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Control of an Autonomous Hybrid System Using a Nonlinear Model Predictive Controller

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Abstract: State estimation and estimator based predictive control of nonlinear autonomous hybrid systems poses a challenging problem as these systems involve discontinuities that are introduced by switching of the discrete variables. In this paper, we propose a state estimation scheme for an autonomous hybrid system using an ensemble Kalman filter (EnKF), which belongs to the class of particle filters and is a derivative free nonlinear state estimator. We then proceed to develop a novel nonlinear model predictive control scheme that inherits the approach used in EnKF formulation for future trajectory predictions. The efficacy of the proposed state estimation and control scheme is demonstrated by conducting simulation studies on a benchmark hybrid three-tank system.

Keywords: Ensemble Kalman Filter, Autonomous Hybrid System, Nonlinear Model Predictive Controller

1. INTRODUCTION

Model predictive control (MPC) has become a major research area over the last few decades. It is generally accepted that the reason for this success are the ability of MPC to optimally control multivariable systems under various constraints. Unlike many other advanced techniques, it has been successfully applied in process industry for controlling complex unit operations in continuously operated plants. However, dynamic systems that involve continuous and discrete states, broadly classified as hybrid systems, are often encountered in engineering applications. Thus, model predictive control of hybrid system has gained increased attention in the recent years (Bemporad and Morari, 1999).

The dynamic model used for state estimation and prediction is the key component of any MPC schemes. Conventional state observer based MPC formulations make use of the Kalman filter (KF) and extended Kalman filter (EKF) as a state estimator. For linear systems, Kalman filters generate optimal estimates of state from observations when uncertainties in state dynamics and measurement can be adequately modelled as Gaussian white noise processes. The Kalman filter has attracted widespread attention of engineering community because of the recursive nature of its computational scheme. For nonlinear systems, the EKF is a natural extension of the linear filter to the nonlinear domain through local linearization. In EKF formulations, the state covariance propagation is carried out using Taylor series expansion of the nonlinear state transition operator. This step requires analytical computation of Jacobians at each time step. This can prove to be prohibitively complex and computationally demanding for high dimensional systems. Moreover, this also implies that nonlinear function vectors $F[.]$ and $H[.]$ should be smooth and at least once differentiable. However, the dynamic models for autonomous hybrid systems involve discontinuities, which are introduced by switching of the discrete variables. Therefore, the EKF cannot be used for state estimation of nonlinear autonomous hybrid systems particularly in the operating regimes where discrete variables undergo frequent transitions. Thus, state estimation and estimator based control of autonomous hybrid systems poses a challenging problem. In recent years, a number of derivative free nonlinear filtering techniques have been proposed in the literature (Patwardhan et al., 2007). For example, the unscented Kalman filter (UKF) has been proposed as an alternative to the EKF where the above limitations has been overcome using the concept of sample statistics (Julier and Uhlmann, 2004). Also, a new class of filtering technique, called particle filtering, can deal with state estimation problems arising from multimodal and non-Gaussian distributions (Arulampalam et al., 2002). A particle filter (PF) approximates multi-dimensional integration involved in the propagation and update steps using Monte Carlo sampling. The Ensemble Kalman filter (EnKF), originally proposed by Evensen (Burger et al., 1998), belong to the class of particle filters. In the EnKF formulation, similar to the EKF or UKF, the observer gain is computed using second order moments of state error and innovations. However, the main difference is that the covariance information is generated using Monte Carlo sampling without making any assumption on the nature of underlying distributions of state estimation error. In addition, the EnKF formulation can deal with state and measurement noise with non-Gaussian and multimodal distributions in contrast to UKF where the Gaussian assumption is implicit.

In this work, we propose a novel nonlinear model predictive control (NMPC) scheme for dealing with servo and regulatory control problems associated with nonlinear autonomous hybrid system. The salient features of the proposed NMPC scheme are as follows: (a) We propose to use the EnKF for state estimation of nonlinear autonomous hybrid system; and (b) for future trajectory predictions we develop a prediction scheme based on Monte Carlo simulation approach similar to the prediction step in the EnKF formulation. The efficacy of the proposed state estimation and control scheme is demonstrated by conducting simulation studies on the benchmark three-tank hybrid system. It may be noted that, while many approaches are now available in the literature for control of hybrid systems based on linear hybrid models, not much work has been reported based on state estimation and control of nonlinear hybrid models.

