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Integrated Analysis of Quality and Production Logistics Performance in Asynchronous Manufacturing Lines

M. Colledani*.

**Politecnico di Milano, Via la Masa 1, 20158, Milan Italy (Tel: 0039-02-23998530; e-mail: marcello.colledani@polimi.it).*

Abstract: The evaluation of quality and production logistics performance measures in production lines has been traditionally considered by two separate research areas. However, the industrial reality shows that quality control strategies have an impact on the system production logistics performance as well as the system configuration has an impact on the quality control system reactivity. Therefore, in order to support the phase of design, operation and management of production lines, integrated methodologies and tools able to capture these bilateral relations and to evaluate the overall system performance are needed. The paper presents a new approximate analytical method developed to estimate the quality and productivity performance measures of asynchronous production lines in which quality control chart are present. The control action performed to prevent machines from working out of control is integrated in the manufacturing system model and the delay in the transmission of the quality information, due to the system architecture, is directly taken into account. The method is proved to be accurate and useful to derive new insights regarding the behavior of the considered systems. The proposed approach paves the way to the development of innovative methodologies to design production systems, jointly achieving the required product quality and system productivity performance.

1. INTRODUCTION

Quality is an important issue in real production lines that has been rarely studied at a system level. This leaning presupposes independence of the quality and production logistics performance in production lines. However, the industrial practice shows that there are several decisions, drawn by quality and productivity needs, which are, in the same time, affected both by quality control and production line variables (Inman, *et al*., 2003). Consider the decision concerning the allocation of buffer storages in a production line. This problem has been widely studied in literature both using analytical techniques (Gershwin *et al*., 2000) and simulation (Powell, 1994). The adopted formulation of the buffer allocation problem only considers the impact of buffer modules on the production line throughput (in some cases included in a cost function). The implication of this decision with the quality of produced parts is neglected. However, larger buffers mean higher throughput but also larger time parts spend in the system. Therefore, the presence of buffers between the monitored machines and the inspection station interferes with the transmission of the quality information, creating a delay in the quality control feedback. As a consequence, the allocation of buffers in the line does have an impact on the quality of products. In (Colledani et al., 2007) it was shown that, for a line in the automotive sector, the large buffers included for decoupling sub-lines while executing maintenance operations had a negative impact on the logistics performance, in terms of WIP and lead time, thus should be reduced. Consider the decision regarding the control chart fundamental parameters setting. This problem has attracted great attention among researchers for decades. It has been addressed by providing methodologies for the economic design (Lorenzen *et al.*, 1986), for the robust design (Linderman *et al.*, 2002) and simple ad hoc rules of thumb for the control chart design (Ishikawa, 1976). In these methods, only the impact of the control chart parameters on the quality of products is considered. The effect on the system throughput is neglected and the fact that the monitored machines are often integrated in a production line is not considered. However, since on-line inspection of parts requires time, the number of parts in a sample also affects the logistics performance of the system. Moreover, since the action triggered by the control chart consists in stopping the machine that is supposed to work out of control, the operational time is influenced by the reactivity attributed to the control action. These and similar considerations highlight the importance of an integrated analysis of production lines performance, jointly considering quality and quantity aspects.

Recent literature started addressing the problem of evaluating quality and logistic performance of production lines in an integrated framework. In (Gershwin *et al.*, 2005) the problem of evaluating the performance of production lines involving inspection of parts is considered. However, statistical quality control is neglected. Previous works by the author of this paper addressed the problem of evaluating the performance of production lines in which on-line (Colledani *et al.* 2006a) and off-line inspections (Colledani *et al.* 2006b) were included. Authors showed that there are cases in which the throughput of conforming products in a line monitored by SPC (Statistical Process Control) presents a maximum for a particular buffer capacity, which depends on the system parameters. However, only synchronous lines characterized by machines having identical processing times were considered and inspection time was not directly taken into account.

