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Compliant coordination and control of multiple vehicles with discrete-time periodic communications [1]

João Almeida * **Carlos Silvestre** * **António Pascoal** *

* *Institute for Systems and Robotics, Instituto Superior Técnico, Av. Rovisco Pais, 1049-001 Lisboa, Portugal (e-mails: {jalmeida,cjs,antonio}@isr.ist.utl.pt)*

Abstract: This paper addresses the problem of coordinated path-following of networked autonomous vehicles with discrete-time periodic communications. The objective is to steer a group of autonomous vehicles along given spatial paths, while holding a desired inter-vehicle formation pattern. For a class of vehicles, we show how Lyapunov based techniques, graph theory, and results from networked control systems can be brought together to yield a decentralized control structure where the dynamics of the cooperating vehicles and the constraints imposed by the topology of the inter-vehicle communications network are explicitly taken into account. Compliant vehicle coordination is achieved by adjusting the speed of each vehicle along its path according to information exchanged periodically on the positions of a subset of the other vehicles, as determined by the communications topology adopted. The closed loop system, obtained by putting together the path-following and coordination strategies, takes an interconnected feedback form where both systems are input-to-state stable (ISS) with respect to the outputs of each other. Stability and convergence of the overall system are guaranteed for an adequate choice of gains.

1. INTRODUCTION

Multi-vehicle systems have received much attention by the research community over the last few years. Applications include such diverse scenarios as seabed surveying, traffic monitoring, and forest fire detection, among others. In all scenarios, coordination among the vehicles is essential in order to accomplish the mission goals in an efficient and robust manner. In most cases, coordination is achieved through the exchange of information among vehicles, implying that control mechanisms must be designed taking into account such practical constraints as limited bandwidth and intermittent communication failures.

As a contribution to the study of these issues, this paper addresses the problem of coordinated path-following (CPF, see, e.g, Ghabcheloo et al. [2006a], Ghabcheloo et al. [2006b], Ihle et al. [2006]) with discrete-time periodic communications, whereby a set of vehicles is required to follow pre-defined spatial paths while keeping a desired inter-vehicle formation pattern in time. This problem arises, for example, in the operation of multiple autonomous underwater vehicles for fast acoustic coverage of the seabed. By imposing constraints on the inter-vehicle formation pattern, the efficacy of the task can be largely improved. Also, due to the environment involved (water) the bandwidth for communications is severely reduced, allowing only for communications in short and small bursts.

We solve the CPF problem for a class of fully-actuated autonomous vehicles moving in two-dimensional space. Nevertheless, the results presented here can be extended to the three-dimensional case. The solution adopted is based on Lyapunov stability theory and addresses explicitly the vehicle dynamics as well as the constraints imposed by the topology of the inter-vehicle communications network. The latter are tackled in the framework of graph theory, where we consider communication topologies with unidirectional or directed links: one vehicle sends information to its neighbors but does not necessarily receive information back. Each vehicle is equipped with a controller that makes the vehicle follow a predefined path (see, e.g, Encarnação and Pascoal [2001], Skjetne et al. [2005]). The speed of each vehicle is then adjusted so that the whole group keeps a desired formation pattern. A supporting communications network provides the vehicles with the medium over which to exchange the information required to synchronize them. Because of bandwidth constraints, the information exchange between vehicles takes place at discrete time instants which occur with a fixed frequency. We assume that the transmission delay can be neglected and that there are no packet collisions when the vehicles communicate simultaneously. Due to the absence of information in the intervals between transmission times, the control action of each vehicle runs in open loop, based on a simple model that predicts the evolution of its neighbors (as in Model-Based Networked Control Systems, Montestruque and Antsaklis [2003]). At transmission times, each vehicle sends information through the network that is used to achieve coordination and to update the models.