The organization of the paper is as follows. Section 2 discusses the details of state estimation in autonomous hybrid systems. Section 3 presents the design of a nonlinear model predictive control scheme for autonomous hybrid systems. The process considered for simulation study is discussed in section 4. Simulation results are presented in section 4 followed by concluding remarks in section 5.

2. STATE ESTIMATION OF AUTONOMOUS HYBRID SYSTEMS

A particular class of discontinuous systems, namely autonomous hybrid systems, is of interest in this work. These systems can be represented by the following set of differential algebraic equations:

$$\mathbf{x}(k) = \mathbf{x}(k-1) + \int_{(k-1)T}^{kT} F[\mathbf{x}(\tau), \mathbf{u}(k-1), \mathbf{d}(k-1) + \mathbf{w}(k-1), \mathbf{z}(\tau)]d\tau \tag{1}$$

$$\mathbf{z}(\tau) = G\left[\mathbf{x}(\tau)\right] \tag{2}$$

$$\mathbf{y}(k) = H[\mathbf{x}(k), \mathbf{v}(k)] \tag{3}$$

In the above process model, $\mathbf{x}(k)$ is the system state vector $(\mathbf{x} \in R^n)$, u(k) is known system input $(\mathbf{u} \in R^m)$, $\mathbf{d}(k) \in R^p$ is the unknown system input, $\mathbf{w}(k)$ is the state noise ($\mathbf{w} \in R^p$) with known distribution, $\mathbf{y}(k)$ is the measured state variable ($\mathbf{y} \in R^r$) and $\mathbf{v}(k)$ is the measurement noise ($\mathbf{v}(k) \in R^r$) with known distribution. The parameter k represents the sampling instant, $F[.]$ and $H[.]$ are the nonlinear process model and nonlinear measurement model respectively. It may be noted that we are interested in the most general case whereby state and measurement noise processes may have arbitrary (but known) distributions. Also, they can influence the system dynamics and measurement map in a non-additive manner.

In equation 2, $\mathbf{z} \in R^h$ represents discrete variables such that it can take only finite integer values, such as {-1, 0, 1} depending on some events, which are functions of continuous state variables in the case of autonomous hybrid systems. The function vector $G[.]$ can be expressed using a combination of Dirac delta functions and logical operators, such as AND, OR, XOR etc. The first step in the development of any NMPC scheme is the formulation of a scheme for the state estimation.

State estimation in autonomous hybrid systems is a challenging problem due to the discontinuities introduced by switching of the discrete variables. In this work, we intend to use the derivative free nonlinear state estimation scheme, namely, the Ensemble Kalman filter (Gillijns et al. (2006)) for estimation of state variables in autonomous hybrid systems. The major advantage of using EnKF is that it avoids explicit computation of Jacobian matrices. The second order statistics necessary for estimation of observer gain is generated using sample points (particles). The sample points straddle the discontinuity and, hence, can approximate the effect of discontinuity (Julier and Uhlmann, 2004). In the following section we present the EnKF algorithm for state estimation of autonomous hybrid systems in detail.

2.1 Ensemble Kalman Filter

The filter is initialized by drawing N particles $\{\mathbf{x}^{(j)}(0|0)\}$ from a suitable distribution. At each time step, N samples $\{\mathbf{w}^{(j)}(k-1), \mathbf{v}^{(j)}(k) : j = 1,..N\}$ for $\{\mathbf{w}(k)\}$ and $\{\mathbf{v}(k)\}$ are drawn randomly using the distributions of state noise and measurement noise. These sample points together with particles $\{\hat{\mathbf{x}}^{(j)}(k-1|k-1) : j = 1,..N\}$ are then propagated through the system dynamics to compute a cloud of transformed sample points (particles) as follows:

$$\hat{\mathbf{x}}^{(i)}(k|k-1) = \hat{\mathbf{x}}^{(i)}(k-1|k-1) + \int_{(k-1)T}^{kT} F\left[\mathbf{x}(\tau), \mathbf{u}(k-1), \mathbf{d}(k-1) + \mathbf{w}^{(i)}(k-1), G\left[\mathbf{x}(\tau)\right]\right]d\tau$$

