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Energy-Insensitive Guidance of Solid Motor Propelled Long Range Flight Vehicles Using MPSP and Dynamic Inversion

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Abstract: A energy-insensitive explicit guidance design is proposed in this paper by appending newly-developed nonlinear model predictive static programming technique with dynamic inversion, which render a closed form solution of the necessary guidance command update. The closed form nature of the proposed optimal guidance scheme suppressed the computational difficulties, and facilitate real-time solution. The guidance law is successfully verified in a solid motor propelled long range flight vehicle, for which developing an effective guidance law is more difficult as compared to a liquid engine propelled vehicle, mainly because of the absence of thrust cutoff facility. The scheme guides the vehicle appropriately so that it completes the mission within a tight error bound assuming that the starting point of the second stage to be a deterministic point beyond the atmosphere. The simulation results demonstrate its ability to intercept the target, even with an uncertainty of greater than 10% in the burnout time.

1. INTRODUCTION

Guidance of solid motor propelled long range flight vehicles with no thrust cutoff facility is a challenging task. A flight vehicle with no thrust termination facility, must expend a fixed amount of energy and still intercept the desired target. As no thrust termination is available, the flight vehicle must consume all available energy. The objective of satisfying the free-flight equation in order to free-fall to an inertial target point must be met at the time of fuel depletion. The uncertainty in the energy content of the solid motor limit the knowledge of the actual burnout time and this may lead to the failure of the mission. To achieve the goal of intercepting, there is a need of taking consideration of this uncertainty in the guidance design.

Since the early development of long range flight vehicle (i.e. launch vehicles and missiles), various guidance schemes have been developed. To name a few, one can find *delta guidance* Battin (1982), *cross-product steering* Battin (1982), *Lambert guidance* Zarchan (1997), *general energy management steering* Zarchan (1997), *optimal control theory based guidance* Padhi (1999), Bryson (1975) in the literatures.

Optimal control theory based guidance schemes are available for many real-life problems. However, for the general class of nonlinear system, the optimal control formulation has not been that successful. The main reason for this is the intensive computational requirement to obtain the optimal solution, which makes it practically useless for any real time application. There has been a wide variety of techniques and approximations used to overcome the difficulties associated with the nonlinear optimal control problem. To make them suitable for online usage (like explicit guidance design), this critical issue must be addressed, which is the primary objective of the recently-developed MPSP design Padhi (2006) used in this paper.

In broad sense, closed loop guidance of flight vehicles can be divided into two groups; namely *implicit guidance* and *explicit guidance*. In case of implicit guidance, the deviation of the actual trajectory from a predefined nominal trajectory is minimized at each instant of flight. The main advantages of this technique are simplified guidance logic and the use of onboard computers with lesser speed. In the case of explicit guidance, on the other hand, the complete set of trajectory equations are solved onboard and new optimum trajectories are computed on flight. The advantages of explicit guidance over implicit guidance can be summarized as: (i) The mission objective can be redefined on flight (consequently, one can opt for complicated missions) and, more important, (ii) The control force and moment requirements are normally not as severe as in implicit guidance.

With liquid engines, the *boost phase guidance* problem for flight vehicles gets greatly simplified because of the thrust cutoff facility. However, a vehicle propelled by liquid engine is normally not preferable over vehicle propelled by solid motor(s), because it gives rise to *sloshing* and *tail wags dog* problems Greensite (1970), which are difficult to handle. More important, the structural weight is higher (which compromised the payload weight) and, owing to its many components, the reliability is also lesser. Moreover, preparation time before launch is also substantially high, which is undesirable in case of ballistic missiles. On the other hand, vehicles propelled by solid motors have many desirable properties like instant firing (less preparation time before launch), low cost, low structural weight, absence of sloshing and tail-wags-dog problems etc. However the guidance problem is more difficult, mainly because of the absence of the thrust cutoff facility, which brings in a stringent requirement that the vehicle must be put in the required target-intercepting free-flight trajectory exactly at the burnout time.

