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Neurocontrollers for Trajectory Tracking Problem of a Nonholonomic Mobile Robot

Nardênio A. Martins * **Douglas W. Bertol** *
Warody C. Lombardi * **Edson R. De Pieri** *
Eugênio de B. C. Neto *

* *Automation and Systems Departament, Federal Univ. Santa Catarina*
P.O. Box 476, 88040-900, Florianópolis, SC, Brazil
(+554837217573;nardenio,dwbertol,warody,edson,eugenio@das.ufsc.br)

Abstract: In this paper, a trajectory tracking control for a nonholonomic mobile robot by the integration of a kinematic controller and a torque controller is investigated. The proposed torque controllers (PTCs) are based on a Gaussian radial basis function neural network (RBFNN) modeling technique, which are used to compensate the mobile robot dynamics, significant uncertainties and disturbances. Also, the PTCs are not dependent of the robot dynamics neither requires the off-line training process. The stability analysis and the convergence of tracking errors to zero, as well as the learning algorithms (for weights, centers, and widths) are guaranteed with basis on Lyapunov's theory. In addition, the simulation results shows the efficiency of the PTCs.

1. INTRODUCTION

Mobile robots, which can move with intelligence without any human intervention, have attracted the interest of many researchers due to their extensive field of applications. Several control methods have been proposed for motion control of a mobile robot under nonholonomic constraints. Some nonlinear feedback controllers have been suggested [Kanayama et al., 1990], which consider only the kinematic model of a mobile robot and suppose 'perfect velocity tracking'. Also, some researches consider the dynamics of mobile robot to achieve 'perfect velocity tracking' [Yang and Kim, 1999]. There are few results on the problems, regarding the integration of both, kinematic and neural dynamic controllers for a mobile robot.

Controlling a mobile robot efficiently, with unknown dynamics, and subjected to the uncertainties and/or unmodeled significant disturbances is a field that has been motivated various researches. The computed torque control approach is able to accomplish the control of mobile robot, but it demands the exact dynamics model that, in fact, is impossible in practice. Adaptive controllers [Fukao et al., 2000] can perform the control of mobile robots even with partially unknown dynamics, however, a complicated online estimation of such unknown dynamics is necessary. Fierro and Lewis [1998] developed a neural network based model by combining the backstepping tracking technique with a torque controller, using a multilayer feedforward neural network (known as MLP), that can learn the mobile robot's dynamics, the bounded unmodeled disturbances and the unstructured dynamics, through its on-line learning. However, the control and neural network learning algorithms are very complicated and it is computationally expensive.

In this paper, the RBFNN is applied to control a dynamic system, since the structure of an RFBNN is simpler that a multi-layer perceptron (MLP), the learning rate of RBFNN is generally faster that a MLP, and a RBFNN is mathematically tractable easily [Seshagiri and Khalil, 2000]. Thus, the neural control method presented by Martins and de Alencar [2003], Martins et al. [2005] is extended to the controlling a nonholonomic mobile robot, considering the trajectory tracking problem. Differently from other investigations with mobile robots [Hu and Yang, 2001, Oh et al., 2003], the implementation of the PTCs is based on the partitioning the RBFNN into several smaller subnets in order to obtain more efficient computation. Moreover, the values of centers m and widths σ of Gaussian radial basis functions of the RBFNN are adjusted on-line. Also, need neither an off-line training process nor the priori information of the robot dynamics. Stability and convergence of the robot control system, as well as the learning algorithms (for weights, centers, and widths) are proved by using Lyapunov's theory, considering the presence of bounded unstructured and unmodeled dynamics.

The present paper is organized as follows. In Section 2 the nonholonomic mobile robot dynamics, its structural properties and the neural networks modeling, for mobile robots, are shown. The kinematic controller for a reference trajectory tracking, and the PTCs are described in Section 3. Section 4 shows the results of numeric simulations and, finally, in Section 5 the conclusions are presented.

2. DYNAMICS AND NEURAL NETWORKS MODELING FOR MOBILE ROBOTS

2.1 A Mobile Robot's Dynamics and Structural Properties

In Fierro and Lewis [1998], the dynamic equation of the nonholonomic mobile robot are:

$$\dot{q} = S(q)v(t) \tag{1}$$

$$\begin{aligned} S^T(q)H(q)S(q)\dot{v} \; + \; & S^T(q)\left[H(q)\dot{S}(q) + C(q,\dot{q})S(q)\right]v + \\ + \; & S^T(q)F(\dot{q}) + S^T(q)\tau_d = S^T(q)B(q)\tau \end{aligned} \tag{2}$$

where q and $\dot{q}$ are constrained positions and velocities in cartesian coordinates, respectively; $S(q)$ is a jacobian matrix; v is the actual velocity of the mobile robot; $H(q)$ is a symmetric, positive definite inertia matrix; $C(q,\dot{q})$ is the centripetal and Coriolis matrix; τ_d denotes the bounded unknown disturbances including unstructured and unmodeled dynamics; $B(q)$ is the input transformation matrix; and τ is the input vector. Disregarding surface friction $F(\dot{q})$ and $G(q)=0$, one can rewritten (2) as follows:

$$\textbf{Dynamics 1}\ \ \bar{H}(q)\dot{v}+\bar{C}(q,\dot{q})v+\bar{\tau}_d=\bar{B}(q)\tau=\bar{\tau} \tag{3}$$

The following pattern properties must be emphasized: *Property 1*: *Boundedness* $\rightarrow \bar{H}(q)$, the norm of the $\bar{C}(q,\dot{q})$ and $\bar{\tau}_d$ are bounded.

Property 2: *Skew-symmetry* $\rightarrow$ The matrix $\dot{\bar{H}}(q)-2\bar{C}(q,\dot{q})$ is skew symmetric. This property is particularly important in the stability analysis of the control system.

To avoid the estimation of positions and orientation, due to the nonholonomic constraints pertinent of mobile robots, (3) can be rewritten as:

$$\textbf{Dynamics 2}\ \ \bar{H}(\dot{v})+\bar{C}(v)+\bar{\tau}_d=\bar{B}(q)\tau=\bar{\tau} \tag{4}$$

Mobile robot's dynamics, (4), also it can be rewritten in a linear form,

$$\textbf{Dynamics 3}\ \ \bar{H}(\dot{v})+\bar{C}(v)+\bar{\tau}_d=\Psi(v,\dot{v})\phi+\bar{\tau}_d \tag{5}$$

where $\Psi(v,\dot{v})$ is a coefficient matrix consisting of the known functions of robot velocity v and acceleration $\dot{v}$, which is referred as the robot regressor; and ϕ is a vector consisting of the known and unknown robot dynamic parameters, such as geometric size, mass, moments of inertias, etc.

2.2 Neural Networks Modeling

Based on (3)-(5), it can be verified that $\bar{H}(q)$, $\bar{H}(\dot{v})$ and $\bar{C}(v)$ are functions of q, $\dot{v}$ and v only, respectively, thus, static neural networks are enough to model them. Assuming that $\bar{h}_{kj}(q)$, $\bar{h}_k(\dot{v})$ and $\bar{c}_k(v)$ can be modeled as:

$$\bar{h}_{kj}(q)=\sum_l X^T_{\bar{h}_{kjl}}\xi_{\bar{h}_{kjl}}(q)+\varepsilon_{\bar{h}_{kj}}(q)=X^T_{\bar{h}_{kj}}\xi_{\bar{h}_{kj}}(q)+\varepsilon_{\bar{h}_{kj}}(q) \tag{6}$$

$$\bar{h}_k(\dot{v})=\sum_l W^T_{\bar{h}_{kl}}\xi_{\bar{h}_{kl}}(\dot{v})+\varepsilon_{\bar{h}_k}(\dot{v})=W_{\bar{h}k}{}^T\xi_{\bar{h}_k}(\dot{v})+\varepsilon_{\bar{h}_k}(\dot{v}) \tag{7}$$

