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Controller Synthesis of an Uncertain Three Tank System Using Polytypic System Approach

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Abstract: Many of the systems in process and aerospace industry can be modeled as a three tank system. LMI and NLMI based controller design methods have been applied to three tank system in literature. Nonlinear methods suffer from lack of ease for implementation, on the other hand, LMI based methods have to be unduly robust to cater for significant uncertainties arising from linearization around an equilibrium point. This paper presents an LMI based controller design method, which specifically uses second or higher order terms of nonlinear three tank model, thus resulting in a robust controller with no compromise on performance. A second degree error model is derived using small signal linearization method about an arbitrary operating point. A set of LMI's are formulated for each polytopic region and solved to obtain the corresponding state feedback controller gains for that particular polytopic region. The series of designed controllers drives the system states from starting point to the desired state through a series of connected polytopic regions. The controller is robust with respect to parameter variations due to the combined polytopic-LMI problem formulation. The effects of parametric variations and disturbances are accounted for in such a formulation via appropriate bounds. The performance of the designed controller is also compared with LMI based H_∞ controller.

1. INTRODUCTION

Three tank system represents a typical process system in chemical industry, fuel management systems of air planes and space vehicles. Many of the systems can be modeled as a three tank system. Most of the three tank system parameters for example flow coefficients are uncertain because of change of liquid or liquid density and aging effects (corrosion, scaling etc). Linearization techniques also introduce approximation errors. It is generally well known that a parametric model does not exactly describe the physical system due to un-modeled dynamics and uncertainty in parameters. These un-modeled dynamics and parametric uncertainties are handled by robust control schemes. Robust control laws cover larger regions of systems around an operating point. Robust control theory utilizes available information about model uncertainties, performance and stability requirements. Many recent works have developed controls for linear and nonlinear systems including three tank system which may contain uncertain parameters, see for example (Cedric and Ramirez 2005), (Hahn et al, 2004), and (AlSwailem, 2004) and the references therein.

A relative new area for control of linear as well as nonlinear systems is based on representing the systems in terms of Linear Matrix Inequalities (LMIs) and solving these LMI's using convex optimization techniques (Boyd et al, 1994), (Scherer and Weiland, 2005). A number of LMI based approaches for the control and observer design of three tank system and other nonlinear systems are discussed in (Rodrigues, 2005), (Armeni, 2004), (Rajamani and Cho, 1995), (Uthaichana, et al. 2003). In (Rodrigues, 2005) an active Fault Tolerant Control (FTC) strategy is developed for three tank system described by multiple linear models to prevent the system deterioration by the synthesis of adapted controller gains through LMIs. A Polytopic Unknown Input Observer (PUIO) is synthesized for actuator fault estimation and controller gains are adjusted to preserve the system performances over a wide operating range (Rodrigues, 2005). Nonlinear three tank system is represented by multiple models (Abdelkader et al, 2003), where a part of its inputs is unknown. The state variables are estimated by the synthesis of a multiple observer based on the elimination of the unknown inputs. The gains of the local observers are determined as a solution to a set of LMIs. (Rajamani and Cho, 1995) LMI based observer design methodology is presented for nonlinear system. In (Armeni, 2004) FDI filter based on residual generation with fault sensitivity constraint is proposed. An integrated controller and observer for LTI systems with model uncertainties and linear parameter varying is designed. FDI scheme is proposed based on solving a family of LMI optimization problems, which guarantees detection and isolation of smaller fault signals with good disturbance attenuation, in the presence of multiple simultaneous faults.

