Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Modelling Dynamic Urban Road Networks Performance under Congestion Pricing Strategies

Loukas Dimitriou*and Antony Stathopoulos**

Laboratory of Railways and Transport, School of Civil Engineering, National Technical University of Athens , 5, Iroon Polytechniou Str, 15773(Tel: ++30-210-7721370; e-mail:lucdimit@central.ntua.gr).
*** Laboratory of Railways and Transport, School of Civil Engineering, National Technical University of Athens, 5, Iroon Polytechniou Str, 15773(Tel: ++30-210-7721288; e-mail:a.stath@transport.ntua.gr).*

Abstract: The development of modern information systems able to process real-time data concerning the traffic state network-wide is making possible the implementation of new forms of strategies concerning urban transportation networks management. The success of the implementation of such strategies is heavily depended on the detailed design of them. In the current study, an evaluation framework of the application of a congestion pricing strategy based on the marginal travel cost is presented, able to identify the dynamic impact of congestion pricing on the performance of the system, by taking into consideration behavioral characteristics of multiple user classes. This framework builds on tactics of variable road congestion pricing, based on theories of stochastic optimal control.

1. INTRODUCTION

The advent of new technologies in information systems is giving rise to the implementation of new forms of strategies concerning urban transportation networks management. The social awareness concerning the negative impacts of the rapidly increasing mobility on the performance of economic activities, has been adding a pressure for the introduction of effective strategies for the management of urban transportation infrastructure. The success of the implementation of such strategies is heavily depended on the detailed design of them, a complex task by itself since urban transportation networks lie on the cutting edge of interrelations among alternative social groups, each having their own objectives.

Congestion pricing of urban road networks has been identified as an effective tool for the management of the increasing demand for mobility in congested parts of metropolitan areas. Although the fundamental theories of road network congestion pricing have been proposed for several decades by the seminal works of Pigou (1920) and Vickrey (1969), it was until recently that these have been applied in metropolitan areas, providing encouraging results on the effective organization of urban mobility demand.

In the current study, an evaluation framework for the application of a congestion pricing strategy based on the marginal travel cost is studied, able to identify the dynamic impact of congestion pricing on the performance of the system, by taking into consideration behavioural characteristics of multiple user classes. This framework builds on tactics of variable road congestion pricing, based on theories of stochastic optimal control. In the next section a brief background of the models used for evaluating the system performance under congestion pricing schemes will be presented. Next follows the architecture of the proposed model with a detailed description of its underlying parts. The fundamental characteristics of the proposed framework will be analyzed by applying it to a hypothetical network able to provide useful insights concerning its features, while results of the model application to a realistic network representing a part of Athens, Greece, Central Business Area will be presented, and finally the last section concludes.

2. BACKGROUND

The main scope of congestion pricing strategies aims on one to the charge of the congested road infrastructure use (by treating congested transportation infrastructure as a public good on scarcity) and on the other to alter the behaviour of network users that leads to the generation of congestion phenomena by internalizing the cost of congestion on the generalized travel cost. Various economic theories and concepts on road pricing and specifically on congestion pricing have been proposed over the years. One of the most widely adopted strategies of congestion pricing relies on the concept of the Marginal Cost Pricing (MCP) initially studied by Walters (1961). Under this assumption, road users should by charged as much as the cost of the '*marginal*' user, i.e. the cost that the last user (or an additional user) imposes on the others. Such pricing strategy provides the framework for treating the transportation infrastructure in an economic efficient (in terms of public economics) yet social equitable manner, increasing the acceptability of the introduction of such management policies.

Many models have been presented for estimating the impacts of MCP congestion charging policies on the transportation systems. Alternative models of the static case, examining the performance of large systems at the peak hour, have been extensively studied (Dafermos and Sparrow, 1971) considering the influence on both the total demand level and at the route choice process (Yang *et al.* 2004) while identifying multiple users classes and stochastic route choice process (Zhao and Kockelman 2006).