$$i = 1, 2, ...N \tag{4}$$

These particles are then used to estimate sample mean and covariance as follows:

$$\overline{\mathbf{x}}^{(i)}(k|k-1) = \frac{1}{N}\sum_{i=1}^{N}\hat{\mathbf{x}}^{(i)}(k|k-1) \tag{5}$$

$$\overline{\mathbf{y}}^{(i)}(k|k-1) = \frac{1}{N}\sum_{i=1}^{N} H\left[\hat{\mathbf{x}}^{(i)}(k|k-1), \mathbf{v}^{(i)}(k)\right] \tag{6}$$

$$P_{\boldsymbol{\varepsilon},\mathbf{e}}(k) = \frac{1}{N-1}\sum_{i=1}^{N}\left[\boldsymbol{\varepsilon}^{(i)}(k)\right]\left[\mathbf{e}^{(i)}(k)\right]^T \tag{7}$$

$$P_{\mathbf{e},\mathbf{e}}(k) = \frac{1}{N-1}\sum_{i=1}^{N}\left[\mathbf{e}^{(i)}(k)\right]\left[\mathbf{e}^{(i)}(k)\right]^T \tag{8}$$

Where,

$$\boldsymbol{\varepsilon}^{(i)}(k) = \hat{\mathbf{x}}^{(i)}(k|k-1) - \overline{\mathbf{x}}^{(i)}(k|k-1) \tag{9}$$

$$\mathbf{e}^{(i)}(k) = H\left[\hat{\mathbf{x}}^{(i)}(k|k-1), \mathbf{v}^{(i)}(k)\right] - \overline{\mathbf{y}}^{(i)}(k|k-1) \tag{10}$$

The Kalman gain and cloud of updated samples (particles) are then computed as follows:

$$L(k) = P_{\varepsilon,e}(k)\left[P_{e,e}(k)\right]^{-1} \quad (11)$$

$$\Upsilon^{(i)}(k|k-1) = \left\{\mathbf{y}(k) - H\left[\hat{\mathbf{x}}^{(i)}(k|k-1), \mathbf{v}^{(i)}(k)\right]\right\} \quad (12)$$

$$\hat{\mathbf{x}}^{(i)}(k|k) = \hat{\mathbf{x}}^{(i)}(k|k-1) + L(k)\Upsilon^{(i)}(k|k-1) \quad (13)$$

$$\Upsilon^{(i)}(k|k) = \left\{\mathbf{y}(k) - H\left[\hat{\mathbf{x}}^{(i)}(k|k), \mathbf{v}^{(i)}(k)\right]\right\} \quad (14)$$

The updated state estimate is computed as the mean of the updated particles cloud, i.e.

$$\begin{aligned} \hat{\mathbf{x}}(k|k) &= \frac{1}{N}\sum_{i=1}^{N}\hat{\mathbf{x}}^{(i)}(k|k) \\ \hat{\mathbf{z}}(k|k) &= G\left[\hat{\mathbf{x}}(k|k)\right] \end{aligned} \quad (15)$$

The above equation gives an estimate of the discrete state variables $\{\hat{\mathbf{z}}(k|k)\}$ at the sampling instant k. However, it may be noted that transition(s) of the discrete state variables can occur within a sampling interval, which is captured in the propagation step for each particle. The accuracy of the estimates depends on the number of data points (N). Gillijns et al. (2006) have indicated that ensemble size between 50 and 100 suffices even for large dimensional systems.

3. NONLINEAR MPC FORMULATION FOR AN AUTONOMOUS HYBRID SYSTEM

The first step in the development of NMPC formulation is generation of model based future predictions. At this stage, it becomes necessary to incorporate measures for dealing with plant-model mismatch. There are two strategies available in the literature for dealing with plant-model mismatch. They are as follows:

State Augmentation: By this approach, artificial states, say $\boldsymbol{\eta}(k)$, equal to number of outputs are introduced in the state dynamics as follows

$$\begin{aligned} &\mathbf{x}(k) = \mathbf{x}(k-1) + \\ &\int_{(k-1)T}^{kT} F[\mathbf{x}(\tau), \mathbf{u}(k-1), \boldsymbol{\eta}(k-1), \mathbf{d}(k-1) + \mathbf{w}(k-1), \mathbf{z}(\tau)]d\tau \\ &\boldsymbol{\eta}(k) = \boldsymbol{\eta}(k-1) + \mathbf{w}_\eta(k-1) \quad (16) \\ &\mathbf{z}(\tau) = G\left[\mathbf{x}(\tau)\right] \\ &\mathbf{y}(k) = H[\mathbf{x}(k), \mathbf{v}(k)] \end{aligned}$$

Here $\mathbf{w}_\eta(k)$ represents zero mean Gaussian white noise process with covariance $\mathbf{Q}_\eta$. The state observer is designed for the augmented system by treating entries in $\mathbf{Q}_\eta$ as tuning parameters. Typically $\boldsymbol{\eta}(k)$ is chosen as bias terms in the inputs or drift in some model parameters. The main difficulty with this approach is that the closed loop performance is a very complex function of $\mathbf{Q}_\eta$ and the values have to be selected by trail and error. In addition, state augmentation implies that a large number of particles have to be used in EnKF formulation, thereby increasing the computational burden.

Innovation Based Correction: Recently, Srinivasrao et al. (2006) have proposed a plant-model mismatch compensation strategy, which is similar to the scheme developed by Ricker (1990). By this approach, filtered values of model residuals, which are defined as

$$\Upsilon(k|k-1) = \mathbf{y}(k) - \hat{\mathbf{y}}(k|k-1) \quad (17)$$

$$\Upsilon(k|k) = \mathbf{y}(k) - \hat{\mathbf{y}}(k|k) \quad (18)$$

are directly used for correction of future state and output predictions without requiring any state augmentation. Note that these filters are identical to that of filters in the feedback path of the nonlinear internal model control scheme and provide extra degrees of freedom for achieving robustness against plant-model mismatch. Srinivasrao et al. (2006) have shown that it is relatively easy to tune these filters than choosing elements in $\mathbf{Q}_\eta$ for the artificial states. In this work we develop a nonlinear MPC scheme using the later approach.

3.1 *Multi-step ahead future Prediction using EnKF*

Consider the problem of generating multi-step ahead predictions over time horizon $\left[k+1, k+N_p\right]$, where N_p represent the prediction horizon. The two sources of uncertainties that arise while performing such predictions are (a) uncertainty in initial state at the beginning of the horizon and (b) unknown disturbances $\{w(k+j): j = 1,2...N_p\}$ that may occur in future. To overcome the difficulties arising from the uncertainties in the initial state, we propose to carry out predictions by propagating all particles over the prediction horizon. The effect of unknown disturbances on the future predictions can be estimated by drawing samples from the known distribution of $\{w(k)\}$ at each prediction step. Thus, given future manipulated input moves $u(k|k).......u(k+1|k).......u(k+N_p-1|k)$, the future state predictions are generated as follows:

$$\begin{aligned} &\hat{\mathbf{x}}^{(i)}(k+j+1|k) = \hat{\mathbf{x}}^{(i)}(k+j|k) \\ &+ \int_{(k+j)T}^{(k+j+1)T} F\left[\mathbf{x}(\tau), \mathbf{u}(k+j|k), \bar{\mathbf{d}}(k-1) + w^{(i)}(k+j|k), G\left[\mathbf{x}(\tau)\right]\right]d\tau \\ &i = 1,2,....N \text{ and } j = 0,1,....N_p - 1 \end{aligned} \quad (19)$$

where $\{w^{(i)}(k+j|k)\}$ represent the samples are drawn from the distribution of the state noise. The predicted mean trajectory is then computed as follows

$$\hat{\mathbf{x}}(k+j+1|k) = \frac{1}{N}\sum_{i=1}^{N}\left[\hat{\mathbf{x}}^{(i)}(k+j+1|k) + L(k)\Upsilon_f^{(i)}(k|k-1)\right] \quad (20)$$

$$\begin{aligned} &\hat{\mathbf{y}}(k+j+1|k) = \\ &\frac{1}{N}\sum_{i=1}^{N}\left[H\left[\hat{\mathbf{x}}^{(i)}(k+j+1|k), \mathbf{v}^{(i)}(k+j+1|k)\right] + \Upsilon_f^{(i)}(k|k)\right] \end{aligned} \quad (21)$$