$$\bar{c}_k(v)=\sum_l W^T_{\bar{c}_{kl}}\xi_{\bar{c}_{kl}}(v)+\varepsilon_{\bar{c}_k}(v)=W_{\bar{c}k}{}^T\xi_{\bar{c}_k}(v)+\varepsilon_{\bar{c}_k}(v) \tag{8}$$

where $X_{\bar{h}_{kjl}}$, $\mathrm{W}_{\bar{c}_{kl}}$, $\mathrm{W}_{\bar{c}_{kl}}$ $\in R$ are weights of the neural networks; $\xi_{\bar{h}_{kjl}}(q)$, $\xi_{\bar{h}_{kl}}(\dot{v})$, $\xi_{\bar{c}_{kl}}(v)$ $\in R$ are Gaussian radial basis functions with their respective input vectors q, $\dot{v}$ and v only, as well as $\varepsilon_{\bar{h}_{kj}}(q)$, $\varepsilon_{\bar{h}_k}(\dot{v})$, $\varepsilon_{\bar{c}_k}(v)$ $\in R$ are modeling errors of $\bar{h}_{kj}(q)$, $\bar{h}_k(\dot{v})$ and $\bar{c}_k(v)$, respectively, and are assumed to be bounded. Bearing in mind that $\bar{C}(q,\dot{q})$, (4), and $\bar{N}(v,\dot{v})=\Psi(v,\dot{v})\phi$, (5), are dynamic neural networks, since they are functions of q and $\dot{q}$, v and $\dot{v}$, respectively, its modeling are required. Assuming that $\bar{c}_{kj}(q,\dot{q})$ and $\bar{n}_k(v,\dot{v})$ can be modeled as:

$$\bar{c}_{kj}(q,\dot{q})=\sum_l X_{\bar{c}_{kjl}}\xi_{\bar{c}_{kjl}}(z)+\varepsilon_{\bar{c}_{kj}}(z)=X^T_{\bar{c}_{kj}}\xi_{\bar{c}_{kj}}(z)+\varepsilon_{\bar{c}_{kj}}(z) \tag{9}$$

$$\begin{aligned}\bar{n}_k(v,\dot{v})&=\sum_l W^T_{\bar{n}_{kl}}\xi_{\bar{n}_{kl}}(v,\dot{v})+\varepsilon_{\bar{n}_k}(v,\dot{v})\\&=W^T_{\bar{n}_k}\xi_{\bar{n}_k}(v,\dot{v})+\varepsilon_{\bar{n}_k}(v,\dot{v})\end{aligned} \tag{10}$$

where $z=\begin{bmatrix}q^T & \dot{q}^T\end{bmatrix}^T \in R^{2n}$, $X_{\bar{c}_{kjl}}$, $W_{\bar{n}_{kl}}$ $\in R$ are weights vectors; $\xi_{\bar{c}_{kjl}}(z)$, $\xi_{\bar{n}kl}(v,\dot{v})$ $\in R$ are Gaussian radial basis functions with their respective input vectors z, v and $\dot{v}$; $\varepsilon_{\bar{c}_{kj}}(z)$, $\varepsilon_{\bar{n}_k}(v,\dot{v})$ $\in R$ are modeling errors of $\bar{c}_{kj}(q,\dot{q})$ and $\bar{n}_k(v,\dot{v})$, which are also assumed to be bounded.

Foregrounded in (3)-(5), the mobile robot's dynamics can be expressed by (6) and (9); (7)-(8); (10), respectively. Therefore, the stability of the neural networks can be analyzed, where matrix Ge-Lee (GL) [Ge, 1996], defined by $\{.\}$, and its product operator '$\bullet$' are used. The ordinary matrix and vector are denoted by $[.]$.

Thus, $\bar{H}(q)$, $\bar{C}(q,\dot{q})$, $\bar{H}(\dot{v})$, $\bar{C}(v)$ and $\bar{N}(v,\dot{v})$ can be expressed as:

$$\textbf{Dynamics 1}\quad \begin{cases}\bar{H}(q)=\left[\{X_{\bar{H}}\}^T\bullet\{\xi_{\bar{H}}(q)\}\right]+E_{\bar{H}}(q)\\ \bar{C}(q,\dot{q})=\left[\{X_{\bar{C}}\}^T\bullet\{\xi_{\bar{C}}(z)\}\right]+E_{\bar{C}}(z)\end{cases} \tag{11}$$

$$\textbf{Dynamics 2}\quad \begin{cases}\bar{H}(\dot{v})=\left[\{W_{\bar{H}}\}^T\bullet\{\xi_{\bar{H}}(\dot{v})\}\right]+E_{\bar{H}}(\dot{v})\\ \bar{C}(v)=\left[\{W_{\bar{C}}\}^T\bullet\{\xi_{\bar{C}}(v)\}\right]+E_{\bar{C}}(v)\end{cases} \tag{12}$$

$$\textbf{Dynamics 3}\quad \bar{N}(v,\dot{v})=\left[\{W_{\bar{N}}\}^T\bullet\{\xi_{\bar{N}}(v,\dot{v})\}\right]+E_{\bar{N}}(v,\dot{v}) \tag{13}$$

where $\{X_{\bar{H}}\}$, $\{\xi_{\bar{H}}(q)\}$, $\{X_{\bar{C}}\}$ and $\{\xi_{\bar{C}}(z)\}$ are GL matrices, whereas $\{W_{\bar{H}}\}$, $\{\xi_{\bar{H}}(\dot{v})\}$, $\{W_{\bar{C}}\}$, $\{\xi_{\bar{C}}(v)\}$, $\{W_{\bar{N}}\}$ and $\{\xi_{\bar{N}}(v,\dot{v})\}$ are GL vectors; and their respective elements are $X_{\bar{h}_{kj}}$, $\xi_{\bar{h}_{kj}}(q)$, $X_{\bar{c}_{kj}}$, $\xi_{\bar{c}_{kj}}(z)$, $W_{\bar{h}_k}$, $\xi_{\bar{h}_k}(\dot{v})$, $W_{\bar{c}_k}$, $\xi_{\bar{c}_k}(v)$, $W_{\bar{n}_k}$ and $\xi_{\bar{n}_k}(v,\dot{v})$. $E_{\bar{H}}(q)\in R^{n\,\mathrm{x}\,n}$ and $E_{\bar{C}}(z)\in R^{n\,\mathrm{x}\,n}$ are matrices and $E_{\bar{H}}(\dot{v})\in R^n$, $E_{\bar{C}}(v)\in R^n$ and $E_{\bar{N}}(v,\dot{v})\in R^n$ are vectors, and their modeling error elements $\varepsilon_{\bar{h}_{kj}}(q)$, $\varepsilon_{\bar{c}_{kj}}(z)$, $\varepsilon_{\bar{h}_k}(\dot{v})$, $\varepsilon_{\bar{c}_k}(v)$ and $\varepsilon_{\bar{n}_k}(v,\dot{v})$, respectively.

3. CONTROL DESIGN

For the mobile robot, the controllers design problem can be described as: given the reference position $q_r(t)$ and the velocity $\dot{q}_r(t)$, design control laws (C1, C2 and C3 - as PTCs will be stated later) for the actuator torques, which drive the mobile robot to move, so the mobile robot velocity tracking a smooth velocity control input and the reference position.