Nevertheless static models are inadequate to capture the system responses to transient phenomena and thus the dynamic influence of MCP on the system performance (Henderson, 1974). Models of the dynamic form of the problem of system performance under MCP have been proposed based on the optimal control theory (Carey and Srinivasan 1993, Huang and Yang 1996, Wie and Tobin 1998). Sandholm (2002) provided an evolutionary game-theoretic mechanism for applying congestion pricing while results demonstrating that a system with users following a myopic adjustment process (in terms of identifying optimal paths and incorporating real-time information) that eventually converges to steady-state conditions. A model of dynamic congestion pricing aiming at the maximization of the net economic benefit, identifying disequilibrium states based on the theory of optimal control has been presented by Friesz *et al.* (2004). De Palma *et al.* (2005) provides results on the effects of time-varying congestion pricing strategies based on a dynamic network model able to incorporate such strategies and treating endogenously departure time, mode and route choices in a detailed environment of microscopic simulation.

The aforementioned models of dynamic congestion pricing rely on *'hard'* assumptions regarding the information acquisition rate of the users. Also, although dynamic models -especially those based on optimal control theory- identify and model an adaptive process related to a learning mechanism, most of them do not rely on behavioural assumptions regarding the users learning process. In the next section a network model able to endogenously treat the dynamic interaction between network users and network operator responsible for the road charging is presented. The model is based on behavioural assumptions regarding the route choice process, based on an evolutionary equilibrium-tending learning mechanism. In that model, multiple user classes are identified, distinguished by their Value of Travel Time (VOTT) and the information acquisition process using for making choices. These features of the model are accompanied by adjustment mechanisms controlling the mix of users of each class participating into the network.

3. A DYNAMIC SYSTEMS MODEL OF URBAN NETWORKS

Dynamic transportation systems, like those of urban road networks, constitute an area of recursive interrelationships, among users, management authorities and prevailing traffic conditions. The proposed model of dynamic urban road network is based on an architecture of discrete-time optimal control in cascade (Fig. 1), at which users are making choices tending to optimize their objectives (outer loop), while the authority responsible for the network management is intervening by setting congestion charges (inner loop).

The proposed dynamic system in order to identify multiple user classes and to model their dynamic trip choices, is composed of a network model able to provide traffic conditions (link-path travel times, travel cost per user class, etc) subject to network loading, four distinct sub-systems, namely the route choice, the information provision, the demand simulation and the MCP sub-system, and finally a historic information data storage feature. In particular the route choice sub-system provides estimations regarding the route choices of the multiple user classes. The information provision sub-system distinguishes users in classes according to the information that will utilize in order to make route choices. The demand simulator provides the demand pattern of each user class and time period, based on a elastic relationship between demand and travel cost. Finally the MCP sub-system provides the reaction of an authority imposing MCP to prevailing conditions. The connections among all parts of the dynamic system are made in a manner such as to replicate the actual process of the evolutionary interaction among the users and the charging authority. In the next sub-sections the description of the sub-systems will follow.

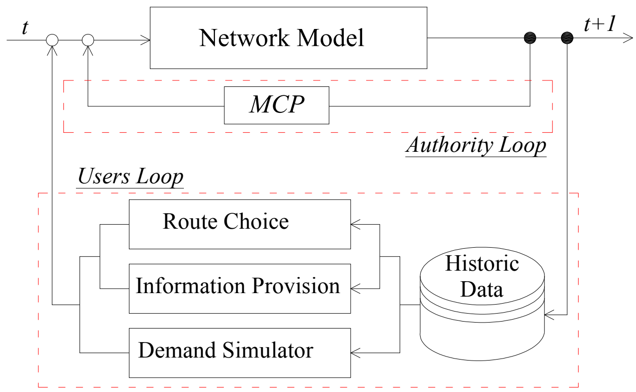

Fig. 1. The architecture of the discrete-time dynamic model of urban road network

