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A Time Delay Estimation Method for MIMO Systems

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Abstract: A time delay estimation(TDE) method for MIMO systems, using combined continuous wavelet transform(CWT) and cross correlation method, is proposed. By calculating and handling the cross correlation between the CWT coefficients of system input and output data, a series of time delays over scales (frequencies) are calculated and an unbiased estimation is deduced from them. The TDE method under closed loop case is also studied. The numerical examples with simulation as well as experimental data verify that the procedure works well.

Keywords: Time-delay estimation; MIMO; Cross correlation; Wavelet transform; Closed-loop.

1. INTRODUCTION

Time delays exist in most practical systems. For instance, a range radar emits a narrow-band signal and consequently receives the delayed reflection to detect the object location; in chemical processes, a product line built up of capacities and pipes has a *dead time*, making the response of the plant to be slow. Time delays significantly affect the performance of the control system since they introduce significant phase lags in the process. There are many methods available for estimating delays in univariate processes. However there are relatively few methods for estimating time delays in multivariate systems under open loop and closed loop conditions. (See Bjöklund 2003 , Richard 2003)

Two TDE methods in literature are due to Zheng and Tjeng (2003) based on signal processing techniques and Isaksson (2001) based on classical control methods. In the latter case, not only the delays , but the system dynamics may also affect the output signal. We focus on the estimation of the actual delay for MIMO systems.

A method is developed for TDE, based on a combination of continuous wavelet transform(CWT) and cross correlation. Cross correlation are computed between the CWT coefficients of the input and output data. Comparison with other methods is given in the numeric examples. The newly proposed method gives interesting insight in the meaning and role of a delay for MIMO systems.

2. TIME DELAY PARAMETRIZATION

A continuous-time model is assumed to describe MIMO systems. Data with uniformly sampled single rate or non-uniformly sampled multirate signals are acceptable. A variety of model structures can be chosen, since we consider a model free method that the TDE algorithm is insensitive to the system parameterization. Without loss of generality, a continuous state space model is used to describe the data:

$$\begin{cases} \dot{\boldsymbol{x}}(t) = & \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}_d(t) + \boldsymbol{\omega}(t) \\ \boldsymbol{y}_d(t) = & \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}_d(t) + \boldsymbol{v}(t) \end{cases} \quad (1)$$

where time delays are parameterized by *input delay* and *output delay* vectors, defined as follows:

$$\boldsymbol{d}^u = [d_1^u \ d_2^u \cdots d_q^u], \quad \boldsymbol{d}^y = [d_1^y \ d_2^y \cdots d_p^y].$$

More clearly, the delayed data are defined as

$$\boldsymbol{u}_d(t) = [u_1(t-d_1^u) \ u_2(t-d_2^u) \ \cdots \ u_q(t-d_q^u)], \quad \boldsymbol{y}_d(t) = [y_1(t+d_1^y) \ y_2(t+d_2^y) \ \cdots \ u_p(t+d_p^y)].$$

The physical meanings are clearly evident: the *input/output delay* vectors split the whole delay into two parts that are contributed by the inputs and outputs respectively. For example, the (i,j)th element of the delay matrix Δ denotes the delay from jth input to ith output with input and output delay contributions given by:

$$\Delta_{ij} = d_i^y + d_j^u$$

In the frequency domain, the corresponding transfer function will be:

$$h_{ij}^d(s) = h_{ij}^0(s)e^{-s\Delta_{ij}},$$

where the transfer function $h^d(s)$ is decomposed into a delay free factor $h^0(s)$, and pure delay term.