Let velocity and position of a reference robot are given as:

$$\begin{array}{lll} q_r=\begin{bmatrix}x_r & y_r & \theta_r\end{bmatrix}^T & & v_{ref}=\begin{bmatrix}v_r & \omega_r\end{bmatrix}^T\\ \dot{x}_r=v_r\cos(\theta_r) & \dot{y}_r=v_r sin(\theta_r) & \dot{\theta}_r=\omega_r\end{array} \tag{14}$$

where $v_r>0$ for all t is the reference linear velocity and ω_r is the reference angular velocity. Thus, the position tracking error vector is expressed in the basis of a frame linked to the mobile robot platform as:

$$e_q=\begin{bmatrix}e_1\\e_2\\e_3\end{bmatrix}=\begin{bmatrix}\cos(\theta) & sin(\theta) & 0\\ -sin(\theta) & \cos(\theta) & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_r-x\\y_r-y\\\theta_r-\theta\end{bmatrix} \tag{15}$$

The position error dynamics can be obtained from the time derivative of (15) as:

$$\dot{e}_q=\begin{bmatrix}\dot{e}_1\\\dot{e}_2\\\dot{e}_3\end{bmatrix}=\begin{bmatrix}\omega e_2-v_1+v_r\cos(e_3)\\-\omega e_1+v_r sin(e_3)\\\omega_r-\omega\end{bmatrix} \tag{16}$$

An auxiliary velocity control input v_c that achieves tracking for (1) that is given by [Kanayama et al., 1990]:

$$v_c=\begin{bmatrix}v_r\cos(e_3)+k_1e_1\\\omega_r+k_2v_re_2+k_3v_r sin(e_3)\end{bmatrix} \tag{17}$$

where k_1, k_2 and k_3 are positive parameters.

Given the desired velocity v_c, one define now the auxiliary velocity tracking error as:

$$e_c = v_c - v = \begin{bmatrix} v_{c_1} - v_1 \\ v_{c_2} - \omega \end{bmatrix} = \begin{bmatrix} e_4 \\ e_5 \end{bmatrix} \quad (18)$$

Differentiating (18) and substituting the result in (3)-(5), respectively, the mobile robot dynamics using the velocity tracking error can be rewritten as:

$$\bar{H}(q)\dot{e}_c = -\bar{C}(z)e_c - \bar{\tau} + \Omega + \bar{\tau}_d \quad (19)$$

where the important nonlinear mobile robot function is:

$$\textbf{C1}\ \Omega = \bar{H}(q)\dot{v}_c + \bar{C}(z)\,v_c \quad (20)$$

$$\textbf{C2}\quad \Omega = \bar{H}(\dot{v}_c) + \bar{C}(v_c) \quad (21)$$

$$\textbf{C3}\qquad \Omega = \Psi(v,\dot{v})\phi \quad (22)$$

Function Ω contains all the mobile robot parameters, such as masses, moments of inertias, friction coefficients, etc, which are quantities often imperfectly unknown and difficult to determine.

Thus, the suitable control input for (3)-(5), respectively, is given as:

$$\bar{\tau} = \hat{\tau}_{NN} + k_4 e_c - \gamma \quad (23)$$

where k_4 is a diagonal positive definite design matrix, and γ a robustifying term to compensate the unmodeled and unstructured disturbances, $\hat{\tau}_{NN}$ is the generated torque from the sum of output torque of the neural networks, that is given by:

$$\textbf{C1}\ \hat{\tau}_{NN} = \left[\left\{\hat{X}_{\bar{H}}\right\}^T \bullet \left\{\hat{\xi}_{\bar{H}}(q)\right\}\right]\dot{v}_c + \left[\left\{\hat{X}_{\bar{C}}\right\}^T \bullet \left\{\hat{\xi}_{\bar{C}}(z)\right\}\right]v_c \quad (24)$$

$$\textbf{C2}\ \ \hat{\tau}_{NN} = \left[\left\{\hat{W}_{\bar{H}}\right\}^T \bullet \left\{\hat{\xi}_{\bar{H}}(\dot{v}_c)\right\}\right] + \left[\left\{\hat{W}_{\bar{C}}\right\}^T \bullet \left\{\hat{\xi}_{\bar{C}}(v_c)\right\}\right] \quad (25)$$

$$\textbf{C3}\qquad \hat{\tau}_{NN} = \left[\left\{\hat{W}_{\bar{N}}\right\}^T \bullet \left\{\hat{\xi}_{\bar{N}}(v_c,\dot{v}_c)\right\}\right] \quad (26)$$

where $\left\{\hat{X}_{\bar{H}}\right\}$, $\left\{\hat{X}_{\bar{C}}\right\}$, $\left\{\hat{W}_{\bar{H}}\right\}$, $\left\{\hat{W}_{\bar{C}}\right\}$, $\left\{\hat{\xi}_{\bar{H}}(.)\right\}$, $\left\{\hat{\xi}_{\bar{C}}(.)\right\}$, $\left\{\hat{W}_{\bar{N}}\right\}$, and $\left\{\hat{\xi}_{\bar{N}}\right\}$ represent estimates of true parameters of the weights $\{X_{\bar{H}}\}$, $\{X_{\bar{C}}\}$, $\{W_{\bar{H}}\}$, $\{W_{\bar{C}}\}$, $\{\xi_{\bar{H}}(.)\}$, $\{\xi_{\bar{C}}(.)\}$, $\{W_{\bar{N}}\}$, and $\{\xi_{\bar{N}}\}$ of (11)-(13), respectively. Three controllers structures are different in terms of input vectors of the RBFNN, that is, C1 is formed by static and dynamic neural networks, C2 is formed by static neural networks, and C3 is formed by dynamic neural networks.

Substituting (23) into (19), and doing some mathematical manipulations, the closed-loop system error dynamics can be expressed as:

$$\bar{H}(q)\dot{e}_c = -(k_4 + \bar{C})e_c + \delta + \bar{\tau}_d + E_{NN} + \gamma \quad (27)$$

where:

$$\textbf{C1}\begin{cases} \delta = \left[\left\{\tilde{X}_{\bar{H}}\right\}^T \bullet \left\{\hat{\xi}_{\bar{H}}(q)\right\}\right]\dot{v}_c + \left[\left\{\tilde{X}_{\bar{C}}\right\}^T \bullet \left\{\hat{\xi}_{\bar{C}}(z)\right\}\right]v_c + \\ + \left[\{X_{\bar{H}}\}^T \bullet \left\{\tilde{\xi}_{\bar{H}}(q)\right\}\right]\dot{v}_c + \left[\{X_{\bar{C}}\}^T \bullet \left\{\tilde{\xi}_{\bar{C}}(z)\right\}\right]v_c \\ E_{NN} = E_{\bar{H}}(q)\,\dot{v}_c + E_{\bar{C}}(z)\,v_c \end{cases} \quad (28)$$

$$\textbf{C2}\begin{cases} \delta = \left[\left\{\tilde{W}_{\bar{H}}\right\}^T \bullet \left\{\hat{\xi}_{\bar{H}}(\dot{v}_c)\right\}\right] + \left[\left\{\tilde{W}_{\bar{C}}\right\}^T \bullet \left\{\hat{\xi}_{\bar{C}}(v_c)\right\}\right] + \\ + \left[\{W_{\bar{H}}\}^T \bullet \left\{\tilde{\xi}_{\bar{H}}(\dot{v}_c)\right\}\right] + \left[\{W_{\bar{C}}\}^T \bullet \left\{\tilde{\xi}_{\bar{C}}(v_c)\right\}\right] \\ E_{NN} = E_{\bar{H}}(\dot{v}) + E_{\bar{C}}(v) \end{cases} \quad (29)$$

$$\textbf{C3}\begin{cases} \delta = \left[\left\{\tilde{W}_{\bar{N}}\right\}^T \bullet \left\{\hat{\xi}_{\bar{N}}(v_c,\dot{v}_c)\right\}\right] + \left[\{W_{\bar{N}}\}^T \bullet \left\{\tilde{\xi}_{\bar{N}}(v_c,\dot{v}_c)\right\}\right] \\ E_{NN} = E_{\bar{N}}(v,\dot{v}) \end{cases} \quad (30)$$

being that $(\tilde{.}) = (.) - (\hat{.})$ define the error vector in the parameters, and E_{NN} define the neural network modeling error.

Assuming that the unmodeled and unstructured disturbances are bounded, as well as the neural networks modeling errors, such that $\|\bar{\tau}_d\| \le b_d$ and $\|E_{NN}\| \le e_{NN}$, the robustifying term is defined as:

$$\gamma = -(k_d + I_n)e_c \quad (31)$$

where k_d is a diagonal positive definite matrix, and I_n is the identity matrix.