3.1 Route choice sub-system

The route choice sub-system provides the network loading pattern. An extended perception of the classical user equilibrium (UE) assumption is adopted here, modelling the route choice as a repeated game with memory. At each repetition of the route choice *'game'*, users are evolving (adjusting) their route choices with respect to predictions (or beliefs) regarding the behaviour of all other participants in the game (rest of the network users), in order to optimize their objectives (typically the minimization of travel time or cost). Under this evolutionary game-theoretic perception of the route choice process it is possible to endogenously consider long-run relationships among the game participants (network users) identifying and taking into account the impact of the behaviour of alternative player classes. Evolutionary game theory (Smith, 1982) provides the framework to investigate the evolution of rational players behaviour into a constantly changing environment (like the conditions in urban road networks) based on their performance in previews stages of the game, forming long-run relationships (or experience, or reputation) with co-participants in the game (Mailath and Samuelson 2006).

Although the incentives of network users is the collective minimization of users belonging in class A travel cost $C_{A,p}^{ij,t}$ using path $p \in P_{ij}$ between an O-D ($i-j$) travelled by at time interval t, here users perception regarding path costs are considered as myopic, giving rise to treating users behaviour under the framework of bounded rationality. Then the route choice probability $\pi_{A,k,p}^{t}$ that path p is chosen by users of class A at interval t can be expressed as a discrete-time stochastic dynamical system of the following evolutionary logit logic:

$$\pi_{A,k,p}^{t} = \frac{\exp(-\theta^{t-k} C_{A,p}^{ij,t-k})}{\sum_{A}\sum_{p} \exp(-\theta^{t-k} C_{A,p}^{ij,t-k})} \tag{1}$$

where $k < t$ is a time interval whose path cost information is utilized by a user to choose path p at interval t and θ^{t-k} is the user's cost perception parameter at interval $t-k$.

3.2 Demand level sub-system

In order to capture users trip departure time choices and the elasticity of the demand to travel cost, the evolution of the demand is modelled as a discrete-time dynamical system of the form:

$$d_{A}^{ij,t} = D_{A}^{ij,t} \exp^{a C_{A}^{ij,t-k}} \tag{2}$$

where $d_{A}^{ij,t}$ stands for demand of users belonging to class A and wishing to move between $i-j$ pair during interval t, while $D_{A}^{ij,t}$ stands for the maximum (desired) demand for travel between $i-j$ and a is a scale parameter. This sub-system allows the redistribution of multiple user classes demand by a desired time resolution t. Nevertheless other dynamic demand simulators can be adopted in order to estimate the demand level by taking into account alternative features of trip scheduling like late/early arrival costs etc.

3.3 Information acquisition sub-system

The system performance is heavily depended by the availability and the quality of information that users utilizing in order to make choices and their switching activity (i.e. the willingness of the users to immediately respond moving to the best route). Simulation tests demonstrated that the increase of real-time reliable information penetration among the user groups, up to a certain threshold (fraction of users) would be beneficial to the system performance. On the other hand, if all users had access to the same real-time information and immediately respond to that by a self-optimizing manner, the system performance would be severely dropped (Mahmassani and Jayakrishnan 1991).

In the current study a dynamic sub-system is incorporated into the proposed model for estimating the users information acquisition process in order to make choices, by controlling the penetration of reliable real-time information into user classes. The users information sources is divided in two classes, namely those utilizing historic information of path travel cost and those that are utilizing reliable real-time information like that provided by online dynamic traffic maps (Athens Real-Time Traffic Map). The concept that is used here in order to estimate the penetration of real-time information is based on the perception that the less the users are choosing the optimal path the more users are seeking for improved travel cost information. Under this assumption users are seeking for improved information as they become inadequate to identify optimal choices (in a proportional rule), forming a dynamical process that provides a desirable real-time information acquisition level, leading towards the system-optimal penetration rate. This dynamical sub-system has the following form:

$$u_{A}^{t-1} = I \times \frac{\left(\sum_{p} x_{p}^{ij,t-1} C_{A,p}^{ij,t-1}\right) - x_{p*}^{ij,t-1} C_{A,p*}^{ij,t-1}}{\sum_{p} x_{p}^{ij,t-1} C_{A,p}^{ij,t-1}} \tag{3}$$

where u_{A}^{t-1} stands for the percentage of users of class A that are accessing reliable real-time information concerning travel cost, $x_{p*}^{ij,t-1} C_{A,p*}^{ij,t-1}$ stands for the total optimal path, $p*$,travel cost and I for the maximum real-time information penetration rate.