It may be noted that $\Upsilon_f^{(i)}(k|k-1)$ and $\Upsilon_f^{(i)}(k|k)$ are filtered values of signals $\Upsilon^{(i)}(k|k-1)$ and $\Upsilon^{(i)}(k|k)$, respectively, which are defined by equation (12) and equation (14). These filtered signals are computed as follows:

$$\Upsilon_f^{(i)}(k\,|\,k-1) = \Phi_e \Upsilon_f^{(i)}(k-1\,|\,k-2) + [I-\Phi_e]\Upsilon^{(i)}(k\,|\,k-1) \quad (22)$$

$$\Upsilon_f^{(i)}(k\,|\,k) = \Phi_d \Upsilon_f^{(i)}(k-1\,|\,k-1) + [I-\Phi_d]\Upsilon^{(i)}(k\,|\,k) \quad (23)$$

Φ_d and Φ_e are diagonal matrices of the form

$\Phi_d = \mathrm{diag}[\lambda_1 \quad \lambda_2 \quad .... \quad \lambda_n]$

$\Phi_e = \mathrm{diag}[\alpha_1 \quad \alpha_2 \quad .... \quad \alpha_n]$

$0 \le \lambda_i < 1$ and $0 \le \alpha_i < 1$

where, λ_i and α_i are tuning parameters.

3.2 *NMPC Formulation*

At any sampling instant k, the NMPC is formulated as a constrained nonlinear optimization problem where the future manipulated inputs are determined by minimizing the following objective function

$$\min_{u(k|k).......u(k+N_p-1|k)} \sum_{j=1}^{N_P} E(k+j|k)^T W_E \, E(k+j|k) + \sum_{j=0}^{N_C-1} \Delta u(k+j|k)^T W_U \, \Delta u(k+j|k) \quad (24)$$

Subject to the

Prediction given by equations (19)-(23)

$$u(k+N_c/k) = u(k+N_c+1/k) = u(k+N_p\text{-}1/k) = u(k+N_c\text{-}1/k) \quad (25)$$

$$x^L \le \hat{x}(k+j|k) \le x^U \qquad (j=1.....N_P) \quad (26)$$

$$y^L \le \hat{y}(k+j|k) \le y^U \qquad (j=1.....N_P) \quad (27)$$

$$u^L \le u(k+j|k) \le u^U \qquad (j=0....N_c\text{-}1) \quad (28)$$

$$\Delta u^L \le \Delta u(k+j|k) \le \Delta u^U \quad (j=0....N_C\text{-}1) \quad (29)$$

Where,

$$E(k+j|k) = y_r(k+j|k) - \hat{y}(k+j|k) \quad (30)$$

$$\Delta u(k+j|k) = u(k+j|k) - u(k+j-1|k) \quad (31)$$

where $y_r(k+j\,|\,k)$, in equation (30) represents future setpoint trajectory. The cost function is minimized subject to constraints on the state variables, output variables, manipulated variables and change in manipulated variables. Equation 25 together with equation 31 states that no future control moves are planned beyond the control horizon of N_c steps. The desired closed loop performance of the proposed NMPC scheme can be achieved by appropriately selecting the prediction horizon N_P, control horizon N_c, the error weighting matrix (W_E), input move weight matrix (W_U) and filter matrices (Φ_d, Φ_e). Further, the NMPC scheme is implemented in a moving horizon framework i.e. only the first move u(k|k) is implemented on the plant and the constrained optimization problem is reformulated at the next sampling instant based on the updated information from the plant.

