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Sliding Sector Design for Nonlinear Systems ⋆

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Abstract: The main contribution of this paper is a new method for sliding surface sector design to reduce the chattering. A new approach enables a systematic design based on the key idea that the Tensor Product (TP) model transformation is capable of decomposing sectors, furthermore it can define a High Order Singular Value Decomposition (HOSVD)-based canonical sector system. It is partially combination of sector sliding mode control, the classical manifold design for the linear system and HOSVD-based canonical description of a wide class of nonlinear systems. Experimental results of a DSP-controlled single-degree-of-freedom motion-control system are presented.

1. INTRODUCTION

Sliding mode control of variable structure systems has a special role in the field of robust control. On one hand, the exact description of sliding mode needs advanced mathematics, which was established by Filippov [1960], Filippov [1964] in the early sixty's. On the other hand, it is quite easy to implement in most engineering systems (Lin et al. [2007]), a simple relay is necessary in most cases. The main utility of sliding mode in control design problems is to decouple the highly coupled nonlinear dynamics, and to desensitize the performance to variations of the unknown system parameters.

The initial works on sliding mode control were followed by a large number of research papers in robotic manipulator control, in motor drive control and in the field power electronics since they are typical variable structure systems. Nowadays sliding mode control is one of the most popular robust control methods for the engineering systems (Cheng et al. [2007]). However, despite the theoretical predictions of superb closed-loop system performance of sliding mode, some of the experimental work indicated that sliding mode has limitations in practice, due to the need for a high sampling frequency to reduce the high-frequency oscillation phenomenon about the sliding mode manifold, collectively referred to as "chattering". In most of the experimental work involving sliding mode (Yildiz et al. [2007]), the effort spent on understanding the theoretical basis of sliding mode control is generally minimized, while a great deal of energy was invested in empirical techniques to reduce chattering.

Sliding sector was introduced by Furuta and Pan [2000], Suzuki et al. [2005] as a promising method to reduce the chattering. Another approach of sliding sector is proposed by Xu et al. [1996], Korondi et al. [1998]. They are valuable but they cannot be applied in a systematic way. The systematic sliding manifold design for linear systems was proposed by Utkin [1992]. It is interesting but the main challenge is the nonlinear systems. A new HOSVD-based canonical description of a wide class of nonlinear systems was proposed by Baranyi [2004] which enables a systematic controller design for a wide class of nonlinear systems.

A new approach of sliding sector design is proposed in this paper enables a systematic design based on the key idea that the TP model transformation is capable of decomposing sectors, furthermore it can define a HOSVD-based canonical sector system. It is partially combination of sector sliding mode control Korondi et al. [1998], the classical manifold design for the linear system Utkin [1992] and HOSVD-based canonical description of a wide class of nonlinear systems Baranyi [2004]. The basic steps of the new design strategy are:

1) Description of the LPV system by convex TP model form (using TP model transformation Baranyi [2004]).
2) Design a sliding surface is for each component of the TP model, using the systematic design method of Utkin [1992].
3) Constructing sliding sector from the set of surfaces defined in the previous step.
4) Selecting a TP model transformation based control law for the TP model transformation based sliding sector.

2. TENSOR PRODUCT BASED MODEL TRANSFORMATION

This section is intended to discuss the fundamental of tensor product models. Consider a parametrically varying dynamical system

$$\begin{aligned}\dot{x}(t) &= A(p(x))x(t) + B(p(x))u(t)\\ y(t) &= C(p(x))x(t) + D(p(x))u(t)\end{aligned} \tag{1}$$

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with input $u(t)$, output $y(t)$ and state vector $x(t)$. The system matrix

$$S(p(x)) = \begin{pmatrix} A(p(x)) & B(p(x)) \\ C(p(x)) & D(p(x)) \end{pmatrix} \in \Re^{OxI} \quad (2)$$

is a parameter-varying object, where $p(x) \in \Omega$ is time varying N-dimensional parameter vector, and is an element of the closed hypercube $\Omega = [a_1, b_1] \times [a_2, b_2] \times \cdots \times [a_N, b_N] \in \Re^N$. The parameter $p(x)$ includes some elements of $x(t)$.