3.4 Dynamic congestion pricing sub-system

A fair congestion pricing strategy should respect the pay-as-you-drive principle, implying that users are charged for the part of the network (under MCP strategy) utilizing to execute a trip. Thus, the proposed MCP scheme is estimated here by imposing a time-varying charge to each link of the part of the network where the congestion pricing strategy will be implemented. In order to introduce such time-varying MCP strategies the model assumes that the part of the network that the road pricing is introduced should be under real-time surveillance in order for the authority responsible for the MCP charges to have knowledge concerning the traffic state. Such an assumption is valid since model-based tools able to provide system-wide real-time surveillance are already available (Papageorgiou *et al.* 2005).

Taking that f_{m}^{t} is the flow at link m during interval t and $c_{m}^{t}(f_{m}^{t})$ is the corresponding average link cost (AC_{m}^{t}), the total cost (TC_{m}^{t}) of that link is equal to $TC_{m}^{t} = f_{m}^{t}\, c_{m}^{t}(f_{m}^{t})$. Hence, the cost of the marginal user (one additional user entering the link) or the marginal cost (MC_{m}^{t}) (see Figure 1) is estimated as the derivative of the TC_{m}^{t} with respect to flow f_{m}^{t}, as follows:

$$MC_{m}^{t} = \left(TC_{m}^{t}\right)' = \left(f_{m}^{t}\, c_{m}^{t}(f_{m}^{t})\right)' = c_{m}^{t} + f_{m}^{t}\left(c_{m}^{t}(f_{m}^{t})\right)' = AC_{m}^{t} + \tau_{m}^{t} \tag{4}$$

where τ_m^t denotes the marginal cost of congestion. The average cost c_m^t at link m during interval t is composed of the value of travel time $VOTT_A$ of user class A and other costs g_m^t, which are irrelevant to the congestion price (for instance, flat toll fares), i.e. $c_m^t(f_m^t) = VOTT_A\ t(f_m^t) + g_m^t$, where $t(f_m^t)$ denotes the travel time at the specific link and interval. Provided that multiple user classes are identified with respect to their $VOTT$, the marginal congestion cost τ_m^t is weighted with the flow of users from different classes sharing link m:

$$\tau_m^t = \frac{\sum_{ijpA} x_p^{ij,t} \delta_{mp,A}^{ij,t}\ VOTT_A\ f_m^t\ t_m'(f_m^t)}{\sum_{ijpA} x_p^{ij,t} \delta_{mp,A}^{ij,t}} \tag{5}$$

where $x_p^{ij,t}$ refers to the path flow given by the evolutionary learning system and $\delta_{mp,A}^{ij,t}$ stands for a incidence variable characterizing whether link m is included in path p connecting $i-j$ pair for user class A at interval t ($\delta_{mp,A}^{ij,t} = 1$) or not ($\delta_{mp,A}^{ij,t} = 0$), such that $\sum_{ijpA} x_p^{ij,t}\ \delta_{mp,A}^{ij,t} = f_m^t$.

3.5 System evolution

The evolutionary models of the sub-systems presented in previews sections consists a cascade discrete-time dynamical system which allow the loading of the network through taking into account different user classes, in terms of the information that users possess in making their trip choices (departure time and route). The connections between them substantiate by the following formula:

$$x_p^{ij,t} = \sum_A \sum_k u_A^{t-k}\ \pi_{A,k,p}^t\ d_A^{ij,t} \tag{6}$$

where u_A^{t-k} denotes the percentage of users of class A processing the information corresponding to interval $t-k$ regarding the cost of path p. The proposed stochastic evolutionary learning model provides an equilibrium-tending dynamical system, whose fixed point (attractor) is the stochastic user equilibrium. Such a system recognizes that the trajectory of the state of urban road networks moves towards equilibrium, but it rarely equilibrates in practice. This is because the traffic conditions required to ensure the system stability rarely can be met in real-world situations.