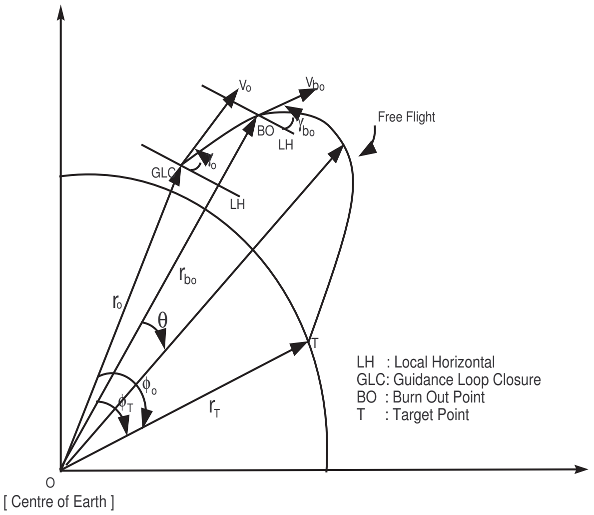

Fig. 1. Schematic view of mission

In this paper, we use a recently developed nonlinear sub-optimal control design technique, named as Model Predictive Static Programming (MPSP) Padhi (2006), in conjunction with the "dynamic inversion approach" Enns (1994),Slotine (1991),Khalil (1996) to develop a energy-insensitive explicit guidance scheme, which leads to a closed form guidance history update. Owing to the closed form nature, the technique is computationally very efficient and can be implemented online. Note that the uncertainty in the energy content implies that the total impulse (area under the thrust-time curve) is not constant and the burnout time of the motor is uncertain. The MPSP technique in conjunction with dynamic inversion approach is used to design a hybrid energy-insensitive guidance scheme, which makes sure that the free-flight equation is satisfied everywhere in a finite time segment towards the predicted burnout time. The forced assurance of satisfying the free-flight equation for a finite time segment leads to the energy-insensitive design.

2. SYSTEM DYNAMICS AND MISSION OBJECTIVE

The schematic view of a typical ballistic missile mission is shown in Figure 1. In the first stage of the two stage flight vehicle, the vehicle follows a fairly mission-independent deterministic path. After the first stage burnout, it has crossed the effective atmosphere and the aerodynamic loading is negligible. After sufficient build up of the thrust, the second stage guidance logic is initiated, which is depicted as the GLC (guidance loop closure) point in Figure 1. Note that because the second stage is beyond atmosphere, the vehicle can be turned significantly as compared to the manoeuver within the atmosphere. This guided second stage ends at the *burnout* point (the point BO in Figure 1), after which the vehicle follows an unpowered free flight path to hit the target 'T'.

With the assumption that the earth is spherical and non-rotating, the point-mass dynamics of the vehicle beyond atmosphere is given by the following set of differential equations Padhi (2005)

$$\begin{bmatrix} \dot{r} \\ \dot{V} \\ \dot{\gamma} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} V \sin\gamma \\ \frac{T}{m}\cos\delta - g\, sin\gamma \\ -\frac{T}{m\,V}\sin\delta + \left(\frac{V}{r} - \frac{g}{V}\right)\cos\gamma \\ -\frac{V\cos\gamma}{r} \end{bmatrix} \tag{1}$$

where, $Re = 6378.4 \times 10^3$ m is the radius of earth
r is the height from center of the earth
γ is the flight path angle wrt the local horizontal
T is the thrust level, m is the mass of the vehicle
V is the velocity of the vehicle
g is the acceleration due to gravity
ϕ is the range angle to be covered
δ is the shear angle (angle between thrust vector and velocity vector), which acts as the guidance parameter