The TP model transformation starts with the given LPV model (2). First a numerical discretization is performed over a hyper-rectangular grid on Ω. The system is known in the discrete points and an interpolation technique is necessary between the discrete points. The next step is reduction of the discrete model by High Order Singular Value Decomposition Baranyi [2004], Petres et al. [2004], which results in the TP model representation:

$$S(p(x)) = \sum_{r=1}^{R} w_r(p(x)) S_r \quad (3)$$

where $w_r(p(x))$ are weighting coefficients and

$$S_r = \begin{pmatrix} A_r & B_r \\ C_r & D_r \end{pmatrix} \quad (4)$$

There are several selection $w_r(p(x))$ and S_r, from now on the canonical form is applied Baranyi [2004] when

$$w_r(p(x)) \in [0, 1] \quad \text{and} \quad \sum_{r=1}^{R} w_r(p(x)) = 1 \quad (5)$$

For further details about TP model transformation, refer to Baranyi [2004], Petres et al. [2004].

3. SLIDING MODE DESIGN FOR LINEAR SYSTEMS

The design of a sliding-mode controller consists of three main steps. First is the design of the sliding surface, the second step is the design the control law which holds the system trajectory on the sliding surface, and the third and key step is the chattering-free implementation.

The following linear time invariant (LTI) system is considered; first the reference signal is supposed to be constant and zero. A single input multi output system is discussed. The dimensions of (6) are modified. The system is transformed to a regular form Lukuyanov and Utkin [1981].

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} u \quad \begin{array}{c} x_1 \in \Re^{n-1} \\ x_2 \in \Re \\ u \in \Re \end{array} \quad (6)$$

The switching surfaces, σ of the sliding mode, where the control vector components have discontinuities, can be written in the following form Korondi and Hashimoto [2000], where K is the "surface matrix".

$$\sigma = x_2 + Kx_1 = 0 \quad \sigma \in \Re \ \text{and} \ K \in \Re^{(n-1)} \quad (7)$$

When sliding mode occurs (when $\sigma = 0$ and $x_2 = -Kx_1$), the design problem of the sliding surfaces can be regarded as a linear state feedback control design for the following subsystem:

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 \quad (8)$$

In (8), x_2 can be considered as the input of the subsystem. A state feedback controller $x_2 = -Kx_1$ for this subsystem gives the switching surface of the whole VSS controller. In sliding mode

$$\dot{x}_1 = (A_{11} - A_{12}K)x_1 \quad (9)$$

Various linear control design methods based on state feedback are applicable for (9) to the design of the switching surfaces. **The main problem is that this method cannot be applied to a non-linear system which is the main challenge. The solution can be the Tensor Product model transformation.**

3.1 Control Law

To ensure that the system remains in the sliding mode ($\sigma = 0$) the condition

$$\dot{\sigma}\sigma < 0 \quad (10)$$

should hold. The simplest control law which can lead to sliding mode is the relay:

$$u = M \cdot \text{sign}(\sigma) \quad (11)$$

This is easy to realize by power electronic circuits. The relay type of controller can directly control the semiconductor switching elements, but it does not ensure the existence of sliding mode for the whole state space, and relatively big values of M is necessary which might cause a severe chattering phenomenon. This control law is preferable if the controller's sample frequency is nearly equal to the maximum switching frequency of semiconductor switching elements.