4. APPLICATION OF THE MODEL

In order to get insight regarding the characteristics of the proposed model at first results from its application on a test network will be provided. This test network (Fig. 2) connects a single origin-destination (O-D) pair and it has 23 links composing a number of 25 paths, leading in a rather complicated case concerning the route choice process. The link travel time estimation is based on the standard formulation of the Bureau of Public Roads (BPR), as follows:

$$t_m = t_m^f \left(1 + \beta \left(\frac{f_m}{y_m}\right)^{\mu}\right) \tag{7}$$

where t_m^f stands for the free-flow link travel time and β, μ scale parameters (here β=0.15 and μ=4) of link m. The capacity of each of the existing links is set equal to y_m =30 vehicles per hour (veh/hr). The free-flow travel time, which is proportional to the link length, is set equal to $t_m^f = 1$ min for links 1-17, while $t_m^f = 1.4$ min, for links 18-23.

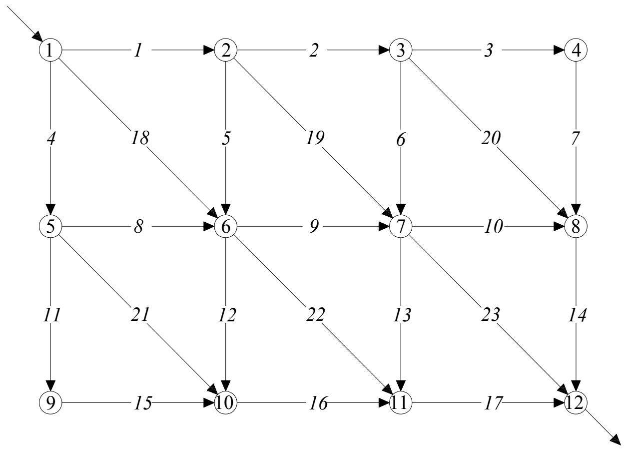

Fig. 2. Network Configuration

By adopting the BPR formula, τ_m^t is calculated as follows:

$$\tau_m^t = \frac{\sum_{ijpA} x_p^{ij,t}\ \delta_{mp,A}^{ij,t}\ VOTT_A \left(\mu \beta t_m^0 \left(\frac{f_m^t}{y_m}\right)^{\mu}\right)}{\sum_{ijpA} x_p^{ij,t}\ \delta_{mp,A}^{ij,t}} \tag{8}$$

The resolution of the time evolution for this example is set to 1 hour, while only one VOTT user class has been considered. The maximum desirable demand is set to $D^{ij,t} = 80$ veh/hr, $\forall t \in T$ while the elasticity on the travel cost scale parameter is set to $a = 0.03$. Also, the maximum real-time information penetration is set equal to *I*=25%. For the users utilizing real-time information a large scale parameter of perception error is preferred ($\theta^{t-1} = 2$) allowing the choices of those users to be concentrating to the shortest path since those users are better informed regarding the network conditions. The rest of the users are utilizing the historic path costs, i.e. path costs for the same period at previous day (*k*=24) and a lower scale parameter of perception error is preferred ($\theta^{t-1} = 0.5$) assuming larger perception error leading to a larger spread of the users among the available paths.