3. REVISITING THE CLASSICAL DELAY ESTIMATE METHODS

The SISO pure delay estimation problem can be considered as follows:

$$\begin{cases} x_1(t) = s(t) + n_1(t) \\ x_2(t) = \alpha s(t-D) + n_2(t) \end{cases},$$

The correlation between the emitted signal x_1 and the received x_2 is given by:

$$R_{x_1,x_2}(\tau) = \mathrm{E}[x_1(t)x_2(t-\tau)].$$

With the assumption of ergodicity, The correlation of $x_1(t)$ and $x_2(t)$ is approximated by

$$\hat{R}_{x_1,x_2}(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} x_1(t)x_2(t-\tau)dt,$$

where $[-T,T]$ is the observation interval. While $x_2(t)$ is proportional to $x_1(t)$, regardless of the noise, the cross correlation is also proportional to the autocorrelation of $x_1(t)$:

$$\hat{R}_{x_1,x_2}(\tau) = \alpha R_{x_1}(\tau - D).$$

The autocorrelation has the property that $R_x(\tau) \le R_x(0)$ for all τ. So one can find the maximum and obtain the delay by locating its abscissa value at $\tau - D = 0$, which solves the problem in SISO case.

Consider the MIMO model in eqn. (1), where in general different delays may exist between distinctly emitted and received signals:

$$y_i(t) = \sum_{k=1}^{q}\int_{-\infty}^{\infty} h_{ik}^0(t-s)e^{-s\Delta_{ik}}u_k(s)ds + v_i(t).$$

For the MIMO case there is no unique maximum correlation due to the effect of other system dynamics on the process output of interest. By simple derivation, we have the following equation:

$$\hat{R}_{u_j,y_i}(\tau) = \sum_{k=1}^{q}\int_{-\infty}^{\infty} h_{ik}^0(s)\hat{R}_{u_j,u_k}(\Delta_{ik}+s+\tau)ds + \hat{R}_{u_j,v_i}(\tau) \tag{2}$$

The following assumptions on the model are needed to further simplify this model:

A1. *All inputs are uncorrelated with each other;*
A2. *All inputs are uncorrelated with each output noise* $v_i(t)$*;*
A3. $h_{ij}^0(t)$ *is a finite impulse response:* $\forall\, \varepsilon > 0$, $\exists\, t_h < \infty$ *that* $|h_{ij}^0(t)| < \varepsilon$ *when* $t > t_h$*.(The duration* t_h *of a system is define as the minimum value of* t_h*.)*

Lemma 1. Under the above stated assumptions(*A1*, *A2* and *A3*), eqn. (2) can be simplified to:

$$\hat{R}_{u_j,y_i}(\tau) \doteq (h_{ij}^0 * \hat{R}_{u_j})(-\tau - \Delta_{ij})|_{[0,t_h]} \tag{3}$$

where "$*$" denotes the convolution operation, and $[0,t_h]$ defines the convolution interval. Here t_h also depends on the channel index i and j, and is omitted for simplicity here and in the ensuing text unless necessary for clarity. Furthermore, the following assumption can be made practically:

A4. *The estimation of time delay according to the maximum value of eqn. (3) is bounded by:*

$$\Delta_{ij} \le \hat{\Delta}_{ij} \le \Delta_{ij} + t_h. \tag{4}$$

This is important to ensure that our theory can be developed.

However, the delay estimate for MIMO systems can only be implemented when t_h is estimated separately. Therefore analysis in the frequency domain is needed to obtain this estimation. For a specific system, the transfer function and its duration t_h are fixed. However if we extract from $h^0(t)$ part of its frequency spectrum by a band-pass filter, then the filtered $h_f^0(t)$ will have a different duration, corresponding to the band frequency. From a series of such analysis, the transfer function is decomposed into a set of filtered transfer functions.

Let $h^d(t)$ be filtered by an ideal narrow band filter $F_b(\omega)$, with the center frequency ω_0 and band width Δ_ω. The time response of the filtered transfer function $h_f^d(t)$ in the frequency domain is given by:

$$h_f^d(\omega)|_{\omega_0} = \begin{cases} h^d(\omega), & |\omega - \omega_0| < \Delta_\omega/2, \quad \Delta_\omega > 0 \\ 0, & \text{otherwise.} \end{cases}$$

since t_{hf} is filtered by a narrow band filter, its impulse response consists of only parts of $h^0(t)$ with frequencies near ω, therefore t_{hf}, the duration of $h_f^0(t)$ under assumption of ideal narrow band is given by:

$$t_{hf}|_{\omega_0} \doteq \left|\frac{\arg[h^d(\omega_0)]}{\omega_0}\right| - \tau = -\frac{\arg[h^0(\omega_0)]}{\omega_0}.$$

Since $\arg[h^0(\omega)]$ is bounded, it follows that when $\omega_0 \to \infty$:

$$\lim_{\omega_0\to\infty} t_{hf}|_{\omega_0} = 0 \tag{5}$$

According to assumption *A4*, it asymptotically suppresses the effect of system dynamics leading to an explicit delay values.