4. SIMULATION STUDIES

The system considered for simulation study has three tanks (Blank et al. 2003), which are connected through the valves u_1 to u_7. The governing equations of the three-tank hybrid system are as follows:

$$A\frac{dh_1}{dt} = q_{max}u_1 - q_2 - q_3 - q_6 \quad (32)$$

$$A\frac{dh_2}{dt} = q_2 + q_3 - q_4 + q_7 - q_5 \quad (33)$$

$$A\frac{dh_3}{dt} = q_{max}u_5 + q_4 + q_7 \quad (34)$$

$$q_3 = k_3\,\mathrm{sgn}(h_1-h_2)\sqrt{|h_1-h_2|}\,u_3 \quad (35)$$

$$q_4 = k_4\,\mathrm{sgn}(h_2-h_3)\sqrt{|h_2-h_3|}\,u_4 \quad (36)$$

$$q_6 = k_6\sqrt{h_1+h_d}\,u_6 \quad (37)$$

$$q_5 = k_5\sqrt{h_2+h_z} \quad (38)$$

$$\begin{aligned} q_2 &= z_1 k_2\sqrt{|(h_1-h_2)|}\,u_2 \quad \{h_1,h_2 \le h_T\} \\ &= z_1 k_2\sqrt{(h_1-h_T)}\,u_2 \quad \{(h_1 \succ h_T)\mathrm{AND}(h_2 \le h_T)\} \\ &= z_1 k_2\sqrt{(h_2-h_T)}\,u_2 \quad \{h_1 \le h_T)\mathrm{AND}(h_2 \succ h_T)\} \\ &= z_1 k_2\sqrt{|(h_1-h_2)|}\,u_2 \quad \{h_1,h_2 \succ h_T\} \end{aligned}$$

$$\begin{aligned} q_7 &= z_2 k_7\sqrt{|(h_2-h_3)|}\,u_7 \quad \{h_2,h_3 \le h_T\} \\ &= z_2 k_7\sqrt{(h_2-h_T)}\,u_7 \quad \{(h_2 \succ h_T)\mathrm{AND}(h_3 \le h_T)\} \\ &= z_2 k_7\sqrt{(h_3-h_T)}\,u_7 \quad \{h_2 \le h_T)\mathrm{AND}(h_3 \succ h_T)\} \\ &= z_2 k_7\sqrt{|(h_2-h_3)|}\,u_7 \quad \{h_2,h_3 \succ h_T\} \end{aligned}$$

$$z_1 = \begin{cases} 0 & \{(h_1 \le h_T)\mathrm{AND}(h_2 \le h_T)\} \\ +1 & \left\{\begin{matrix}[(h_1 \succ h_T)\mathrm{AND}(h_2 \le h_T)]\ \mathrm{OR} \\ [[(h_1 \succ h_T)\mathrm{AND}(h_2 \succ h_T)]\mathrm{AND}(h_1 \succ h_2)]\end{matrix}\right\} \\ -1 & \left\{\begin{matrix}[(h_1 \le h_T)\mathrm{AND}(h_2 \succ h_T)]\mathrm{OR} \\ [[(h_1 \succ h_T)\mathrm{AND}(h_2 \succ h_T)]\mathrm{AND}(h_1 \prec h_2)]\end{matrix}\right\} \end{cases}$$

$$z_2 = \begin{cases} 0 & \{(h_2 \le h_T)\mathrm{AND}(h_3 \le h_T)\} \\ +1 & \left\{\begin{matrix}[(h_2 \succ h_T)\mathrm{AND}(h_3 \le h_T)]\mathrm{OR} \\ [[(h_2 \succ h_T)\mathrm{AND}(h_3 \succ h_T)]\mathrm{AND}(h_2 \succ h_3)]\end{matrix}\right\} \\ -1 & \left\{\begin{matrix}[(h_2 \le h_T)\mathrm{AND}(h_3 \succ h_T)]\mathrm{OR} \\ [[(h_2 \succ h_T)\mathrm{AND}(h_3 \succ h_T)]\mathrm{AND}(h_2 \prec h_3)]\end{matrix}\right\} \end{cases}$$

For the three tank hybrid system z1 and z2 represent discrete variables and they can take only finite integer values, such as {-1,0,1} depending on the level of fluids in the tanks. For example

- If the discrete variable (z1) value is zero, it implies that the level in the first tank and second tank are below the predefined threshold value (h_T).

- If the discrete variable (z1) value is +1, it denotes that either level in the first tank alone is above the threshold value or levels in the first tank and second tank are above the threshold value, with level in the first tank greater than level in the second tank causing flow in the pipe.
- If the discrete variable (z1) value is -1, it indicates that either level in the second tank alone is above the threshold value or levels in the first tank and second tank are above the threshold value, with level in the second tank greater than level in the first tank causing flow in the pipe.