Using a polar co-ordinate system, a *closed-form solution* for the equation of motion for radial distance r beyond the atmosphere assuming a spherical and non-rotating earth is given by Zarchan (1997), Wheelon (1959)

$$\frac{r_{bo}}{r} = \frac{1-\cos\theta}{\lambda\,\cos^2\gamma_{bo}} + \frac{\cos(\theta+\gamma_{bo})}{\cos\gamma_{bo}} \tag{2}$$

where $\lambda = (r_{bo} \times V_{bo}^2)/GM$. Here GM is the product of universal gravitational constant and mass of the earth, r_{bo}, V_{bo} and γ_{bo} are the values of r, V and γ respectively at the burnout time (t_{bo}), θ is the central angle between r_{bo} and r. Equation (2) is known as *free flight equation* (no thrust or aerodynamics forces act in this segment). If it is desired to hit a target that is at a distance x along the surface of earth, then the angular distance (central angle) to be covered is given by $\phi = x/Re$. In that case, $\theta = \phi$ and $r = r_T$, the radial distance at the target. The free flight equation in this case is known as the hit equation Zarchan (1997), Wheelon (1959), which is given by

$$\frac{r_{bo}}{r_T} = \frac{1-\cos\,\phi_{bo}}{\lambda_{bo}\,\cos^2\gamma_{bo}} + \frac{\cos(\phi_{bo}+\gamma_{bo})}{\cos\gamma_{bo}} \tag{3}$$

In the first stage of the flight, the vehicle follows a deterministic path to reach the starting point of second phase. The present guidance scheme is implemented in the second stage. The guided phase ends at the *burnout* time, which is a uncertain, after which the vehicle follows an unpowered free flight path to hit the target. The free flight motion, governed by equation (2), is elliptic in nature so long as $\lambda < 2$. If $\lambda \geq 2$ the vehicle reaches the escape velocity, and leaves the earth permanently. Moreover, for any target intercepting trajectory at a particular position r_{bo} and velocity V_{bo}, the choice of γ_{bo} is not unique. In fact, two choices of γ_{bo} are possible, out of these two choices, one leads to a steep trajectory and the other leads to a shallow trajectory. Here we have aimed for a shallow trajectory (else, we found that it is impossible to design an energy-insensitive guidance scheme without a discontinuity in the shear angle history). The objective of the guidance scheme is to compute a shear angle history to satisfy the hit equation at the exact time of burnout despite the uncertainty in the energy content of solid motors.

3. GUIDANCE DESIGN: GENERIC THEORY

In this section, for better clarity and generality, we discuss the theoretical details of the techniques followed in this paper in a generic sense. The problem specific equation are discussed separately in Section IV.

3.1 Model Predictive Static Programming Design

In this section, for completeness, we present the mathematical details of the newly developed MPSP technique Padhi (2006) in fair detail. In this design, we consider general nonlinear systems in discrete form, the state and output dynamics of which are given by

$$X_{k+1} = F_k(X_k, U_k) \tag{4}$$

$$Y_k = h(X_k) \tag{5}$$

where $X \in \Re^n$, $U \in \Re^m$, $Y \in \Re^p$ and $k = 1, 2, \ldots, N$ are the time steps. The primary objective is to obtain a suitable control history U_k, $k = 1, 2, \ldots, N-1$, so that the output at the final time step Y_N goes to a desired value Y_N^*, i.e. $Y_N \to Y_N^*$. In addition, we aim to achieve this task with minimum control effort.

For the technique presented here, one needs to start from a "guess history" of the control solution. In this section, we present a way to compute an error history of the control variable, which needs to be subtracted from the previous history to get an improved control history. This iteration continues until the objective is met i.e. until $Y_N \to Y_N^*$. To meet the objective $Y_N \to Y_N^*$, first we define the error in the output as $\triangle Y_N \triangleq Y_N - Y_N^*$. Next, using small error approximation (i.e. neglecting higher order terms in the Taylor series expansion) we write

$$\triangle Y_N \cong dY_N = \left[\frac{\partial Y_N}{\partial X_N}\right] dX_N \tag{6}$$