In this paper a new approximate analytical method, based on the line decomposition, is proposed to evaluate the performance of asynchronous production lines monitored by SPC. Asynchronous production lines are characterized by deterministic processing times, which are different at each production stage. In literature, they have been modelled through the use of continuous flow models. These models approximate the behaviour of asynchronous discrete lines by considering machines that act as valves processing the material flow at different speeds (Le Bihan *et al.*, 1997), (Levantesi *et al.*, 2003). However, methods involving quality control issues and SPC are currently not available. The aim of the paper is twofold: firstly, an accurate, fast and integrated methodology to evaluate the overall system performance, which can be used by practitioners during the phase of design, management and control of production systems is provided; secondly, the developed method is used to scientifically derive important insights about asynchronous lines behaviour, exploring the relation among quality and production logistics performance measures.

2. SYSTEM DESCRIPTION

The considered type of production line monitored by SPC is represented in Fig. 1. The system has a linear layout and is composed by K stations, represented as squares, and $(K-1)$ buffers, represented as circles. Buffers are frequently present in real production lines, with the function of decoupling the machines. They can be automatic conveyors, AS/RSs, floor space, etc. Considered stations can be machining stations, inspection stations or integrated stations. Machining stations are those realizing machining operations on parts flowing in the system. Inspection stations are those measuring some quality characteristics of the parts produced at one or more upstream machining stations. Integrated stations are those performing both manufacturing and inspection operations. For instance in Fig. 1, machine M_1, M_2 and M_5 are machining stations, machine M_3 is an inspection station which measures quality characteristics of parts already processed by stations M_1 and M_2 and machine M_4 is an integrated station measuring quality features of parts processed at the same stage.

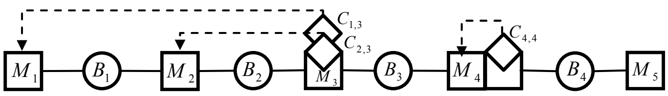

Fig. 1. Representation of the modelled production line

According to the SPC theory, the machining stations can produce being either in control or in out of control state. The in control state is normally characterised by a low fraction of non-conforming parts produced, while the out of control state is normally characterised by a higher fraction of non-conforming parts produced. For instance, the cause for out of control can be the loss of the process settings, the wear of tools or fixtures, the malfunctioning of some components of the machines, etc. Even if, in general, multiple causes for out of control are possible, the commonly adopted assumption of unique out of control mode is considered for each machine.

In order to detect out of control conditions, control charts have been developed in the SPC theory (Montgomery, 1991). Control charts are logical devices that perform statistical tests of hypothesis basing on data measured on the produced parts or on data collected directly from the process. In the model we consider only the first case. In Fig. 1, control charts are represented as rhombus and named $C_{i,j}$, where i refers to the machining station M_i that processed the monitored feature and j is associated to the inspection station M_j which measures the product feature on which the control chart is based. For instance, in Fig. 1, $C_{1,3}$ monitors the behaviour of the machining station M_1 basing on data measured by the inspection station M_3. In this case M_1 is said to be *remotely monitored* by $C_{1,3}$. On the contrary, control chart $C_{4,4}$ is based on data measured at the inspection device in station M_4 monitoring the state of the machining device in the same station. Therefore, station M_4 is said to be *locally monitored* by control chart $C_{4,4}$. In the model, we consider that features machined at different production stages are independent. The competing hypotheses of the statistical test performed by the control chart are H_0, i.e. the monitored machine is in control and H_1, i.e. the monitored machine is out of control. This statistical test is subject to two types of errors, named type I and type II errors. The first error happens with probability α when the hypothesis H_0 is rejected while being true. The type II error happens with probability β when the hypothesis H_1 is accepted while being false.

In order to provide data to be processed by the control charts, inspection plans must be designed. One can design the quality control system to measure all the produced parts, in this case a 100% inspection is performed, or to measure only a fraction of the produced parts, in this case sampling inspection is used. The first policy is normally implemented in those cases in which the time required for inspecting parts is lower than the processing time of productive stations. The second policy is normally followed when inspections are time consuming or are performed manually. Data collected by the inspection stations are normally used also to decide whether the inspected parts can be considered as conforming or non-conforming. Actions which follow this evaluation generally involve scrapping or reworking of defects. The method allows to model scrap even if, to simplify the analysis, in this paper scrapping is not considered.