LMI and NLMI based controller design methods have been applied to three tank system in literature. Nonlinear methods suffer from lack of ease for implementation, on the other hand, LMI based methods have to be unduly robust to cater for significant uncertainties arising from linearization around an equilibrium point. In this paper an LMI based polytopic controller design method (Soren et al, 2002) is employed which specifically uses second or higher order terms of nonlinear three tank model, thus resulting in a robust controller with no compromise on performance. A 2nd degree nonlinear perturbation model is derived about each operating point in a small region bounded by a polytope. A chain of overlapping small regions of the state space is formed so that each region contains an equilibrium point common to the next polytopic region in the chain (Figure 2). It is assumed that these consecutive polytopic regions are continuous. Each vertex of this polytope corresponds to a 2nd degree nonlinear system. Lyapunov stabilizing method is applied to each polytopic region, a feedback controller and a desired ellipsoidal domain of attraction is obtained by solving a set of LMIs. The designed controller moves the state through the associated region to an operating point common to the domain of attraction of the current region and the next region along the chain. The controller for the next equilibrium state is invoked when the system is sufficiently close to the preceding equilibrium state. This technique for the synthesis of controller is good in the sense, that at any particular operating point we have more than one controller available, which can be used to meet the appropriate performance measures. This paper is organized as following: Three tank system is explained in section 2. Section 3 describes LMI method for stabilization of polytopic nonlinear system and polytopic controller design. Simulation results are discussed in section 4 and concluding remarks are in section 5.

2. THE THREE TANK SYSTEM

A Three Tank System shown in Figure 1 is a benchmark system for the development, experimentation and analysis of complex linear as well nonlinear control and diagnosis algorithms. The mathematical model of the three tank system (Cedric and Ramirez 2005) is obtained by "mass balance" equations by:

$$S\frac{dL_1}{dt} = q_1 - q_{13}$$
$$S\frac{dL_2}{dt} = q_2 + q_{23} - q_{20} \qquad (1)$$
$$S\frac{dL_3}{dt} = q_{13} - q_{32}$$

where q_{ij} represents the water flow rates from tank i to j., which, is given by $q_{ij} = \mu_i S_p sgn(L_i - L_j)\sqrt{2g|L_i - L_j|}$, i, j=1,2,3 and q_{20} is the outflow rate with $q_{20} = \mu_2 S_p\sqrt{2gL_2}$, μ_i, S_p are the flow coefficients and cross sectional areas of interconnecting pipes, L_i are water levels in tanks, q_1 and q_2 are flow rates into tank 1 and tank 2 respectively. The full system model is then obtained as follows:

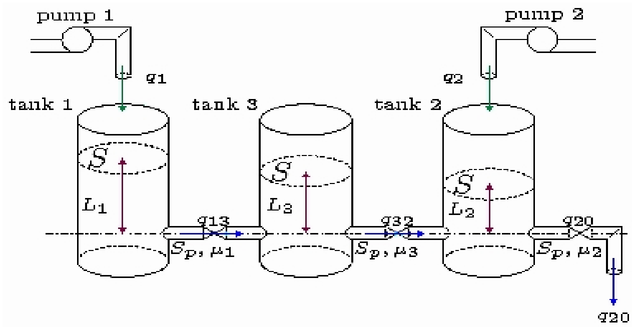

Figure. 1. The Three-Tank System

$$\dot{x}_1 = C_1 sgn(x_1 - x_3)\sqrt{|x_1 - x_3|} + \frac{(q_1 + w_1)}{S}$$
$$\dot{x}_2 = C_3 sgn(x_3 - x_2)\sqrt{|x_3 - x_2|} - C_2 sgn(x_2)\sqrt{|x_2|} + \frac{(q_2 + w_2)}{S}$$
$$\dot{x}_3 = C_1 sgn(x_1 - x_3)\sqrt{|x_1 - x_3|} - C_3 sgn(x_3 - x_2)\sqrt{|x_3 - x_2|}$$
$$y_1 = x_1, y_2 = x_2, y_3 = x_3 \qquad (2)$$

where $x_i(t)$ is the liquid level in tank i and $C_i = \frac{1}{S}\mu_i S_p\sqrt{2g}$. The two control signals are $q_1(t)$ and $q_2(t)$ (input flow rates) respectively, w_1 and w_2 are actuator faults/ disturbances which perturb the behavior of the system. These actuator faults/disturbances must be compensated for graceful operation of the system. The parameters C_i which includes flow coefficient μ_i are uncertain because of change of liquid or liquid density and aging effects (corrosion, scaling etc). These parametric variations and uncertainties must also be accommodated by controller. The typical parameters values of benchmark three tank system are given in Table 1.

Since the system in (2) is inherently unstable, a controller is required to regulate the flow rates and levels of the tanks, to achieve the steady state condition in the presence of disturbances and uncertainties.