However from (4), we can write the error in state at time step $k+$) as

$$dX_{k+1} = \left[\frac{\partial F_k}{\partial X_k}\right] dX_k + \left[\frac{\partial F_k}{\partial U_k}\right] dU_k \tag{7}$$

where dX_k and dU_k are the error of state and control at time step k respectively. Expanding dX_N as in (7) (for $k = N-1$) and substituting it in (6), we get

$$dY_N = \left[\frac{\partial Y_N}{\partial X_N}\right] \left(\left[\frac{\partial F_{N-1}}{\partial X_{N-1}}\right] dX_{N-1} + \left[\frac{\partial F_{N-1}}{\partial U_{N-1}}\right] dU_{N-1} \right) \tag{8}$$

Expanding $dX_k, k = (N-2), (N-3), \ldots, 1$, in a sequential manner arranging the terms, we get

$$dY_N = A\, dX_1 + B_1 dU_1 + \ldots + B_{N-1} dU_{N-1} \tag{9}$$

$$\text{where} \quad A \triangleq \left[\frac{\partial Y_N}{\partial X_N}\right] \left[\frac{\partial F_{N-1}}{\partial X_{N-1}}\right] \cdots \left[\frac{\partial F_1}{\partial X_1}\right]$$

$$B_k \triangleq \left[\frac{\partial Y_N}{\partial X_N}\right] \left[\frac{\partial F_{N-1}}{\partial X_{N-1}}\right] \cdots \left[\frac{\partial F_{k+1}}{\partial X_{k+1}}\right] \left[\frac{\partial F_k}{\partial U_k}\right]$$

where $k = 1, \ldots, N-1$. Since the initial condition is specified, there is no error in the first term; which means $dX_1 = 0$. With this (9) reduces to

$$dY_N = \sum_{k=1}^{N-1} B_k dU_k \tag{10}$$

Here we would like to point out that if one evaluates each of the B_k, $k = 1, \ldots, (N-1)$ as in equation (10), it will be a computationally intensive task (especially when N is high). However, fortunately it is possible to compute them recursively. For doing this, first we define B_{N-1} and B_{N-1}^0 as follows

$$B_{N-1} = \left[\frac{\partial Y_N}{\partial X_N}\right] \left[\frac{\partial F_{N-1}}{\partial U_{N-1}}\right], \quad B_{N-1}^0 = \left[\frac{\partial Y_N}{\partial X_N}\right] \tag{11}$$

Next we compute B_k^0, B_k $k = (N-2), (N-3), \ldots, 1$ as

$$B_k^0 = B_{k+1}^0 \left[\frac{\partial F_{k+1}}{\partial X_{k+1}}\right], B_k = B_k^0 \left[\frac{\partial F_k}{\partial U_k}\right] \tag{12}$$

Equations (11)-(12) provides a *recursive way* of computing B_k, $k = (N-2), (N-3), \ldots, 1$, which leads to substantial saving of the computational time. In equation (10), we have *(N-1)m* unknowns and p equations. Usually $p < (N-1)m$, and hence, it is an under-constrained system of equations. We take advantage of this opportunity and aim to minimize the following objective (performance index)

$$J = \frac{1}{2} \sum_{k=1}^{N-1} dU_k^T R_k dU_k \tag{13}$$

The cost function in (13) needs to be minimized subjected to the constraint in (10). Here $R_k > 0$ (a positive definite matrix) is the weighting matrix. Following the theory of static optimization Bryson (1975), the augmented cost function is given by

$$\bar{J} = \frac{1}{2} \sum_{k=1}^{N-1} dU_k^T R_k dU_k + \lambda^T (dY_N - \sum_{k=1}^{N-1} B_k dU_k) \tag{14}$$

The necessary conditions of optimality are given by

$$\frac{\partial \bar{J}}{\partial dU_k} = R_k dU_k - B_k^T \lambda = 0 \tag{15}$$

$$\frac{\partial \bar{J}}{\partial \lambda} = dY_N - \sum_{k=1}^{N-1} B_k dU_k = 0 \tag{16}$$