If sliding mode exists then there is continuous control, so-called "equivalent" control, u_{eq}, which can hold the system on the sliding manifold. It can be calculated from $\dot{\sigma} = 0$

$$\begin{array}{l} \dot{\sigma} = \dot{x}_2 + K\dot{x}_1 = 0 \\ \dot{\sigma} = A_{21}x_1 + A_{22}x_2 + B_2u + K(A_{11}x_1 + A_{12}x_2) = 0 \end{array} \quad (12)$$

u_{eq} can be expressed from (12)

$$u_{eq} = -\left((A_{21} + KA_{11})x_1 + (A_{22} + KA_{12})x_2\right)/B_2 \quad (13)$$

In the practice, there is never perfect knowledge of the whole system and its parameters. Only $\hat{u}_{eq}$, the estimation of u_{eq}, can be calculated. Since u_{eq} does not guarantee the convergence to the switching manifold in general, a discontinuous term is usually added to $\hat{u}_{eq}$.

$$u = \hat{u}_{eq} + M \cdot \text{sign}(\sigma) \quad (14)$$

The control laws (14) do not control the semiconductor switching elements directly; additional PWM is needed. Usually, this is no problem since the switching frequency of the semiconductor elements can be much higher then the sampling frequency of the fastest digital controller.

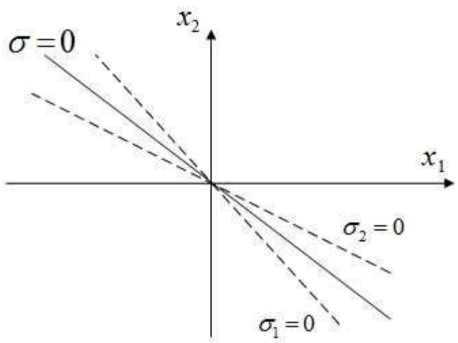

Fig. 1. Sliding sectors (In case of $n = 2$)

3.2 Chattering free implementation, Sector Sliding Mode

The chattering in the basic sliding mode control is essentially due to the requirement that the system state must stick to the switching surface. Obviously this requirement is too restrict when only finite switching rate is available. Replacing the switching surface to the sliding sector may enable the system state to move continuously.

To implement the proposed approach, two sliding surfaces are defined first

$$\sigma_r = x_2 + K_r x_1 = 0 \quad r = 1, 2 \tag{15}$$

Then the two sliding surfaces divide the whole state space into three regions defined as

$$\begin{aligned}\mathcal{R}_1 &= \{\ \mathbf{x}\ |\ \sigma_1(\mathbf{x}) > 0 \text{ and } \sigma_2(\mathbf{x}) > 0\ \}\\ \mathcal{R}_2 &= \{\ \mathbf{x}\ |\ \sigma_1(\mathbf{x}) < 0 \text{ and } \sigma_2(\mathbf{x}) < 0\ \}\\ \mathcal{R}_3 &= \{\ \mathbf{x}\ |\ \sigma_1(\mathbf{x})\sigma_2(\mathbf{x}) \le 0\ \}\end{aligned}$$

Here the region $\mathcal{R}_3$ is the sliding sector.

The control strategy of the proposed modified sliding mode control method is

$$u = u_{eq} + u_d \tag{16}$$

where u_{eq} is the continuous "equivalent" and u_d is defined as

$$u_d = \begin{cases} M\mathrm{sign}\left(\dfrac{\sigma_1 + \sigma_2}{2}\right) & \mathbf{x} \in \mathcal{R}_1 \cup \mathcal{R}_2 \\ M\dfrac{\sigma_1 + \sigma_2}{|\sigma_1| + |\sigma_2|}, & \mathbf{x} \in \mathcal{R}_3 \end{cases} \tag{17}$$

As shown in Fig. 1, let's represent the sector by the a surface of

$$\sigma = x_2 + K x_1 = 0 \text{ where } K = \frac{K_1 + K_2}{2} \tag{18}$$

Robustness of the proposed method The stability of the proposed sliding sector can be checked by the Lyapunov function candidate

$$V = \sigma^2/2 \tag{19}$$

where σ is the middle of the sliding sector. To define $\dot{V}$, the value of $\dot{\sigma}$ is necessary. Outside of the sector (in $\mathcal{R}_1$ and $\mathcal{R}_2$), according to (13) and (17)

$$\dot{\sigma} = -B_2 M\mathrm{sign}\,(\sigma) \tag{20}$$

According to (20), $\dot{V}$ is always negative outside of the sector

$$\dot{V} = \sigma\dot{\sigma} < 0 \tag{21}$$

It means the system trajectory enters into the sector in finite time. In side of the sector

$$\dot{\sigma} = -B_2 M\frac{\sigma_1 + \sigma_2}{|\sigma_1| + |\sigma_2|} \text{ and } \dot{V} = \sigma\dot{\sigma} \le 0 \tag{22}$$