However this can never be implemented by Fourier transform, which loses all the time domain information, since it is impractical to take transform at infinite frequencies. This is the reason for introducing wavelet transform, which preserves time domain information while analyzing the signal in the frequency domain.

4. USING CWT: CORRELATION ANALYSIS IN TIME-FREQUENCY DOMAIN

Based on the preceding analysis, wavelet transform is used to decompose the input and output data, in time and frequency domains. The CWT coefficients appear to be *pseudo pure delay* signals, which means that only a time delay (phase lag) exists between the input/output coefficients. We then change scales (i.e. time) to vary the effect of the system dynamics on the correlation, which gives information of the true delay. This eases the TDE task, and delay estimation with an improved precision is obtained.

4.1 Continuous wavelet transform

The wavelet mother function $\psi(t)$ satisfies the admissibility condition and it is defined as:

$$\psi^{a,b}(t) = \frac{1}{\sqrt{a}}\psi\left(\frac{t-b}{a}\right),$$

with the scale and shift parameter a, b. So the CWT of $f(t)$ is defined as its inner product with $\psi^{a,b}$:

$$W_{f(t)}(a,b) = \int_{-\infty}^{\infty} f(t)\psi^{a,b}(t)\mathrm{d}t,$$

where the superscript "*" denotes the complex conjugate. To change a and b in $\psi^{a,b}$ is equivalent to stretch (or compress) and slide $\psi(t)$ along the time axis. The result can be shown in a 2-D shifting-scaling (similar to time-frequency) graphic, showing the local frequency components in the signal.

Take the 'Morlet' function for an example, which is defined by $\psi_{morl}(t) = 1/\sqrt{2\pi} \cdot e^{(-t^2/2)}e^{j\omega_0 t}$ and $\hat{\psi}_{morl}(\omega) = 1/\sqrt{2\pi} \cdot H(\omega)e^{(\omega-\omega_0)^2/2}$ in time and frequency domain respectively. $H(\omega)$ is the unit Heaviside function. The original input/output is then decomposed, as if being band-pass filtered by the wavelet:

$$h_f^0(\omega)|_{\omega_0} = \begin{cases} h^0(\omega)e^{-(\omega-\omega_0)^2/2}, & |\omega-\omega_0| < \delta/2, \quad \delta > 0 \\ 0, & \text{otherwise.} \end{cases}$$

Although the frequency band of wavelet is not narrow enough, the series of transforms corresponding to different scales (frequencies) will still show the trend where the true t_h supposed to lie. For example, the property *self similarity* given by Tabaru (1997a) forms one of the method.

The inverse transform (see eqn. (6)) given (Daubechies, 1992) will be used to explain the functionality of our algorithm later in this paper.

$$f(t) = C_\psi^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dadb}{a^2} W_{f(t)}(a,b)\psi^{a,b} \qquad (6)$$

4.2 Correlation analysis based on wavelet transform

If we collect all of $W_{u_j(t)}(a,b)$ with a_0 as the $\psi^{a_0,b}(t)$-filtered signal, and take b as the time index, we can get correlations of wavelet filtered signal as follows (See derivation in Appendix B.):

$$R_{[W_{u_j(t)}(a_0,b),W_{y_i(t)}(a_0,b)]}(\tau) = \sum_{k=1}^{q} \int_{-\infty}^{\infty} h_{ik}^0(r) R_{[W_{u_j(t)}(a_0,b),W_{u_k(t)}(a_0,b)]}(r+\tau+\Delta_{ik})dr.$$

Denote $RW(x,y,t) = R_{[W_x(a_0,b),W_y(a_0,b)]}(t)$ and $RW(x,t) = R_{W_x(a_0,b)}(t)$ so the above equation becomes:

$$RW(u_j,y_i,\tau) = \sum_{k=1}^{q} \int_{-\infty}^{\infty} h_{ik}^0(r) RW(u_j,u_k,r+\tau+\Delta_{ik})dr.$$

