}^i_3 \\ \Delta \dot{x}^i_4 \\ \Delta \dot{x}^i_5 \end{bmatrix} = \tag{23}$$

$$a \begin{bmatrix} \begin{pmatrix} -2(h+b)- \\ 8c_1 F_{T_i} x^i_{1eq}{}^3 \\ +k_1 \end{pmatrix} & 2b & 0 & 0 & 0 \\ b & -2b & b & 0 & 0 \\ 0 & b & -2b & b & 0 \\ 0 & 0 & b & -2b & b \\ 0 & 0 & 0 & 2b & \begin{pmatrix} -2(h+b)- \\ 8c_1 F_{B_i} x^i_{5eq}{}^3 \\ +k_2 \end{pmatrix} \end{bmatrix} \begin{bmatrix} \Delta x^i_1 \\ \Delta x^i_2 \\ \Delta x^i_3 \\ \Delta x^i_4 \\ \Delta x^i_5 \end{bmatrix}$$

Since in this thermoforming process model there is only one varying parameter a, the vector θ in (4) reduces to the scalar a. Referring to (2), the original varying parameter is C_p, which causes the parameter a to vary. Therefore, the interval limits introduced in (6) and (7) should be defined for the parameter a according to expected variations in C_p. Based on our experience these limits for high-density polyethylene (HDPE) are as:

$$\begin{aligned} C_p &\in [C_{p\min}, C_{p\max}], \\ \dot{C}_p &\in [C^d_{p\min}, C^d_{p\max}], \end{aligned} \tag{24}$$

where

$$\begin{aligned} C_{p\min} &= 1500 \text{ J/(K}\cdot\text{kg)}, \\ C_{p\max} &= 7500 \text{ J/(K}\cdot\text{kg)}, \\ C^d_{p\min} &= \text{-}10 \text{ J/(K}\cdot\text{kg}\cdot s), \\ C^d_{p\max} &= 10 \text{ J/(K}\cdot\text{kg}\cdot s). \end{aligned} \tag{25}$$

Consequently, the ranges of variation of parameter a and its derivative can be obtained as follows:

$$\begin{aligned} a &\in [a_{\min}, a_{\max}], \\ \dot{a} &\in [a^d_{\max}, a^d_{\min}], \end{aligned} \tag{26}$$

where:

$$\begin{aligned} a_{\min} &= 4.6784\times 10^{-5}, \\ a_{\max} &= 2.3392\times 10^{-4}, \\ a^d_{\min} &= -5.8480\times 10^{-4}, \\ a^d_{\max} &= 5.8480\times 10^{-4}. \end{aligned} \tag{27}$$

The vertices of the corresponding parameter boxes, for the parameter a and its derivative, are defined as:

$$\begin{aligned} V_a &:= \{\omega : \omega \in \{a_{\min},\ a_{\max}\}\}, \\ R_a &:= \{\tau : \tau \in \{a^d_{\max}, a^d_{\min}\}\}. \end{aligned} \tag{28}$$

Now that a linear model is obtained, we transform its state equations to the standard form required in the previous section. Equation (23) can now be written as:

$$\Delta \dot{x}^i = A(a)\Delta x^i\,, \tag{29}$$

where:

$$\Delta x^i = \begin{bmatrix} \Delta x^i_1 \\ \Delta x^i_2 \\ \Delta x^i_3 \\ \Delta x^i_4 \\ \Delta x^i_5 \end{bmatrix},$$

$$A(a)=a\begin{bmatrix} \begin{pmatrix}-2(h+b)-\\ 8c_1F_{T_i}{x^i_{1eq}}^3\\ +k_1\end{pmatrix} & 2b & 0 & 0 & 0\\ b & -2b & b & 0 & 0\\ 0 & b & -2b & b & 0\\ 0 & 0 & b & -2b & b\\ 0 & 0 & 0 & 2b & \begin{pmatrix}-2(h+b)-\\ 8c_1F_{B_i}{x^i_{5eq}}^3\\ +k_2\end{pmatrix}\end{bmatrix} \quad (30)$$

Since $\theta=a$, Equation (5) becomes:

$$A(a)=A_0+aA_1\,, \quad (31)$$

in which

$$A_0=0$$

$$A_1=\begin{bmatrix} \begin{pmatrix}-2(h+b)-\\ 8c_1F_{T_i}{x^i_{1eq}}^3\\ +k_1\end{pmatrix} & 2b & 0 & 0 & 0\\ b & -2b & b & 0 & 0\\ 0 & b & -2b & b & 0\\ 0 & 0 & b & -2b & b\\ 0 & 0 & 0 & 2b & \begin{pmatrix}-2(h+b)-\\ 8c_1F_{B_i}{x^i_{5eq}}^3\\ +k_2\end{pmatrix}\end{bmatrix}. \quad (32)$$