Table 1 Typical Parameter Values

Parameter	Value
S, Area of the Tanks	0.0154 m^2
S_n, Area of pipes, n=1,2,3	5×10^{-5} m^2
q_{1max}, q_{2max} (input flow rates)	100 ml/s
$L_{i\text{-}max}$, x_i, Level in Tanks, i=1,2,3	0.62 m
C_1, C_3 and C_2	0.0072, 0.0097
Operating point x_{10}, x_{20}, and x_{30}	0.60, 0.40, 0.25
Initial conditions	0,0,0

The four operating regions corresponding to combination of states are $x_1 \geq x_3$, $x_1 < x_3$, $x_2 \geq x_3$ and $x_2 < x_3$. Let us introduce constants as: $s_{13} = sign(x_1 - x_3)$, $s_{32} = sign(x_3 - x_2)$ and $s_{02} = sign(x_2)$. These constant represent the signs of the differences of states. These constants are not continuous functions, they act as switching function and switches their value between [-1, 1] depending upon the sign of difference of two corresponding states. For simplicity their values can be fixed by taking hierarchical cases study (for example $x_1 > x_2 > x_3$ leading to $s_{13} = s_{02} = 1$ and $s_{32} = -1$). Using these constants, the system represented by (2) can be written as:

$$\dot{x}_1 = C_1 s_{13}\sqrt{|x_1 - x_3|} + \frac{(u_1+w_1)}{S}$$
$$\dot{x}_2 = C_3 s_{32}\sqrt{|x_3 - x_2|} - C_2 s_{02}\sqrt{|x_2|} + \frac{(u_2+w_2)}{S}$$
$$\dot{x}_3 = C_1 s_{13}\sqrt{|x_1 - x_3|} - C_3 s_{32}\sqrt{|x_3 - x_2|} \quad (3)$$

The system (3) will be used for the synthesis of polytopic system based controller using small signal linearization.

3. THE POLYTOPIC CONTROLLER DESIGN

For a nonlinear system of the form:
$$\dot{x}(t) = A(t,x)x(t) + B(t,x)u(t), x(t_0) = x_0 \quad (4)$$
where the time/state dependent matrices $A(t,x)$ and $B(t,x)$ has the following structure.
$$A(t,x) = A_0 + \sum_{i=1}^{l} \Delta A_i \psi_i(t,x) \quad (5)$$
$$B(t,x) = B_0 + \sum_{i=1}^{l} \Delta B_i \psi_i(t,x) \quad (6)$$
where $\psi_i(t,x)$ scalar valued are functions of t and x, $A_0, \Delta A_{1,\dots,}\Delta A_i$ are constant $n \times n$ dimensional matrices and $B_0, \Delta B_{1,\dots,}\Delta B_i$ are $n \times m$ matrices. Additionally we require that, whenever $\|C_i x\| \le \xi$, $a_i \le \psi_i(t,x) \le b_i$ for constant matrices C_i, a positive scalar ξ, constants a_i and b_i, for $i = 1 \dots l$, which are the sufficient LMI conditions for the system to be uniformly exponentially stable (Soren et al, 2005).

For a closed loop system $\dot{x}(t) = (A(t,x) + B(t,x)K)x(t)$ is obtained from (4) with the linear state feedback $u(t) = Kx(t) = LS^{-1}x(t)$, the origin is uniformly exponentially stable equilibrium point with $\Omega = \{x \in R^n : x^T S^{-1} x < \xi^2\}$ if the matrices L and a symmetric positive definite S and the scalar ξ satisfy
$$AS + SA^T + BL + L^T B^T < 0 \quad (7a)$$
$$CSC^T \le I$$
for all
$$C \in \{C_i | i = 1, \dots, l\} \quad (7b)$$
And matrix pairs
$$(A,B) \in \{(A_0 + \textstyle\sum_{i=1}^{l} \Delta A_i \psi_i(t,x)),\ (B_0 + \sum_{i=1}^{l} \Delta B_i \psi_i(t,x)) | \psi_i = a_i \text{ or } b_i,\ i = 1, \dots l\} \quad (7c)$$