Since $\dot{V} = 0$ implies $\sigma = 0$, the sector sliding mode inherits the most important characteristic of classical sliding mode. Note, that at the boundary of the sector

$$M\mathrm{sign}\,(\sigma) = -M\frac{\sigma_1 + \sigma_2}{|\sigma_1| + |\sigma_2|} \tag{23}$$

As the system state approaches the middle of the sector the absolute value of the discontinuous term is getting smaller that ensures the chattering free applications.

4. SLIDING MODE DESIGN BASED ON TENSOR PRODUCT TRANSFORMATION

The sliding sector design method can be extended for nonlinear systems given in the form of (1). Firs it is transformed to the form of (3) and a sliding surface is designed for each system S_r.

$$\sigma_r = x_2 + K_r x_1 = 0 \tag{24}$$

The definition of the three regions can be extended in the following way

$$\begin{aligned}&\mathcal{R}_1 \in \{x|\bigcap_{r=1}^{R}\sigma_r(x) < 0\} \quad \mathcal{R}_2 \in \{x|\bigcap_{r=1}^{R}\sigma_r(x) > 0\}\\ &\mathcal{R}_3 \in \{x|\bigcup_{i,j}^{R}\sigma_i(x)\sigma_j(x) \le 0\}\end{aligned} \tag{25}$$

Here the region $\mathcal{R}_3$ is a sliding sector. $R = 2$ in case of Fig. 1.

A modified version of (17) is applied

$$u = u_c + u_d \tag{26}$$

where u_c is a feed forward compensation term based on the estimation of the "equivalent" control, u_d is a switching term to suppress the system parameter variations and disturbances.

$$\begin{aligned}&u_c = \hat{u}_{eq}\\ &u_d = \begin{cases} -M\ rmsign\,(\sigma_1) \quad if\ \ x \in \mathcal{R}_1 \cup \mathcal{R}_2 \\ -M\left(\dfrac{\sum\limits_{r=1}^{R} w_r((p(X_s))\sigma_r}{\sum\limits_{r=1}^{R} w_r((p(X_s))\,|\sigma_r|}\right) \quad if\ \ x \in \mathcal{R}_3 \end{cases}\end{aligned} \tag{27}$$

where X_s is the value of x at a properly selected point of the sliding sector. There are three cases

- The nonlinearity is only inside of subspace (8)
- The nonlinearity is only outside of subspace (8)
- The nonlinearity is inside and outside of subspace (8)