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This paper continues the work started in Almeida et al. [2007] by introducing a feedback signal from the coordination control subsystem to the path-following controller, thus yielding smoother convergence of the vehicles to the paths. The system that is obtained by putting together the path-following and vehicle coordination strategies takes an interconnected form, where both systems are input-to-state stable (ISS) with respect to the outputs of each other. With the control structure adopted, path-following (in space) and inter-vehicle coordination (in time) become essentially decoupled. In fact, both control subsystems are designed separately, ignoring the signals interconnecting them, and then stability and convergence of the closed loop system are analyzed and guaranteed using the small-gain theorem (see, e.g, Isidori [1999]).

The paper is organized as follows. Section 2 describes the dynamic model of the autonomous vehicles considered. In Section 3, key concepts from graph theory are introduced that are necessary to formally state the coordinated path-following problem in Section 4. Section 5 illustrates the general structure of the proposed CPF controller that is formed by a path-following controller presented in Section 6, and a coordination controller operating with periodic communications developed in Section 7. Section 8 gives an illustrative example where simulation results are presented. Finally, Section 9 contains the conclusion and directions of future research.

The proofs of all lemmas and theorems have been omitted and can be found in Almeida [2007].

2. VEHICLE MODELING

The kinematic and dynamic equations of a large class of vehicles are summarized next (see, e.g, Fossen [1994]). To this effect, consider an autonomous vehicle modeled as a rigid body subject to external forces and torques moving in a two dimensional space, as, for example, a surface vessel moving at sea. Let $\{\mathcal{I}\}$ be an inertial coordinate frame and $\{\mathcal{B}\}$ a body-fixed coordinate frame whose origin is located at the center of mass of the vehicle. The generalized position of the vehicle is $\boldsymbol{\eta} = (x, y, \psi)$, where (x, y) are the coordinates of the origin of $\{\mathcal{B}\}$ in $\{\mathcal{I}\}$ and ψ is the orientation of the vehicle (yaw angle) that parameterizes the body-to-inertial coordinate transformation matrix

$$J := J(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Denoting by $\boldsymbol{\nu} := (u, v, r)$ the generalized velocity of the vehicle relative to $\{\mathcal{I}\}$ expressed in $\{\mathcal{B}\}$, the following *kinematic* relations apply:

$$\dot{\boldsymbol{\eta}} = J\boldsymbol{\nu}\,, \qquad \dot{J} = rJS\,, \tag{1}$$

where S is a skew-symmetric matrix. We consider fully-actuated vehicles with *dynamic* equations of motion of the form

$$M\dot{\boldsymbol{\nu}} = \boldsymbol{f}(\boldsymbol{\nu}, \boldsymbol{\eta}) + \boldsymbol{\tau}\,, \tag{2}$$

where $M \in \mathbb{R}^{3\times3}$ denotes a constant symmetric positive definite mass matrix, $\boldsymbol{\tau} = (\tau_u, \tau_v, \tau_r)$ is the generalized control input, and $\boldsymbol{f}(\boldsymbol{\nu}, \boldsymbol{\eta})$ represents all the remaining equivalent forces and torques acting on the body, related to Coriolis, centripetal and hydrodynamic damping effects. For marine vessels, M also includes the so-called hydrodynamic added-mass M_A, i.e., $M = M_{RB} + M_A$, where M_{RB} is the rigid-body mass matrix.

3. GRAPH THEORY

This section contains some key concepts and results in graph theory that play an important role in what follows. See Godsil and Royle [2001] for an in-depth presentation of this subject. A directed graph or digraph $\mathcal{G} = \mathcal{G}(\mathcal{V}, \mathcal{E})$ consists of a finite set $\mathcal{V} = \{1, 2, \ldots, n\}$ of n vertices and a finite set $\mathcal{E}$ of m ordered pairs of vertices $(i, j) \in \mathcal{E}$ named arcs. Given an arc $(i, j) \in \mathcal{E}$, its first and second elements are called the tail and head of the arc, respectively. The out-degree of a vertex i is the number of arcs with i as its tail. If (i, j) belongs to $\mathcal{E}$ then we say that i is adjacent to j. A path from i to j is a sequence of distinct vertices starting with i and ending with j such that consecutive vertices are adjacent. If there is a path in $\mathcal{G}$ from vertex i to vertex j, then j is said to be reachable from i. A vertex i is globally reachable if it is reachable from every other vertex in $\mathcal{G}$. The adjacency matrix of a digraph, denoted A, is a square matrix with rows and columns indexed by the vertices, such that the i, j-entry of A is 1 if $(i, j) \in \mathcal{E}$ and 0 otherwise. The out-degree matrix D of a digraph is a diagonal matrix where the i, i-entry is equal to the out-degree of vertex i.

