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Fractional II$^{\alpha}$ controller combined with a Smith predictor for effective water distribution in a main irrigation canal pool

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Abstract: In this paper, a new fractional order II$^{\alpha}$ controller combined with a Smith predictor for effective water distribution in a main irrigation canal pool is designed. A new tuning method, based on frequency techniques, is proposed, providing to the controlled system the same nominal behavior than a conventional PI controller with Smith Predictor and more robustness to variations in the system gain.

1. INTRODUCTION

Water is becoming a precious and very scarce resource in many countries due to the increase of industrial and agricultural demands, as well as of the population. Irrigation is the main water consuming activity in the world, as it represent about the 80 % of the available fresh water consumption. There is a growing interest for the application of advanced management methods that prevent wastage and facilitate the efficient use of this vital resource (Clemmens and Schuurmans, 2004; Litrico, 2001).

It is widely accepted that automation could lead to a better efficiency of water management in many irrigation systems with open canals, which are subject to large losses. The used of specific control method depends of canal construction properties (including slope, structures, and storage volumes), the rules for delivery service, the type of water delivery, the availability of communication between the control center and automatic structures, the expectation of water users and operation staff, and economic considerations (Kovalenko, 1983). Automatic control of water distribution in main irrigation canals can be justified by improved service to clients, improved efficiency in water distribution, reduced overall operation costs, considerable decreased in water losses and increased safety exploitation (Malatere, 1998).

Designing a control strategy leading to a practical and effective controller of water distribution is an arduous task because the hydraulic behavior of irrigation canals shows that these systems are distributed over long distances, with dynamics characterized by important varying time delays, strong nonlinearities, numerous interactions between different consecutive sub-systems and the existence of others dynamic parameters that change over time during operation (Malaterre, 1998). Thus, the whole irrigation canal has to be regarded as a system with complex dynamical behavior (Litrico, 2001).

The main objective of the control strategies is to satisfy, in spite of uncertainties, the water demand of each consumer while guaranteeing a minimum discharge all along the canal and spending a minimum water volume from the upstream reservoir (Litrico, 2001). A usual solution to reach these control objectives has been the use of conventional PI controllers (Litrico X., Fromion V. Baume J.P. and Rijo M., 2003; Baume J.P., Malaterre P.O. and Sau J., 1999). Nevertheless, many studies have shown that these classical regulators seem to be unsuitable to solve the problem of effective control of water distribution in main irrigation canals due to the difficult dynamical behavior that characterizes these processes (Malaterre, Rogers and Schuurmans, 1998; Montazar, Overloop and Brouwer, 2005; Rivas et al. 2002; Wahlin, 2004).

Other control schemes widely proposed to modify the dynamics of system characterized by large time delays are based on the Smith predictor, which simplifies the closed loop transfer function of the system, removing the time delay of the denominator and provides robustness to system parameters changes (Feliu, V., Rivas R. and Castillo F.J. 2005).Recently fractional operators have been applied by different authors, e.g. Podlubny, 1999, to model and control difficult dynamical behavior processes. An interesting feature of fractional-order controllers is that they exhibit some advantages when designing robust control systems in the frequency domain for processes whose parameters vary in a large range (Feliu, Rivas, Sanchez, 2007; Feliu, Rivas and Castillo, 2005). In this paper these characteristics are explored in order to design robust controllers to solve the problem of effective water distribution control in a main irrigation canal pool. In particular this paper is focused on the design of a fractional integral-integral$^{\alpha}$ controller (FII$^{\alpha}$) for a main irrigation canal pool.

This paper is organized as follows. A model for the main irrigation canal pool to be controlled is proposed in Section II. Section III develops the method for designing the FII$^{\alpha}$ controller. Section IV compares the designed controller with other standard ones. Finally some conclusions are drawn in Section V.

2. MAIN IRRIGATION CANAL POOL MODEL FOR CONTROL

An irrigation main canal is an open hydraulic system, whose objective is mainly to convey water from its source down to its final users (farmers). This system is integrated by several pools separated by cross structures (mainly hydraulic gates), which are operated for regulating the water levels (flows), discharges and/or volumes from one pool to the next one. Fig. 1 shows a schematic representation of an open irrigation main canal.