This is a sum of q SISO cases, analogous to eqn. (2), hence it can have more than one peak indicating delays in different channels. Assumption *A1* is practically a strong condition, since redundant signals often appear in the data collection. As the data are all decomposed by wavelets, only the noncorrelation of the wavelet coefficients are necessary. The following assumption is then considered to replace assumption *A1*:

A5.*(replace A1): Assume that the inputs vary and are sustained for sufficiently long periods so that their CWT coefficients at part of the scales are uncorrelated.*

In data preprocessing, only data segments with enough excitation are chosen from the sampled data set (Isaksson, 2001). Since a series of delay estimations are obtained with respect to different scales, only part of the CWT coefficients have to be uncorrelated. This will be a weaker condition than to require that all u_k and u_j uncorrelated in *A1*.

Then the above result of delay estimation still holds based on assumption *A5*:

$$RW(u_j,y_i,\tau) \doteq (h_{ij,W}^0 * RW(u_j))(-\tau-\Delta_{ij})|_{[0,t_{hW}]} \quad (7)$$

where t_{hW} is the duration of $h_{ij,W}^0(t)$, filtered $h_{ij}^0(t)$, by the wavelet $\psi^{a_0,b}(t)$.

When we view the equation in the frequency domain, it is clear that $W_{u_j(t)}(a_0,b)$ and $W_{y_i(t)}(a_0,b)$ will both have the same narrow band centered at $\omega = \omega_0/a_0$. Recall the relationship in assumption *A4* and eqn. (7), where we have the delay estimate for each scale such that:

$$\Delta_{ij} \le \hat{\Delta}_{ij}(a_0) \le \Delta_{ij} + t_{hW}(a_0,b) \qquad (8)$$

Based on eqn. (5) and the fact $a_0 \to 0 \Rightarrow \omega_0/a_0 \to \infty$, it results in $t_{hW}(a_0,b) \to 0$, hence leads to our most important conclusion:

Theorem 1. Assuming that the spectrum of each input varies enough with time, we then have

$$a_0 \to 0 \Rightarrow \hat{\Delta}_{ij} \to \Delta_{ij} \qquad (9)$$

Remark 1. Data with enough excitation are needed to satisfy the assumption for this theorem. As can be seen from further analysis, non-stationary signals may give a better result.

Remark 2. It will be noticed that the wavelet transform is not available at or too close to $a_0 = 0$, due to noise corruption and numerical computation. Then this limitation is circumvented by a deduction from estimation at some higher scales.

Visually, the algorithm begins with the cross correlation between CWT coefficients of input/output data, in a series of scales. A 2-D correlation is obtained, where we search at each scale for several peak values (maximum and sub-maximum), whose location will most possibly refer to the delay (We use classical correlation TDE method to identify them). Therefore more than one candidate value for each scale is chosen, which is shown graphically with respect to scale (see fig. 1, the correlation of 4th input and first output of a MIMO system. Details about the concerned system are described in subsection 5.1). We can see that the delay candidates appear as almost straight lines! In addition, with the scale decreasing, they point to the true time delay.

Such figures give the estimation of one element in the delay matrix. In most cases, not all elements are well estimated. Some of the delays are allowed to be listed as *missing*. According to eqn. (2), we can calculate input delay and output delay vectors by LS solution as follows:

