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Sliding Mode Controllers for Active Suspensions

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Abstract: This paper presents a method for controlling a quarter-car hydraulic active suspension system using SMC techniques. The method guarantees the stability of the existence of a sliding mode and also the stability of the overall system. The proposed method considers many phases. In the first phase, the suspension dynamics is controlled via the actuator between the sprung and unsprung masses. Then the spool valve displacement dynamics is considered to control the current of the servo valve. Since there is an unknown parameter in the system an adaptation law is proposed to yield an appropriate estimate.

1. INTRODUCTION

A car suspension system is designed to reduce the vibrations resulting from road roughness and load on board when a vehicle moves along a road. Suspension system design relies on mathematical models of a vehicle. Suspension performance depends upon the type of suspension system and control strategy. Various suspension systems have been designed: active, semi-active and passive. The passive or semi-active suspension systems have been widely used to reduce the effect of the external disturbance including road disturbance, improve the maneuverability for various riding conditions and comfortability of passengers. A passive suspension system, which has usually fixed parameters, stores the energy using one or two springs and dissipates it via a damper. Its parameters are selected based upon a trade-off between road holding, load carrying (passengers) and comfort. To improve the suspension performance, the active suspensions are used. An actuating force within an active suspension system enables the system to suppress the effect of the disturbance on the vehicle body and passengers. A hydraulic active suspension is essentially a nonlinear system.

The aim of designing a suspension control is to improve the ride comfort and compromises between ride comfort and road holding. A suitable vehicle suspension system should be able to enhance road holding and passenger comfort. Compromise between these two issues is a major problem for designing and controlling suspension systems. In general, vehicle body vibrations depend on the speed of the vehicle, the road condition, the weight of the vehicle and load. Some vehicles have stiff suspensions with poor passenger comfort and many others have softer suspensions with poor road holding capabilities. Passive suspension systems provide only fixed rates for spring and damping coefficients. Therefore, they are efficiently not able to compromise between ride comfort and road holding. Comfortability of a vehicle's passengers depends on the vertical motion and angular motion of a vehicle. If a force is the only control input which is applied between the sprung and unsprung masses, vibration of the sprung mass at the wheel frequency modes cannot be reduced. Active suspension systems are used to reduce vibration of the sprung mass, and provide an appropriate response depending on many factors including road conditions and vehicle speed.

However, an active suspension system is expensive, complicated and requires an external power source. Therefore, the semi-active suspension system has been widely developed (Goodall, and Kortum, 1983). A semi-active suspension requires no hydraulic power supply, and implementation is simpler and cheaper than a full active system and only obeys damping or spring laws. The forces in the damper are generated by modulating its orifice for fluid flow and the performance of a damper is controlled via this modulation. Therefore, it can only dissipate or store energy (Alanoly and Sankar, 1988). The new technology enables a designer to propose a new control method for designing active suspensions with less expense and high quality performances and responses. The new methods may need further equipment because they are more sensitive with higher performance. In fact, an active suspension system has the ability to store, dissipate and to introduce energy to the system and consists of a forcing element with a spring and a damper. Its parameters may vary depending upon operating conditions and can have knowledge other than the strut deflection the passive system is limited to. The force is varied by the forcing elements based upon a control law.

Suspension systems can be obtained from bond graphs which are due to Paynter (1959). Bond graphs provide a graphical description of dynamic behaviour in terms of power bonds, connecting the elements of physical systems. Most work has been done for design and analysis of passive, semi-active and active suspension control systems, based upon linear suspension model or linearising the components around the equilibrium points. The system is controlled by applying an optimal control, H_∞ and most recently intelligent control including fuzzy control, neural networks, genetic algorithms and adaptive control techniques. However, a suspension system has a nonlinear behaviour. An optimal control for suspensions systems with ideal actuators has been studied by Ulsoy *et al.* (1994). The proposed optimal control guarantees the stability margin of the system. Hrovat (1997) has studied optimal PID controllers for semi-active suspensions. Srinivasa and Teja (1996) and Alleyne and Hedrick (1995) have designed an adaptive controller with a modified adapting scheme to reduce the model error and cope with the system uncertainties.