The Laplacian of a digraph is defined as $L = D - A$. By definition, the Laplacian satisfies $L\mathbf{1} = \mathbf{0}$ with $\mathbf{1} = [1]_{n\times1}$, therefore 0 is eigenvalue of L with $\mathbf{1}$ being its associated right eigenvector. In Lin et al. [2005], it is shown that a digraph has at least one globally reachable vertex if and only if zero is a simple eigenvalue of L. This implies that $\operatorname{rank} L = n - 1$ for a graph with one globally reachable vertex. Hence, there exist matrices $F \in \mathbb{R}^{n\times(n-1)}$ and $G \in \mathbb{R}^{(n-1)\times n}$ such that $L = FG$, where $\operatorname{rank} F = \operatorname{rank} G = n - 1$. The matrix G also satisfies $G\mathbf{1} = \mathbf{0}$.

4. PROBLEM STATEMENT

We now consider the problem of coordinated path-following (CPF). In the most general setup, we are given a set of $n \geq 2$ autonomous vehicles and a set of n spatial paths $\boldsymbol{\eta}_{di} = (x_{di}, y_{di}, \psi_{di})$ for $i = 1, 2, \ldots, n$. Each path $\boldsymbol{\eta}_{di}(\gamma_i)$ is parameterized by a continuous variable γ_i and it is required that vehicle i follow path $\boldsymbol{\eta}_{di}$. Following Ghabcheloo et al. [2006a], the CPF problem is separated in two subproblems: a path-following problem for each vehicle, where we require the vehicle to follow a predefined desired spatial path, and a coordination problem for all vehicles addressing the constraints imposed by the topology of the communications network. The problem of path-following is formally stated next.

Path-following problem: Let $\boldsymbol{\eta}_{di}(\gamma_i) \in \mathbb{R}^3$ be a desired smooth path parameterized by a continuous variable $\gamma_i \in \mathbb{R}$ and $v_{di} \in \mathbb{R}$ a desired, constant speed assignment for vehicle i. For each vehicle with equations of motion given by (1) and (2), design a feedback control law for $\boldsymbol{\tau}_i$ such that all closed-loop signals are bounded, and, as $t \to +\infty$, the position of the vehicle converges to the desired path, i.e., $\|\boldsymbol{\eta}_i(t) - \boldsymbol{\eta}_{di}(\gamma_i(t))\| \to 0$.

For the coordination problem, we start by introducing a measure of the degree of coordination of a fleet of vehicles. As in Ghabcheloo et al. [2006b], this is done by reparameterizing each path $\boldsymbol{\eta}_{di}(\gamma_i)$ in terms of a conveniently defined variable ξ_i such that coordination is said to be achieved along the paths if and only if $\xi_1 = \xi_2 = \ldots = \xi_n$. At this point, we formally define the "along-path" distances between vehicle i and j as $\xi_{i,j} = \xi_i - \xi_j$. Then, coordination is achieved if and only if $\xi_{i,j} = 0$ for all $i, j \in \{1, 2, \ldots, n\}$. Let the reparameterization of the path be represented by $\gamma_i = \gamma_i(\xi_i)$ and define $R_i(\xi_i) := \partial\gamma_i/\partial\xi_i$, which is assumed to be positive and bounded for all ξ_i. The dynamics of ξ_i and γ_i are related by

$$\dot{\gamma}_i = R_i(\xi_i)\dot{\xi}_i \,. \tag{3}$$

This paper considers the case where $R_i(\xi_i)$ is constant. While restrictive, it still allows us to consider paths where there is no need for reparameterization ($\xi_i = \gamma_i$) or it amounts to a constant scaling.