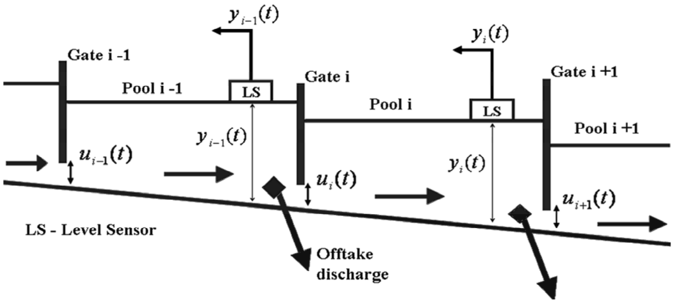

Fig. 1. Scheme of an open irrigation main canal with gates.

Physical dynamics of an open canal have traditionally been modeled by the Saint-Venant equations, which are nonlinear hyperbolic partial differential equations (a distributed parameters model). These equations are derived from mass and momentum balances and are given by (Chaudhry, 1993):

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q; \quad \frac{\partial Q}{\partial t} + \frac{\partial Q^2 / A}{\partial x} + gA\frac{\partial y}{\partial x} = -gAS_f, \qquad (1)$$

where A, the canal cross-section area; Q, the discharge (flow) across section A; q, the lateral discharge (inflow or outflow); x, the longitudinal abscissa in the direction of the flow; y, the water surface absolute elevation; g, the gravitational acceleration; S_f, the friction slope; t, the time variable.

Nowadays different methods exist for the solution of Saint-Venant's equations, all of them exhibiting large mathematical complexities. These equations are also very difficult to use for prediction and control. Often, an equivalent first order systems plus a delay are used to model the canal dynamic behavior (Rivas Perez et al., 2003; Weyer, 2001).

The irrigation canal considered in this paper is the Guira de Melena Main Irrigation Canal (GMMIC), in Havana County, Cuba. It is a cross structure canal of 60 km long with design head discharge of 15 m^3/s and normal water depth varying from 2.0 m to 1.5 m. This canal has a trapezoidal cross section, the bed width varies from 5 m to 3 m, and the average slope is 0.15 m per km. It has seven pools of different lengths separated by undershot gated cross structures. Water is delivered to the secondary network through 12 gated offtakes structures. The irrigated area of GMMIC is about 10 000 hectares. This canal is mainly operated in a downstream end control mode with data communication via a radio network.

The last three pools of GMMIC supply water to three big reservoirs (Guira 1-Guira 3) of 20 000 cubic meters (supplier reservoirs). These reservoirs provide water to other five reservoirs (R1-R5) of smaller capacity (5 000 cubic meters). The reservoirs of smaller capacity are those that supply water to the cultivated areas. A schematic representation of these canal pools is shown in Fig. 2.

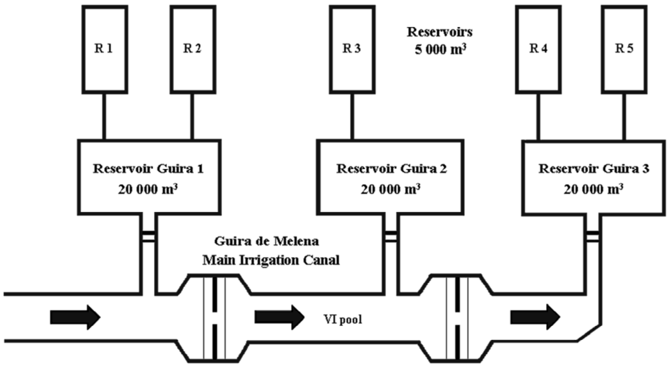

Fig. 2. Schematic representation of the last pools of Guira de Melena Main Canal.

The water level sensors are installed within offline stilling wells at the downstream end of the pool. These sensors are of float and counter-weight type, attached to a stainless steel tape which runs over a sprocket wheel. The wheel movements are transferred to a potentiometer that transmits the analogical inputs corresponding to the water surface to a PLC.

The offtakes are located at the downstream end of each pool, approximately at 7 m upstream of the next gate. These offtakes can withdraw water from the main canal to supply their corresponding users. These flows represent the main external perturbation ($q_i(t)$) acting on each canal pool.

Considering that the downstream end water level is the controlled variable and the gate position the manipulated variable, the mathematical model for control will consider the water level as its output and the gate position as its input.
The data and results reported in this paper are from the VI pool of GMMIC, which is one of the last pools of this canal and whose length is 10 km (see Fig. 2). This canal pool is operated by means of the downstream end water level regulation method (Kovalenko, 1983; Malaterre, Rogers and Schuurmans, 1998). The available measurements are the downstream end water levels ($y(t)$) and the gate position ($u(t)$) as sketched in Fig. 1.

