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The Holding and the Transportation Costs Optimization in a Simple Supply Chain : The multiple transporters case

Salah Eddine MERZOUK * **Olivier GRUNDER** **
Mohammed EL BAGDOURI ***

* *Systems and Transport Laboratory, University of Technology Belfort Montbelliard*
Tel: +33(0)3 84-58-38-14
salah-eddine.merzouk@utbm.fr
** *olivier.grunder@utbm.fr*
*** *mohammed.elbagdouri@utbm.fr*

Abstract: In this paper, we consider the optimization of the physical flows along a simple supply chain in a single-product and multiple transporters context. The studied problem can be assimilated to the lot-sizing problem (LSP) while having its own specificities. A mathematical formulation of the problem is given and a just in time policy is applied to calculate the different dates at each entity of the supply chain. Then, we propose to generalize the Branch and Bound Procedure (BBP) developed in previous work for the single transporter case to the multiple transporters case. The aim is to find both the optimal sequence of lots size and the optimal sequence of the transporters that have to deliver these lots from the supplier to the customer. These sequences have on the one hand to satisfy all system constraints including the final customer due dates, and on the other hand, to minimize the total cost induced by the various operations of production, holding and transportation.

1. INTRODUCTION

Today the requirements of customers in terms of costs and delays are in constant increase. The simultaneous optimization of the production, transport and holding activities become thus a key factor in the success of a company in a particular way, and of the whole supply chain in a general way. Indeed, the world competition led the majority of the industrial companies to recognize the need for taking into account all the activities of the supply chain in order to reduce their costs and to increase their reactivity vis-a-vis the perpetual trends in the market.

The aim of this paper is to propose an exact method to resolve the Lot Sizing and Delivery Problem (LSDP) applyed to a simple Supply Chain. This supply chain is mainly characterized by the single product and the multiple transporters context. Two different sites that we call respectively "supplier" and "customer" are considered and a fleet of various transporters are used to deliver products from the supplier to the customer. The customer needs a given number of products at given due dates. To satisfy this request and to minimize the related costs (production, holding and transportation), the different products have to be gathered in lots before being transported.

The studied problem can be assimilated to the class of Lot-Sizing Problems (LSP) while having its own specificities. Indeed, the LSP is one of the most challenging subjects that arise in production planning for the supply chain managers. The LSP is a production problem in which there is a time varying demand over T periods. The objective is to determine those periods where production will take place and the quantities that have to be produced in each period. As each production series involves an additional cost of setup, then, the global production quantity has to satisfy the total demand of the customer and has to minimize the global cost of the system. Several lot-sizing problems are accosted with various exact or approached methods [13], [15] and [10]. When only one item can be produced in each period, the problem is called the Discrete lot sizing Problem (DLSP) [6]. Studies have been carried out to consider new extensions of the LSP such as the capacity constraints [10], or the LSP with sequence dependent setup times and setup [7], [3] and [14]. The reader is referred to several papers such as [1], [5], [15] and [16] for detailed surveys of LSP.

Basing on the analogy between the LSDP and the LSP, we can deduce that the transportation activities of the LSDP can be assimilated to the setup activities of the LSP since they are in both cases inserted between the production of two consecutive lots. However, in the LSDP, even if a transportation time is needed between two deliveries, the production can be occurred during these time. Hence, contrary to the LSP, for which the setup and production tasks are sequential, in the LSDP we observe a both sequential and parallel aspect for the production and the transportation tasks. These tasks are sequential when the same lot is considered and parallel for different lots.

In the following section, we present the studied system and the constraints related to its dynamic. In Order to support our approach, the mathematical model is analysed and several properties are provided in section 3. Then, an efficient Branch and Bound Procedure (BBP) is proposed in section 4 which allows to find the optimal solution of the problem. illustrative experimental results show the efficiency of the proposed algorithm in section 5. Finally, section 6 wraps up the paper with a short conclusion and future research directions.

2. STUDIED SYSTEM

2.1 Structure

In this paper, a simple supply chain where two manufacturing sites and a fleet of transporter is considered. The first site is called the supplier and the second one is called the customer. Several parameters characterize each transporter : the loading capacity, the unit cost per delivery, the duration to load one unit at the supplier, the duration to unload one unit at the customer and the transportation duration between the two sites. The supplier is characterized by a given production duration per unit. The products have to be available in the customer stock, at the latest, at given due dates. A unit holding cost per product and per time unit is considered for both the supplier and the customer.