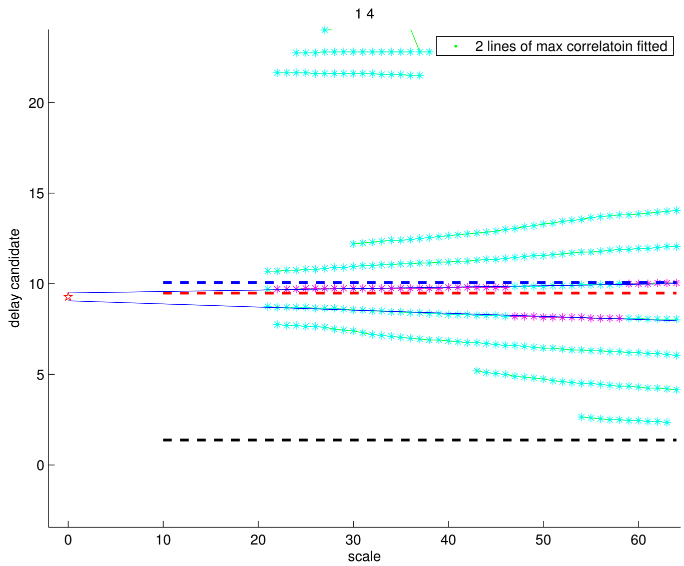

Fig. 1. *Possible delay values from correlation. (Middle of the three dash lines (red): true delay; upper (blue) dash line: result from classical correlation TDE method; lower (black) dash line: specified lower bound for estimation (upper bound invisible); dark (magenta) stars: max correlation for each scale; light (cyan) star: sub-max correlations for each scale; thin (and blue) lines: straight line fitting; pentagon: final estimation.)*

$$\begin{cases} \hat{d}_1^u + \hat{d}_1^y = \hat{\Delta}_{11} \\ \hat{d}_2^u + \hat{d}_1^y = \hat{\Delta}_{12} \\ \vdots (\textit{missing} \text{ equations}) \\ \hat{d}_q^u + \hat{d}_p^y = \hat{\Delta}_{pq} \end{cases}$$

The delay matrix will be reconstructed with the estimated values: $\hat{d}_1^u, \cdots, \hat{d}_q^u, \hat{d}_1^y, \cdots, \hat{d}_p^y$.

4.3 Comparison with existing method

Tabaru (1997a, b) also applied wavelet transform and cross correlation. Both of his proposed methods share the same idea of taking wavelet transform on the correlation of input and output data. These two methods are compared here.

It is hard to tell which solution is the better from the frequency analysis, according to the Wiener-Khintchine equation

$$RW(u_j, y_i, \tau) = \int_{-\infty}^{\infty} SW(u_j, y_i, \omega) e^{j\omega\tau} d\omega,$$

where $SW(u_j, y_i, \omega) = S_{[W_{u_j(t)}(a,b), W_{y_i(t)}(a,b)]}(\omega)$. So it is found that

$$RW(u_j, y_i, \tau) = \int_{-\infty}^{\infty} (\hat{\psi}^{a,b}(\omega))^2 S_{u_j, y_i}(\omega) e^{j\omega\tau} d\omega.$$

Compared with Tabaru's algorithm (while denoting that $WR(u_j, y_i, \tau) = W_{R_{[u_j(t), y_i(t)]}(\tau)}(a, b)$):

$$WR(u_j, y_i, \tau) = \int_{-\infty}^{\infty} \hat{\psi}^{a,b}(\omega) S_{u_j, y_i}(\omega) e^{j\omega\tau} d\omega,$$

the two have nearly the same frequency spectrum, Since $(\hat{\psi}^{a,b}(\omega))^2$ and $\hat{\psi}^{a,b}(\omega)$ are both Gaussian with different parameters. So why should the former have a better performance, according to our numerical example (see numeric examples below)?

This can be explained by the local and global properties. The correlation is a global function that ignores the time varying property, and the wavelet function is well aware of that. We can see that our method calculates the correlation from the wavelet decomposed signals:

$$RW(u_j, y_i, \tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} W_{u_j(t)}(a_0, b) W_{y_i(t)}(a_0, b-\tau) db \tag{10}$$

while in Tabaru's method, correlation is carried out before wavelet transform:

$$WR(u_j, y_i, \tau) = \int_{-\infty}^{\infty} R_{[u_j(t), y_i(t)]}(\tau) \cdot \frac{1}{\sqrt{a_0}} \psi^*(\frac{\tau - b}{a_0}) d\tau.$$