For the controllers (C1, C2, and C3) or (23)-(26), and (31), learning algorithms for the neural networks should to be developed, so that the control system can be stable, and both the position and velocity tracking errors converge to zeros.

Let us consider Lyapunov's candidate function:

$$\begin{cases} V = V_1 + k_1(e_1^2 + e_2^2) + 2(k_1/k_2)(1 - \cos(e_3)) \\ V_1 = (1/2)(e_c^T \bar{H}(q)e_c + V_2) \end{cases} \quad (32)$$

where:

$$\textbf{C1}\begin{cases} V_2 = \sum_{k=1}^{n} \tilde{X}_{\bar{H}_k}^T \Gamma_{\bar{H}_k}^{-1} \tilde{X}_{\bar{H}_k} + \sum_{k=1}^{n} \tilde{X}_{\bar{C}_k}^T \Gamma_{\bar{C}_k}^{-1} \tilde{X}_{\bar{C}_k} + \\ \quad + \sum_{k=1}^{n} \tilde{m}_{\bar{H}_k}^T \Gamma_{m\bar{H}_k}^{-1} \tilde{m}_{\bar{H}_k} + \sum_{k=1}^{n} \tilde{m}_{\bar{C}_k}^T \Gamma_{m\bar{C}_k}^{-1} \tilde{m}_{\bar{C}_k} + \\ \quad + \sum_{k=1}^{n} \tilde{\sigma}_{\bar{H}_k}^T \Gamma_{\sigma\bar{H}_k}^{-1} \tilde{\sigma}_{\bar{H}_k} + \sum_{k=1}^{n} \tilde{\sigma}_{\bar{C}_k}^T \Gamma_{\sigma\bar{C}_k}^{-1} \tilde{\sigma}_{\bar{C}_k} \end{cases} \quad (33)$$

$$\textbf{C2}\begin{cases} V_2 = \sum_{k=1}^{n} \tilde{W}_{\bar{H}_k}^T \Gamma_{\bar{H}_k}^{-1} \tilde{W}_{\bar{H}_k} + \sum_{k=1}^{n} \tilde{W}_{\bar{C}_k}^T \Gamma_{\bar{C}_k}^{-1} \tilde{W}_{\bar{C}_k} + \\ \quad + \sum_{k=1}^{n} \tilde{m}_{\bar{H}_k}^T \Gamma_{m\bar{H}_k}^{-1} \tilde{m}_{\bar{H}_k} + \sum_{k=1}^{n} \tilde{m}_{\bar{C}_k}^T \Gamma_{m\bar{C}_k}^{-1} \tilde{m}_{\bar{C}_k} + \\ \quad + \sum_{k=1}^{n} \tilde{\sigma}_{\bar{H}_k}^T \Gamma_{\sigma\bar{H}_k}^{-1} \tilde{\sigma}_{\bar{H}_k} + \sum_{k=1}^{n} \tilde{\sigma}_{\bar{C}_k}^T \Gamma_{\sigma\bar{C}_k}^{-1} \tilde{\sigma}_{\bar{C}_k} \end{cases} \quad (34)$$

$$\textbf{C3}\begin{cases} V_2 = \sum_{k=1}^{n} \tilde{W}_{\bar{N}_k}^T \Gamma_{\bar{N}_k}^{-1} \tilde{W}_{\bar{N}_k} + \sum_{k=1}^{n} \tilde{m}_{\bar{N}_k}^T \Gamma_{m\bar{N}_k}^{-1} \tilde{m}_{\bar{N}_k} + \\ \quad + \sum_{k=1}^{n} \tilde{\sigma}_{\bar{N}_k}^T \Gamma_{\sigma\bar{N}_k}^{-1} \tilde{\sigma}_{\bar{N}_k} \end{cases} \quad (35)$$

with $\Gamma_{.k}$, $\Gamma_{m.k}$ and $\Gamma_{\sigma.k}$ being dimensional compatible symmetric positive definite matrices. Clearly, $V \ge 0$, and $V = 0$ if only if $e_q = 0$, $e_c = 0$, $\left\{\tilde{X}_{\bar{H}}\right\} = 0$, $\left\{\tilde{X}_{\bar{C}}\right\} = 0$, $\left\{\tilde{W}_{\bar{H}}\right\} = 0$, $\left\{\tilde{W}_{\bar{C}}\right\} = 0$, $\left\{\tilde{W}_{\bar{N}}\right\} = 0$, $\tilde{m}_{\bar{H}} = 0$, $\tilde{m}_{\bar{C}} = 0$, $\tilde{m}_{\bar{N}} = 0$, $\tilde{\sigma}_{\bar{H}} = 0$, $\tilde{\sigma}_{\bar{C}} = 0$, and $\tilde{\sigma}_{\bar{N}} = 0$. Differentiating V, (32), and substituting the error dynamics, (27), $\dot{V}$ is given as:

$$\begin{cases} \dot{V} = 2k_1e_1\dot{e}_1 + 2k_1e_1\dot{e}_2 + 2(k_1/k_2)\dot{e}_3\sin(e_3) + \dot{V}_1 \\ \dot{V}_1 = -e_c^T k_4 e_c + e_c^T \bar{\tau}_d + e_c^T E_{NN} + e_c^T \gamma + \dot{V}_2 \end{cases} \quad (36)$$

where, using the property 2, $\dot{V}_2$ stays:

$$\mathbf{C1}\begin{cases}\dot{V}_2 = e_c^T(\left[\left\{\tilde{X}_{\bar{H}}\right\}^T \bullet \left\{\hat{\xi}_{\bar{H}}(q)\right\}\right] + \left[\{X_{\bar{H}}\}^T \bullet \left\{\tilde{\xi}_{\bar{H}}(q)\right\}\right])\dot{v}_c + \\ + e_c^T(\left[\left\{\tilde{X}_{\bar{C}}\right\}^T \bullet \left\{\hat{\xi}_{\bar{C}}(z)\right\}\right] + \left[\{X_{\bar{C}}\}^T \bullet \left\{\tilde{\xi}_{\bar{C}}(z)\right\}\right])v_c + \\ + \sum_{k=1}^{n} \tilde{X}_{\bar{H}_k}^T \Gamma_{\bar{H}_k}^{-1} \dot{\tilde{X}}_{\bar{H}_k} + \sum_{k=1}^{n} \tilde{X}_{\bar{C}_k}^T \Gamma_{\bar{C}_k}^{-1} \dot{\tilde{X}}_{\bar{C}_k} + \\ + \sum_{k=1}^{n} \tilde{m}_{\bar{H}_k}^T \Gamma_{m\bar{H}_k}^{-1} \dot{\tilde{m}}_{\bar{H}_k} + \sum_{k=1}^{n} \tilde{m}_{\bar{C}_k}^T \Gamma_{m\bar{C}_k}^{-1} \dot{\tilde{m}}_{\bar{C}_k} + \\ + \sum_{k=1}^{n} \tilde{\sigma}_{\bar{H}_k}^T \Gamma_{\sigma\bar{H}_k}^{-1} \dot{\tilde{\sigma}}_{\bar{H}_k} + \sum_{k=1}^{n} \tilde{\sigma}_{\bar{C}_k}^T \Gamma_{\sigma\bar{C}_k}^{-1} \dot{\tilde{\sigma}}_{\bar{C}_k}\end{cases} \quad (37)$$