Suppose one vehicle, henceforth referred to as vehicle $\mathcal{L}$, is elected as the "leader" and let the corresponding path $\eta_{d\mathcal{L}}$ be parameterized by $\gamma_{\mathcal{L}} = \xi_{\mathcal{L}}$. For this vehicle, $R_{\mathcal{L}} = 1$. Let $v_{\mathcal{L}}$ be the desired constant speed assigned to the leader in advance, that is $\dot{\xi}_{\mathcal{L}} = v_{\mathcal{L}}$ in steady-state, known to all vehicles. From (3), it follows that the desired "along-path" speeds for the vehicles are $v_{di} := R_i v_{\mathcal{L}}$. It is important to point out that $\mathcal{L}$ can always be taken as a "virtual" vehicle that is added to the set of "real" vehicles as an expedient to simplify the coordination strategy.

So far, the problem of coordination has been reduced to that of "aligning" the coordination states ξ_i. To go from this alignment to a more complex spatial configuration, we introduce appropriate offsets in the desired positions of the vehicles relative to the mean point of the formation as defined with respect to the paths. To this effect, let $\boldsymbol{\xi} := [\xi_i]_{n\times 1}$ and define the formation mean point and offsets as $\bar{\xi} := \frac{1}{n}\mathbf{1}^\top\boldsymbol{\xi}$ and $\boldsymbol{\delta} := \boldsymbol{\xi} - \bar{\xi}\mathbf{1}$, respectively. Notice that $\mathbf{1}^\top\boldsymbol{\delta} = 0$. Let $\boldsymbol{\phi} \in \mathbb{R}^n$ represent a desired constant formation pattern that verifies $\mathbf{1}^\top\boldsymbol{\phi} = 0$. The problem of coordination with pattern tracking is reduced to that of making $(\boldsymbol{\delta} - \boldsymbol{\phi}) \to \mathbf{0}$ as $t \to +\infty$.

From a graph theoretical point of view, each vehicle is represented by a vertex and a communication link between two vehicles is represented by an arc. The communication links are assumed unidirectional, thereby inducing a directed graph. We consider time-invariant communication topologies and assume that the induced graph has at least one globally reachable vertex. The flow of information in an arc is directed from its head to its tail. The set of neighbors of vertex i is represented by $\mathcal{N}_i$ and contains all vertices j such that $(i, j) \in \mathcal{E}$. In other words, it is the group of vehicles from which vehicle i receives information.

For design purposes, we will take each $\dot{\xi}_i$ as a control input of the coordination dynamics (3). In order to satisfy the constraints imposed by the communication network, the control law for vehicle i must be decentralized, i.e, it may only depend on local states and/or on information exchange with its neighbors as specified by $\mathcal{N}_i$. The coordination problem is formulated as:

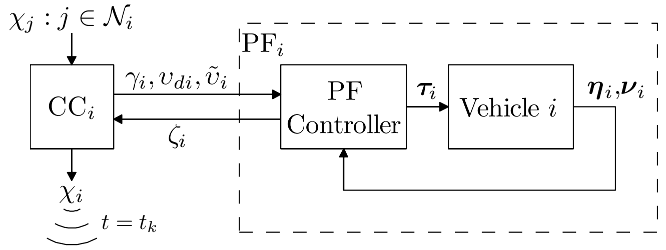

Fig. 1. Proposed control structure for coordinated path-following, represented for vehicle i.

Coordination problem: Design a decentralized feedback control law for $\dot{\boldsymbol{\xi}}$ such that $\boldsymbol{\delta} - \boldsymbol{\phi}$ tends asymptotically to zero and all vehicles travel at the desired "along-path" speed, that is, $\dot{\gamma}_i \to v_{di}$ for all $i \in \{1, 2, \ldots, n\}$.

The coordination part of our CPF problem is closely related to agreement problems (see, e.g., Olfati-Saber and Murray [2004]). In our case, all vehicles must agree on a common value, the mean point of the formation pattern, while they follow a desired speed profile under the constraint that the communications between vehicles be discrete-time periodic.