Solving for dU_k from (15), we get

$$dU_k = R_k^{-1} B_k^T \lambda \tag{17}$$

Substituting for dU_k from (17) into (16), it leads to

$$A_\lambda \lambda = dY_N \tag{18}$$

where $A_\lambda \triangleq \left[\sum_{k=1}^{N-1} B_k R_k^{-1} B_k^T\right]$. Note that A_λ is a $p \times p$ matrix. Assuming A_λ to be nonsingluar, the solution for λ is given by

$$\lambda = A_\lambda^{-1} dY_N \tag{19}$$

Using (19) in (17), it leads to

$$dU_k = R_k^{-1} B_k^T A_\lambda^{-1} dY_N \tag{20}$$

Hence, the updated control at time step $k = 1, 2, \ldots, (N-1)$ is given by

$$U_k = U_k^0 - dU_k = U_k^0 - R_k^{-1} B_k^T A_\lambda^{-1} dY_N \tag{21}$$

It is clear from (21) that the updated control history solution in (21) is a *closed form solution*. Hence, this formulation is suitable for online implementation. Here, we wish to point out that to save computational time, only one update can be done at a time step and the next improvement (iteration) can be carried at the next time step. More details about this newly developed technique can be found in Padhi (2006).

3.2 Dynamic Inversion

A relatively simpler and popular method of nonlinear control design is the technique of dynamic inversion, which is essentially based on the philosophy of feedback linearization Enns (1994),Slotine (1991),Khalil (1996).

Here we focus on a class of nonlinear systems that are represented by the following system dynamics

$$\dot{X} = f(X, U)$$

$$Y = h(X) \tag{22}$$

where $X \in \Re^n$, $U \in \Re^m$, $Y \in \Re^p$ are the state, control and output vectors of the system respectively. We assume that the system is pointwise controllable. The objective is to design a controller U so that $Y \rightarrow Y^*$ as $t \rightarrow \infty$, where $Y^*(t)$ is the commanded signal for Y to track. We assume that $Y^*(t)$ is bounded, smooth and slowly-varying.

To achieve the above objective, we first notice that from (22), using the chain rule of derivative, the expression for $\dot{Y}$ can be written as

$$\dot{Y} = f_Y(X,U) \tag{23}$$

where $f_Y \triangleq \left[\frac{\partial h}{\partial X}\right] f(X,U)$. Next, defining $E \triangleq (Y - Y^*)$ the controller is synthesized such that the following stable linear error dynamics is satisfied

$$\dot{E} + KE = 0 \tag{24}$$

where K is chosen to be a positive-definite gain matrix. Next, using the definition of E and substituting the expression for $\dot{Y}$ from (23) in (24) to obtain

$$\left(f_Y(X,U) - \dot{Y}^*\right) + K\left(h(X) - Y^*\right) = 0 \tag{25}$$

When $f_Y(X,U)$ is not linear in control, there is no straight forward way to get solution of (25), however, depends on the form of (25) various approaches can be taken up to get a closed form solution of control variables. This completes an overview of the basic steps of the dynamic inversion controller.

4. GUIDANCE DESIGN: PROBLEM SPECIFIC EQUATIONS

As mentioned in Section I, the objective here is to develop a robust "energy-insensitive" guidance design scheme to intercept the target without the actual knowledge of burnout time. In other words, the guidance design scheme must make sure that $dY_N \rightarrow 0$ as $t \rightarrow t_{bo}$, and this goal should be achieved in the presence of uncertainty in the energy content of solid motor. The guidance scheme is designed in two steps. First, we assume that the motor is guaranteed to burn up to a certain duration of the predicted burnout time (say 90%) and design the guidance scheme using the MPSP technique Padhi (2006). Next, we switch over to dynamic inversion approach Enns (1994),Slotine (1991),Khalil (1996), which assures that the free flight equation is satisfied throughout for the remaining time. Based on the generic theoretical details presented in Section III, problem specific equations are outlined in this section.