Rajamani and Hedrick (1995) developed an adaptive observer for the parameter identification of an active suspension system. Kim (1996) proposed an indirect adaptive controller for a vehicle active suspension with a nonlinear hydraulic actuator. Chantranuwathana and Peng (1999) designed an adaptive robust force controller to tackle the actuator uncertainties of active suspension systems. Fukao *et al.* (1999) and Nguyen *et al*. (2001) used a H_∞ controller to tackle the road surface disturbance and an adaptive backstepping control method to deal with the actuator nonlinearities. Smith and Wang (2002) derived a parameterised stable controller with a fixed prespecified closed-loop transfer function for a vehicle active suspension system. Fialho and Balas (2002) combined a linear parameter-varying control and a nonlinear backstepping technique to design a road adaptive active suspension system. The control performance of the adaptive control scheme fully depends on the accuracy of the dynamic model.

Sliding mode control (SMC) is a method for designing a control for the system with disturbances and unmodelled uncertainties. Various SMC algorithms and controllers have been proposed to control active suspension systems with random disturbances (Sam, *et al.,* 2004; Kim *et al.*, 1999; Kim and Ro 1998). Huang and Kuo (1999) proposed an adaptive sliding controller for nonlinear systems containing time-varying uncertainties by using the functional approximation technique.

Al-Holou *et al.* (1999) designed a fuzzy logic SMC for an active vehicle suspension system in which the fuzzy rules and the control parameters are designed based on a trial and error method. Huang and Lin (2004) proposed a pole assignment adaptive control strategy for a hydraulic active suspension system and introduced a fuzzy logic control loop to compensate the modelling error.

Note that designing a control for nonlinear semi-active and active suspension system is required to improve the stability of vehicle motion, safety of its journey and passengers. For simplicity, a quarter car model is considered. A quarter common suspension system of a car model, comprises of a sprung mass supported on a suspension system, which has stiffness and damping characteristics. A suspension system is connected to the unsprung mass of the axle. An active suspension control usually benefits from different energy domains such as electrical, mechanical, and hydraulic.

The control of active suspension system is addressed in this paper. The suspension model is nonlinear with road disturbances. In this model there are four dynamics including the displacement of the car body, the displacement of the wheel, the pressure drop across the piston (or the hydraulic actuator) and the displacement of the spool valve. The actual control is obtained after three different processes.

The paper is organised as follows: The mathematical model and system description are presented in Section 2. Section 3 deals with designing of an SMC and tracking the command signal actuator. The simulation results are given in Section 4. Conclusions are expressed in Section 5.

2. ACTIVE SUSPENSION

Consider the nonlinear dynamics of a quarter-car suspension system

$$
\begin{aligned}
M_s\ddot{x}_1 &= K_a^l(x_1-x_2)+C_a^l(\dot{x}_2-\dot{x}_1)+C_a^s\left|\dot{x}_2-\dot{x}_1\right|+\\
&\quad C_a^n\sqrt{\left|\dot{x}_2-\dot{x}_1\right|}\,\mathrm{sgn}\left(\dot{x}_2-\dot{x}_1\right)+K_a^n(x_1-x_2)^3+F\\
M_u\ddot{x}_2 &= -K_a^l(x_1-x_2)-C_a^l(\dot{x}_2-\dot{x}_1)-C_a^s\left|\dot{x}_2-\dot{x}_1\right|-\\
&\quad C_a^n\sqrt{\left|\dot{x}_2-\dot{x}_1\right|}\,\mathrm{sgn}\left(\dot{x}_2-\dot{x}_1\right)-K_a^n(x_1-x_2)^3-K_t(x_2-r)+F
\end{aligned}
\tag{1}
$$

where

M_s	: The sprung mass
M_u	: The unsprung mass
r	: The road elevation profile:
x_1	: Displacement of the car body
x_2	: Displacement of the wheel
K_a^l	: The linear part of the spring coefficient
K_a^n	: The nonlinear part of the spring coefficient
K_t	: The stiffness of the unsprung element
C_a^l	: The linear part of the damper coefficient
C_a^n	: The nonlinear part of the damper coefficient
C_a^s	: The coefficient related to symmetric behaviour of the damper
F	: The force from the hydraulic actuator