5. CONTROL STRUCTURE

In this section, we propose a control structure for the CPF problem. It is based on the work presented in Ghabcheloo et al. [2006a] and is illustrated in Fig. 1 for the ith vehicle. The structure proposed shows an interconnection of two subsystems: the path-following (PF) subsystem and the coordination control (CC) subsystem.

The PF subsystem is formed by the vehicle itself and a feedback controller designed to guide it to the desired path. The PF controller drives the vehicle through its command inputs $\boldsymbol{\tau}_i$ using a control law that depends on the vehicle's position $\boldsymbol{\eta}_i$ and velocity $\boldsymbol{\nu}_i$, and on signals provided by the CC subsystem. It also provides the CC subsystem with a feedback signal ζ_i.

The CC subsystem handles all the communications with neighboring vehicles and provides the PF subsystem with the inputs γ_i, v_{di}, and $\tilde{v}_i$. The exchange of information occurs at discrete-time instants $\{t_k\}$, that will henceforth be referred to as *update times*, of the form $\{t_k = hk + t_0 : k \in \mathbb{N}\}$ where h is the communication period in seconds. At these update times, a certain information variable χ_i, to be defined later, is sent by vehicle i to its adjacent vehicles and information variables $\chi_j : j \in \mathcal{N}_i$ from its neighbors are received. These information variables are the only data that must be exchanged between vehicles to achieve coordination. The dynamic equation of γ_i is of the form

$$\dot{\gamma}_i = R_i v_{\mathcal{L}} + \tilde{v}_i + w_i\zeta_i = v_{di} + \tilde{v}_i + w_i\zeta_i \tag{4}$$

where v_{di} is the desired speed, $\tilde{v}_i$ is a control signal to the chosen in order to solve the coordination problem (as a function of χ_i and $\chi_j : j \in \mathcal{N}_i$), ζ_i is a signal from the PF subsystem that will be treated as an external input, and w_i is an adjustable gain.

The PF subsystem will be shown to be input-to-state stable (ISS) with respect to state $\boldsymbol{x}_{Pi}$ (which represents the state of the PF subsystem of vehicle i) and input $\tilde{v}_i$.

The CC subsystem will be shown to be ISS with respect to state $\boldsymbol{x}_{Ci}$ (which represents the state of the CC subsystem of vehicle i) and input ζ_i.

A sampled-data based approach to the problem of coordinated path-following was proposed in Ihle et al. [2006], where the variables parameterizing the paths (γ_i) evolve in a discrete fashion, and therefore the coordination control problem is posed in discrete-time. However, the authors only considered communication topologies with bidirectional links.

6. PATH-FOLLOWING

Central to the development of CPF strategies is the derivation of appropriate path-following control laws for each vehicle. In this section, we present a path-following controller for an autonomous vehicle described by the equations of motions introduced in Section 2, that follows the lines of those developed in Encarnação and Pascoal [2001], Skjetne et al. [2005]. The controller is local to each vehicle so the index i will be omitted for the sake of clarity.

Define the position error in the body-fixed frame as $\boldsymbol{z}_1 := J^\top(\boldsymbol{\eta} - \boldsymbol{\eta}_d)$, and let $\zeta := \dot{\gamma} - \upsilon_d$ denote the "along-path" speed tracking error. The goal of the path-following controller is to drive $\boldsymbol{z}_1$ and ζ to zero. Applying backstepping design procedures (see, e.g, Krstić et al. [1995]), the Lyapunov function

$$V := \frac{1}{2}\boldsymbol{z}_1^\top \boldsymbol{z}_1 + \frac{1}{2}\boldsymbol{z}_2^\top M \boldsymbol{z}_2 + \frac{1}{2}\zeta^2 , \tag{5}$$

yields the feedback control laws

$$\boldsymbol{\tau} = -\boldsymbol{z}_1 - K_2\boldsymbol{z}_2 - \boldsymbol{f} + M(\boldsymbol{\alpha}^t + \boldsymbol{\alpha}^\gamma\dot{\gamma}) , \tag{6}$$