For water distribution control in main irrigation canal pools it is not necessary to know the water level variations along the whole pool, but only at specific points which depend on the canal operation method that is being used (Kovalenko, 1983). Considering this, a linear model with concentrated parameters and a time delay can adequately characterize the dynamical behaviour of an irrigation canal pool at specific points (Clemmens and Schuurmans, 2004; Rivas Perez, 1990).

Experiments based on the response to a step like input were carried out in the VI canal pool of GMMIC in order to obtain a mathematical model that describes its dynamic behavior. For step test the downstream gate was kept in a fixed position, the upstream gate was stimulated with a step signal and the downstream end water level was measured with a level sensor. The experimental response of this canal pool to a step command is exhibited in Fig. 3.

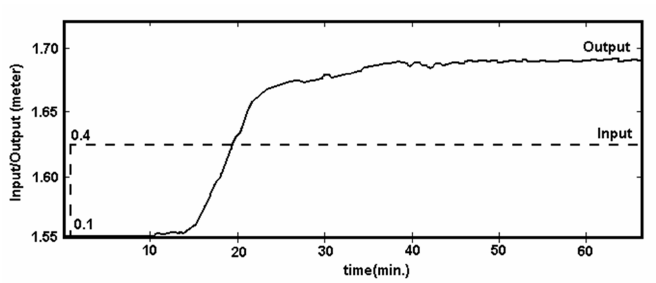

Fig. 3. Step test of the VI pool of GMMIC.

Such response shows that the dynamic behavior of a single canal pool can be represented by expression:

$$G(s) = \frac{\Delta y_1(s)}{\Delta u_1(s)} = \frac{K}{(T_1 s + 1)(T_2 s + 1)} e^{-\tau s} \qquad (2)$$

where K is the static gain; T_1, T_2 are time constants; τ is the time delay. We consider that T_1 is the dominant time constant, associated to the dynamics of the canal pool, while T_2 is the smaller time constant that represents the motors + gates dynamics, which is much faster than the canal pool dynamics.

Experiments reported in previous works (Rivas, Feliu and Sanchez, 2007) on identification of this canal pool showed that all dynamic parameters of mathematical model (2) may exhibit wide and non predictable variations when the discharge regimes change across the gates in operation range $(Q_{\min}, Q_{max})$. For this reason any controller that is designed for this canal pool should be robust in front of the dynamic parameters variations. An ARMAX model structure was used to identify the system. The resulting nominal parameters of the system were the following: K_p=0.466, T_1=550 s., T_2=46.4 s. and τ=600 s.

3. CONTROL DESIGN

3.1 Smith Predictor Scheme.

Fig. 4, represents the usual control scheme for any linear system.

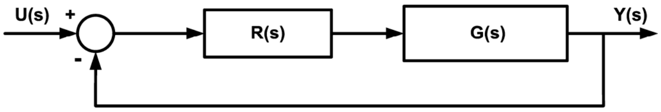

Fig. 4. Conventional control scheme.

With this scheme, in systems with time delay of the form $G(s)=G'(s)\cdot e^{-\tau s}$, the closed loop transfer function denominator also presents a time delay term, becoming difficult the control task.

As it was explained in the introduction section, a control scheme that improves the behavior of systems with time delay, as irrigation canals, is the Smith predictor (SP), which has been widely combined with conventional controllers (Owens D.H., Raya A., 1982; Marquez, Fliess and Mounier, 2001; Kaya I., 2003).This control scheme is shown in Fig. 5.

Supposing that the SP is perfectly tuned, the equivalent closed loop transfer function of the system is:

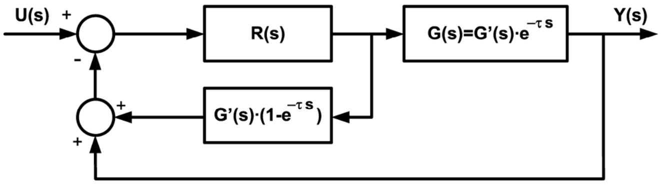

Fig. 5. Control scheme of a Smith predictor.

$$M(s) = \frac{R(s)G'(s)}{1 + R(s)G'(s)} e^{-\tau s} \qquad (3)$$

where the time delay of the denominator of the closed loop transfer function has been removed.

In the present work the *R(s)* controller is tuned using frequency specifications, that is, gain crossover frequency, ω_c (rapidity of the system) and phase margin, φ_m (overshoot).