The hydraulic actuator is a four valve-piston system which is mounted parallel to the passive suspension system and is controlled by electro-hydraulic servo-valves. Therefore, the forces between sprung and unsprung masses can be generated. The actuator force F is given by

$$F = AP_p$$

where A is the area of the piston and P_p is the pressure drop across the piston with respect to the front and rear of the suspensions. The dynamics of P_p may be expressed as

$$\frac{V_t}{4\beta_m}\dot{P}_p = -C_{il}P_p - A\left(x_2 - x_4\right) + H_{lf} \qquad (2)$$

with

$$H_{lf} = C_d S\, x_{sv}\sqrt{\tfrac{1}{\rho}\left|P_s - \mathrm{sgn}(x_{sv})P_p\right|}\,\mathrm{sgn}\left(P_s - \mathrm{sgn}(x_{sv})P_p\right)$$

where C_d , S , x_{sv} , P_s and ρ are, respectively, the discharge coefficient, the spool valve area gradient, the displacement of the spool valve, the supply pressure and the density of the hydraulic fluid. H_{lf} is the hydraulic load flow. The displacement of the spool valve x_{sv} is controlled by the servo-valve current u and its dynamics is approximately given by

$$\dot{x}_{sv} = \tfrac{1}{\tau}\left(-x_{sv} + ku\right) \qquad (3)$$

where τ is the time constant and k is a gain. See Fig. 1 in which P_r is the return pressure going out of the spool valve.

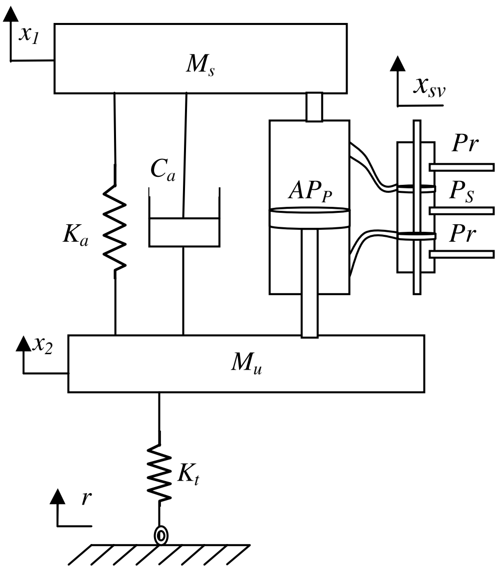

Fig. 1. A suspension model of a quarter of a vehicle with electro-hydraulic actuator

Consider $\dot{x}_1 = x_3$, $\dot{x}_2 = x_4$, $x_5 = P_p$, $x_6 = x_{sv}$. Then the state space representation of the system excluding the actuator force, outer-loop controller, is

$$\begin{aligned}
\dot{x}_1 &= x_3 \\
\dot{x}_2 &= x_4 \\
\dot{x}_3 &= \frac{1}{M_s}\Big(K_a^l(x_1 - x_2) + C_a^l(x_4 - x_3) + C_a^s\left|x_4 - x_3\right| + \\
&\quad C_a^n\sqrt{\left|x_4 - x_3\right|}\,\mathrm{sgn}\left(x_4 - x_3\right) + K_a^n(x_1 - x_2)^3 + Av_c\Big) \\
\dot{x}_4 &= \frac{1}{M_u}\Big(-K_a^l(x_1 - x_2) - C_a^l(x_4 - x_3) - C_a^s\left|x_4 - x_3\right| - \\
&\quad C_a^n\sqrt{\left|x_4 - x_3\right|}\,\mathrm{sgn}\left(x_4 - x_3\right) - K_a^n(x_1 - x_2)^3 - K_t(x_2 - r) - Av_c\Big)
\end{aligned} \qquad (4)$$