Next, in order to analyze the stability of this system, we must find the Lyapunov matrix *P(a)*. The Lyapunov function corresponding to *P(a)* has the form:

$$V(\Delta x^i,a):=\Delta x^{iT}P(a)\Delta x^i, \quad (33)$$

where

$$P(a)=P_0+aP_1\ . \quad (34)$$

Considering the structure of the *A* matrix in the model, the set of LMIs (14) and (15) can be written as follows:

$$A_i^TP_i+P_iA_i\ \geq 0 \quad i=0,1\ , \quad (35)$$

$$L(\omega,\tau):=\omega(A_1^TP_0+P_0A_1)+\omega^2(A_1^TP_1+P_1A_1)+\tau P_1<0\,,$$
$$(\omega,\tau)\in V_a\times R_a\,. \quad (36)$$

P_0 and P_1 are the symmetric matrices to be found as the solutions of this LMI problem. The two inequalities in (35) ensure the multi-convexity. Since $A_0=0$, the inequality related to $i=0$, holds trivially. As can be seen in (36), the derivative of the Lyapunov function with respect to time, *L*, is only considered on the vertices of the parameter boxes, V_a and R_a. As introduced in (28), four vertices exist, resulting in four inequalities in (36). Therefore, the problem of finding an affine parameter-dependent Lyapunov function is reduced to five LMIs defined by (35) and (36). Using the LMI toolbox of MATLAB, the following P_0 and P_1 can be obtained:

$$P_0=\begin{bmatrix}186.4818 & 251.9727 & 191.3607 & 145.3557 & 51.1843\\ 251.9727 & 655.2064 & 528.8390 & 409.7752 & 145.3557\\ 191.3607 & 528.8390 & 667.6532 & 528.8390 & 191.3607\\ 145.3557 & 409.7752 & 528.8390 & 655.2064 & 251.9727\\ 51.1843 & 145.3557 & 191.3607 & 251.9727 & 186.4818\end{bmatrix},$$

$$P_1=-\begin{bmatrix}0.0302 & 0.0409 & 0.0310 & 0.0236 & 0.0083\\ 0.0409 & 0.1063 & 0.0858 & 0.0665 & 0.0236\\ 0.0310 & 0.0858 & 0.1083 & 0.0858 & 0.0310\\ 0.0236 & 0.0665 & 0.0858 & 0.1063 & 0.0409\\ 0.0083 & 0.0236 & 0.0310 & 0.0409 & 0.0302\end{bmatrix}. \quad (34)$$

These symmetric matrices satisfy the LMIs (35) and (36). Therefore, the LMI problem is feasible and the system is AQS. The next section discusses some of the simulation results which confirm the stability of the system.

5. SIMULATION RESULTS

In the thermoforming reheat process, each plastic sheet first enters the oven. The sheet is heated to a particular temperature and is then sent to the thermoforming station. The desired trajectory for the temperatures at the two nodes on the top and bottom surfaces of the sheet is a ramp function leveling off after achieving a desired final value. Figure 3 shows this desired trajectory. Figure 4 demonstrates the temperatures of the nodes on the five layers of the plastic sheet. The curves show temperature trajectories for different constant as well as varying values for parameter C_p. In the case that C_p varies during simulation (dashed curve), it is assumed that when temperature increases to the glass transition temperature (150°C), C_p will increase from 1500 to 7500. If the temperature increases more, C_p decreases. It can be seen that the surface temperatures follow the desired trajectory in all scenarios, considered for the varying parameter C_p. Finally, Figure 5 shows the error between the desired value and the actual temperature on the node of the first layer. These curves consistently confirm the robust stability of the system with respect to variations in the parameter C_p.

6. CONCLUSION

In this paper, the problem of stability analysis of a feedback-controlled thermoforming sheet reheat process was addressed. Since most thermoforming processes are not currently operated autonomously, stability analysis can be considered as an important step towards automatic control of such systems. The nonlinearity and parameter-varying nature of these processes makes this analysis more difficult. Searching for a parameter-dependent Lyapunov function was chosen as a strategy to test the stability of the system. This methodology suggests a number of linear matrix inequalities to check the stability of the system. In models that possess a large number of varying parameters this approach may be hard to apply. However, the thermoforming model applied in this paper is dependent on only one varying parameter. Therefore, it can be considered as a suitable application of this approach. Simulation results were also provided to support the theoretical findings.