This gives a family of LMI's and will be used to derive a linear state feedback for the 2nd degree model of Three Tank System. The existence of L and S and ξ in the LMI (7) provides state feedback that is sufficient for stability. Theoretical, sufficient conditions in terms of system structure guaranteeing the existence of solution to the family of LMI's remains an open question (Soren et al, 2005).
Two more constraints are imposed on L and S for feasible solution to our control problem. The 1st constraint allows the inclusion of a starting point x_0, the initial water level in a tank, in the invariant ellipsoid Ω centered at x_1^d, and can be expressed as an LMI using the Schur's complement (Scherer and Weiland, 2005):
$$(x_0 - x_1^d)^T S^{-1}(x_0 - x_1^d)^T < \xi^2 \Leftrightarrow \begin{bmatrix} \xi^2 & (x_0 - x_1^d)^T \\ (x_0 - x_1^d)^T & S \end{bmatrix} > 0 \quad (8)$$

and 2nd constraint defines upper bound of the control input, $u(t)$, the water being pumped into the system, by γ. This γ can be chosen to impose a maximum energy constraint on $u(t)$ over the interval [0, T] (Uthaichana et al, 2003). This bound for $x(t) \in \Omega$ requires that for all t
$$\|u(t)\|^2 = \|Kx(t)\|^2 = x^T(t)K^T K x(t) \le \gamma^2 \quad (9)$$
If $K^T K \le \frac{\gamma^2}{\xi^2} S^{-1}$ holds, then $\|u(t)\|_2 \le \gamma$ as desired. This equation can be converted to LMI as
$$K^T K \le S^{-1} L^T L S^{-1} \le \frac{\gamma^2}{\xi^2} S^{-1} \Leftrightarrow L^T L \le \frac{\gamma^2}{\xi^2} S \quad (10)$$
Using Schur's complement, this can be written as an LMI
$$\begin{bmatrix} S & L^T \\ L & \frac{\gamma^2}{\xi^2} I \end{bmatrix} > 0 \quad (11)$$

The system of LMI's formed by inequalities (7), (8) and (11) are used to design a gain scheduled controller. Theorem 2 from (Soren et al, 2005) is used for the design of controller for the 2nd degree nonlinear three tank system model. At each equilibrium point from starting point to the desired operating point, a set of LMI's is generated, the solution to these LMI's gives a desired controller which drives the system states to the next region of attraction.

3.1 Derivation of Nonlinear Perturbation (Error) Model

A 2nd degree nonlinear perturbation model of (3) about an equilibrium point (x_{10}, x_{20}, x_{30}) using small signal linearization is derived. Generally, while linearizing, 1st order terms are included to ensure the linear behavior of the linearized system, for our case we have included 2nd order terms to capture 2nd order nonlinearities. Taking $\dot{x}_1$ in (3), applying Taylor series expansion and rearranging, we get:

$$\dot{x}_1 = -S_{13}C_1\sqrt{x_1}\left(1 - \frac{x_3}{x_1}\right)^{1/2} + (u_1 + w_1)/S$$
$$= -S_{13}C_1\left[\sqrt{x_1} - 1/2 x_3 x_1^{-1/2}\right] + (u_1 + w_1)/S$$
Now applying perturbation theory, we obtain,
$$\dot{x}_{10} + \delta\dot{x}_1 = -S_{13}C_1\left[(x_{10} + \delta x_1)^{\frac{1}{2}} - \frac{1}{2}(x_{30} + \delta x_3)(x_{10} + \delta x_1)^{-\frac{1}{2}}\right] + (u_1 + w_1)/S$$
$$\dot{x}_{10} + \delta\dot{x}_1 = -S_{13}C_1\left[(x_{10})^{\frac{1}{2}}\left(1 + \frac{\delta x_1}{x_{10}}\right)^{\frac{1}{2}} - \frac{1}{2}x_{30}\left(1 + \frac{\delta x_3}{x_{30}}\right)(x_{10})^{-\frac{1}{2}}\left(1 + \frac{\delta x_1}{x_{10}}\right)^{-\frac{1}{2}}\right] + (u_1 + w_1)/S$$
Using Taylor series expansion, keeping 1st, 2nd order terms, neglecting 3rd and higher order terms, we get after simplification:

$$\delta\dot{x}_1 = S_{13}C_1\left[\left\{-\frac{1}{2\sqrt{x_{10}}} - \frac{x_{30}}{4\sqrt{x_{10}^3}}\right\}\delta x_1 + \frac{1}{2\sqrt{x_{10}}}\delta x_3\right] + \frac{u_1+w_1}{S}$$
$$+ S_{13}C_1\left\{-\frac{1}{8\sqrt{x_{10}^3}} - \frac{3x_{30}}{8\sqrt{x_{10}^5}}\right\}\delta x_1^2 - S_{13}C_1\frac{1}{4\sqrt{x_{10}^3}}\delta x_1 \delta x_3 \quad (12)$$

which is 2nd order nonlinear perturbation equivalent of $\dot{x}_1$ in (3) about x_{10}. Similarly, following similar steps for $\dot{x}_2$ and $\dot{x}_3$ in (3), we get:

$$\delta\dot{x}_2 = S_{32}C_3\left\{\frac{1}{2\sqrt{x_{30}}}+\frac{x_{20}}{4\sqrt{x_{30}^3}}\right\}\delta x_3 - \left\{\frac{S_{32}C_3}{2\sqrt{x_{30}}}+\frac{S_{02}C_2}{2\sqrt{x_{20}}}\right\}\delta x_2 + \frac{S_{20}C_2}{8\sqrt{x_{20}^3}}\delta x_2^2 + \left\{-\frac{S_{32}C_3}{8\sqrt{x_{30}^3}}+\frac{3\,S_{32}C_3x_{30}}{16\sqrt{x_{30}^5}}\right\}\delta x_3^2 - \frac{S_{32}C_3}{8\sqrt{x_{20}^3}}\delta x_2\delta x_3 + \frac{u_2+w_2}{S} \tag{13}$$

$$\delta\dot{x}_3 = S_{13}C_1\left[\left\{\frac{1}{2\sqrt{x_{10}}}+\frac{x_{30}}{4\sqrt{x_{10}^3}}\right\}\delta x_1 - \frac{1}{2\sqrt{x_{10}}}\delta x_3\right] + S_{32}C_3\left\{\frac{1}{2\sqrt{x_{30}}}+\frac{x_{20}}{4\sqrt{x_{30}^3}}\right\}\delta x_3 - \frac{S_{32}C_3}{2\sqrt{x_{30}}}\delta x_2 - S_{13}C_1\left\{\frac{1}{8\sqrt{x_{10}^3}}+\frac{3x_{30}}{8\sqrt{x_{10}^5}}\right\}\delta x_1^2 + \frac{1}{4\sqrt{x_{10}^3}}\delta x_1\delta x_3 + \left\{\frac{S_{32}C_3}{8\sqrt{x_{10}^3}}+\frac{3\,S_{32}C_3x_{20}}{8\sqrt{x_{10}^5}}\right\}\delta x_3^2 + \frac{S_{32}C_3}{2\sqrt{x_{10}^3}}\delta x_2\,\delta x_3 \tag{14}$$

The perturbation error model given by (12) ~ (14) is used to develop polytopic controller in next section.

3.2 Polytopic form of 2nd degree Perturbation (Error) Model

The polytope Ω is determined by the convex hull of the set of matrix vertices i.e. $\Omega = \mathrm{Co}\{[\mathrm{A_i\ B_i}]\}$. The nonlinear perturbation (error) model satisfies the sufficient conditions of polytopic form (Boyd et al, 1994). The state space representation of the linearized model represented by (12) ~ (14) can be written as:

$$\Delta\dot{x}(t) = A(\Delta x)\Delta x + B(\Delta x)\Delta u \tag{15}$$

where system matrices *A, B* are defined by (5), (6) and $\Delta x, \Delta \mathrm{u}$ are given as under:

$\Delta x = [\delta x_1 \;\; \delta x_2 \;\; \delta x_3]^T$ and $\Delta u = [u_1 \;\; u_2]$