The inner-loop controller can be expressed as

$$\begin{aligned}
\dot{v} &= \theta\Big(-\beta v - 4A\left(x_3 - x_4\right) + \gamma x_6\sqrt{\left|P_s - \mathrm{sgn}(x_6)v\right|}\,\mathrm{sgn}\left(P_s - \mathrm{sgn}(x_6)v\right)\Big) \\
\dot{x}_6 &= \tfrac{1}{\tau}\left(-x_6 + ku\right)
\end{aligned} \qquad (5)$$

with $\theta = \frac{\beta_m}{V_t}$, $\beta = 4C_{il}$ and $\gamma = 4C_d S\sqrt{\frac{1}{\rho}}$. Note that $\theta = \frac{\beta_m}{V_t}$ is an unknown parameter.

The main objectives of designing an appropriate control are to improve the vehicle stability and maximise the passenger comfort by controlling the suspension system under road disturbances. The road disturbances magnitude does not reach to the suspension travel limits at any time of travel. In addition, a suitable control is expected to minimise the car body accelerations under the road disturbances. The control of suspension is achieved at two main phases. Using the systems (4) the force command v is designed to achieve the performance requirement and reduce the affect of the disturbance including the road disturbances. A suitable SMC is able to reduce the disturbances even if the disturbances are relatively large. Using the force command v_c and inner-loop (5) the servo-valve command u is determined. Note that the suspension travel x_1 is physically constrained, i.e. for any t , $\left|x_1\right| \le \bar{x}_1$.

The disturbance for a smooth road is small and usually is neglected. The road disturbances may be considered as a piecewise continuous function such as

$$r = \begin{cases} a_k\left(1 - \sin(b_k \pi t)\right) & \text{if } t_k^d \le t \le t_{k+1}^d \\ 0 & \text{Otherwise} \end{cases} \qquad (6)$$

where t_k^d and t_{k+1}^d are the times that road disturbance is nonzero, a_k and b_k are related to the disturbance magnitude and amplitude, respectively.

3. SMC DESIGN

An SMC is a robust control which is designed for a system with uncertainties. SMCs are able to reject the matched disturbance completely and reduce the effect of unmatched disturbance on the system. This important property is the main reason for the wide use of SMCs. In this section an SMC is designed for stabilising the system (4). Assume that $\left|r\right| \le h$ where h is a known real function or a constant. Define the sliding hyperplane as

$$s = x_4 + \phi\left(x_1, x_2, x_3\right) = 0 \tag{7}$$

The function $\phi\left(x_1, x_2, x_3\right)$ is selected so that the reduced-order system (system in the sliding mode) is asymptotically stable. $\phi\left(x_1, x_2, x_3\right)$ is normally selected as a linear combination of the states. However, in many SMC design methods, a nonlinear function may be considered. This function may include an integral term or it is a piecewise continuous function. The selection of the sliding function depends essentially on the nature of the system model. Therefore, for designing an SMC two phases are considered: control design and sliding surface (hyperplane) design. In many cases the structure of the sliding surface is first selected and then an SMC is designed so that the system trajectories move on the sliding surface in a finite time and remain on it thereafter. Finally the sliding function parameters are identified such the system stability is assured. So for a particular problem, various methods based on SMC may be designed. Dynamic, integral and proportional-integral (PI) SMC approaches are a variety of SMCs. The most important problem is to select an appropriate SMC approach for the system.

The control v_c is designed so that the sliding mode reaching condition $\dot{s}s < 0$ is fulfilled. Consider the control

$$\begin{aligned} v_c = \frac{-1}{A}\Bigg[& \Big(K_a^l(x_1 - x_2) + C_a^l(x_4 - x_3) + C_a^s\left|x_4 - x_3\right| + \\ & C_a^n\sqrt{\left|x_4 - x_3\right|}\,\mathrm{sgn}\left(x_4 - x_3\right) + K_a^n(x_1 - x_2)^3 \Big) + \\ & \frac{M_u M_s}{\left(\frac{\partial\phi}{\partial x_3}M_u - M_s\right)}\left(\frac{-K_t}{M_u}x_2 + \frac{\partial\phi}{\partial x_1}x_3 + \frac{\partial\phi}{\partial x_2}x_4 + W\,\mathrm{sgn}(s) \right) \Bigg] \end{aligned} \tag{8}$$