Subsequently, the tuning process is described in detail for both, a conventional PI controller + SP and the new FII$^{\alpha}$ controller + SP, proposed in this article.

3.2 PI + SP tuning.

Let it be the usual PI controller with the following transfer function, $R_1(s)$:

$$R_1(s) = K_{p1}\left(1 + \frac{1}{T_{i1} s}\right) \qquad (4)$$

The gain crossover frequency, ω_c, can be expressed in terms of the settling time, t_s, and the time delay, τ, as follows:

$$\omega_c \approx 3/(t_s - \tau) \qquad (5)$$

The obtained results verify that this hypothesis of design is correct.

Assuming a perfect tuning of the SP, the equivalent open loop transfer function we use to obtain the tuning rules for the controllers is:

$$H(s) = R(s)G'(s) \tag{6}$$

Where the equations that govern the frequency behavior are:

$$\left|H(j\omega_c)\right| = 1\ ;\ \angle\left\{H(j\omega_c)\right\} = \pi + \varphi_m \tag{7}$$

Dividing $G'(s)$ in its real and complex parts:

$$G'(j\omega) = \chi + j\delta \tag{8}$$

And operating in (7), next expressions are obtained:

$$\left|\left(\chi\omega_c + \frac{\delta}{T_{i1}}\right) + j\left(\delta\omega_c - \frac{\chi}{T_{i1}}\right)\right| = \frac{\omega_c}{K_{p1}} \tag{9}$$

$$\frac{\delta\omega_c - \dfrac{\chi}{T_{i1}}}{\chi\omega_c + \dfrac{\delta}{T_{i1}}} = \tan^{-1}(\varphi_m) \tag{10}$$

Rearranging terms in (9) and (10), the equations that determine the controller parameters, K_{p1} y T_{i1}, turn out to be:

$$T_{i1} = \frac{\chi + \delta\tan^{-1}(\varphi_m)}{\omega_c\left[\delta - \chi\tan^{-1}(\varphi_m)\right]} \tag{11}$$

$$K_{p1} = \frac{1}{\sqrt{\left(\chi + \dfrac{\delta}{T_{i1}\omega_c}\right)^2 + \left(\delta - \dfrac{\chi}{T_{i1}\omega_c}\right)^2}} \tag{12}$$

In this manner, controller parameters can be calculated as functions of the system parameters and the frequency specifications.

3.3 FII^{α} + SP tuning.

In previous works, a FPI^{α} controller + SP was used (Feliu, Rivas and Castillo, 2005) with the next transfer function, $R_2'(s)$:

$$R_2'(s) = K_{p2}'\left(1 + \frac{1}{T_{i2}'s^{\alpha}}\right) \tag{13}$$

The use of this controller presents advantages in robustness when the system parameters vary, nevertheless, due to the non integer integral action of the controller, the steady state error tends slowly to zero.

With the object to avoid this undesired effect, the present paper proposes a FII^{α} controller $R_2(s)$, with an integer and a fractional integral action:

$$R_2(s) = K_{p2}\left(\frac{T_{d2}s^{\alpha} + 1}{s}\right) \tag{14}$$

This controller has no proportional, if $\alpha \neq 1$, or derivative action and, once fixed α parameter, it depends of the same number of parameters that a PI controller. In the case of $\alpha = 1$, $R_2(s)$ becomes to (4), the case of the PI controller, with $K_{p1} = K_{p2}/T_{d2}$ and $T_{d1} = T_{d2}$.

Using the same tuning method of the PI one, and applying equation (14) over the equations that establish the frequency behaviour (7), we can obtain the next expressions:

$$\left|\left[\left(T_{d2}\omega_c^{\alpha}\sin\left(\alpha\frac{\pi}{2}\right)\right) - j\left(T_{d2}\omega_c^{\alpha}\cos\left(\alpha\frac{\pi}{2}\right) + 1\right)\right]\cdot\left[\chi + j\delta\right]\right| = \frac{\omega_c}{K_{p2}} \tag{15}$$

$$\frac{\delta T_{d2}\omega_c^{\alpha}\sin\left(\alpha\frac{\pi}{2}\right) - \chi T_{d2}\omega_c^{\alpha}\cos\left(\alpha\frac{\pi}{2}\right) - \chi}{\chi T_{d2}\omega_c^{\alpha}\sin\left(\alpha\frac{\pi}{2}\right) + \delta T_{d2}\omega_c^{\alpha}\cos\left(\alpha\frac{\pi}{2}\right) + \delta} = \tan^{-1}(\varphi_m) \tag{16}$$