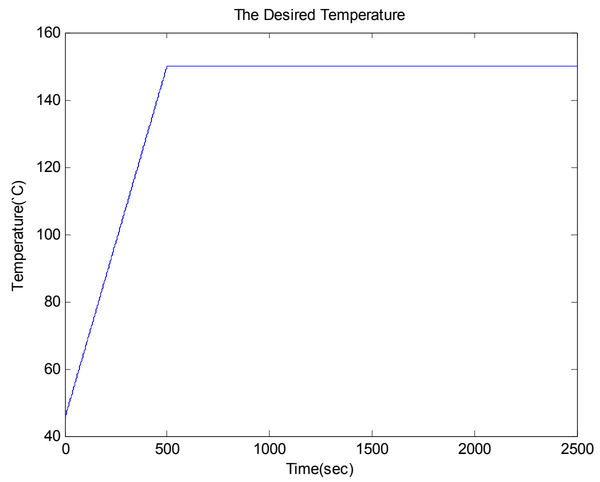

Figure 3: The desired trajectory for the surface temperatures

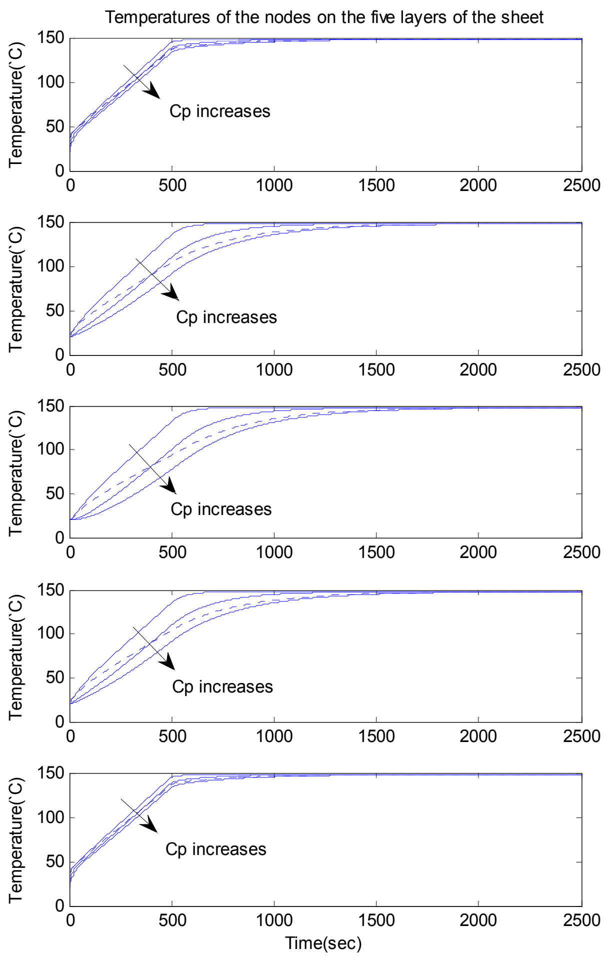

Figure 4: Temperatures of the nodes on the five layers
Solid Curve: For different values of the parameter C_p (1500, 4500, 7500)
Dashed Curve: For C_p rising from 1500 to 7500 during the first 500 seconds.

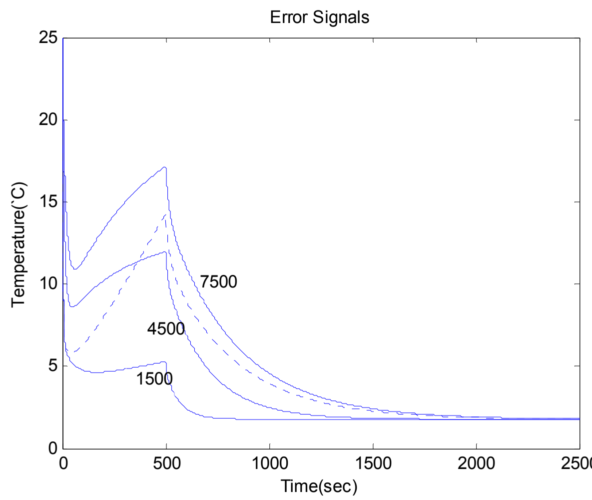

Figure 5: The error between the temperature of the first layer node and the desired trajectory
Solid Curve: For three different values of the parameter C_p (1500, 4500, 7500)
Dashed Curve: For C_p rising from 1500 to 7500 during the first 500 seconds.

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