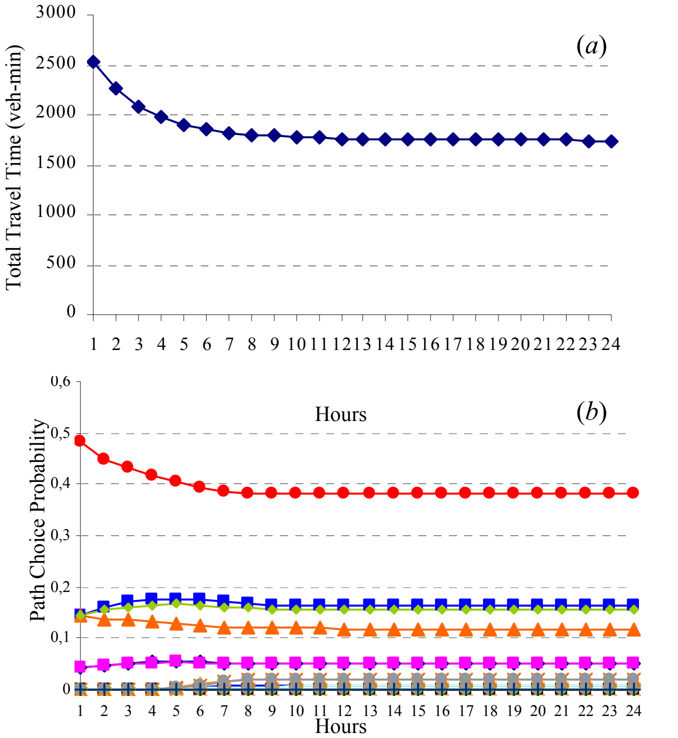

Fig. 3. Evolution of Total Travel Time (*a*) and Path Choice Probabilities (*b*) of the Information Constraint Setup

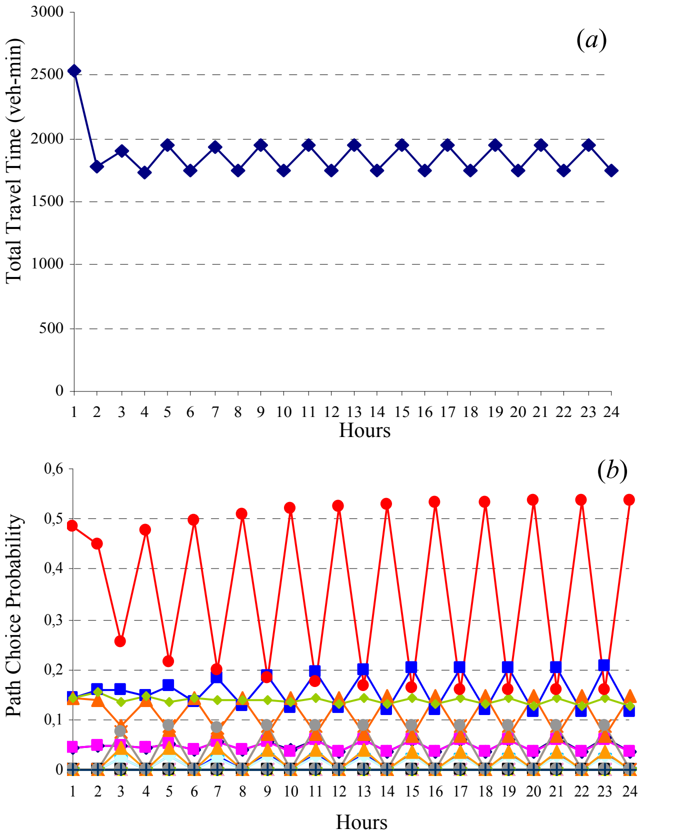

Fig. 4. Evolution of Total Travel Time (*a*) and Path Choice Probabilities (*b*) under the Extreme Case

Under this assumption the model reaches the stochastic user equilibrium conditions after a warm-up period of few hours (Fig 3*a* and Fig 3*b*). In the extreme case where *all* users are reconsidering their route choice based on the same real-time information, then the network conditions oscillates failing to converge (approach optimal conditions) since the interaction among the users is constant and no steady state can be achieved (Fig. 4*a* and Fig. 4*b*). As it can be observed, users path choices oscillate between the optimum path (red line) and the alternative paths. This happens since users identifies optimal paths at the beginning, then they tend to choose it collectively increasing path travel cost and thus at the next evolution phase they are choosing alternative paths (collectively again) making the previous path costs to be reduced again creating an oscillation among paths.