3. MODELLING ASSUMPTIONS

A detailed list of the method assumptions follows, highlighting both the quality and the productivity aspects.

Continuous flow model: the asynchronous production line behaviour is modelled through a continuous flow model. A continuous flow of material from outside is supposed to enter the system at the first station, then moves to the first buffer, visits the other machines and buffers in sequence until it reaches the last machine and leaves the system. There is always available material at the input of the system (i.e. the first machine is never starved) and available space for material storage at the output of the system (i.e. the last machine is never blocked).

Unreliable machines: the stochastic behaviour of failures is modelled by considering exponentially distributed times to failure and times to repair. The failure and repair rates for machine M_i are respectively $p_{i,fi}$ and $r_{i,fi}$, where $f_i=1,\dots,F_i$ is the total number of failure modes. Failures are Operation Dependent Failures (ODF).

Out of control state: for the machining station M_i the transition to the out of control state is assumed to happen with rate $p_i^{quality}=1/MTOC_i$ with Time to Out of Control (TOC) exponentially distributed. The in control conditions are reset with rate $r_i^{quality}$. Not all the machining stations in the model are necessarily subject to out of control.

Fraction of non conforming products: according to the specification limits, the fractions of non-conforming parts produced are γ_i^W and γ_i^O, respectively when the process performed by station M_i is in control and out of control.

Control charts: parameters related to control chart $C_{i,q}$ are the sample size $m(C_{i,q})$, the number of not measured parts between samples $h(C_{i,q})$, the probability of type I error $\alpha(C_{i,q})$ and the probability of type II error $\beta(C_{i,q})$.

Machining stations processing rates: machining station M_i processes parts at deterministic processing rate μ_i when it is not starved, not blocked and not slowed down (Levantesi *et. al* 2003) by other upstream and downstream stations.

Inspection rate: the inspection of the product feature monitored by control chart $C_{i,q}$ is performed at rate $\mu_{i,q}$ by machine M_q. Since sampling inspection can be performed, thus not all the parts are always measured, the average inspection rate can be approximated with the following equation:

$$\mu_q^{-1} = \sum_{i \le q} \mu_{i,q}^{-1} \frac{m(C_{i,q})}{m(C_{i,q}) + h(C_{i,q})} \qquad (1)$$

If the inspection station is integrated downstream the machining station, the overall station processing rate is the sum of the contribution of both machining and the inspection rates.

Finite capacity buffers: the capacity of buffer B_i is N_i.

The problem addressed in this paper can be formalized as follows: given the system described in Section 2 and the modelling assumptions provided in this section evaluate the following system performance measures:

- P^{Tot} the average total production rate of the system, including both conforming and non-conforming parts;
- P^{Eff} the average effective production rate of the system, including only conforming parts;
- Y^{Sys}, the system yield, that is the fraction of conforming parts produced by the system; it is given by the ratio between the average effective and total production rates;
- n_i the average level of *WIP* in buffer B_i;

4. DESCRIPTION OF THE METHOD

The approximate analytical method proposed in this paper uses an improvement of the decomposition technique originally introduced in (Gershwin, 1994) and extended to the case of multiple failure mode machines in (Levantesi *et al.* 2003), named Two-Level Decomposition (Colledani, *et al.*, 2005).