Defining the following functions:

$$\psi(t,x) = [\psi_1(x) \;\; \psi_2(x) \;\; \psi_3(x)]^T = [\delta x_1 \;\; \delta x_2 \;\; \delta x_3]^T \tag{16}$$

the matrices *A,B* can be written as (5),(6):

$$A(t,x) = A_0 + \textstyle\sum_{i=1}^{3}\Delta A_i\psi_i(t,x)$$

$$A(t,x) = A_0 + \Delta A_1\psi_1(x) + \Delta A_2\psi_2(x) + \Delta A_3\psi_3(x) \tag{17}$$

$$B(t,x) = B_0 + \textstyle\sum_{i=1}^{3}\Delta B_i\psi_i(t,x)$$

$$B(t,x) = B_0 + \Delta B_1\psi_1(x) + \Delta B_2\psi_2(x) + \Delta B_3\psi_3(x) \tag{18}$$

Comparing (12)-(14) with (17)-(18) the following matrices are obtained.

$$A_0 = \begin{bmatrix} -\frac{S_{13}C_1}{2\sqrt{x_{10}}}-\frac{S_{13}C_1x_{30}}{4\sqrt{x_{10}^3}} & 0 & \frac{S_{13}C_1}{2\sqrt{x_{10}}} \\ 0 & \frac{S_{32}C_3}{2\sqrt{x_{30}}}+\frac{S_{20}C_2}{2\sqrt{x_{20}}} & \frac{S_{32}C_3}{2\sqrt{x_{30}}}+\frac{S_{32}C_3x_{20}}{4\sqrt{x_{30}^3}} \\ \frac{S_{13}C_1}{2\sqrt{x_{10}}}+\frac{S_{13}C_1x_{30}}{8\sqrt{x_{10}^3}} & -\frac{S_{32}C_3}{2\sqrt{x_{30}}} & \frac{S_{32}C_3}{2\sqrt{x_{30}}}+\frac{S_{32}C_3x_{20}}{4\sqrt{x_{30}^3}}-\frac{S_{13}C_1}{2\sqrt{x_{10}}} \end{bmatrix}$$

$$\Delta A_1 = \begin{bmatrix} -\frac{S_{13}C_1}{8\sqrt{x_{10}^3}}-\frac{3S_{13}C_1x_{30}}{8\sqrt{x_{10}^5}} & 0 & -\frac{S_{13}C_1}{4\sqrt{x_{10}^3}} \\ 0 & 0 & 0 \\ -\frac{S_{13}C_1}{8\sqrt{x_{10}^3}}-\frac{3S_{13}C_1x_{30}}{8\sqrt{x_{10}^5}} & 0 & \frac{S_{13}C_1}{4\sqrt{x_{10}^3}} \end{bmatrix}$$

$$\Delta A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{S_{20}C_2}{8\sqrt{x_{20}^3}} & -\frac{S_{32}C_3}{8\sqrt{x_{30}^3}} \\ 0 & 0 & \frac{S_{32}C_3}{8\sqrt{x_{10}^3}} \end{bmatrix}$$

$$\Delta A_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{S_{32}C_3}{8\sqrt{x_{30}^3}}+\frac{3\,S_{32}C_3x_{30}}{16\sqrt{x_{30}^5}} \\ 0 & 0 & \frac{S_{32}C_3}{8\sqrt{x_{10}^3}}+\frac{3\,S_{32}C_3x_{20}}{8\sqrt{x_{10}^5}} \end{bmatrix}$$

$$B_0 = B = \begin{bmatrix} \frac{1}{S} & 0 \\ 0 & \frac{1}{S} \\ 0 & 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{19}$$

and $\Delta B_i = 0, i = 1,2,3$. Equations (17) - (19) represent the polytopic form of the 2nd degree model of three tank system. For a small $\xi > 0$, the constants a_i and b_i can be explicitly chosen such that $a_i \le \psi_i(t,x) \le b_i$ whenever $\|C_i x\| \le \xi$ for $i = 1,2,3$. The bounds a_i and b_i are obtained from the physical conditions of the systems. Equations (7), (8) and (11) specify the polytopic form of the perturbation model. The linear state feedback controllers K_i ($\Delta u_i = K_i\Delta x_i$) can be obtained as a solution to system of LMI's represented by equations (7), (8) and (11).