In the correlation

$$R_{[u_j(t), y_i(t-\tau)]}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} u(t) y(t-\tau) dt,$$

we recall the inverse CWT in eqn. (6)

$$R_{[u_j(t), y_i(t-\tau)]}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} \left(C_\psi^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} W_{u(t)}(a, b) \psi^{a,b} \frac{dadb}{a^2} \right) y(t-\tau) dt,$$

and found the following relationship by exchanging the integration order:

$$R_{[u_j(t), y_i(t-\tau)]}(\tau) = C_\psi^{-1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left(\lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} W_{u(t)}(a, b) y(t-\tau) dt \right) \psi^{a,b} \frac{dadb}{a^2},$$

from which Tabaru's method is interpreted as

$$WR(u_j, y_i, \tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} W_{u(t)}(a_0, b) y(t-\tau) dt \tag{11}$$

Then the difference is clarified by comparing eqs. (10) and (11). Since $y(t-\tau)$ has not been decomposed in the latter case, the correlation in Tabaru's method is "contaminated" by the system dynamics while our method is not. Therefore a better result is expected from our method.

4.4 Closed loop case

In practical cases, closed loop systems are more likely to be considered. The transfer function of the controller is defined as $\boldsymbol{H}^c(s)$ in s-domain without delay. So the closed loop system can be expressed as follows:

$$\boldsymbol{y}(s) = (\boldsymbol{I} + \boldsymbol{H}(s)\boldsymbol{H}^c(s))^{-1} \boldsymbol{H}(s)\boldsymbol{H}^c(s)\boldsymbol{r}(s),$$

where $\boldsymbol{r}(s)$ is the reference input vector. And the controller output vector $\boldsymbol{e}(t)$ is defined as:

$$\boldsymbol{e}(t) = \boldsymbol{H}^c(t) * (\boldsymbol{r}(t) - \boldsymbol{y}(t)),$$

$\boldsymbol{y}(t)$ and $\boldsymbol{e}(t)$ have the same relationship as the open loop system:

$$\boldsymbol{y}(t) = \boldsymbol{H}(t) * \boldsymbol{e}(t).$$

The time delays are estimated in the same manner. However, since $\boldsymbol{e}(t)$ is now correlated with $\boldsymbol{y}(t)$, eqn. (7) will change accordingly:

$$RW(e_j, y_i, \tau) = \sum_{k=1}^{p} RW(h^c_{jk}(t) * r_k(t), y_i, \tau) - \sum_{k=1}^{p} RW(h^c_{jk}(t) * y_k(t), y_i, \tau),$$

In a closed loop system $\boldsymbol{y}(t)$ will follow $\boldsymbol{r}(t)$, so uncorrelation of different $y_i(t)$ is assumed because $r_k(t)$ is uncorrelated with each other. Similarly, $y_i(t)$ and $r_k(t)$ in different channels are considered as uncorrelated. As a result, the above equation is reduced to two main terms:

$$RW(e_j, y_i, \tau) \doteq RW(h^c_{ji}(t) * r_i(t), y_i, \tau) - RW(h^c_{ji}(t) * y_i(t), y_i, \tau).$$

The second term will degrade the precision of the estimation, which is determined by the strength of the feedback link. Therefore the algorithm is applicable in the closed loop case, but will not yield as good results compared with the open loop case, as is shown in the simulation in the next section.

5. SIMULATION AND EXPERIMENTAL EXAMPLES

We illustrate the applicability of the proposed methods, with data from numerical simulation and a laboratory experiment. Both open loop and closed loop results are shown. The results are verified from knowledge of the true parameters and/or simulation.

5.1 Simulation example

In order to show the effect of the wavelet method, we apply the algorithm to a simulated system, which is a 3rd order state space system consisting of 4 inputs and 3 outputs. The settling times of all of the subsystems' impulse responses are less than 20s. The total simulation time is 400s. The input delay and output delay vectors are set as $\boldsymbol{d}^u$ =[4.43 5.24 3.54 4.27]s and $\boldsymbol{d}^y$ =[5.22 3.37 7.43]s respectively. Therefore the true delay matrix is constructed from the two vectors as:

$$\Delta = \begin{bmatrix} 9.65 & 10.46 & 8.76 & 9.49 \\ 7.80 & 8.61 & 6.91 & 7.64 \\ 11.86 & 12.67 & 10.97 & 11.70 \end{bmatrix}$$

A cluster of sinusoidal waves with frequencies restricted in [0, 5]rad/s generate the input signals, discretized to T=0.02s. At t=200s, a change of frequency spectrum occurs.