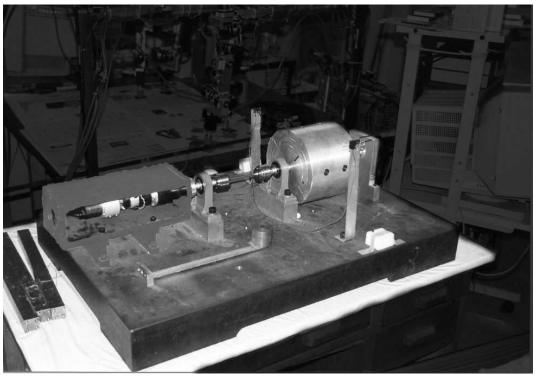

Fig. 2. The experimental system

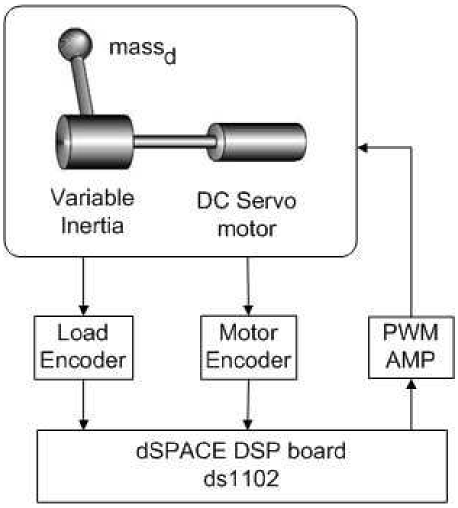

Fig. 3. System structure

Here only the second case is discussed. The sliding surface is designed in such a way that

$$\hat{u}_{eq} = u_{eq1} = u_{eq2} \cdots = u_{eqR} \tag{28}$$

where u_{eqi} is calculated for each system S_r according to (13)

$$u_{eqi} = -\left((A_{21i} + K_i A_{11i})x_1 + (A_{22i} + K_i A_{12i})x_2\right)/B_{2i} \tag{29}$$

Note, u_{eqi} is not the real equivalent control signal, and (28) itself cannot definite the sliding surface.

5. APPLICATIONS

The experimental system consists of a conventional DC servo gear motor with encoder feedback and variable inertia load coupled by a relatively rigid shaft, as shown in Fig. 2 and the structure can be seen in Fig. 3. The controller is implemented using a DSP as the computation engine.

5.1 System equations

In the course of control design, a reduced-order model is used, in which the armature inductance and the flexibility of the shaft are ignored. The state variables are the shaft position, θ and the shaft angular velocity, ω, the control signal u is the motor voltage. The effect of $mass_d$ is considered as a disturbance. The model calculated from the nominal parameters of the system is as follows (when the friction is ignored):

$$\begin{pmatrix} \dot{\theta} \\ \dot{\omega} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & -76 \end{pmatrix} \begin{pmatrix} \theta \\ \omega \end{pmatrix} + \begin{pmatrix} 0 \\ 18 \end{pmatrix} u \tag{30}$$

The harmonic gear connected to the motor has relative big friction. Coulomb has nonlinear characteristic, which is modeled in the following way:

$$u = u' \left(1 - \frac{1}{|u'| + 2.7}\right) \tag{31}$$

where u' is the control signal of the original linear system. The first part of the correction of the control signal in (31) is achieved empirically. It is quite straightforward to explain. The Coulomb friction torque is independent of the input voltage of the motor. If the input voltage is small the effect of the Coulomb friction is relative big. As you increase the absolute value of the motor voltage, the effect of Coulomb friction is getting relatively smaller and smaller. The second (dynamic) term is necessary because of TP model transformation. There is a Matlab toolbox for Tensor Product model transformation. The toolbox is available for download together with documentation and examples at http://tptool.sztaki.hu/.

The system matrix

$$\mathbf{S}(p(x)) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -76 & p(x) \\ 1 & 0 & 0 \end{pmatrix} \tag{32}$$

$$where\ \ p(x) \equiv p(u) = 18 * \left(1 - \frac{1}{2.7 * \mathrm{abs}(u) + 1}\right) \tag{33}$$

The parameter vector is

$$\Omega = [u_{\min}, u_{\max}] = [-22, 22] \tag{34}$$

Since equidistant sampling is applied and the sampling density must be high around zero voltage, the interval Ω is sampled at 137 grid points. The sampled system is arranged into a tensor

$$S_u = \left(S_1 \ S_2 \ \cdots \ S_{137} \right) \in \Re^{137\mathrm{x}(3\mathrm{x}3)} \tag{35}$$

where tensor, S_u, has only two singular values (1061.6 and 31.7). That is why the above nonlinear system can be modelled by two linear systems:

$$S_1 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -76 & 17.7 \\ 1 & 0 & 0 \end{pmatrix} \ , \ S_2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -76 & 13.9 \\ 1 & 0 & 0 \end{pmatrix} \tag{36}$$

The two weighing coefficients as a function of the control are shown in Fig. 4.