From the equations (15) and (16), it is easy to determine the FII^{α} controller parameters:

$$T_{d2} = \frac{\chi + \delta\tan^{-1}(\varphi_m)}{\omega_c^{\alpha}\left[\sin\left(\alpha\frac{\pi}{2}\right)\left(\delta - \chi\tan^{-1}(\varphi_m)\right) - \cos\left(\alpha\frac{\pi}{2}\right)\left(\chi + \delta\tan^{-1}(\varphi_m)\right)\right]} \tag{17}$$

$$K_{p2} = \frac{\omega}{\sqrt{\xi^2 + \sigma^2}} \tag{18}$$

Where

$$\xi = \chi T_{d2}\omega_c^{\alpha}\sin\left(\alpha\frac{\pi}{2}\right) + \delta\left(T_{d2}\omega_c^{\alpha}\cos\left(\alpha\frac{\pi}{2}\right) + 1\right) \tag{19}$$

$$\sigma = \delta T_{d2}\omega_c^{\alpha}\sin\left(\alpha\frac{\pi}{2}\right) - \chi\left(T_{d2}\omega_c^{\alpha}\cos\left(\alpha\frac{\pi}{2}\right) + 1\right) \tag{20}$$

In this manner, controller parameters can be calculated as functions of the system parameters and the frequency specifications.

3.4 FII^{α} controller implementation.

The derivative fractional operator has been implemented using the numerical approximation of Grunwald-Letnikov, successfully applied in previous works (Feliu, Rivas and Castillo, 2005). The approximation used is:

$$D_t^{\alpha} f(t) = \frac{1}{h^{\alpha}}\sum_{k=0}^{n}\omega_k^{\alpha} f(t - hk) \tag{21}$$

where,

$$\omega_k^{\alpha} = (-1)^k \binom{\alpha}{k} \quad (22)$$

and n is the truncation order of the numerical approximation. (fixed to 1000).

4. COMPARISON OF CONTROLLERS

The response of the open loop system exhibits a settling time of approximately 2500 s. As an example of controller tuning, a settling time of 1000 s. has been set for the closed loop system, becoming 2.5 times faster. By means of (5) the crossover frequency is set to ω_c = 0.0075 rad/s. On the other hand, a standard phase margin has been chosen, φ_m = 60º, allowing approximately an overshoot of 25%.

The transfer function of the PI controller is determined by (11) and (12):

$$R_1(s) = 6.761\left(1 + \frac{1}{131.093\,s}\right) \quad (23)$$

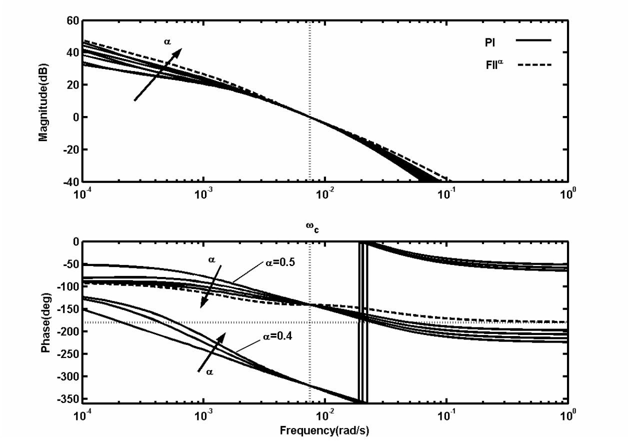

Fig. 6. Bode diagram: PI + SP vs. FII$^{\alpha}$ + SP (α parameter variation).

The FII$^{\alpha}$ controller tuning also depends of the selection of α parameter. In order to compare the results and to choose the best value of α parameter, Fig. 6 displays the Bode diagram of $H(s)$ for PI and FII$^{\alpha}$ controllers, varying α. Fig. 6 shows that the frequency response fits to ω_c and φ_m for PI controller and FII$^{\alpha}$ controller in a range of values of $\alpha \geq \alpha_c$. If $\alpha < \alpha_c$ FII$^{\alpha}$ controller becomes unstable (φ_m<0). In addition, when α parameter decreases, the phase crossover frequency, ω_l, decreases too, increasing the gain margin. Therefore the value of α must be chosen to minimize the phase crossover frequency, ω_l. Fig. 7 represents the variation of ω_l in the range of $\alpha \in [0,1]$. Within the stability zone, the value of α parameter that provides the minimum value of ω_l matches with the results obtained by means of the Bode diagram, i.e. $\alpha = \alpha_c = 0.495$.