$$\mathbf{C2}\begin{cases}\dot{V}_2 = e_c^T(\left[\left\{\tilde{W}_{\bar{H}}\right\}^T \bullet \left\{\hat{\xi}_{\bar{H}}(\dot{v}_c)\right\}\right] + \left[\{W_{\bar{H}}\}^T \bullet \left\{\tilde{\xi}_{\bar{H}})(\dot{v}_c)\right\}\right]) + \\ + e_c^T(\left[\left\{\tilde{W}_{\bar{C}}\right\}^T \bullet \left\{\hat{\xi}_{\bar{C}}(v_c)\right\}\right] + \left[\{W_{\bar{C}}\}^T \bullet \left\{\tilde{\xi}_{\bar{C}}(v_c)\right\}\right]) + \\ + \sum_{k=1}^{n} \tilde{W}_{\bar{H}_k}^T \Gamma_{\bar{H}_k}^{-1} \dot{\tilde{W}}_{\bar{H}_k} + \sum_{k=1}^{n} \tilde{W}_{\bar{C}_k}^T \Gamma_{\bar{C}_k}^{-1} \dot{\tilde{W}}_{\bar{C}_k} + \\ + \sum_{k=1}^{n} \tilde{m}_{\bar{H}_k}^T \Gamma_{m\bar{H}_k}^{-1} \dot{\tilde{m}}_{\bar{H}_k} + \sum_{k=1}^{n} \tilde{m}_{\bar{C}_k}^T \Gamma_{m\bar{C}_k}^{-1} \dot{\tilde{m}}_{\bar{C}_k} + \\ + \sum_{k=1}^{n} \tilde{\sigma}_{\bar{H}_k}^T \Gamma_{\sigma\bar{H}_k}^{-1} \dot{\tilde{\sigma}}_{\bar{H}_k} + \sum_{k=1}^{n} \tilde{\sigma}_{\bar{C}_k}^T \Gamma_{\sigma\bar{C}_k}^{-1} \dot{\tilde{\sigma}}_{\bar{C}_k}\end{cases} \quad (38)$$

$$\mathbf{C3}\begin{cases}\dot{V}_2 = e_c^T \left[\left\{\tilde{W}_{\bar{N}}\right\}^T \bullet \left\{\hat{\xi}_{\bar{N}}(v_c, \dot{v}_c)\right\}\right] + \\ + e_c^T \left[\{W_{\bar{N}}\}^T \bullet \left\{\tilde{\xi}_{\bar{N}}(v_c, \dot{v}_c)\right\}\right] + \sum_{k=1}^{n} \tilde{W}_{\bar{N}_k}^T \Gamma_{\bar{N}_k}^{-1} \dot{\tilde{W}}_{\bar{N}_k} + \\ + \sum_{k=1}^{n} \tilde{m}_{\bar{N}_k}^T \Gamma_{m\bar{N}_k}^{-1} \dot{\tilde{m}}_{\bar{N}_k} + \sum_{k=1}^{n} \tilde{\sigma}_{\bar{N}_k}^T \Gamma_{\sigma\bar{N}_k}^{-1} \dot{\tilde{\sigma}}_{\bar{N}_k}\end{cases} \quad (39)$$

Emphasizing that:

$$\mathbf{C1}\begin{cases}e_c^T \left[\left\{\tilde{X}_{\bar{H}}\right\}^T \bullet \left\{\hat{\xi}_{\bar{H}}(q)\right\}\right] \dot{v}_c = \sum_{k=1}^{n} \left\{\tilde{X}_{\bar{H}_k}\right\}^T \bullet \left\{\hat{\xi}_{\bar{H}_k}(q)\right\} \dot{v}_c e_{c_k} \\ e_c^T \left[\left\{\tilde{X}_{\bar{C}}\right\}^T \bullet \left\{\hat{\xi}_{\bar{C}}(z)\right\}\right] v_c = \sum_{k=1}^{n} \left\{\tilde{X}_{\bar{C}_k}\right\}^T \bullet \left\{\hat{\xi}_{\bar{C}_k}(z)\right\} v_c e_{c_k} \\ \left\|\left[\{X_{\bar{H}}\}^T \bullet \left\{\tilde{\xi}_{\bar{H}}(q)\right\}\right] \dot{v}_c\right\| \le \alpha \\ \left\|\left[\{X_{\bar{C}}\}^T \bullet \left\{\tilde{\xi}_{\bar{C}}(z)\right\}\right] v_c\right\| \le \beta\end{cases} \quad (40)$$

$$\mathbf{C2}\begin{cases}e_c^T \left[\left\{\tilde{W}_{\bar{H}}\right\}^T \bullet \left\{\hat{\xi}_{\bar{H}}(\dot{v}_c)\right\}\right] = \sum_{k=1}^{n} \left\{\tilde{W}_{\bar{H}_k}\right\}^T \bullet \left\{\hat{\xi}_{\bar{H}_k}(\dot{v}_c)\right\} e_{c_k} \\ e_c^T \left[\left\{\tilde{W}_{\bar{C}}\right\}^T \bullet \left\{\hat{\xi}_{\bar{C}}(v_c)\right\}\right] = \sum_{k=1}^{n} \left\{\tilde{W}_{\bar{C}_k}\right\}^T \bullet \left\{\hat{\xi}_{\bar{C}_k}(v_c)\right\} e_{c_k} \\ \left\|\left[\{W_{\bar{H}}\}^T \bullet \left\{\tilde{\xi}_{\bar{H}})(\dot{v}_c)\right\}\right]\right\| \le \alpha \\ \left\|\left[\{W_{\bar{C}}\}^T \bullet \left\{\tilde{\xi}_{\bar{C}}(v_c)\right\}\right]\right\| \le \beta\end{cases} \quad (41)$$

$$\mathbf{C3}\begin{cases}e_c^T \left[\left\{\tilde{W}_{\bar{N}}\right\}^T \bullet \left\{\hat{\xi}_{\bar{N}}(v_c, \dot{v}_c)\right\}\right] = \sum_{k=1}^{n} \tilde{W}_{\bar{N}_k}^T \hat{\xi}_{\bar{N}_k}(v_c, \dot{v}_c) e_{c_k} \\ \left\|\left[\{W_{\bar{N}}\}^T \bullet \left\{\tilde{\xi}_{\bar{N}}(v_c, \dot{v}_c)\right\}\right]\right\| \le \lambda\end{cases} \quad (42)$$

and also, it can be seen that $\left\{\tilde{X}_{.}\right\} = \{X_{.}\} - \left\{\hat{X}_{.}\right\}$, $\left\{\tilde{W}_{.}\right\} = \{W_{.}\} - \left\{\hat{W}_{.}\right\}$, $\tilde{m}_{.} = m_{.} - \hat{m}_{.}$, and $\tilde{\sigma}_{.} = \sigma_{.} - \hat{\sigma}_{.}$, then $\left\{\dot{\tilde{X}}_{.}\right\} = -\left\{\dot{\hat{X}}_{.}\right\}$, $\left\{\dot{\tilde{W}}_{.}\right\} = -\left\{\dot{\hat{W}}_{.}\right\}$, $\dot{\tilde{m}}_{.} = -\dot{\hat{m}}_{.}$, and $\dot{\tilde{\sigma}}_{.} = -\dot{\hat{\sigma}}_{.}$. Therefore, the learning laws for the neural networks are obtained as:

$$\mathbf{C1}\begin{cases}\dot{\hat{X}}_{\bar{H}_k} = \Gamma_{\bar{H}_k} \bullet \left\{\hat{\xi}_{\bar{H}_k}(q)\right\} \dot{v}_c e_{c_k} - K_{\bar{H}_k}\Gamma_{\bar{H}_k} \|e_c\| \hat{X}_{\bar{H}_k} \\ \dot{\hat{X}}_{\bar{C}_k} = \Gamma_{\bar{C}_k} \bullet \left\{\hat{\xi}_{\bar{C}_k}(z)\right\} v_c e_{c_k} - K_{\bar{C}_k}\Gamma_{\bar{C}_k} \|e_c\| \hat{X}_{\bar{C}_k} \\ \dot{\hat{m}}_{\bar{H}_k} = -\Gamma_{m\bar{H}_k} \|e_c\| \hat{m}_{\bar{H}_k}; \quad \dot{\hat{m}}_{\bar{C}_k} = -\Gamma_{m\bar{C}_k} \|e_c\| \hat{m}_{\bar{C}_k} \\ \dot{\hat{\sigma}}_{\bar{H}_k} = -\Gamma_{\sigma\bar{H}_k} \|e_c\| \hat{\sigma}_{\bar{H}_k}; \quad \dot{\hat{\sigma}}_{\bar{C}_k} = -\Gamma_{\sigma\bar{C}_k} \|e_c\| \hat{\sigma}_{\bar{C}_k}\end{cases} \quad (43)$$