5.2 Sliding surface design

Since a second-order model is applied, the sliding "surface" is a sliding line that can be described by a scalar parameter K in (24).

$$\sigma_r = \omega + K_r\theta = 0 \ \text{ where } \ r = 1, 2 \tag{37}$$

According to (13) and (30),

$$u_{eq} = -\left((-76 + K_r)x_2\right)/B_{2r} \tag{38}$$

where the values of B_{2r} can be read from (36). One equation cannot definite two parameters. The sliding surface

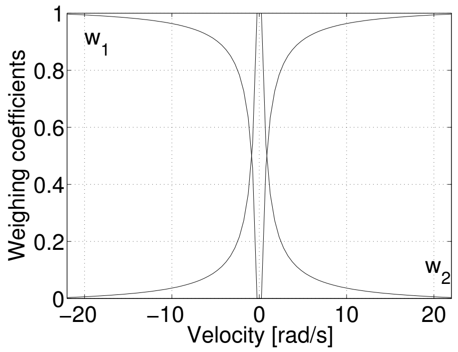

Fig. 4. The weighing coefficients as a function of the control signal

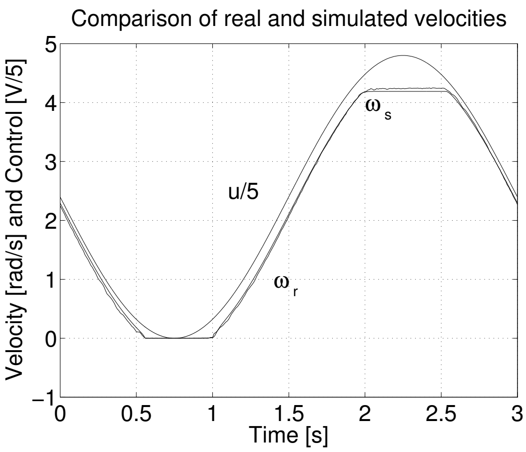

Fig. 5. The open loop response

is carefully designed (i.e. pole K is selected according to Korondi and Hashimoto [2000]) to satisfy both fast response both vibration suppression for the both systems. $K = 15$ in Korondi and Hashimoto [2000], the sector around that value is selected. If $K_1 = 8$ than $K_2 = 22$ according to (38) which is acceptable. X_p is selected in such a way that $w_1(p(X_s)) = 0.5$, $w_2(p(X_s)) = 0.5$.

5.3 Experimental results

The experimental results of the proposed and a conventional controller are compared in Fig. 5-Fig. 10.

The nonlinearity of the system is borne from the huge friction of the harmonic gear. To verify the friction model, the real and simulated velocities (ω_r, ω_s) are compared in Fig. 5, where the input voltage of the motor is a shifted sinusoid with amplitude of 12 V (open loop response). Note, the voltage input is divided by 5 to use the same scale as the speed. It can be seen in the Fig. 5, if the motor is in standstill, at least 2 V should be switched across the motor to start it. On the other hand, the motor is stick, if the input voltage is under 1.2 V. The power electronic PWM unit is saturated at 22 V. It is also a kind of nonlinearity which could be handled using a TP model. Because this paper concentrates on sliding sector design, only the nonlinearity of the friction is handled by TP model.