In all the simulation runs, the process is simulated using the nonlinear first principles model (32 -34) and the true state variables are computed by solving the nonlinear differential equations using differential equation solver in Matlab 6.5. The control problem is to track levels h_1 and h_2 by manipulating valves u_1 and u_5. The NMPC scheme (EnKF based NMPC) for the three-tank system has been developed with the prediction horizon of $N_P = 10,$ and control horizon of $N_c = 1.$ The error weighting matrix and the controller weighting matrix used in the NMPC formulations are $W_E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $W_U = \bar{0}.$ The following constraints on the manipulated inputs are imposed $0 \le u_1 \le 1$ and $0 \le u_5 \le 1$.

4.1 ***Servo response of Three-tank System with EnKF based NMPC Scheme:***

In order to assess the tracking capability of the proposed EnKF based NMPC scheme setpoint variations in h_1 and h_2 as shown in Figures 1(a) and 1(b) are introduced. The sample points used to estimate the statistics of the estimated state of the model in EnKF is 10. The covariance matrices of measurement noise and state noise are assumed as

$$R = \begin{bmatrix} 6.2e-5 & 0 & 0 \\ 0 & 4.8e\text{-}5 & 0 \\ 0 & 0 & 5.4e\text{-}5 \end{bmatrix} \text{ and } Q = \begin{bmatrix} 3.1e-4 & 0 & 0 \\ 0 & 2.4e\text{-}4 & 0 \\ 0 & 0 & 2.7e-4 \end{bmatrix}$$

The initial value of the error covariance matrix $P(0/0)$ in EnKF is assumed equal to that of covariance matrix of state noise. The closed loop performances of EnKF based NMPC scheme is shown in Figures 1(a) & 1(b). From Figure 1, it is inferred that EnKF based NMPC formulation is able to track the setpoint variations effectively. For the case of setpoint tracking we observed that the estimated state variables and true state variables are found to be close (see Figure 2). The controller outputs are shown in Figure 1(c). The evolution of true and estimated value of discrete variables (z_1 and z_2) of three-tank hybrid system with EnKF based NMPC scheme is shown in figure 3.

4.2 ***Performance of EnKF based NMPC Scheme in the presence of Plant-model mismatch***

In order to assess the performance of the proposed EnKF based control scheme, in the presence of plant-model mismatch, we performed simulation studies, by deliberately changing the process parameter k_5 (see equation 38). The closed loop response (EnKF based NMPC scheme) is shown in figures 4a and 4b. The manipulated variable profiles of EnKF based NMPC scheme is shown in figure 4c. The following observation can be drawn from the simulation studies:

- a step change in the setpoint has been introduced at the 25th sampling instant and it can be noted that the controller is able to maintain the process variables (h_1 and h_2) at the desired setpoint, as evident from 50th sampling instant to 60th sampling instant in figures 4a and 4b
- With the setpoint values being persistent, the process parameter value k_5 is changed from 2e-4 to 1.75e-4 at the 60th sampling instant and it can be seen that the EnKF based MPC control scheme is still able to maintain the process variables at the desired setpoint values, as evident from the 80th sampling instant to 100th sampling instant in figures 4a and 4b.

5. CONCLUSION

In this paper, we have proposed a state estimation scheme for an autonomous hybrid system using derivative free nonlinear state estimator. Further, state estimation based nonlinear model predictive control scheme for an autonomous hybrid system has been proposed. The efficacy of the proposed state estimation and control scheme has been demonstrated on the three-tank hybrid benchmark system. From the extensive simulation studies on the three-tank hybrid system, it can be concluded that the proposed nonlinear state estimation based NMPC scheme has good setpoint tracking and disturbance rejection capabilities.