This polytopic approach allows reasonable variation in plant states around each operating point. These variations or uncertainties $\psi_i(t,x)$ depend on the states of the system and has local bounds $a_i \le \psi_i(t,x) \le b_i$. These bounds double the vertices of a polytope represented by the set of LMI's represented by (7), (8) and (11).

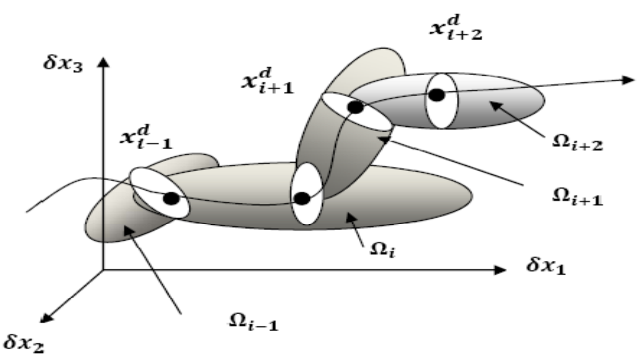

Figure 2. Working principle polytopic controller design for the three tank system.

4. SIMULATION RESULTS

The state feedback controller is designed for the 2nd degree perturbation model as a solution set of LMI's (7), (8) and (11) and is evaluated for the nonlinear model of three tank systems. The simulation parameters used for the performance analysis of designed controller are given in Table 1. A hierarchical case study for the simulations is considered i.e. operating conditions with system states $x_1 > x_2 > x_3$ (level in tank-1 is higher than tank-2 which is higher than tank-3). The simulation results for tracking are given in Figure 3. These results are compared with LMI based H_∞ controller (Iqbal et al, 2008) under similar operating conditions of Table 1 shown in Figure 4. It is clear that the settling time for the case of polytopic controller is much better than the corresponding LMI based H_∞ controller using multi-objective approach. Figure 5 and Figure 6, shows the performance comparison for the case of input disturbance and plant parametric variations (uncertainties) along with control effort. The detailed analysis shows that the disturbance rejection of polytopic controller in the presence of parametric uncertainties is reasonably good.

Different controllers were designed for different number of polytopic regions from starting point to the desired points. It is noted that, number of polytopic regions can affect the convergence of the solution to set of LMI's. With larger polytopic regions (less number of intermediate points), the solution of set of LMI's might not converge (i.e., controller will not be able to drive the system to desired point), for such case smaller step sizes in that polytopic region can be chosen so that feasible solution to set of LMI's for that point shall converge. The convergence time can be adjusted by allowing reasonable variations in control inputs, but within acceptable limits. This can be managed by allowing norm of the controllers to smaller or larger values. This gives flexibility in the controller design to achieve specific performance parameters. The associated computational issues using LMI toolbox in MATLAB are discussed in (Soren et al, 2005).

5. CONCLUSIONS

A polytopic system based state feedback controller is designed for the 2nd degree perturbation model and evaluated on the nonlinear model of the three tank system. The designed controller drives the states from starting point to the desired state through a series of connected polytopic regions. A set of LMI's are formulated for each polytopic region and solved to obtain the corresponding state feedback controller gains for that particular polytopic region. The feasibility of each LMI system written for each polytope vertex implies the existence of a stabilizing quadratic Lyapunov function for each 2nd degree perturbation model. The approach amounts to a gain scheduled implementation of the controllers. The controller stabilizes the system about each operating point as none of the 2nd degree perturbation systems is locally stable about any of the operating points. Also, the controller is robust with respect to parameter variations due to the combined polytopic-LMI problem formulation and solution. The effects of parametric variations and disturbances are accounted for in such a formulation via appropriate bounds. This work can be extended by implementing the designed controller on the actual three tank system, along with LMI based observer for the sensor and actuator fault diagnosis.