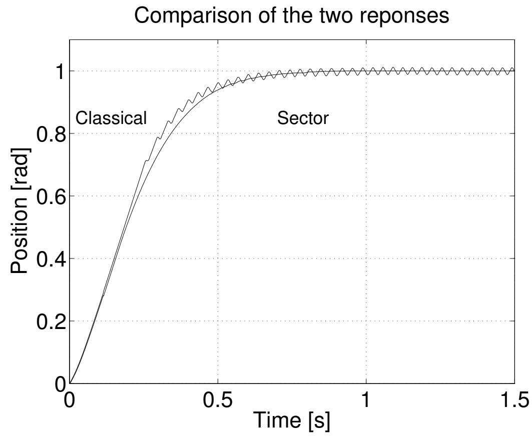

Fig. 6. Comparison of the position responses

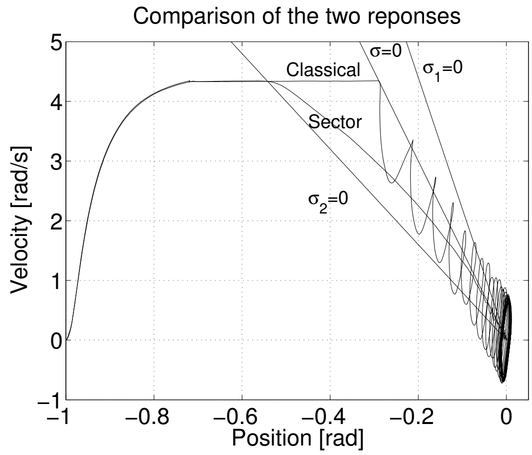

Fig. 7. Comparison of the trajectories

The chattering of the classical sliding mode and the chattering free response of the sector sliding mode control can be compared in Fig. 6. and Fig. 7.

After entering into the sector, the trajectory reaches the surface ($\sigma = 0$) gradually and smoothly, in case of sector sliding mode. The phase trajectory of the conventional sliding mode controller reaches the sliding surface directly and earlier than that of the sector sliding mode controller. After reaching the surface, the trajectory chatters around the surface. In Fig. 10, the system enters into the sliding sector approximately at $t = 0.5s$ ($\sigma_1 = 0$) in case of the sector sliding mode control. The main difference appears in the control activity. The conventional sliding mode is very robust but it needs intensive control action (see in Fig. 8), which causes significant audio noise as well. The sampling rate was quite rare. $T_{sampling} = 10ms$. The chattering could be reduced by increasing the sampling frequency but this paper demonstrates that the reduction of chattering (the intensity of the control action and the audio noise) is significant at the same sampling rate, if the TP based sector sliding mode (Fig. 9) is applied instead of the traditional sliding mode control. The oscillation in the control signal is caused by the friction (Fig. 9. $t = 0.5 - 08s$).