4.1 Guidance Design Using MPSP

The objective of the guidance scheme is to compute the guidance command input to satisfy the hit equation at switch over time (t_{so}). At the switch over time (t_{so}), the guidance algorithm is switched over to other guidance scheme which is based on dynamic inversion approach. In other words, the guidance scheme must make sure that $Y_N \rightarrow Y_N^*$ at $t = t_{so}$. To achieve this objective, we guide the flight vehicle following the development in Section III.A by taking the help of the flight vehicle dynamics in (1). In this problem the state vector $X \triangleq [r\ V\ \gamma\ \phi]^T$ and the control input $U \triangleq \delta$ (guidance command). Using the Euler method, discretized state dynamics is given by

$$X_{k+1} = F_k(X_k,U_k) = X_k + \Delta t\, f(X_k,U_k) \tag{26}$$

where $f(X_k,U_k)$ is right hand side expression of state equation in (1). From (3), the discretized output at $k = N$ (i.e $t = t_{so}$) is given by

$$y_N = r_T = \frac{r_N^2\ V_N^2 \cos^2 \gamma_N}{GM(1-\cos\phi_N) + r_N V_N^2 \cos(\phi_N + \gamma_N)\cos\gamma_N} \tag{27}$$

Here r_T is the radial distance from the center of the earth at target point. As mentioned before the actual motor burnout time is unknown, and hence, the guidance scheme should be robust against the uncertainty. To achieve this objective, we have carried out following algebra to come up with meaningful condition which make guidance scheme robust against uncertainty. Expanding the hit equation about the switch over time using Taylor series and neglecting higher order terms, we get

$$y \cong y_{so} + \dot{y}_{so} \triangle t \tag{28}$$

To make guidance scheme insensitive to actual burnout time, we force the sensitive term $\dot{y}_{so}$ to zero. If the height derivative term stay near zero, the change in the height will be the minimum. Taking the advantage of this fact, we have chosen output $Y_N = [y\ y_{so}]^T$. The error in outputs are calculated as difference as follows

$$dY_N = Y_N - Y_N^* \tag{29}$$

where $Y_N^* = [r_T^*\ 0]^T$ is desired output at target position, r_T^* is aimed radial distance at the target. The aim here is to compute the guidance command sequence δ_k, $k = 1,\ldots,(N-1)$ so that $dY_N \rightarrow 0$. To achieve this objective, the coefficients B_1 to B_{N-1} are evaluated using (10). Finally, the guidance command sequence $\delta_k, k = 1,\ldots,(N-1)$ is updated by using (17).

4.2 Guidance Design Using Dynamic Inversion Approach

The steering command obtained from MPSP guidance design is applied till switch over time t_{so} to satisfy the hit equation. After switch over time, the occurrence of burnout time is uncertain. However, for the mission to be successful, the hit equation should satisfy at the time of burnout. This can be guaranteed using dynamic inversion approach which forces the hit equation to be satisfied at each step of time. We shall first define height error at the target position from (23), as

$$h_{T_e} = y - y^* \tag{30}$$

where y is same as y_N in (27) and the tracking command is $h_{T_e}^* = Re + 65$ km. Following the philosophy presented above, the goal here is to compute control histories δ such that

$$\dot{h}_{T_e} + k\ h_{T_e} = 0 \tag{31}$$

where $k > 0$ is the chosen gain value. Substituting the required expression in (31), we get

$$\dot{y} + k(y - y^*) = 0 \tag{32}$$

where

$$\dot{y} = \frac{\partial y}{\partial r}\dot{r} + \frac{\partial y}{\partial V}\dot{V} + \frac{\partial y}{\partial \gamma}\dot{\gamma} + \frac{\partial y}{\partial \phi}\dot{\phi} \tag{33}$$