where $W > K_t h$. The condition $W > K_t h$ is vital for ensuring the stability of the sliding mode, and without this condition the sliding mode stability may not be achieved. Using the derivative of the sliding function s defined as in (7) and substituting the control (8) yields

$$\begin{aligned} s\dot{s} &= s\left(\dot{x}_4 + \frac{\partial\phi}{\partial x_1}x_3 + \frac{\partial\phi}{\partial x_2}x_4 + \frac{\partial\phi}{\partial x_3}\dot{x}_3 \right) \\ &= s\Bigg[\frac{\partial\phi}{\partial x_1}x_3 + \frac{\partial\phi}{\partial x_2}x_4 + \frac{1}{M_s}\Big(K_a^l(x_1 - x_2) + C_a^l(x_4 - x_3) + \\ &\quad C_a^s\left|x_4 - x_3\right| + C_a^n\sqrt{\left|x_4 - x_3\right|}\,\mathrm{sgn}\left(x_4 - x_3\right) + K_a^n(x_1 - x_2)^3 + Av \Big)\times \\ &\quad \frac{\partial\phi}{\partial x_3} + \frac{1}{M_u}\Big(-K_a^l(x_1 - x_2) - C_a^l(x_4 - x_3) - C_a^s\left|x_4 - x_3\right| - \\ &\quad C_a^n\sqrt{\left|x_4 - x_3\right|}\,\mathrm{sgn}\left(x_4 - x_3\right) - K_a^n(x_1 - x_2)^3 - K_t(x_2 - r) - Av \Big) \\ &\le (-W + K_t h)\left|s\right| \end{aligned} \tag{9}$$

Therefore the control (8) enforces the state trajectories onto the sliding surface (7). The main problem is now to select the function $\phi\left(x_1, x_2, x_3\right)$ for ensuring the asymptotic stability of the system. During the sliding mode $s = 0$ and $\dot{s} = 0$. From $\dot{s} = 0$ the equivalent control is obtained

$$\begin{aligned} v_c^{eq} = \frac{-1}{A}\Bigg[& \Big(K_a^l(x_1 - x_2) + C_a^l(x_4 - x_3) + C_a^s\left|x_4 - x_3\right| + \\ & \Big(C_a^n\sqrt{\left|x_4 - x_3\right|}\,\mathrm{sgn}\left(x_4 - x_3\right) + K_a^n(x_1 - x_2)^3 \Big) \\ & + \frac{M_u M_s}{\left(\frac{\partial\phi}{\partial x_3}M_u - M_s\right)}\left(\frac{-K_t}{M_u}\left(x_2 - r\right) + \frac{\partial\phi}{\partial x_1}x_3 + \frac{\partial\phi}{\partial x_2}x_4 \right) \Bigg] \end{aligned} \tag{10}$$

Note that the equivalent control is not usually accessible because the disturbance r is not measurable. This control is normally used for analysing the system in the sliding mode. Substituting (10) into (4) implies the sliding mode system

$$\begin{aligned} \dot{x}_1 &= x_3 \\ \dot{x}_2 &= x_4 \\ \dot{x}_3 &= \frac{-M_u}{\left(\frac{\partial\phi}{\partial x_3}M_u - M_s\right)}\left(\frac{-K_t}{M_u}(x_2 - r) + x_3\frac{\partial\phi}{\partial x_1} + x_4\frac{\partial\phi}{\partial x_2} \right) \\ \dot{x}_4 &= \frac{M_s}{\left(\frac{\partial\phi}{\partial x_3}M_u - M_s\right)}\left(x_3\frac{\partial\phi}{\partial x_1} + x_4\frac{\partial\phi}{\partial x_2} - \frac{K_t}{M_s}\left(x_2 - r\right)\frac{\partial\phi}{\partial x_3} \right) \end{aligned} \tag{11}$$