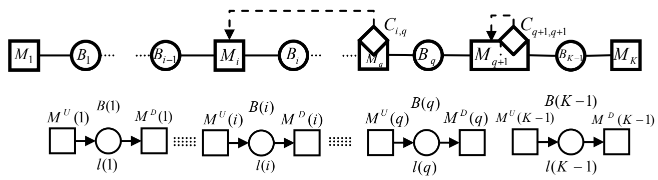

Fig. 2. Buffer Level Decomposition

The original multiple failure modes decomposition consists in decomposing the system with K machines into *(K-1)* subsystems formed by two pseudo-machines, $M^u(i)$ and $M^d(i)$, and one buffer, $B(i)$, easy to analyse with exact analytical techniques (Levantesi et al. 1999). Since the attention is focused on the flow of parts through the buffers of the decomposed lines we named this level Buffer Level Decomposition (BLD), Fig. 2. For each two-machine line $l(i)$ performance measures can be calculated, such as the production rate of the sub-system $P(i)$, the average buffer level $n(i)$, the probability of the upstream and downstream machines being operative, $E^u(i)$ and $E^d(i)$, the probabilities of starvation of the downstream machine $Ps_{j,f}(i)$ and the probabilities of blocking of the upstream machine $Pb_{j,f}(i)$, linked to the failures which caused the starvation and blocking. Moreover, the probability of the upstream machine being down in mode f, $D^u_f(i)$ and the probability of the downstream machine being down in mode f, $D^d_f(i)$ can be obtained. If the machines are characterized by multiple failure modes, then the BLD is enough to accurately evaluate the performance of the asynchronous lines. However, when the machines behaviour is more complex an additional level of analysis of the system focused on machines must be considered. This analysis is named Machine Level Decomposition (MLD). The MLD consists in modelling the behaviour of quality monitored machines through a continuous time - discrete state Markov chain. The Markov chain representing the behaviour of the general machining station M_i, remotely monitored by the control chart $C_{i,q}$ is reported in Fig. 3

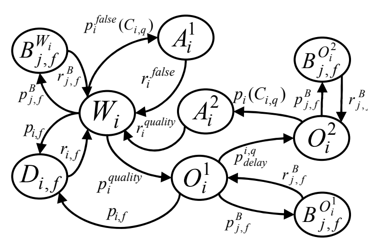

Fig. 3: Quality monitored machine model

The states in which the M_i can be found are (starvation states are not represented in the Fig. 3 for avoiding confusion. They are symmetrical to blocking states):

- W_i: the in control state;
- O_i^1: the out of control state, not observable by the control chart, since parts produced out of control have not already been measured by the inspection device;
- O_i^2: is the out of control state, not detected but observable to the control chart;
- A_i^1: is the false alarm state;
- A_i^2: is the detected out of control state;
- $D_{i,fi}$: is the operation dependent failure state;
- $B_{j,f_j}^{W_1}$ ($S_{t,f_t}^{W_2}$): is the blocking (starvation) in control state;
- $B_{j,f_j}^{O_1^1}$ / $B_{j,f_j}^{O_1^2}$ ($S_{t,f_t}^{O_1^1}$ / $S_{t,f_t}^{O_1^2}$) is the blocking (starvation) out of control state.

The machine has three operative states, W_i, O_i^1, O_i^2. In these states, the processing rate is greater than zero, while in all the other starvation, blocking and failure states, the processing rate is set to zero, according to the continuous flow model. However, given the fact that the machine can be slowed down by other slower machines in the line, the resulting processing rate is lower than the maximal processing rate μ_i. The processing rate μ_i^{sd} adjusted considering the slow down phenomenon must be calculated. Also, the rates at which the machine goes failed, starved and blocked, related to the logistic impact of other machines and buffers on the considered station are unknown. These parameters are calculated by the decomposition equations in Appendix A.

Other unknown transition rates are those related to quality control. We name the equations that allow calculating these transition rates *quality link equations*:

$$p_i^{false}(C_{i,q}) = \frac{1}{MTTFA(C_{i,q})} = \frac{\mu_i^{sd}}{\mu_i} \frac{1}{ARL_0(C_{i,q})[h(C_{i,q}) + m(C_{i,q})]} \quad (2)$$

$$p_i(C_{i,q}) = \frac{1}{MTTD(C_{i,q})} = \frac{\mu_i^{sd}}{\mu_i} \frac{1}{ARL_1(C_{i,q})[h(C_{i,q}) + m(C_{i,q})]} \quad (3)$$