Substituting for $\dot{y}$ in (32) and after rearranging, we get

$$\frac{\partial y}{\partial V}\dot{V} + \frac{\partial y}{\partial \gamma}\dot{\gamma} = \beta \tag{34}$$

where

$$\beta \triangleq -\left(k(y - y^*) + \frac{\partial Y}{\partial r}\dot{r} + \frac{\partial Y}{\partial \phi}\dot{\phi}\right) \tag{35}$$

Substituting for $\dot{V}$ and $\dot{\gamma}$ in (34) from (1) and again rearranging, we get

$$\frac{\partial y}{\partial V}\ \frac{T}{m}\ \cos\delta + \frac{\partial y}{\partial \gamma}\left(-\frac{T}{m\,V}\right)\sin\delta = \beta + \beta_a \tag{36}$$

where

$$\beta_a \triangleq \frac{\partial y}{\partial V}\ g\ \sin\gamma - \frac{\partial y}{\partial \gamma}\left(\frac{V}{r} - \frac{g}{V}\right)\cos\gamma \tag{37}$$

Using trigonometric relations, after some algebra, a closed form expression for guidance command to intersect the target is given by

$$\delta = \sin^{-1}\left(\frac{\beta+\beta_a}{A}\right) - \alpha \tag{38}$$

where

$$A = \left[\frac{\partial y}{\partial V}\,\frac{T}{m} + \frac{\partial y}{\partial \gamma}\,\left(-\frac{T}{m\,V}\right)\right]^{0.5}$$

$$\alpha = tan^{-1}\left(\frac{\partial y/\partial V\;V}{\partial y/\partial \gamma}\right)$$

5. SIMULATION STUDIES

5.1 Data Generation and Assumptions

As mentioned in Section II, a long range ballistic missile having a two stage solid-solid motor has been considered for our simulation studies. From numerical experiment, a research vehicle was designed from preliminary *staging calculation* Padhi (1999). Starting point for the second stage is assumed to be a deterministic point. The task is to guide the vehicle in the thrusted region of the second stage so that the missile falls within the required error bound at the desired target. The guidance is started at the *guidance loop closure (GLC)* time, which is assumed to be 1 *sec* after the starting of the second stage. This was done so that sufficient amount of thrust is built up by that time for the guidance to be effective.

From the results of the staging calculations, the first stage burn out velocity is fixed at 3300 m/s. Similarly the gross mass of the vehicle at the starting point of the second stage (which is assumed to be the point of guidance loop closure) is fixed at 13.1×10^3 kg. The initial *flight path angle* is assumed to be 50^0 (wrt. to the local horizontal). The range covered from the launch point is assumed to be 30 km. The target for the boost guidance scheme is assumed to be at a height of $r_T^* = Re + 65km$. As a simplified yet realistic assumption, the GLC point is assumed to be sufficiently above the effective atmosphere. The guidance cycle time is assumed to 100 *msec*. A height error of ±1 *m* at the target is chosen to be the *convergence criterion* to terminate the MPSP algorithm.

5.2 Numerical Results

Simulations are carried out for three different missions at the ranges of 3000, 3500 and 3500 km. The guidance command histories are shown in figure 2, along with the complete mission trajectories in figure 3. It can be seen from figure 3 that the proposed technique successfully guides the missile at the corresponding target position. The height errors are shown in figure 4 and it can be observed from the figure that the height errors stay near zero from switch over time t_{so} to final time t_f. If burnout occurs within this time period, the missile will intersect the target accurately. These results demonstrate that the energy-insensitive guidance scheme can successfully be applied with the uncertainty in the burnout time.

Note that the core idea behind the energy-insensitive guidance design presented in the paper relies on the technique of dynamic inversion, which continuously enforces $h_{T_e} \to 0$ and $\dot{h}_{T_e} \to 0$.