2.2 Functional description

To satisfy the customer requirements, the supplier has to produce and has to deliver the products in lots with different sizes. When the production of one lot at the supplier is finished, the products are loaded according to a loading duration of one unit in a chosen transporter. When this transporter arrives at the customer, the products are unloaded according to the unloading duration of one unit of the considered transporter. Once unloaded, the transporter returns empty to the supplier. During the loading / transport / unloading tasks, the production of the following lots can be occured. The same transporter used previously or another one can be used for the delivery of these lots.

The sequence of used transporters and the sequence of the delivery lots have to satisfy the total customer demand and have to minimize the global cost of the system.

3. OPTIMIZATION PROBLEM

We note α (respectively β) the holding cost per product and per time unit at the supplier (respectively the customer). A unit transportation cost per delivery, noted γ_j is associated to each transporter j independently of the delivery lot size. Under these conditions, one has to find the different sizes of production and delivery lots, and the sequence of the transporters who have to deliver these lots from the supplier to the customer. These sequences aim to minimize the generated global cost of the system (holding and transportation costs).

However, we realize that the sequence of lots sizes and the sequence of the transporters are not sufficient to evaluate a given solution. Indeed, to calculate the holding cost at the supplier and the customer, we have to calculate the relating products dates (the production dates, the loading dates and the unloading dates), and the arrival and the departure dates of the transporters.

With an aim of simplification, we assume that the unit holding cost at the customer is greater than the unit holding cost at the supplier. This assumption is often corresponds to the reality and ensures that a Just in time policy of delivery will be optimal for a given solution. Hence, for a given solution, the different products dates can be formulated using recursive equations and the global cost obtained corresponds to the evaluation of this solution.

3.1 Mathematical Model

First, we give the notations used throughout this paper :

- n : total customer demand.
- l : total number of transporters
- c_m : loading capacity of the transporter m.
- t_p : production duration of one product at the supplier.
- t_c^j and t_d^j : respectively the loading and the unloading duration of one product of the transporter j.
- t_t^j : transport duration between the supplier and the customer for the transporter j.
- α and β : respectively the unit holding cost at the supplier and the customer.
- γ_j : transportation cost for each delivery of the transporter j independently of the lot size.
- σ : a given solution of the problem.
- σ' : a partial solution of the problem for the last n' products.
- $w_i(\sigma)$ and $x_i(\sigma)$: respectively the end of production date and the loading date of the unit i at the supplier, obtained for the solution σ.
- $y_i(\sigma)$ and yd_i : respectively the unloading date and the due date of the unit i at the customer, obtained for the solution σ.
- $C_f(\sigma)$ and $C_c(\sigma)$: total holding cost at respectively the supplier and the customer obtained for the solution σ.
- $C_t(\sigma)$: total transportation cost obtained for the solution σ.

A solution of the problem σ can be defined as a series $(\sigma_{j,k})_{1\leq k\leq K_\sigma}$ which satisfies the following conditions :

(1) $\sigma_{j,k}$: The lot k which contains $\sigma_{j,k}$ products is delivered using the transporter j.
(2) $\forall k \in [1, K_\sigma] \quad , \forall j \in [1, l] \quad , \quad 0 < \sigma_{j,k} \leq c_j$.
(3) $\sum_{k=1}^{K_\sigma} \sigma_{j,k} = n$.
(4) K_σ is a finite integer not equal to zero, which is depending of the sequence σ. It corresponds to the number of lots in solution σ.