Where ARL_0 and ARL_1 are the average run lengths, i.e. the number of samples to be processed by the control chart before detecting, respectively, a false alarm or a real out of control state. According to the SPC theory, they are related to Type I and Type II error probabilities. Moreover, in case the machine M_i is not locally monitored, produced parts have to be stored into the buffers included among M_i and M_q before being measured; therefore a delay in the quality information transmission is observed. It can be approximately estimated by using the following additional *quality link equation*:

$$p_{delay}^{i,q} = \frac{1}{delay(i,q)} = \frac{1}{\sum_{j=i}^{q-1} \frac{\bar{n}(j)}{P(j)}} \quad (4)$$

which is basically the inverse of the average time parts spend in the sub-systems included among the monitored and the inspection station, calculated with the Little's law. When all the transition rates are known the Markov chain can be solved. The calculated steady state probabilities can be used to transfer to the BLD the failure rates of the pseudo-machines. The interruption of flow due to the need of restoring the machine after an out of control is detected is modelled as a failure for the pseudo-machine as well as the interruption of flow due to false alarms of the control chart. In case the upstream pseudo-machine $M^{U(i)}$ of the sub-system *L(i)* is under consideration, these transition rates can be respectively estimated as:

$$p^{u(i)}(C_{i,q}) = \frac{\pi(A_i^2)}{\pi(W_i) + \pi(O_i^1) + \pi(O_i^2)} r_i^{quality} \quad (6)$$

$$p_{false}^{U(i)}(C_{i,q}) = \frac{\pi(A_i^1)}{\pi(W_i) + \pi(O_i^1) + \pi(O_i^2)} r_i^{false} \quad (5)$$

Moreover, from the MLD it is possible to estimate the machine yield:

$$Y_i = \frac{\pi(W_1)(1-\gamma_i^{W_i}) + (\pi(O_i^1) + \pi(O_i^2))(1-\gamma_i^{O_i})}{\pi(W_i) + \pi(O_i^1) + \pi(O_i^2)} = Y^{U(i)} \quad (7)$$

The developed method follows an iterative scheme that alternatively visits the MLD and the BLD. The algorithm is reported in Appendix A. The system level performance measures can be estimated as follows:

$$P^{tot} = P(i) \qquad \forall i = 1,..,K\text{-}1 \quad (8)$$

$$Y^{sys} = \prod_{i=1}^{K} Y_i \quad (9)$$

$$P^{Eff} = P^{tot} Y^{sys} \quad (10)$$

5. NUMERICAL RESULTS

The accuracy of the proposed approximate analytical method has been tested by comparing results with those provided by discrete event simulation. The simulation model was developed in ARENA 7.01. The asynchronous line model is based on the system description provided in Section 2. In order to test the assumption of assimilating the monitored machine to a Markov process, the quality control loop is considered in details, directly simulating the control feedback of statistical control charts, in line with the SPC theory. A set of 65 randomly generated cases have been studied, exploring a wide set of possible system architectures and parameters. Test systems were characterized by different number and location of inspection devices, number of machining stations varying from 2 to 10, operational failure and repair rates, varying from 0.0005 to 0.5, out of control rates varying from 0.0001 to 0.1 and processing and inspection rates, varying from 0.5 to 2. Moreover, sample size varied from 4 to 25 and buffer capacities varied from 3 to 40. The error of the proposed method, $\varepsilon_{method,}$ in the estimation of the generic system performance measure, θ, has been calculated by using the following equation:

$$\varepsilon_{method} = \frac{\left(\theta_{method} - \bar{\theta}_{simulation}\right)}{\bar{\theta}_{simulation}} \cdot 100 \quad (11)$$

where $\overline{\theta}_{simulation}$ is the average value of the outputs $\theta^{r}_{simulation}$ of the replicates of the simulation test. Simulation runs length was set to 1.000.000 time units and 10 replicates for each case were performed; this leads to a maximum half width of the confidence interval on the effective throughput of 0.003. The maximum error in the evaluation of the effective throughput is 3.24% but in the 72% of the cases the error is lower than 2%. The maximum error in the estimation of the system yield is 1.3 % and it is lower than 1% in the 92% of cases. The maximum error in the buffer level estimation is 5.3%; in the 64% of cases it is lower than 3%.