CWT is carried out on every input and output in scales from 10 to 65, and delay estimation is obtained from each channel from the maximum correlation and linear regression (refer to fig. 1).

$$\hat{\Delta} = \begin{bmatrix} 9.64 & 10.42 & 8.72 & 9.28 \\ 7.93 & \text{xxxx} & 6.99 & 7.60 \\ 11.89 & 12.49 & 10.89 & 11.74 \end{bmatrix}$$

where "xxxx"s indicates failure of the algorithm in some channels, due to its poor performance, we reconstruct the delay matrix accordingly:

$$\hat{\Delta}^R = \begin{bmatrix} 9.65 & 10.34 & 8.70 & 9.37 \\ 7.92 & 8.60 & 6.96 & 7.64 \\ 11.89 & 12.58 & 10.93 & 11.61 \end{bmatrix}$$

this is very close to the true delays, with a mean absolute error of 0.06s (around 0.6%) and max absolute error 0.12s (around 1.2%).

We compare this result with Tabaru's method, by carrying out their algorithm channel by channel:

$$\hat{\Delta}_{Tabaru} = \begin{bmatrix} 9.80 & 10.48 & 8.62 & 9.75 \\ 7.28 & 7.96 & 6.10 & 7.22 \\ 11.52 & 12.20 & 10.34 & 11.46 \end{bmatrix}$$

whose mean and max absolute error are 0.39s(around 3.9%) and 0.81s(around 8.1%) respectively.

The conventional correlation method gives even worse results, which is also indicated in fig. 1. It yields the mean and max absolute error as 6.71s and 20.37s. The reason is that some of the channels are estimated incorrectly and can not be detected at all, so that the errors spread by linear regression to all the channels.

The closed loop case is also studied under similar conditions, where the process consists of a 3×3 system with delays as:

$$\Delta_c = \begin{bmatrix} 16.65 & 27.46 & 21.76 \\ 7.80 & 18.61 & 12.91 \\ 11.86 & 22.67 & 16.97 \end{bmatrix}$$

The feedback is given by a random state space model. The delay matrix is estimated as:

$$\hat{\Delta}_c = \begin{bmatrix} 26.64 & 27.35 & 27.25 \\ 7.97 & 18.56 & 6.61 \\ 11.83 & 13.98 & \text{xxxx} \end{bmatrix}$$

while finally the reconstructed delay matrix is obtained:

$$\hat{\Delta}^R_c = \begin{bmatrix} 25.91 & 30.39 & 24.94 \\ 9.87 & 14.35 & 8.91 \\ 10.67 & 15.15 & 9.71 \end{bmatrix}$$

with mean and max absolute error as 4.62s(∼26%) and 9.26s(∼53%) respectively. Since the mutual correlation between channels is large due to the feedback , the estimated delay matrix is not as good compared with the open loop case.

5.2 Experimental data

We have also collected data from a four tank water level system with two adjustable valves. The flow rate through the valves to tanks are set as two inputs, while the outputs are two of the four tank water levels. Data from this equipment is sampled at $T = 1$s, and the experiment takes 25000s in total. Rectangle waves with random period are given as the inputs. The system delays were estimated in advance by other methods, and then manually set to $[72, 126]$s for input delays and $[225, 117]$s for output delays. Hence the true delay matrix is

$$\Delta = \begin{bmatrix} 297 & 351 \\ 189 & 243 \end{bmatrix}.$$

From the wavelet method, We get the following result:

$$\hat{\Delta}^R = \begin{bmatrix} 303 & 357 \\ 195 & 249 \end{bmatrix}.$$

Each of the channel has an error of 6s (or around 2%), larger than the true value. No closed loop data were available from this experimental setup for the specified delays and hence closed loop evaluation is not possible.