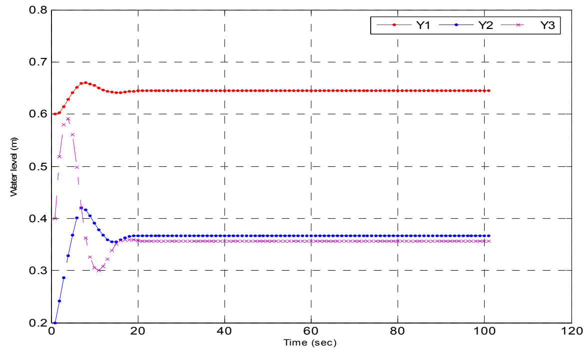

Figure 3. Performance of Polytopic Controller for regulation. The desired water levels to be maintained are (x1=0.65, x2= 0.4)

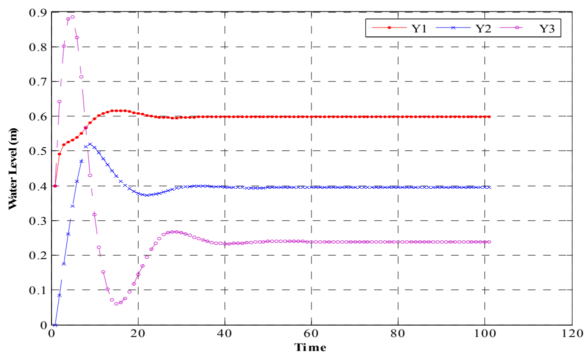

Figure 4. Performance of LMI based H_∞Controller for regulation. The desired water levels to be maintained are (x1=0.60, x2= 0.4)

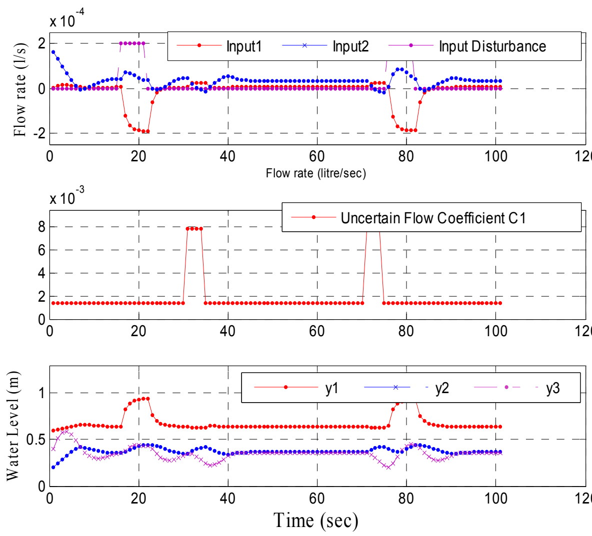

Figure 5. Disturbance rejection in presence of parametric uncertainty for polytopic controller [uncertainty: 20% in c1 (flow coefficient), disturbance: 10 times nominal (steady state) value of control input u1]. The desired water levels to be maintained are (x1=0.65, x2= 0.4)

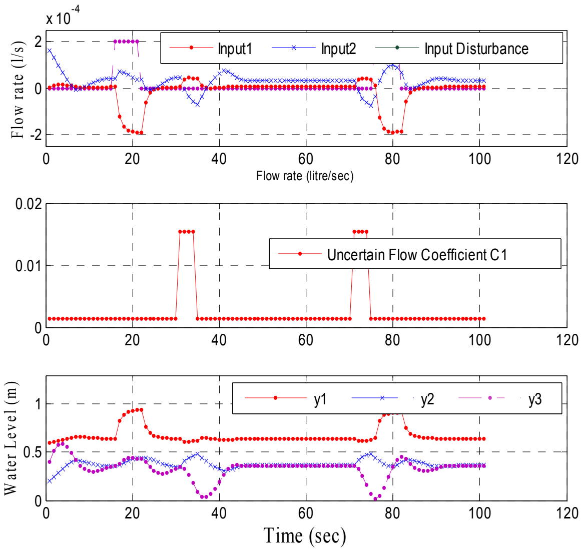

Figure 6. Disturbance rejection in presence of parametric uncertainty for H_∞ Controller [Uncertainty: 50% in C_1 (flow coefficient), Disturbance: 10 times nominal (steady state) value of control input u_1]. The desired water levels to be maintained are (x1=0.65, x2= 0.4)

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