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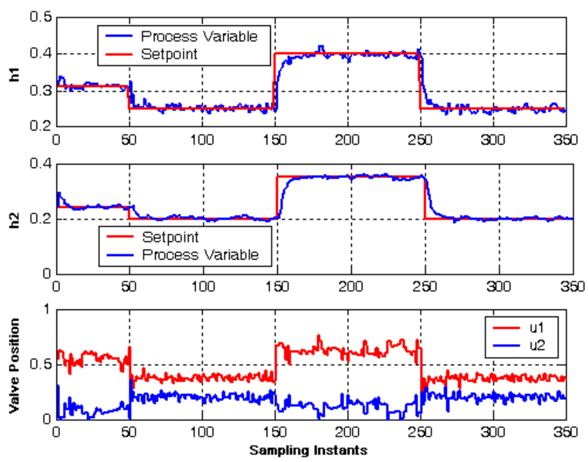

Figure 1: Servo response of three-tank hybrid system (a) Process output (h1) (b) Process output (h2) (c) Controller outputs

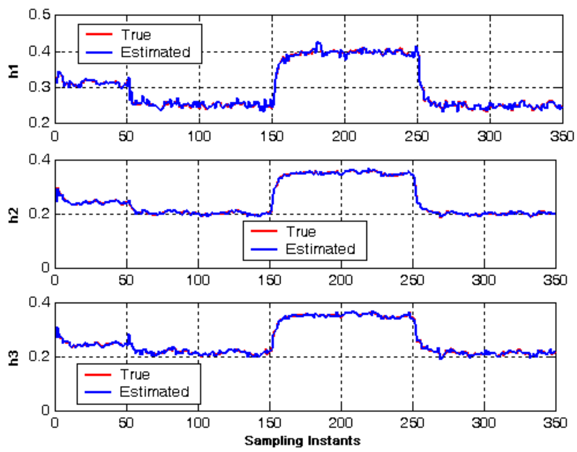

Figure 2: Evolution of true and estimated states of Three-tank hybrid system (a) Level in Tank1 (b) Level in Tank2 (c) Level in Tank3

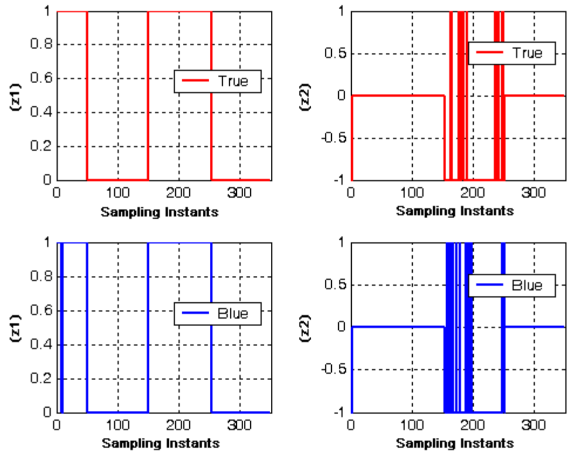

Figure 3: Evolution of True and Estimated values of Discrete Variables

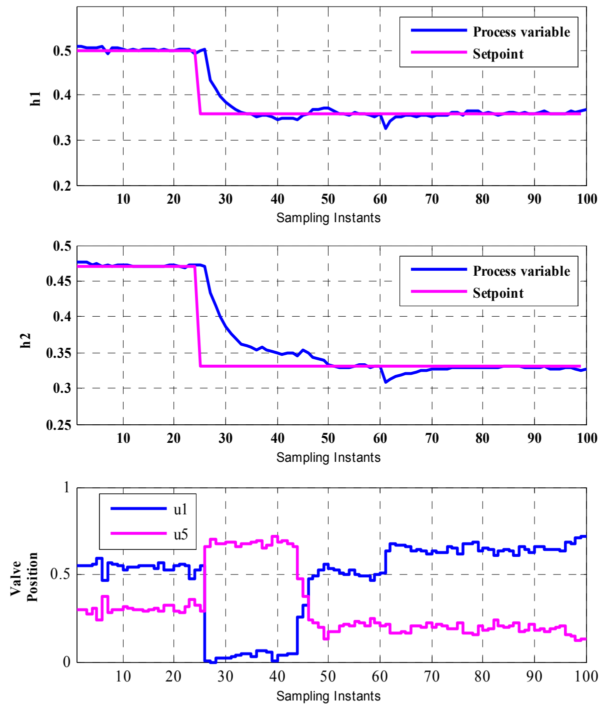

Figure 4: Closed loop response in the Presence of Plant model mismatch (a) Process output (h1) (b) Process output (h2) (c) Controller outputs