Given the good accuracy observed, a more detailed analysis has been performed to investigate whether the accuracy of the method depends on some factors, with the objective of identifying possible directions for future improvements. Therefore a 2^5 factorial plan has been designed. The system layout is reported in Fig. 4; Xbar control chart was considered (Montgomery 1991).

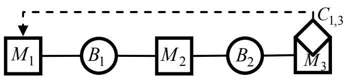

Fig. 4: Considered system layout

The five factors have been both quality control and production line variables. In particular the efficiency in isolation (Levantesi *et al.* 1999) of the machine M_2, named e_2, has been considered with levels 0.85 and 0.9. This allowed including balanced and un-balanced lines. The capacity of the buffer B_1, N_1, has been considered with levels 4 and 50; the processing rate of M_1, μ_1, with levels 0.8 and 1.3; the entity of the shift of the quality characteristic distribution δ_i, with levels 1.2 and 1.8 (in standard deviation units); the value of the sample size $m(C_{1,3})$ with levels 4 and 15. A full experimental plan has been carried out with five replicates; as a consequence the analysis required 5x32 experimental conditions to be tested. The response was the error of the method in the estimation of the effective production rate of the system. The output of the ANOVA, with $\alpha_{family}=0.05$, highlights that all the factors, both those related to quality and production logistics, except the shift δ_i are significant. Moreover, by observing the main effect plots, the error increases when the sample size is higher. This is due to the approximation introduced while averaging the inspection rate, thus the method can be improved by detailing this modelling aspect. Moreover, for lower speed of the first machine the method performs better, since the lead time and the quality feedback delay is less influent. This aspect is visible also looking at the significant interaction among N_1 and μ_1. For high values of these parameters, the buffer is highly populated in average and the delay increases, thus reducing the accuracy of the method. This suggests spending more effort in the quality feedback delay modelling, in order to improve the method accuracy.

6. SYSTEM BEHAVIOUR

The developed method has been used to study the behaviour of the asynchronous production lines monitored by SPC, with the aim of deriving new rules to manage and design such type of production systems, very common in industrial practice. In this paragraph, some of the most important results leading new insights on the topic are summarized. The first result applies to the system in Fig. 4. The behaviour of the system effective production rate while varying the capacity of the buffer B_1 has been observed. Results are reported in Fig. 5, for 7 different values of the processing rate of M_1, μ_1. As it can be noticed there are cases in which the P^{eff} curve differs from the well known throughput curve, since it presents a maximum. This means that there is a particular buffer capacity which maximizes the production of conforming products. This behaviour is due to the quality and production logistics trade-off. It is known that increasing the buffers increases the system production rate. However, given the quality information feedback, increasing the buffer also increases the delay of the control action, which is directly influenced by the average number of parts in the system (WIP). When the first effect weights more then the second the P^{eff} curve increases, when weights change, the P^{eff} curve decreases.

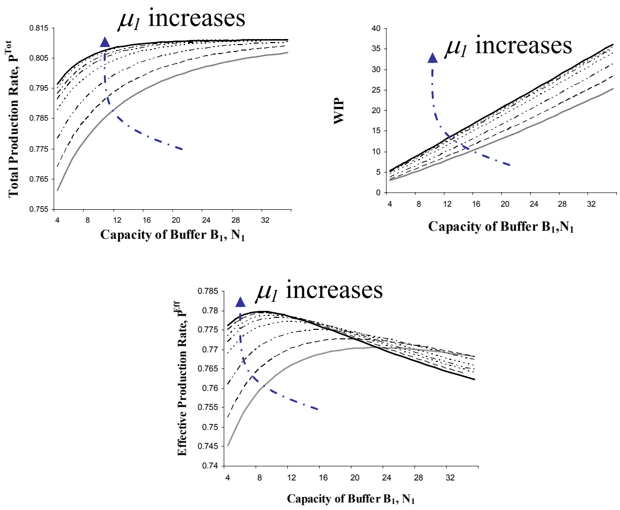

Fig. 5. WIP, Effective and Total Production Rate Curves as a Function of the Buffer Capacity