The objective is to obtain the solution (σ^*) which minimize the total holding cost at the supplier and the customer and the total transportation cost.

$$\text{Min} z(\sigma) = C_f(\sigma) + C_c(\sigma) + C_t(\sigma) \qquad (1)$$

The waiting time of a given unit i at the supplier is the length of time passes between the end of production date of this item, noted $x_i(\sigma)$ and its loading date at the transporter, noted $w_i(\sigma)$), calculated for the solution σ. Then, the total holding cost at the supplier is obtained by the multiplication of the total waiting times of the whole products and the unit holding cost α for the supplier.

$$C_f(\sigma) = \alpha \cdot \sum_{i=1}^{n} (x_i(\sigma) - w_i(\sigma)) \tag{2}$$

In the same way, the waiting time of a given unit i at the customer is explained by the term: $(yd_i - y_i(\sigma))$, where yd_i and $y_i(\sigma))$ are respectively the due date and the unloading date at the customer of the unit i, obtained for the solution σ. Hence, the total holding cost for the customer is expressed by :

$$C_c(\sigma) = \beta \cdot \sum_{i=1}^{n} (yd_i - y_i(\sigma)) \tag{3}$$

A fixed cost γ_j is considered for each delivery of the transporter j, independently of the transportation amount. If the total number of deliveries for this transporter is v_j, the total transportation cost of the system is thus expressed by:

$$C_t(\sigma) = \sum_{j=1}^{l} \gamma_j \cdot v_j \tag{4}$$

Note that $\sum_{j=1}^{l} v_j = K_\sigma$

3.2 Dates formulation

The formulation of these costs leads to apply a strategic policy of production and delivery in order to calculate the different dates of products at the supplier and the customer. In our study, the production policy is dictated by the due dates of the customer demand. Then, for a defined solution of the problem, the just-in-time (JIT) policy is the most appropriate policy to satisfy the system constraints [9] and [4]. The dates are formulated using retropropagation equations. Indeed, as proved in [11], the products dates of the last lot have to be calculated first and then, go up recursively to calculate the dates of the lots upstream. For each lot, the unloading dates at the customer are expressed, then the loading dates in the transporter, and finally, the end of production dates at the supplier.

With an aim of simplicity, we note ι_k the index of the first product of the lot k of the solution σ. Then :

$$\iota_k = 1 + \sum_{m=1}^{k-1} \sigma_{j,m} \tag{5}$$

Hence, the different dates of the lot k can be formulated as follows :

- The unloading dates of the products at the customer have to be late in order to minimize the global waiting times. The products of the same lots will have their unloading dates separated by a time interval equal to the unloading duration t_d^j of one product of the used transporter j. If the same transporter has to deliver several lots, the transportation, the loading and the production durations of two successive lots must be taken into account to calculate the unloading dates of these lots upstream.

Hence, the unloading date of the product ι_k, noted $y_{\iota_k}(\sigma)$ is formulated as follows :

$$y_{\iota_k}(\sigma) = \min\left[\min_{i=0}^{\sigma_{k,j}-1}\left(yd_{j,(\iota_k+i)} - i \cdot t_d^j\right), \Upsilon_k(\sigma)\right] \tag{6}$$

Where :

$$\Upsilon_k(\sigma) = \begin{cases} \infty & \text{If k is the last delivery of the transporter j} \\ y_{j,\iota_m}(\sigma) - 2 \cdot t_t^j - \sigma_{j,m} \cdot t_c^j - \sigma_{j,k} \cdot t_d^m & \text{else} \end{cases}$$

m : the next lot that is planned to be delivered by the transporter j.

$\sigma_{j,m}$: the size of the lot m.

$y_{j,\iota_m}(\sigma)$: the unloading date of the first product of the lot m.

The sequence of the unloading dates of the other products of the lot k is an arithmetic series for which the constant difference between terms is equal to t_d^j. Then:

$$\forall i \in [1, \sigma_{j,k}] \ , \ y_{\iota_k+i}(\sigma) = y_{\iota_k}(\sigma) + i \cdot t_d^j \tag{7}$$

- The transporter j must arrive at the customer one unloading unit time before the unloading date of the product ι_k. Then, its departure date from the supplier, that we note $d_{j,k}(\sigma)$ is obtained by :

$$d_{j,k}(\sigma) = y_{\iota_k}(\sigma) - t_d^j - t_t^j$$

The transporter j can not leave the supplier if the loading of the whole products of the lot k is not finished. Then, the departure date $d_{j,k}(\sigma)$ corresponds to the end of loading date of the last product of this lot. Hence, The loading dates, noted $x_{\iota_k+i}(\sigma)$ of the products of the lot k are formulated as follows :

$$\forall\, i \in [0, \sigma_{j,k} - 1] \ , \ x_{\iota_k+i}(\sigma) = d_{j,k}(\sigma) - (\sigma_{j,k} - i + 1) \cdot t_c^j \tag{8}$$