$$\mathbf{C2}\begin{cases}\dot{\hat{W}}_{\bar{H}_k} = \Gamma_{\bar{H}_k} \bullet \left\{\hat{\xi}_{\bar{H}_k}(\dot{v}_c)\right\} e_{c_k} - K_{\bar{H}_k}\Gamma_{\bar{H}_k} \|e_c\| \hat{W}_{\bar{H}_k} \\ \dot{\hat{W}}_{\bar{C}_k} = \Gamma_{\bar{C}_k} \bullet \left\{\hat{\xi}_{\bar{C}_k}(v_c)\right\} e_{c_k} - K_{\bar{C}_k}\Gamma_{\bar{C}_k} \|e_c\| \hat{W}_{\bar{C}_k} \\ \dot{\hat{m}}_{\bar{H}_k} = -\Gamma_{m\bar{H}_k} \|e_c\| \hat{m}_{\bar{H}_k}; \dot{\hat{m}}_{\bar{C}_k} = -\Gamma_{m\bar{C}_k} \|e_c\| \hat{m}_{\bar{C}_k} \\ \dot{\hat{\sigma}}_{\bar{H}_k} = -\Gamma_{\sigma\bar{H}_k} \|e_c\| \hat{\sigma}_{\bar{H}_k}; \dot{\hat{\sigma}}_{\bar{C}_k} = -\Gamma_{\sigma\bar{C}_k} \|e_c\| \hat{\sigma}_{\bar{C}_k}\end{cases} \quad (44)$$

$$\mathbf{C3}\begin{cases}\dot{\hat{W}}_{\bar{N}_k} = \Gamma_{\bar{N}_k} \hat{\xi}_{\bar{N}_k}(v_c, \dot{v}_c) e_{c_k} - K_{\bar{N}_k}\Gamma_{\bar{N}_k} \|e_c\| \hat{W}_{\bar{N}_k} \\ \dot{\hat{m}}_{\bar{N}_k} = -\Gamma_{m\bar{N}_k} \|e_c\| \hat{m}_{\bar{N}_k}; \dot{\hat{\sigma}}_{\bar{N}_k} = -\Gamma_{\sigma\bar{N}_k} \|e_c\| \hat{\sigma}_{\bar{N}_k}\end{cases} \quad (45)$$

where $K_{.k} > 0$.

Then $\dot{V}_1$ of (36) can be simplified as:

$$\begin{cases}\dot{V} \le 2k_1 e_1 \dot{e}_1 + 2k_1 e_1 \dot{e}_2 + 2(k_1/k_2)\dot{e}_3 \sin(e_3) + \dot{V}_1 \\ \dot{V}_1 \le -e_c^T k_4 e_c + e_c^T \bar{\tau}_d + e_c^T E_{NN} + e_c^T \gamma + \dot{V}_2\end{cases} \quad (46)$$

where:

$$\mathbf{C1}\begin{cases}\dot{V}_2 \le K_{\bar{H}} \|e_c\| \sum_{k=1}^{n} \tilde{X}_{\bar{H}_k}^T \hat{X}_{\bar{H}_k} + K_{\bar{C}} \|e_c\| \sum_{k=1}^{n} \tilde{X}_{\bar{C}_k}^T \hat{X}_{\bar{C}_k} + \\ + \|e_c\| \sum_{k=1}^{n} \tilde{m}_{\bar{H}_k}^T \hat{m}_{\bar{H}_k} + \|e_c\| \sum_{k=1}^{n} \tilde{m}_{\bar{C}_k}^T \hat{m}_{\bar{C}_k} + \\ + \|e_c\| \sum_{k=1}^{n} \tilde{\sigma}_{\bar{H}_k}^T \hat{\sigma}_{\bar{H}_k} + \|e_c\| \sum_{k=1}^{n} \tilde{\sigma}_{\bar{C}_k}^T \hat{\sigma}_{\bar{C}_k} + \|e_c\| \mu\end{cases} \quad (47)$$

$$\mathbf{C2}\begin{cases}\dot{V}_2 \le K_{\bar{H}} \|e_c\| \sum_{k=1}^{n} \tilde{W}_{\bar{H}_k}^T \hat{W}_{\bar{H}_k} + K_{\bar{C}} \|e_c\| \sum_{k=1}^{n} \tilde{W}_{\bar{C}_k}^T \hat{W}_{\bar{C}_k} + \\ + \|e_c\| \sum_{k=1}^{n} \tilde{m}_{\bar{H}_k}^T \hat{m}_{\bar{H}_k} + \|e_c\| \sum_{k=1}^{n} \tilde{m}_{\bar{C}_k}^T \hat{m}_{\bar{C}_k} + \\ + \|e_c\| \sum_{k=1}^{n} \tilde{\sigma}_{\bar{H}_k}^T \hat{\sigma}_{\bar{H}_k} + \|e_c\| \sum_{k=1}^{n} \tilde{\sigma}_{\bar{C}_k}^T \hat{\sigma}_{\bar{C}_k} + \|e_c\| \mu\end{cases} \quad (48)$$

$$\mathbf{C3}\begin{cases}\dot{V}_2 \le K_{\bar{N}} \|e_c\| \sum_{k=1}^{n} \tilde{W}_{\bar{N}_k}^T \hat{W}_{\bar{N}_k} + \|e_c\| \sum_{k=1}^{n} \tilde{m}_{\bar{N}_k}^T \hat{m}_{\bar{N}_k} + \\ + \|e_c\| \sum_{k=1}^{n} \tilde{\sigma}_{\bar{N}_k}^T \hat{\sigma}_{\bar{N}_k} + \|e_c\| \mu\end{cases} \quad (49)$$

being that $K_{.} = K_{.k}$, $\mu = \alpha + \beta$ of (47)-(48), $\mu = \lambda$ of (49). Observing that $tr\left(\hat{X}_{.}\tilde{X}_{.}^T\right) = \sum_{k=1}^{n} \tilde{X}_{.k}^T \hat{X}_{.k}$, $tr\left(\hat{W}_{.}\tilde{W}_{.}^T\right) = \sum_{k=1}^{n} \tilde{W}_{.k}^T \hat{W}_{.k}$, $tr\left(\hat{m}_{.}\tilde{m}_{.}^T\right) = \sum_{k=1}^{n} \tilde{m}_{.k}^T \hat{m}_{.k}$, $tr\left(\hat{\sigma}_{.}\tilde{\sigma}_{.}^T\right) = \sum_{k=1}^{n} \tilde{\sigma}_{.k}^T \hat{\sigma}_{.k}$, and replacing the robustifying term, (31), in $\dot{V}_1$ of (46), one obtain:

$$\begin{cases}\dot{V} \le 2k_1 e_1 \dot{e}_1 + 2k_1 e_1 \dot{e}_2 + 2(k_1/k_2)\dot{e}_3 \sin(e_3) + \dot{V}_1 \\ \dot{V}_1 \le -e_c^T e_c - e_c^T k_4 e_c - e_c^T (k_d e_c - \bar{\tau}_d - E_{NN}) + \dot{V}_2\end{cases} \quad (50)$$

where:

$$\mathbf{C1}\begin{cases}\dot{V}_2 \le K_{\bar{H}} \|e_c\| tr\left(\hat{X}_{\bar{H}}\tilde{X}_{\bar{H}}^T\right) + K_{\bar{C}} \|e_c\| tr\left(\hat{X}_{\bar{C}}\tilde{X}_{\bar{C}}^T\right) + \\ + \|e_c\| tr\left(\hat{m}_{\bar{H}}\tilde{m}_{\bar{H}}^T\right) + \|e_c\| tr\left(\hat{m}_{\bar{C}}\tilde{m}_{\bar{C}}^T\right) + \\ + \|e_c\| tr\left(\hat{\sigma}_{\bar{H}}\tilde{\sigma}_{\bar{H}}^T\right) + \|e_c\| tr\left(\hat{\sigma}_{\bar{C}}\tilde{\sigma}_{\bar{C}}^T\right) + \|e_c\| \mu\end{cases} \quad (51)$$