More interesting, by decreasing the processing rate of the first machine M_1, the buffer will be less full and the lead time will decrease. Therefore, also the negative effect of the quality information feedback delay will decrease and the P^{Eff} curve behaves as the known monotonically increasing production rate curve. It is very interesting to notice that, for small buffer capacities, effort in increasing the machine speed has a positive impact on the system performance; however, for large buffers, a local machine improvement, i.e. higher machine speed, has a negative impact on the overall system performance. This behaviour is counterintuitive as it is in contrast with the known monotonicity properties of asynchronous production lines. However, it is supported by a scientific analysis which grounds on the modelled interaction among quality and production logistics performance.

The second experiment had the objective of investigating the impact of the inspection station allocation on the system performance. The analyzed asynchronous line was formed by 4 machining stations, all subject to out of controls; therefore 4 control charts were present in the system. All the possible allocations of inspection stations were considered. This resulted in a set of 8 system configurations to be analyzed. The case with 4 inspection stations and all the machines locally monitored is represented in Fig. 6. The effective production rate as a function of the number of inspection stations in the system is reported in Fig.7.

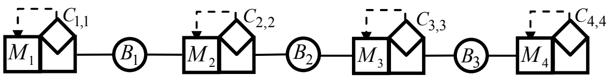

Fig. 6. System Configuration with 4 Inspection Stations

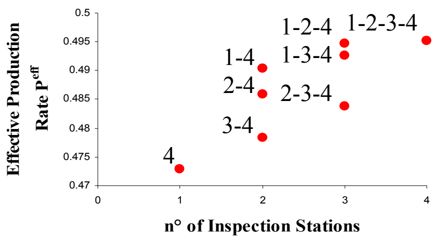

Fig. 7. Effective Production Rate as a Function of the Number of Inspection Stations in the System

By observing the graph, three important considerations can be done. Firstly, a lower number of inspection stations coherently positioned performs better than a higher number of stations poorly allocated. Consider the system configuration 1-4, in which machines M_1 and M_4 are locally monitored and machines M_2 and M_3 are remotely monitored by control charts $C_{2,4}$ and $C_{3,4}$. This solution provides a higher effective production rate than the solution 2-3-4, which involves an additional inspection station. This phenomenon is due to the high impact of the station M_1 quality feedback delay on the overall line performance. Therefore, in this case, the money investment for the additional inspection device is not correctly exploited. Secondly, the inspection station allocation strongly impacts on the production rate of the system. Indeed, the maximum difference observed between the cases characterized by 1 and 4 inspection stations is 4.72%. Thirdly, the big improvement in the effective production rate is obtained by shifting from one inspection device to two inspection devices, correctly allocated (+3.55%). The further addition of inspection devices only marginally increases the effective production rate of the system. These results highlight the need for adopting an integrated approach while allocating inspection devices in the line, since system configurations provided by neglecting the quality and productivity relations may be sub-optimal and may lead to wrong money investment.

In the third experiment, the setting of the control chart parameters has been taken into account. In particular, the attention was focused on the number of parts in the sample $m(C_{i,q})$. The analyzed system was the three machines line reported in Fig. 4. The considered control chart was the Shewart chart, monitoring the average value of one product feature, measured over a sample of parts. The number of not measured parts between samples $h(C_{1,3})$ was set to 30 and the inspection rate $\mu_{1,3}$ varied among 5 and 9 (the processing rate of M_3 was μ_3=1). In the analysis, the dependency of the control chart type II error probability and that of the Control Limit values on the sample size was directly taken into account (Montgomery, 1991). The effective production rate of the system as a function of the sample size, varying from 1 to 15, is reported in Fig. 8.