Now the objective is to find a function ϕ such that the stability of (11) is guaranteed. The system (11) is linear if ϕ is defined as a linear function. Define, the function ϕ as

$$\phi = Cx_s = c_1x_1 + c_2x_2 + c_3x_3 \tag{12}$$

where c_1, c_2 and c_3 are selected such that the polynomial

$$\begin{aligned} &\left(c_3M_u - M_s\right)s^3 + \left(M_uc_1 - c_2M_s\right)s^2 + \\ &\qquad c_3K_ts + c_1K_t\left(c_3M_u - M_s\right) = 0 \end{aligned} \tag{13}$$

to be Hurwitz. Then the system in the sliding mode, the reduced order system, is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -c_1 & -c_2 & -c_3 \\ \frac{c_1c_2M_u}{c_3M_u - M_s} & \frac{c_2^2M_u - K_t}{c_3M_u - M_s} & \frac{a\left(c_2c_3 - c_1\right)M_u}{c_3M_u - M_s} \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{-K_t}{c_3M_u - M_s} \end{bmatrix} r \tag{14}$$

The condition on (13) guarantees the stability of the system (14) in the absence of disturbance. However, the design parameters, may be selected such that the effect of the disturbance is reduced.

Now the tracking problem of the actuator is considered. Select

$$w = x_6\sqrt{|P_S - \text{sgn}(x_6)v|}\,\text{sgn}\left(P_S - \text{sgn}(x_6)v\right) \tag{15}$$

Then the inner-loop controller is governed by

$$\begin{aligned}\dot{v} &= \theta\left(-\beta v - 4A\left(x_3 - x_4\right) + \gamma w\right)\\ \dot{x}_6 &= \frac{1}{\tau}\left(-x_6 + ku\right)\end{aligned} \tag{16}$$

It is required to design the new control w such that the actuator force v converges to the command signal v_c. Then the servo-valve current u is obtained using (16). θ is an unknown parameter and should be estimated. Let $\hat{\theta}$ be an estimate of θ. Consider the Lyapunov function

$$V = \frac{1}{2}\left(v - v_c\right)^2 + \frac{\rho}{2}\left(\theta - \hat{\theta}\right)^2 \tag{17}$$

where $\rho > 0$, then

$$\begin{aligned}\dot{V} &= \left(v - v_c\right)\left(\dot{v} - \dot{v}_c\right) - \rho\left(\theta - \hat{\theta}\right)\dot{\hat{\theta}}\\ &= \left[\hat{\theta}\left(-\beta v - 4A\left(x_3 - x_4\right) + \gamma w\right) - \dot{v}_c\right]\left(v - v_c\right)\\ &\quad + \rho\left(\theta - \hat{\theta}\right)\left(\left(-\beta v - 4A\left(x_3 - x_4\right) + \gamma w\right)\left(v - v_c\right) - \dot{\hat{\theta}}\right)\end{aligned}$$

The adaptation law is given by

$$\dot{\hat{\theta}} = \left(-\beta v - 4A\left(x_3 - x_4\right) + \gamma w\right)\left(v - v_c\right) \tag{18}$$

Select

$$v = v_c - \varepsilon\left[\hat{\theta}\left(-\beta v - 4A\left(x_3 - x_4\right) + \gamma w\right) - \dot{v}_c\right] \tag{19}$$

where $\varepsilon > 0$. Then

$$\dot{V} = -\frac{1}{\varepsilon}\left(v - v_c\right)^2$$

In fact, from (16)

$$\dot{v} - \dot{v}_c = -\frac{1}{\varepsilon}\left(v - v_c\right)$$

which guarantees $v \to v_c$ exponentially.

Now to complete the procedure design, the actual control u should be designed. From (15) and (16) the servo-valve current is obtained. An appropriate servo-valve current is

$$u = \frac{w\,\text{sgn}\left(P_S - \text{sgn}(w)v\right)}{k\sqrt{|P_S - \text{sgn}(w)v|}} \tag{20}$$

(Chantranuwathana and Peng, 1999). Since $v \to v_c$ exponentially, one may consider v_c as an estimate of v for evaluating u defined as in (20).