- In order to minimize the holding cost at the supplier, the global waiting time of products between their production dates and their loading dates must be as small as possible. We have assumed that the loading of a given lot can not start before the manufacture of all the products of this lot is finished. Then, the end of production dates of the lot k is given as follows :

$$\forall\, i \in [0, \sigma_{j,k} - 1],$$
$$w_{\iota_k+i}(\sigma) = \min\left\{x_{\iota_k}(\sigma) \ , \ w_{\iota_{k+1}}(\sigma)\right\} - (\sigma_{j,k} - 1 - i) \cdot t_p \tag{9}$$

$x_{\iota_k}(\sigma)$ and $w_{\iota_k}(\sigma)$ are respectively the loading date and the end of production date of the first product of the lot k.

The reader is referred to [12] for detailed proofs of these formulations.

4. OPTIMIZATION METHOD

Before detailing the developed method, we point out that the single-transporter case was studied in previous work

[12]. The optimisation model proposed for this case has showed very interesting mathematical properties that have been used to formulate an efficient dominance relation. This dominance relation which is the basis of the developed BBP is generalized in this paper to the case of multiple transporters.

4.1 Dominance relation

Let n the total demand of the customer and C the capacity vector of the whole available transporters in the supply chain ($C = [c_1, c_2, \cdots, c_l]$). We note $U_{n,C}$ the set of the whole possible solutions for this problem.

Let a partial solution σ' for the n' last products. We define the set $\Psi(\sigma')$ as a subset of $U_{n,C}$ that contains the whole complete solutions built from σ'.

$$\Psi(\sigma') = \left\{\sigma \in U_{n,C} / \forall p < K_{\sigma'}, \sigma'_{j,K_{\sigma'}-p} = \sigma_{j,K_{\sigma}-p}\right\}$$

We note $\sigma^*(\sigma')$ the complete sequence belonging to $\Psi(\sigma')$ which has the smallest global cost.

$$\forall \sigma \in \Psi(\sigma')\ ,\ z(\sigma^*(\sigma')) \leq z(\sigma)$$

A dominance relation can be highlighted for partial solutions of the same number of products :

Proposition : *Let two partial sequences σ' and ω' which belong to the set $U_{n',C}$. We note $a_{j,k}(\sigma')$ (respectively $a_{j,m}(\omega')$) the first arrival date of the transporter j to deliver a given lot k of the sequence σ' (respectively a given lot m of ω').*

If the 2 partial sequences satisfy:

$$\begin{cases} C_c(\sigma') \leq C_c(\omega') \\ C_f(\sigma') \leq C_f(\omega') \\ C_t(\sigma') \leq C_t(\omega') \\ \forall\ j \in \tau \quad , \quad a_{j,k}(\sigma') - \sigma'_k.t_c^j \geq a_{j,m}(\omega') - \omega'_m.t_c^j \end{cases}$$

Then: σ' dominate ω' $(z(\sigma^*(\sigma')) < z(\omega^*(\omega'))$.

where τ is the set of the transporters used in the sequence σ', and ι_k (respctively ι_m) is the index of the first product of the lot k of σ' (respectively the m of ω').

This proposition means that the best solution obtained from σ' is better than the best one obtained from ω'.

Proof :

We assume that :

$$z(\sigma^*(\sigma')) \geq z(\omega^*(\omega')) \tag{10}$$

The sequence $\omega^*(\omega')$ is made up of two parts: the first one is the partial sequence ω'. The second one is another partial sequence which is composed of lots $(\omega"_{j,k})_{1 \leq k \leq K_{\omega"}}$ that complete the customer request.