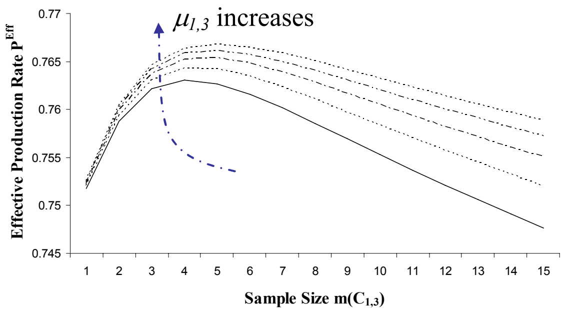

Fig. 8. Effective Production Rate as a Function of the Sample Size

As it can be observed in the graph, the effective production rate is maximized for a particular value of the sample size. This is due to the fact that the higher is the number of parts in a sample, the higher is the probability of the control chart to generate a correct alarm when out of controls are observed. However, higher number of parts in a sample also means higher inspection time, thus affecting the logistic performance of the system in terms of total production rate. Therefore, when the positive impact of the first effect weights more than the second negative effect, the production rate of conforming parts increases to the sample size; however, it decreases when the weights change. Finally, it can be observed that for lower time required to inspect parts, i.e. higher inspection rate, the optimal sample size is larger, since the positive effect due to a more reactive quality control is predominant. These results suggest that also the decisions concerning the control charts parameters should be taken considering the implications with the logistics performance of the system.

7. CONCLUSIONS AND OPERATIVE RULES

The paper proposes an approximate analytical method to estimate the performance of asynchronous production lines monitored by SPC. The proposed method directly takes into account the most important interactions among quality and productivity system variables. In particular, the quality information feedback delay due to the presence of buffers in the line and the corrective action triggered by control charts are modelled. The main results which have practical impact on the way the considered systems are designed and managed, can be summarized in the following list:

- Lower WIP does not always mean higher conforming parts production rate, since the behaviour strictly depends on the buffer capacity;

- The design of buffers should be performed by jointly considering their impact on quality and productivity;
- Local improvements (increasing the machine speed) can have negative impact on the system level performance;
- Allocating higher number of inspection devices does not always mean higher conforming parts production rate;
- The control chart parameters should be set also considering the impact of inspection times on the line production logistic performance.

Future activities will be targeted to the development of an integrated framework to jointly design the production system configuration and the quality control system parameters.

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Appendix A. Solution Algorithm.

Initialization: for each sub-line $l(i)$ in which the system is decomposed, the known parameters are introduced while the unknown parameters are initialized to the corresponding value of the physical machines of the original system. For $i=1,..,K-1$ solve the two-machine lines by using the method in (Levantesi *et al.,* 1999).

Step 1: for $i=1,...,K-1$. If the machine M_i is subject to out of control then go to MLD, otherwise go to BLD.

MLD: Consider the Markov model of Fig. 3. Use *quality link equations* (2), (3) and (4) to calculate unknown transition rates. Solve the Markov chain. Use equations (5) and (6) to calculate pseudo machine $M^u(i)$ new failure rates. Calculate the yield by using equation (7).

BLD: Calculate the two-machine line performance. Use decomposition equations to estimate unknown transition rates for the analysis of M_{i+1}:

$$\mu_{i+1}^{sd} = \frac{P(i)}{E^u(i+1)}$$

$$p_{i+1,f} = \frac{D_f^d(i)}{E^u(i+1)} r_{i+1,f} \qquad \textit{for } f=1,..,F_i$$

$$p_{t,f}^s = \frac{S_{t,f}(i)}{E^u(i+1)} r_{t,f} \qquad \textit{for } t \leq i \textit{ and } f=1,..,F_t$$

$$p_{j,f}^B = \frac{B_{j,f}(i+1)}{E^u(i+1)} r_{j,f} \qquad \textit{for } j>i+1 \textit{ and } f=1,..,F_j$$

Step 2: *for* $i=K,...,2$. Follow MLD and BLD as in Step 1, using similar decomposition equations.

Terminating condition: no changes in the unknown system parameters are identified or conservation of flow is met.