6. CONCLUSION

A method to estimate the time delays of MIMO systems has been discussed, based on the CWT and correlation analysis. For the data with good excitation when the proposed conditions are satisfied, delays of distinct channels in MIMO systems can be separated and identified well. Under ideal conditions based on the principle of the algorithm, the estimation should converge to the true values.

We deal with the closed loop case in the same way, where the simulation example has shown that the estimation is not as good as in open loop case. Nevertheless, we can separately examine the correlation of the wavelet transform and obtain a better solution. The closed loop time delay system still needs much improvement.

The main contributions of this paper are:

(1) CWT is used to decompose the signal spectrum and separate the system dynamics and time delay, making this TDE algorithm applicable to MIMO systems;
(2) Under closed loop systems, the algorithm works well, although the results depend on how strong the feedback link is.
(3) A parametrization scheme of input and output delay is used, reducing effectively the number of delays and thus improving the precision;
(4) The mechanism of CWT giving the time delay estimation, i.e. so called *self similarity* in Tabaru's paper, is resolved theoretically in this paper.

In summary, this TDE algorithm is a nontrivial synthesis and improvement of existing theories and methods, in order to get a effective TDE method for MIMO systems.

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Appendix A. DERIVATION OF EQN. (2)

From definition, the correlation

$$\hat{R}_{u_j,y_i}(\tau) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} u_j(t)y_i(t-\tau)dt$$

is given, as well as the output $y_i(t-\tau)$ is generated by inputs with delays:

$$y_i(t-\tau) = \sum_{k=1}^{q}\int_{-\infty}^{\infty} h_{ik}^0(s)u_k(t-\Delta_{ik}-s-\tau)ds + v_i(t-\tau).$$

By substituting the second equation to the first one, and changing the order of integration (definition integration that belongs to commutative operations) and summation, we have

$$\hat{R}_{u_j,y_i}(\tau) = \sum_{k=1}^{q}\int_{-\infty}^{\infty} h_{ik}^0(s)\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} u_j(t)u_k(t-\Delta_{ik} -s-\tau)\mathrm{d}t\mathrm{d}s + \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} u_j(t)v_i(t-\tau)dt,$$

where the simplified expression becomes

$$\hat{R}_{u_j,y_i}(\tau) = \sum_{k=1}^{q}\int_{-\infty}^{\infty} h_{ik}^0(s)\hat{R}_{u_j,u_k}(\Delta_{ij}+s+\tau)ds + \hat{R}_{u_j,v_i}(\tau).$$

This is the result shown in eqn. (2).

Appendix B. DERIVATION OF EQN. (7)

For a specific scale $a = a_0$, the correlation between the two CWT coefficients of inputs and outputs is given by:

$$RW(u_j, y_i, \tau) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T} W_{u_j(t)}(a_0,b)W_{y_i(t-\tau)}(a_0,b)dt.$$

The wavelet transform of $y_i(t)$ is substituted by the definition:

$$RW(u_j, y_i, \tau) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\left\{W_{u_j(t)}(a_0,b)\cdot\frac{1}{\sqrt{a_0}}\int_{-\infty}^{\infty} y_i(s-\tau)\psi^*(\frac{s-b}{a_0})ds\right\}dt.$$

Moreover, the item $y_i(s-\tau)$ is also replaced by

$$y_i(s-\tau) = \sum_{k=1}^{q}\int_{-\infty}^{\infty} h_{ik}^0(r)u_k(s-\tau-r-\Delta_{ik})dr + v_i(s-\tau),$$

hence it can be derived that

$$RW(u_j, y_i, \tau) = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}\left(W_{u_j(t)}(a_0,b)\int_{-\infty}^{\infty} h_{ik}^0(r)\sum_{k=1}^{q} W_{u_k(t-\tau-r-\Delta_{ik})}(a_0,b)ds + W_{v_i(t-\tau)}(a_0,b)ds\right)dt$$

and finally,

$$RW(u_j, y_i, \tau) = \sum_{k=1}^{q}\int_{-\infty}^{\infty} h_{ik}^0(r)RW(u_j, u_k, r+\tau+\Delta_{ik})dr + RW(u_j, v_i, \tau).$$

Eqn. (7) is derived when noise term is ignored.

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