Note that $\sum_{k=1}^{K_{\omega"}} \omega"_{j,k} = (n - n')$

We complete the partial sequence σ' with the partial sequence $\omega^*(\omega')$. We note the obtained sequence $\sigma"$

$$(\sigma") = \{\omega"_{j,1}, ..., \omega"_{j,K_{\omega"}}, \sigma'_{j,1}, ..., \sigma'_{j,K_{\sigma'}}\}, j \in [1, l]$$

The unloading products dates at the customer for both the solutions $\sigma"$ and $\omega^*(\omega')$ can be calculated. Basing on the last condition of the proposition, it is obvious that these dates will satisfy :

$$\forall i \in [1, \iota_1 - 1]\, , y_i(\sigma") \geq y_i(\omega^*(\omega'))$$

Thus : $\forall i \in [1, \iota_1 - 1]\, , yd_i - y_i(\sigma") \leq yd_i - y_i(\omega^*(\omega'))$

$$\Rightarrow \sum_{i=1}^{\iota_1 - 1} (yd_i - y_i(\sigma")) \leq \sum_{i=1}^{\iota_1 - 1} (yd_i - y_i(\omega^*(\omega')))$$

$$\Rightarrow C_c(\sigma") \leq C_c(\omega^*(\omega')) \tag{11}$$

As $C_t(\sigma') \leq C_t(\omega')$ and $C_f(\sigma') \leq C_f(\omega')$, we obtain :

$$C_t(\sigma") \leq C_t(\omega^*(\omega')) \tag{12}$$

$$C_f(\sigma") \leq C_f(\omega^*(\omega')) \tag{13}$$

Consequently, the addition of equations (11), (12), and (13), gives : $z(\sigma") \leq z(\xi^*(\omega'))$.

This means that we find one sequence belonging to $\Psi(\sigma')$ whose the global cost is smaller than the best complete sequence built from ω'. Hence, the assumption (10) was false. Consequently, $z(\sigma^*(\sigma')) < z(\omega^*(\omega'))$.

4.2 Optimization method

The proposed method can be assimilated to a Branch and Bound Procedure (BBP). The particularity of our BBP is that the width search in the arborescence is favored. The BBP is a general exact algorithmic method for finding optimal solutions of various optimization problems, especially in discrete and combinatorial optimization. It is basically an enumeration approach in a fashion that prunes the "nonpromising" search space [8].

The loading sequences optimization problem can be modelled as a tree search. The nodes correspond to the possible quantities of products that each transporter j can deliver. These quantities are determined by the loading capacity of the transporter and by the quantities of the remaining products. Hence, the sequence of nodes from a current one until the root of the search tree will correspond to a partial solution. This partial solution will be characterized by several parameters such as the holding costs at the customer and the supplier, transportation cost, etc.

When the customer demand (n) is very large, the complete exploration of the whole tree search is impossible since the computing time becomes enormous. Hence, we propose to use the previous proposition to remove the partial sequences that will end inevitably to bad solutions.

The algorithmic representation of the developed method can be resumed as follows:

(1) Construct a starting solution and evaluate it.
(2) $n' = 1$.
(3) while $n' \leq n$:
   - Construct all the partial solutions from the dominant ones obtained in the previous levels.
   - Remove the partial solutions whose global cost is greater than the global cost of the starting solution.
   - Compare the remaining partial solutions to each others in terms of the previous proposition and then remove the non dominant ones.

- $n' = n' + 1$

(4) The sequence whose global cost is the smallest is retained as the solution of the problem.

As in the programming dynamic procedure, the starting solution is constructed successively from the partial solution of one product until the one containing n products. The global cost criterion is considered to compare partial solutions to each others for each products amount n'. Only the partial sequence that has the smallest total cost is retained for this level n'.

For the following $(n' + 1)$ products, all the possible sequences to deliver the $(n'+1)$ last products are considered. These possibilities are:

- The partial sequences of n' products for which 1 product must be added.
- The partial sequences of $(n' - 1)$ products for which 2 products must be added.
- $\cdots$
- The partial sequences of $(n' - c_j + 1)$ products for which c_j products must be added.

This principle is repeated until the level n is reached. The obtained sequence is considered as a starting sequence. The evaluation of this solution is considered as an upper bound of the global cost of the optimal solution. It is important to announce that this starting solution is very interesting since it corresponds to the optimal solution for several problem instances.