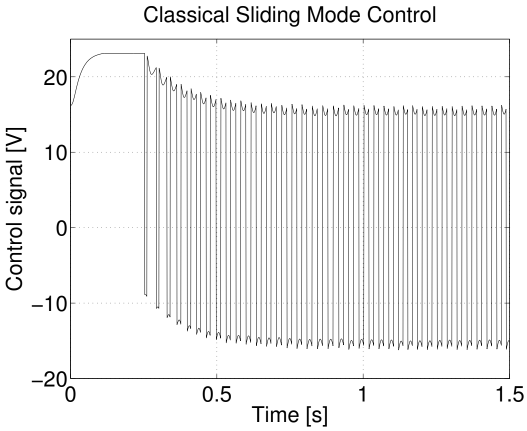

Fig. 8. Control signal in the case of conventional SMC

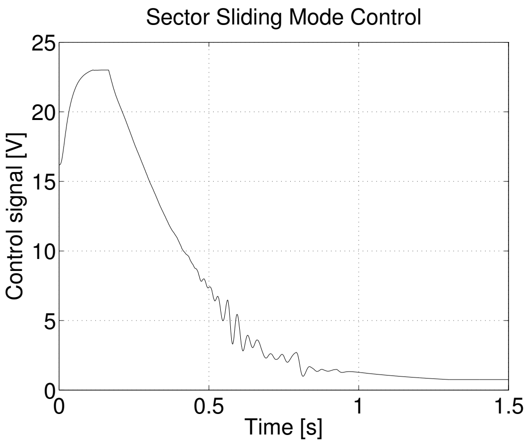

Fig. 9. Control signal in the case of sector SMC

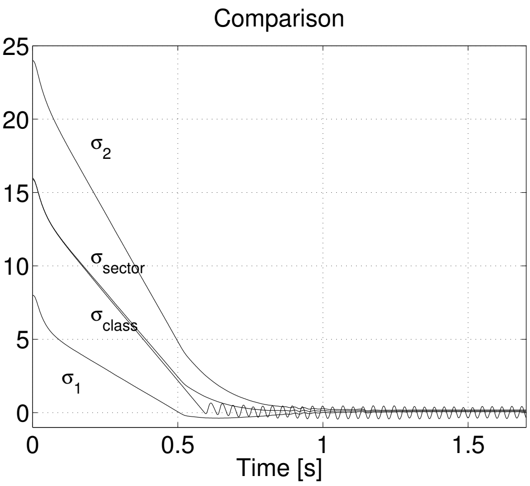

Fig. 10. Distance from the sliding surfaces

6. CONCLUSION

In this paper, a modified variable structure control strategy with continuous switching control has been developed in detail for the nonlinear system with uncertainty. The control strategy can be regarded as the extension of conventional VSS based sliding mode control method through expanding the switching surface to the sliding sector. The sliding sector is designed by a tensor product model transformation. The major advantage of the proposed control scheme is the introduction of the continuous switching control which successfully achieves smooth control response and retains the robustness of sliding mode control simultaneously. Both theoretical analysis and simulations demonstrate the attractiveness and the asymptotic stability of the sliding sector with the use of the proposed switching control which is essentially an interpolated control.

REFERENCES

P. Baranyi. Tp model transformation as a way to lmi based controller design. *IEEE Trans. Ind. Electron.*, 51 (2):387–400, 2004.

K.-H. Cheng, C.-F. Hsu, C.-M. Lin, T.-T. Lee, and C. Li. Fuzzyneural sliding-mode control for dcdc converters using asymmetric gaussian membership functions. *IEEE Trans. Ind. Electron.*, 54(3):1528–1536, nov 2007.

A. G. Filippov. Application of the theory of differential equations with discontinuous right-hand sides to nonlinear problems in automatic control. In *1st IFAC congress*, pages 923–925, 1960.

A. G. Filippov. Differential equations with discontinuous right-hand side. *Ann. Math Soc. Transl.*, 42:199–231, 1964.

K. Furuta and Y. Pan. Variable structure control with sliding sector. *Automatica*, 36:211–228, 2000.

P. Korondi and H. Hashimoto. Sliding mode design for motion control. *Studies in Applied Electromagnetics and Mechanics*, 16:1–12, 2000.

P. Korondi, J-X. Xu, and H. Hashimoto. Sector sliding mode controller for motion control. In *8th Conference on Power Electronics and Motion Control*, volume 5, pages 254–259, 1998.

F.-J. Lin, L.-T. Teng, and P.-H. Shieh. Intelligent sliding-mode control using rbfn for magnetic levitation system. *IEEE Trans. Ind. Electron.*, 54(3):1752–1762, nov 2007.

A. G. Lukuyanov and V. I. Utkin. Method of reducing equations of dynamic systems to regular form. *Transactions on Automation and Remote Control*, 42(1):413–420, 1981.

Z. Petres, B. Resko, and P. Baranyi. Tp model transformation based control of the tora system. In *Production Systems and Information Engineering*, volume 2, pages 159–175, 2004.

Satoshi Suzuki, Yaodong Pan, Katsuhisa Furuta, and Shoshiro Hatakeyama. Invariant sliding sector for variable structure control. *Asian Journal of Control*, 7(2): 124–134, 2005.

V. I. Utkin. *Variable Structure Control Optimization.* Springer-Verlag, 1992.

Jian-Xin Xu, Tong-Heng Lee, Mao Wang, and X. H. Yu. On the design of variable structure controllers with continuous switching control. *International Journal of Control*, 65(5):409–431, 1996.

Y. Yildiz, A. Sabanovic, and K. Abidi. Sliding-mode neuro-controller for uncertain systems. *IEEE Trans. Ind. Electron.*, 54(3):1676–1685, nov 2007.