At the next step, a variable demand pattern has been used in order to test the model performance under more realistic conditions. Two model configurations have been tested for a period of 1 day (24 periods), one with the limited real-time information penetration (I=25%) and other assuming all users receiving real-time information (Fig. 5). The results show that the model with the limited real-time information (red line) is performing better in terms of total travel cost, than that of the fully informed users (blue line), for the time periods of increased congestion where the interaction among the users is high and small perturbations in the loading patterns have a severe impact on the system performance, a result that coincides with that of Mahmassani and Jayakrishnan (1991). On the other hand, at low congestion periods, system performance is almost identical for the two models since, in that case, perturbations in the route choices have little effect on the system performance and thus optimal route is easier to be identified.

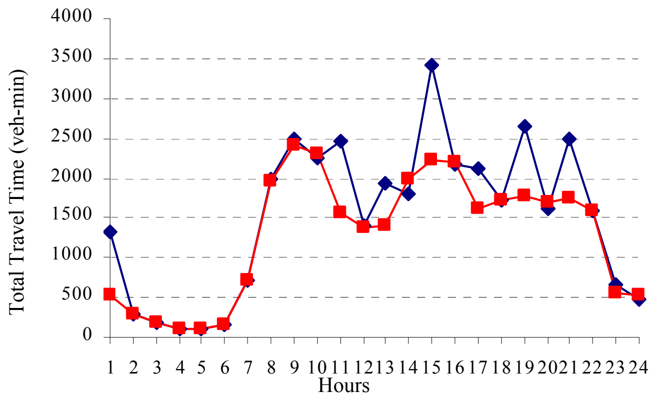

Fig. 5. Evolution of the System Total Travel Time

Next the model is applied in a much more complex case, that of a sparse representation of a realistic network. The particular case stands for a part of the business centre main arterials network, of Athens, Greece (Fig. 6). This network has been coded by a network of 123 links connecting a sum of 506 O-D pairs. Part of the Athens centre is under an access restriction policy (shaded area in Fig. 6). For the current application in the links belonging to the access restricted area, a MCP policy has been investigated by the model of the constrained real-time information penetration, with I=25%. This penetration rate can be considered as a relatively large percentage number of potential users to have real-time information access, even for using it as the upper limit, although Mahmassani and Jayakrishnan (1991) suggested that under these conditions maximum system efficiency is expected to be gained.

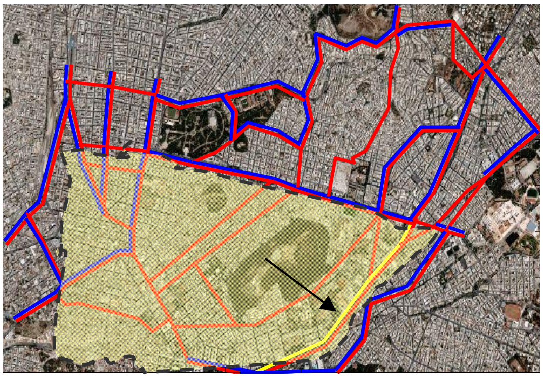

Fig. 6. Central Athens Network Layout

In this case, the time slice evolution has been set at the 15 minutes, a time step able to capture users' choices in a rather dense resolution. Two users classes has been identified with respect to their *VOTT*, user class A with $VOTT_A$=1.50 €/hour and class B with $VOTT_B$=4.00 €/hour. The mixture of the demand by those two user classes is set as 20%-A class and 80%-B class. The BPR formula is also used for the network model. The variable demand pattern for all O-D pairs follows a realistic pattern obtained by field measurements. In order to investigate the system performance, a warm-up period of 20 days of evolution has been utilized, to which period the information tank is able to provide reliable information regarding historical path costs. Under the above assumptions the evolution of MCP charges for a part of the network connecting the boundaries of the restricted area (congestion abatement) with the city centre (indicated in yellow at the middle-down area of Fig. 6) is provided below, for a complete day. This particular section is composed of two links (a part of Vas. Sofias Av.) of two lanes per direction, with length of 2755 metres and the free-flow speed is set up to 50 km/hour. So for that road stretch, the maximum congestion charge equals the amount of 0.35 € for the peak hour period and ranges between 0.25 € - 0.30 € for most of the day.