and

$$\dot{\zeta} = -w\mu - k_\zeta \zeta , \tag{7}$$

where $\boldsymbol{z}_2 := \boldsymbol{\nu} - \boldsymbol{\alpha}$, $\mu := -(\boldsymbol{\eta}_d^\gamma)^\top J \boldsymbol{z}_1$,

$$\boldsymbol{\alpha} := J^\top \boldsymbol{\eta}_d^\gamma \upsilon_d - K_1\boldsymbol{z}_1 , \ \boldsymbol{\alpha}^\gamma := K_1 J^\top \boldsymbol{\eta}_d^\gamma + J^\top \boldsymbol{\eta}_d^{\gamma^2} \upsilon_d ,$$
$$\text{and } \boldsymbol{\alpha}^t := -K_1(\boldsymbol{\nu} - rS\boldsymbol{z}_1) - rSJ^\top \boldsymbol{\eta}_d^\gamma \upsilon_d + J^\top \boldsymbol{\eta}_d^\gamma \dot{\upsilon}_d ,$$

with $\boldsymbol{\eta}_d^{\gamma^p} = \partial^p \boldsymbol{\eta}_d / \partial\gamma^p$ for $p = 1, 2$. The time derivative of (5) along the solutions of (1) and (2), using (6) and (7) as feedback laws, is

$$\dot{V} = -\boldsymbol{z}_1^\top K_1 \boldsymbol{z}_1 - \boldsymbol{z}_2^\top K_2 \boldsymbol{z}_2 - k_\zeta \zeta^2 + \mu\tilde{\upsilon} .$$

The closed loop system with $\tilde{\upsilon}$ as input has the following important property.

Lemma 1. The PF subsystem described is ISS with respect to state $\boldsymbol{x}_P = [\boldsymbol{z}_1^\top\ \boldsymbol{z}_2^\top\ \zeta]^\top \in \mathbb{R}^7$ and input $\tilde{\upsilon}$, that is,

$$\|\boldsymbol{x}_P(t)\| \le \kappa_P e^{-\lambda_P t}\|\boldsymbol{x}_P(t_0)\| + \sigma_P \sup_{s\in[t_0,t]} |\tilde{\upsilon}(s)|$$

for some positive constants κ_P, λ_P, and σ_P. Moreover, σ_P can be made arbitrary small by increasing the PF gains K_1, K_2 and k_ζ.

7. CPF WITH PERIODIC COMMUNICATION

We now offer a solution to the coordination problem. Rewriting (4) in terms of $\dot{\xi}_i$ and using a control law for $\tilde{\upsilon}_i$ adapted from Ghabcheloo et al. [2006b], we obtain a controller that solves the coordination problem under continuous communication:

$$\dot{\xi}_i = \upsilon_{\mathcal{L}} + R_i^{-1}(\tilde{\upsilon}_i + w_i\zeta_i) , \tag{8}$$
$$\tilde{\upsilon}_i = -k_{ci} \sum_{j\in\mathcal{N}_i} (\xi_i - \phi_i - \xi_j + \phi_j) , \tag{9}$$

where $k_{ci} > 0$ is an adjustable control gain and ϕ_i are the components of desired formation pattern represented by $\boldsymbol{\phi}$. Notice that the information required by vehicle i about its neighbors is $\chi_j := \xi_j - \phi_j$, that we refer to as *information state*, and not the coordination state ξ_j itself. The control law (8) can be rewritten as

$$\dot{\chi}_i = \upsilon_{\mathcal{L}} + R_i^{-1}(\tilde{\upsilon}_i + w_i\zeta_i) , \tag{10}$$
$$\tilde{\upsilon}_i = -k_{ci} d_i \chi_i + k_{ci} \sum_{j\in\mathcal{N}_i} \chi_j , \tag{11}$$