$$\mathbf{C2}\begin{cases}\dot{V}_2 \le K_{\bar{H}} \|e_c\| tr\left(\hat{W}_{\bar{H}}\tilde{W}_{\bar{H}}^T\right) + K_{\bar{C}} \|e_c\| tr\left(\hat{W}_{\bar{C}}\tilde{W}_{\bar{C}}^T\right) + \\ + \|e_c\| tr\left(\hat{m}_{\bar{H}}\tilde{m}_{\bar{H}}^T\right) + \|e_c\| tr\left(\hat{m}_{\bar{C}}\tilde{m}_{\bar{C}}^T\right) + \\ + \|e_c\| tr\left(\hat{\sigma}_{\bar{H}}\tilde{\sigma}_{\bar{H}}^T\right) + \|e_c\| tr\left(\hat{\sigma}_{\bar{C}}\tilde{\sigma}_{\bar{C}}^T\right) + \|e_c\| \mu\end{cases} \quad (52)$$

$$\mathbf{C3}\begin{cases}\dot{V}_2 \le K_{\bar{N}} \|e_c\| tr\left(\hat{W}_{\bar{N}}\tilde{W}_{\bar{N}}^T\right) + \|e_c\| tr\left(\hat{m}_{\bar{N}}\tilde{m}_{\bar{N}}^T\right) + \\ + \|e_c\| tr\left(\hat{\sigma}_{\bar{N}}\tilde{\sigma}_{\bar{N}}^T\right) + \|e_c\| \mu\end{cases} \quad (53)$$

or

$$\begin{cases} \dot{V} \leq 2k_1 e_1 \dot{e}_1 + 2k_1 e_1 \dot{e}_2 + 2(k_1/k_2)\dot{e}_3 \sin(e_3) + \dot{V}_1 \\ \dot{V}_1 \leq -e_c^T e_c - k_{4_{\min}} \|e_c\|^2 - \\ \quad - \|e_c\| \left(k_{d_{\min}} \|e_c\| - b_d - e_{NN} - \varphi - \mu\right) \end{cases} \quad (54)$$

with:

$$\mathbf{C1} \begin{cases} \phi = K_{\bar{H}} tr\left(\hat{X}_{\bar{H}} \tilde{X}_{\bar{H}}^T\right) + K_{\bar{C}} tr\left(\hat{X}_{\bar{C}} \tilde{X}_{\bar{C}}^T\right) + \\ \quad + tr\left(\hat{m}_{\bar{H}} \tilde{m}_{\bar{H}}^T\right) + tr\left(\hat{m}_{\bar{C}} \tilde{m}_{\bar{C}}^T\right) + tr\left(\hat{\sigma}_{\bar{H}} \tilde{\sigma}_{\bar{H}}^T\right) + \\ \quad + tr\left(\hat{\sigma}_{\bar{C}} \tilde{\sigma}_{\bar{C}}^T\right) \end{cases} \quad (55)$$

$$\mathbf{C2} \begin{cases} \phi = K_{\bar{H}} tr\left(\hat{W}_{\bar{H}} \tilde{W}_{\bar{H}}^T\right) + K_{\bar{C}} tr\left(\hat{W}_{\bar{C}} \tilde{W}_{\bar{C}}^T\right) + \\ \quad + tr\left(\hat{m}_{\bar{H}} \tilde{m}_{\bar{H}}^T\right) + tr\left(\hat{m}_{\bar{C}} \tilde{m}_{\bar{C}}^T\right) + tr\left(\hat{\sigma}_{\bar{H}} \tilde{\sigma}_{\bar{H}}^T\right) + \\ \quad + tr\left(\hat{\sigma}_{\bar{C}} \tilde{\sigma}_{\bar{C}}^T\right) \end{cases} \quad (56)$$

$$\mathbf{C3} \left\{ \phi = K_{\bar{N}} tr\left(\hat{W}_{\bar{N}} \tilde{W}_{\bar{N}}^T\right) + tr\left(\hat{m}_{\bar{N}} \tilde{m}_{\bar{N}}^T\right) + tr\left(\hat{\sigma}_{\bar{N}} \tilde{\sigma}_{\bar{N}}^T\right) \right. \quad (57)$$

and $\|\phi\| \leq \varphi$, $k_{4_{\min}}$ and $k_{d_{\min}}$ are the minimum singular values of k_4 and k_d, respectively. Substituting the position error dynamics, (16), the time derivative of V results in:

$$\begin{cases} \dot{V} \leq 2k_1 e_1(\omega e_2 - v_1 + v_r \cos(e_3)) + 2k_1 e_2(v_r \sin(e_3) - \omega e_1) + \\ \quad + 2k_3 v_r(\omega_r - \omega)\sin(e_3) + \dot{V}_1 \\ \dot{V}_1 \leq -e_c^T e_c - k_{4_{\min}} \|e_c\|^2 - \\ \quad - \|e_c\| \left(k_{d_{\min}} \|e_c\| - b_d - e_{NN} - \varphi - \mu\right) \end{cases} \quad (58)$$

where $k_3 v_r = (k_1/k_2)$. Substituting e_c and $v = v_c - e_c$, (18), $\dot{V}$ becomes:

$$\begin{cases} \dot{V} \leq -k_1^2 e_1^2 - ((k_1 k_3)/k_2) v_r \sin^2(e_3) - (k_1 e_1 - e_4)^2 - \\ \quad - (k_1/(k_2 k_3 v_r))(k_3 v_r \sin(e_3) - e_5)^2 - k_{4_{\min}} \|e_c\|^2 - \\ \quad - \|e_c\| \left(k_{d_{\min}} \|e_c\| - b_d - e_{NN} - \varphi - \mu\right) \end{cases} \quad (59)$$

Since $\|e_c\| \geq ((b_d + e_{NN} + \varphi + \mu)/k_{d_{\min}})$, $\dot{V}$ is guaranteed negative. According to a standard Lyapunov theory and Barbalat lemma, all signals $\|e_q\|$, $\|e_c\|$, $\left\{\tilde{X}_{\bar{H}}\right\}$, $\left\{\tilde{X}_{\bar{C}}\right\}$, $\left\{\tilde{W}_{\bar{H}}\right\}$, $\left\{\tilde{W}_{\bar{C}}\right\}$, $\left\{\tilde{W}_{\bar{N}}\right\}$, $\tilde{m}_{\bar{H}}$, $\tilde{m}_{\bar{C}}$, $\tilde{m}_{\bar{N}}$, $\tilde{\sigma}_{\bar{H}}$, $\tilde{\sigma}_{\bar{C}}$, and $\tilde{\sigma}_{\bar{N}}$ are bounded.

4. SIMULATION RESULTS

For accomplishing the simulations, the dynamic model described in Fierro and Lewis [1998] is used, with $d = 0.2m$, $v_r = 0.5m/s$, and $\omega_r = 0.0rad/s$.

To illustrate the performance of the three PTCs (C1, C2, and C3 or (23)-(26), and (31)), the robot should to track a straight line with the disturbance of Coulomb friction that subject to sudden changes, as well as load disturbance.