After obtaining the starting solution, the same principle is repeated to explore the search space. However, the partial sequences are compared to each other by using the dominance relation defined by the previous proposition. Only the dominant partial solutions are retained at each level n'. For the level $(n' + 1)$, the adequate products quantities are added successively to each dominant partial solution of the levels n', $(n' - 1), \cdots, (n' - c_j + 1)$. Any obtained partial solution whose the global cost is greater than the global cost of the starting solution is automatically discarded. The remaining partial sequences are compared to each other using the previous proposition in order to discard the non dominant ones.

In the following level $(n' + 1)$, all the dominant sequences of level $(n', n' - 1, \cdots, n' - c + 1)$ are constructed. And so on, the exploration is repeated until the level n of the table. The sequence that has the smallest global cost is retained as the optimal solution of the problem.

In the following section, we provide results from computational tests obtained for several size of problems instance.

5. EXPERIMENTAL RESULTS

The purpose of this computational study is to analyze the efficiency of the developed method for different problem instances. Note that a computer that has a Pentium 4 processor running at 2.66 G-Hz is used, and all simulations are implemented with JAVA language. The aim of these tests is to study the influence of the problem parameters n and l on the quality of the algorithm. Our method, noted BBP-wf is compared to another method that we note BBP-df. The BBP-df is a classical approach of the BBP where the dominance relation is not used. The discareded partial sequences are those which have a global cost greater than the evaluation of the starting solution. Each partial sequence of n' products is compared with the best dominant partial sequence for the same quantity n'. If the current sequence is dominated, it will be automatically removed.

Table 1. Influence of the total demand n on the computational times (ms)

	BBP-wf	BBP-df	Av. profit
pb15p2t20r	476	7657	93.24 %
pb20p2t20r	5750	-	-
pb25p2t20r	17391	-	-
pb15p2t200r	109	242	54.26%
pb20p2t200r	461	8008	94.22%
pb25p2t200r	1297	170725	99.24%
pb50p2t200r	61501	-	-

Table 2. Influence of the number l of the transporters on the computational times (ms)

	BBP-wf	BBP-df	Av. profit
pb20p2t20r	476	7657	93.24%
pb20p3t20r	16531	249265	93.36%
pb20p4t20r	364953	2288055	74.66%

Hence, we have considered two problem classes for which the transportation part is higlighted : in the first one, the transportation unit cost γ_j is 20 times more important than the unit holding costs α and β at the supplier and the customer. For the second class, the ratio transportation/holding costs is over more important (200). For each problem class, we have randomly generated several instances with different parameters. These problems are noted $pbXpYtZr$, where X, Y and Z are respectively the total customer demand, the number of transporters and the ratio transportation / holding costs. The whole problems have been solved with both the two the developed BBP. We resumed in tables 1 and 2 the results of this comparison. For 10 different instances $pbXpYtZr$, the average computing times of each method and the average profit of our method compared to the other BBP are given in these table. The obtained profit of the BBP-wf compared to the BBP-df is calcultaed by the term $(100\%[TE_2 - TE_1]/TE_2)$, where TE_1 and TE_2 are the computational times of respectively BBP-wf and BBP-df.

These results show that the introduction of the dominance relation in the BBP improve considerably the efficiency of our method compared to a classical approach of the BBP. The observed profit in terms of computational time is more important for problem with large size, particularly, when the transportation cost is predominant compared to the holding costs. Indeed, when a new lot is added to a given partial sequence, the global cost increases in a significant manner. Consequently, the solutions that have small lots sizes will have an important cost, and could be thus discarded more quickly than as in the other problem class.

6. CONCLUSION

In this paper, we propose an efficient exact method to optimize deliveries between two factories where a fleet of transporters is used. The aim is to find the optimal solution that satisfies the customer requirement while minimizing the holding and the transportation costs of the whole considered system. The proposition obtained in previous works for the single-transporter case is generalized to taking into account a multi-transporters case. Then, a Branch and bound Procedure is developed to find the optimal solution in very reduced time. The experimental results show the efficiency of the developed algorithm to resolve several problem instances.

This will open the way to various perspectives. Other system constraints as the process setup cost, must be included in the mathematical model and the multi-items case must be studied. The developed BBP has to be further refined in order to obtain an exact method dealing with complex supply chains.

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