4. SIMULATION RESULTS

Consider the following values for simulation:

$M_s = 450$ Kg, $M_u = 35$ Kg, $K_a^l = 1650$ N/m, $C_a^l = 100$ Ns/m, $C_a^n = 7000$ Ns/m, $C_a^s = 300$ Ns/m, $K_a^n = 15000$ N/m, $\bar{x}_1 = 0.1$ m, $K_t = 90000$ N/m, $\gamma = 1.545\times10^9$ (N/m$^{5/2}Kg^{1/2}$) $A = 3.35\times10^{-4}$ m^2.

Also the road elevation profile r is considered as

$$r = \begin{cases} a_0\left(1 - \sin(b_0\pi t)\right) & \text{if } 0.5 < t < 0.75\\ 0 & \text{Otherwise}\end{cases}$$

with $a_0 = 0.5$ and $b_0 = 0.8$. The sliding surface is defined as

$$s = \frac{1}{6}x_1 + x_2 + \frac{11}{6}x_3 + x_4 = 0$$

The SMC is given by (8) with $W = 1$. It is assumed that the suspension travel limits are ±10 cm and spool valve displacement limits are ±1 cm. The simulation results are shown in Figs. 2 and 3. Fig. 2 shows that the displacement of the car body is between −8 and 8 (cm). The chattering is related to the discontinuous controller and the nature of the systems. To reduce the chatting one may use the continuous approximation of discontinuous terms and/or higher order SMC.

5. CONCLUSIONS

A quarter-car hydraulic active suspension system has been controlled using SMC techniques. The proposed SMC guarantees the trajectories move on the sliding surface in finite time and remain there for future time, and also stability of the overall system. The suspension dynamics has been controlled using the actuator between the sprung and unsprung masses. The spool valve displacement dynamics has been considered to control the current of the servo valve. An adaptation law has been presented for estimation of a system with an unknown parameter.

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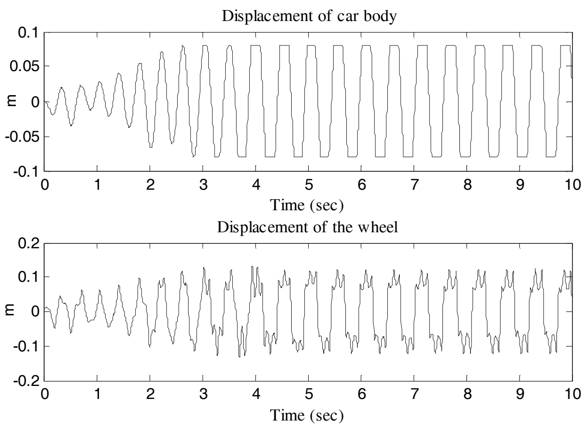

Fig. 2. The responses of the suspension system with SMC

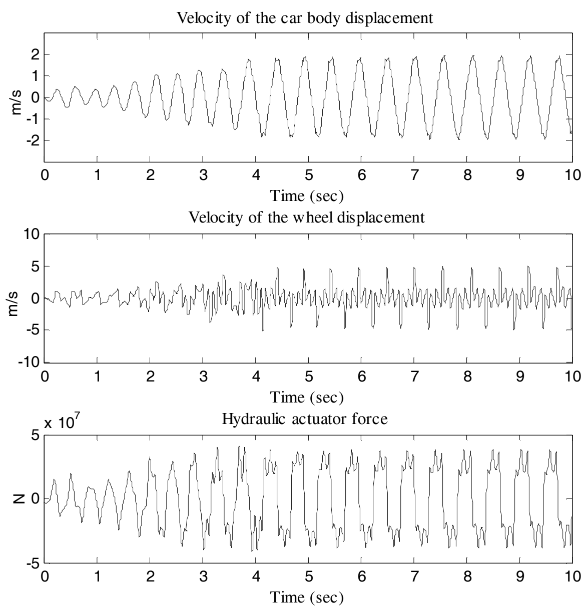

Fig. 3. The behaviour of the hydraulic actuator force of the suspension system and the velocity of the car body and the wheel displacement with SMC