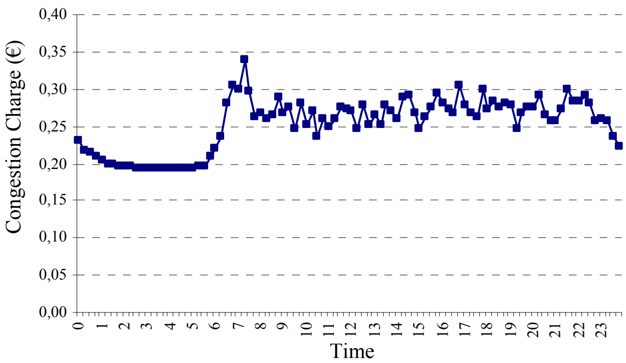

Fig. 7. Evolution of Congestion Charges for the Selected Link

It should be noted that the estimation of the link congestion charges is based on the BPR formula, that tends to under-estimate congestion effects (propagation of queues, intersection delays etc.) on the link travel time.

5. CONCLUSIONS

In the current study, an evaluation framework of the application of a congestion pricing strategy based on the marginal travel cost has been presented, able to identify the dynamic impact of congestion pricing to the performance of the system, by taking into consideration behavioural characteristics of multiple user classes. The proposed framework builds on tactics of variable road congestion pricing, based on theories of evolutionary game theory and stochastic optimal control.

REFERENCES

De Palma, A., Kilani, M., and Lindsey, R. (2005) Congestion pricing on a road network: A study using the dynamic equilibrium simulator METROPOLIS, *Transportation Research Part A*, **39** (7-9), 588-611.

Friesz, T.L., Bernstein, D., and Kydes, N. (2004). Dynamic congestion pricing in disequilibrium. *Networks and Spatial Economics,* **4**, p.p. 181-202.

Henderson, J.V. (1974) Road congestion: A reconsideration of pricing theory, *Journal of Urban Economics*, **1** (3), 346-365.

Papageorgiou, M., Wang, Y., Messmer, A. (2005). RENAISSANCE in motorway surveillance. *ITS International* **11** (4), pp. 55-56.

Laboratory of Railways and Transport, Athens National Technical University of Athens. Athens Real-Time Traffic Congestion Map. Available at: http://www.transport.ntua.gr/map/en/

Mahmassani, H. S., and R. Jayakrishnan (1991). System performance and user response under real-time information in a congested traffic corridor. *Transportation Research Part A,* **25** (5), 293-307.

Maynard Smith, J. (1982) *Evolution and the Theory of Games.* Cambridge University Press, Cambridge, UK.

Pigou, A.C. (1920) *The Economics of Welfare*. Macmillan, London, U.K.

Sandholm W.H. (2002). Evolutionary Implementation and Congestion Pricing. *The Review of Economic Studies*, **69** (3), pp. 667-689(23)

Vickrey, W S. (1969) Congestion theory and transport investment. *Am. Ec. Rev.*, **59** (2), 251-260.

Walters, A.A. (1961) The theory and measurement of private and social cost of highway congestion. *Econometrica,* **29** (4), 676-699.

Zhao, Y. and Kockelman, K.M. (2006) On-line marginal-cost pricing across networks: Incorporating heterogeneous users and stochastic equilibria, *Transportation Research Part B,* **40** (5), 424–435.

Yang, H., Meng, Q. and Lee, D.H. (2004) Trial-and-error implementation of marginal-cost pricing on networks in the absence of demand functions. *Transportation Research Part B,* **38** (6), 477–493.

Dafermos, S. and Sparrow, F. T. (1971) Optimal resource allocation and toll patterns in user-optimized transport networks. *Journ. of Transp. Econ. and Pol.* **5**, 184-200.

Mailath, G.J. and L. Samuelson (2006). *Repeated games and reputations. Long-run relationships.* Oxford University Press.