The reference trajectory is a straight line with initial coordinates $(1, 2)$ and orientation of 26.56°, respectively. The initial position of the robot is $[x_0, y_0, \theta_0] = [2, 1, 10°]$. The design parameters of controllers are chosen as: $k_1 = 1$, $k_2 = 3$, $k_3 = 2$, $k_4 = diag[7]$, $\Gamma_{\cdot k} = 10$, $\Gamma_{m \cdot k} = 0.001$, $\Gamma_{\sigma \cdot k} = 0.001$, $K_{\cdot} = 0.001$, and $k_d = diag[7]$. In this case, the number of hidden neuron is: 20 for C1, 20 for C2 and 16 for C3. The values of centers m and widths σ of Gaussian radial basis functions of the RBFNN are adjusted on-line. Different tracking performance can be achieved by adjusting parameters gains and others factors, such as the size of the RBFNN. A Coulomb friction term unmodeled disturbance, as well as bounded periodic disturbance are added to the robot system as,

$$F = \begin{bmatrix} (f_1 + f_1(t))sgn(v_1) + 0.1\sin(2t) \\ (f_2 + f_2(t))sgn(\omega) + 0.1\cos(2t) \end{bmatrix}$$

where $f_1 = 0.3$, $f_2 = 0.5$. Function $f(t)$ is non-linear, defined as: $[\, f_1(t) \;\; f_2(t)\,] = [\, 0.0 \;\; 0.0\,]^T$ if $t < 8$; $[\, f_1(t) \;\; f_2(t)\,] = [\, 0.2 \;\; 0.2\,]^T$ if $t \geq 8$, respectively. Thus, disturbance is subject to a sudden change at time goes to 8 sec. Moreover, in 8 sec, the mobile robot suddenly dropped of an object of $2.5kg$, that is, a quarter of its original mass.

The tracking performance of the PTCs (C1, C2, and C3) are verified in the tracking of the reference trajectory (Fig. 1), in the tracking errors in the X and Y directions, and in the orientation (Fig. 2), in the total control torques (Fig. 3) and in the actual linear and angular velocities of the mobile robot (Fig. 4). It can be observed that the tracking errors of the PTCs (C1, C2, and C3) tend to zero, as well as the robot velocities and the control torques converge to its steady state. In addition, when the sudden change of friction and load variation occurs, is observed that the tracking errors tend to zero, because the PTCs (C1, C2, and C3) are able to compensate the sudden changes of the robot dynamics through learning mechanism of the RBFNN. Also, for comparison, the trajectory tracking performance of the controllers (only k_4 controllers and PTCs) can be quantified using the mean square quadratic error (MSE), where the results are on the Table 1.

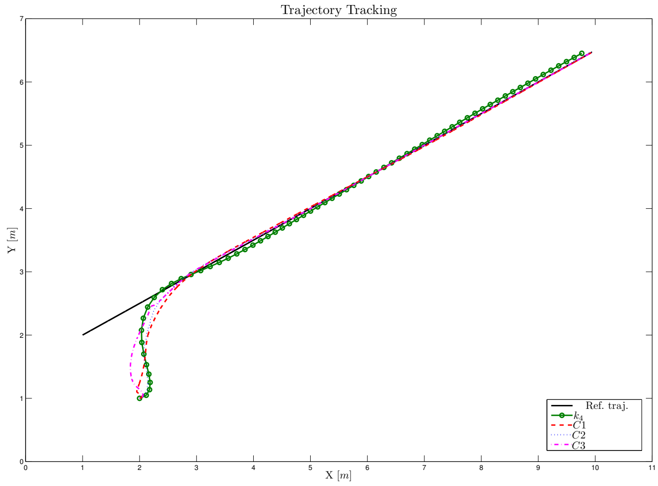

Fig. 1. Reference trajectory and actual trajectory - k_4 controller; $C1$, $C2$, and $C3$ controllers

Table 1. Mean Square Quadratic Error - MSE

Controller	MSE		
	X direction	Y direction	θ orientation angle
k_4	0.0609	0.0930	0.1397
C1	0.0478	0.0463	0.0887
C2	0.0507	0.0573	0.1169
C3	0.0355	0.0623	0.0953

5. CONCLUSIONS

This paper suggests control algorithms for a nonholonomic mobile robot, with a completely unknown robot dynamics, and subject to bounded unknown disturbance including unmodeled and unstructured dynamics.

Since the three dynamic neurocontrollers have different structures in terms of connection of the input vectors to

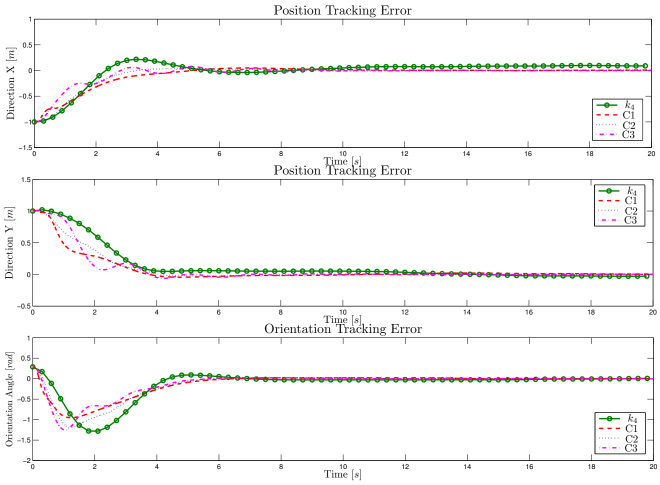

Fig. 2. Tracking errors - k_4 controller; C1, C2, and C3 controllers

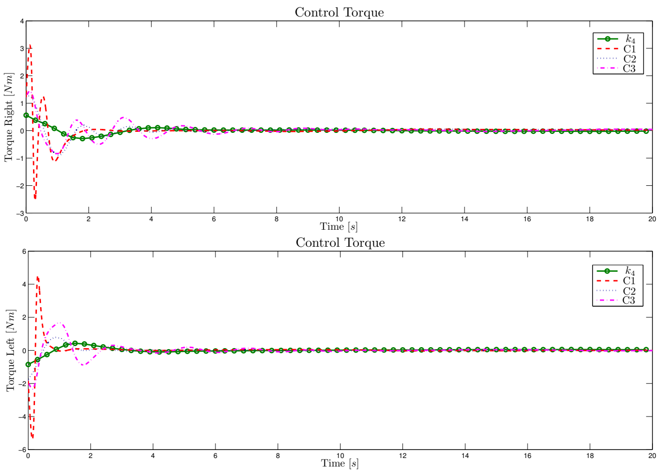

Fig. 3. Control torques - k_4 controller; C1, C2, and C3 controllers

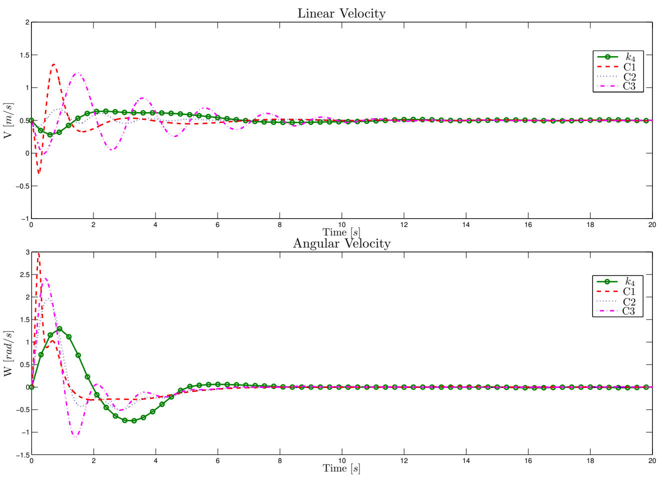

Fig. 4. Velocities - k_4 controller; C1, C2, and C3 controllers

RBFNN, the implementation these controllers is based on the partitioning the neural networks into several smaller subnets in order to obtain more efficient computation, which simplifies the design, gives added controller structure, and also leads to contributes to faster weight tuning algorithms.

The RBFNN used in the PTCs neither requires an off-line training process nor the priori information of the robot dynamics. Stability and convergence of the robot control system, as well as the learning algorithms (for weights, centers, and widths), are proved by using Lyapunov's theory, considering the presence of bounded unstructured and unmodeled dynamics. The simulation results show the efficiency of the PTCs, where it is possible to note different dynamic behaviours, since these controllers have different structures.

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