Then, the controlled system must present the same temporal response in nominal conditions and an increment in the robustness to changes in the system gain.

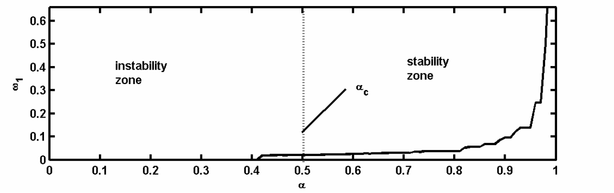

Fig. 7. Variation of ω_l ($\alpha \in [0,1]$).

The new FII$^{\alpha}$ controller tuned using (17) and (18) has the following transfer function:

$$R_2(s) = 3.292 \cdot 10^{-5}\left(\frac{1.281 \cdot 10^4 s^{0.495} + 1}{s}\right) \quad (24)$$

Fig. 8 shows the temporal response of both controllers, under nominal conditions, presenting the same settling time and overshoot.

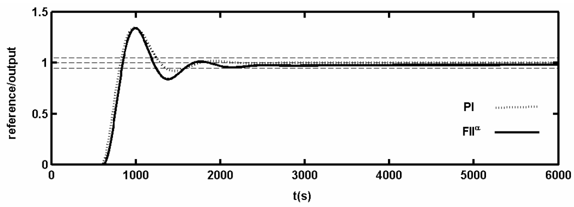

Fig. 8. Temporal response: PI + SP vs. FII$^{\alpha}$+SP (nominal case)

With the object to compare the robustness of the two control schemes proposed, Fig. 9 and 10 display the temporal response of both, PI+ SP and FII$^{\alpha}$ + SP schemes, respectively, when the system gain varies. The new FII$^{\alpha}$+SP controller provides an increment of a 25% in the robustness of the system with respect to the PI+SP scheme.

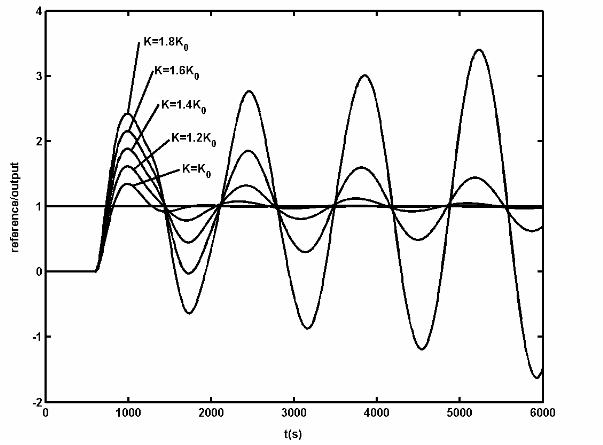

Fig. 9. Temporal response of PI + SP (system gain variation)

5. DISCUSSION AND CONCLUSIONS

The present article has proposed a new fractional II^{α} controller without proportional action combined with a Smith predictor.

With the new tuning method presented, the deduction of the controller parameters becomes straightforward once we have chosen a value for parameter α, which is selected in order to provide the maximum robustness to changes in the system gain.

In comparison with the use of a PI controller combined with a Smith predictor, the same temporal behavior is provided under nominal conditions (same settling time and same overshoot), increasing the robustness to system gain variations. In our case of study, the VI canal pool of GMMIC, the robustness to changes in the system gain has been increased a 25% with respect to the PI+SP scheme.

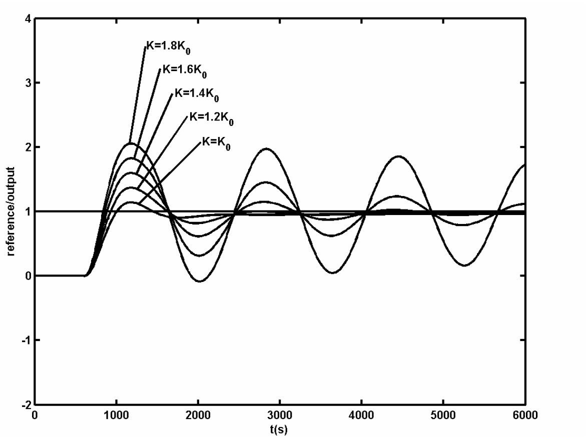

Fig. 10. Temporal response of FII^{α} + SP (system gain variation)

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