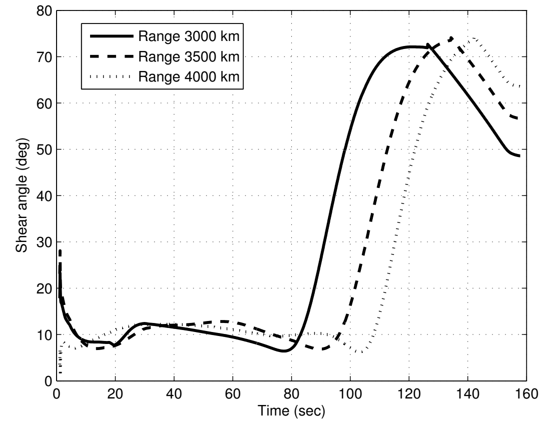

Fig. 2. Guidance command for three different missions

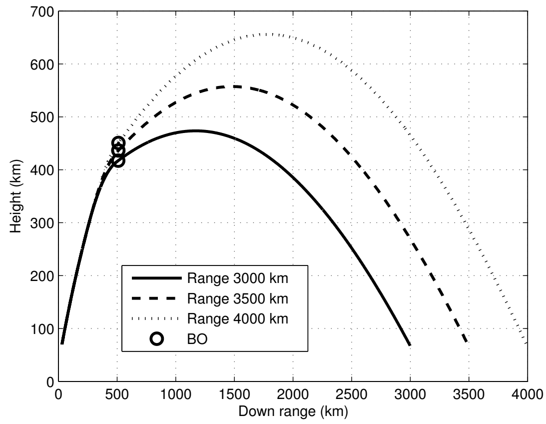

Fig. 3. Final converged trajectories for three different missions

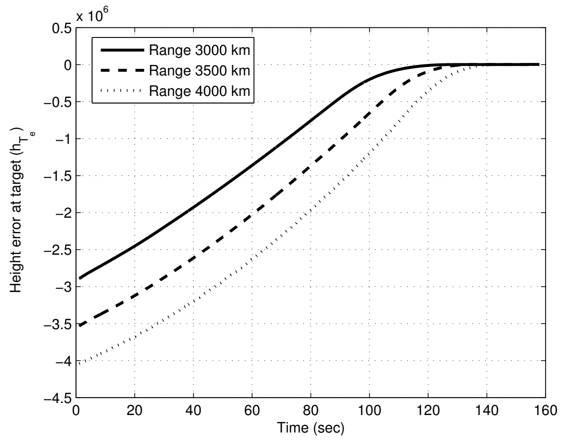

Fig. 4. Height error at target (h_{T_e}) trajectories

Because of this reason, an early switch over is desired. However, a drawback of the dynamic inversion design is the large control magnitude requirement when the error is high. Hence, there should be a judicious choice of the switch over time, taking into account the constraints of the vehicle capability and mission requirement. Keeping this in mind, we carried out a study for the minimum switch over time; the results of which are tabulated in Table 1.

Table 1. Switch over percentage time

Range (km)	Switch over time (t_{so})
3000	0.80 t_f
3500	0.85 t_f
4000	0.90 t_f

It is clear from Table 1 that for smaller ranges, the guidance scheme is more robust in the sense that the amount of uncertainty can be tolerated for a longer duration of time. A visual effect of this can be observed from the height error trajectories from figure 4. From our extensive simulation studies we have observed that for $3000-4000$ km range, the new hybrid guidance scheme assures robustness with respect to 10% (or more) uncertainty in the predicted burnout time.

6. CONCLUSIONS

Combining nonlinear model predictive static programming technique with dynamic inversion approach, a hybrid robust (energy-insensitive) guidance design method is presented in this paper. This nonlinear guidance method provides a sub-optimal solution of the necessary guidance command in the presence of uncertainty in burnout time. The method is used to design an explicit boost phase guidance scheme for long range flight vehicles, which leads to a closed form solution of the necessary guidance command update. The guidance law is successfully tested in a solid motor propelled ballistic vehicle.

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