where d_i is the number of neighbors of vehicle i (out-degree of vertex i), $k_{ci} = 0$ if $d_i = 0$. When using periodic communications, vehicle i does not receive $\chi_j : j \in \mathcal{N}_i$ between update times, so it needs to model their evolution in that interval. Let $\hat{\chi}_j^i$ represent local "replicas" of each χ_j as seen by vehicle i, that we refer to as *predictor states*. Analyzing (10), we see that if a steady-state condition is achieved, then $\dot{\chi}_i = \upsilon_{\mathcal{L}}$ for all i. This suggests that the dynamics of χ_j can be predicted as $\dot{\hat{\chi}}_j^i = \upsilon_{\mathcal{L}}$, thus yielding the controller

$$\tilde{\upsilon}_i = -k_{ci} d_i \chi_i + k_{ci} \sum_{j\in\mathcal{N}_i} \hat{\chi}_j^i , \tag{12}$$
$$\dot{\hat{\chi}}_j^i = \upsilon_{\mathcal{L}} , \text{ for each } j \in \mathcal{N}_i . \tag{13}$$

However, this is not sufficient to achieve coordination due to initial conditions that do not match the desired formation pattern. To overcome this problem, a reset is made to the predictor states when information is exchanged. We therefore add the following condition to the controller:

$$\hat{\chi}_j^i(t_k) = \chi_j^i(t_k^-), \text{ for each } j \in \mathcal{N}_i \qquad \text{(update)} , \tag{14}$$

where the notation $x(t^-)$ stands for the left limit or limit from below, i.e., $x(t^-) = \lim_{s\nearrow t} x(s)$. Because all $\hat{\chi}_j^i$ are initialized with the same value $\chi_j(t_0)$, and because we assume there is absolute synchronization with respect to update times, vehicles that model the same predictor state have equal values, i.e, $\hat{\chi}_j^{i_1} = \hat{\chi}_j^{i_2}$ for all i_1, i_2, and j. Therefore, we do not need to refer to $\hat{\chi}_j^{i_1}$ and $\hat{\chi}_j^{i_2}$ as different states, we simply refer to them as $\hat{\chi}_j$.

Defining $\hat{\boldsymbol{\chi}} := \boldsymbol{\xi} - \boldsymbol{\phi} = [\hat{\chi}_i]_{n\times1}$, $\boldsymbol{\zeta} := [\zeta_i]_{n\times1}$, the diagonal matrices $K_c := \mathrm{diag}[k_{ci}]_{n\times n}$, $W := \mathrm{diag}[w_i]_{n\times n}$, and $C := \mathrm{diag}[R_i^{-1}]_{n\times n}$, equations (12)-(14) can be written in vector form as

$$\dot{\boldsymbol{\chi}} = \upsilon_{\mathcal{L}}\mathbf{1} + C(-K_c D\boldsymbol{\chi} + K_c A\hat{\boldsymbol{\chi}} + W\boldsymbol{\zeta}) , \tag{15a}$$
$$\dot{\hat{\boldsymbol{\chi}}} = \upsilon_{\mathcal{L}}\mathbf{1} , \tag{15b}$$
$$\hat{\boldsymbol{\chi}}(t_k) = \boldsymbol{\chi}(t_k^-) . \tag{15c}$$

The coordination error is defined as

$$\boldsymbol{\theta} := G(\boldsymbol{\xi} - \boldsymbol{\phi}) = G\boldsymbol{\chi} \in \mathbb{R}^{n-1} ,$$

where G is obtained from the decomposition of the Laplacian discussed in Section 3. Since $G\mathbf{1} = \mathbf{0}$ and using the definitions of formation mean point and offsets of Section 4, we have

$$G(\boldsymbol{\xi} - \boldsymbol{\phi}) = G(\boldsymbol{\delta} + \bar{\xi}\mathbf{1} - \boldsymbol{\phi}) = G(\boldsymbol{\delta} - \boldsymbol{\phi}) .$$

Because $\mathbf{1}^\top \boldsymbol{\delta} = \mathbf{1}^\top \boldsymbol{\phi} = 0$, $\boldsymbol{\delta} - \boldsymbol{\phi}$ is normal to the null space of G. We conclude that $\boldsymbol{\theta} = \mathbf{0}$ if and only if $(\boldsymbol{\delta} - \boldsymbol{\phi}) = \mathbf{0}$. Let

$$\tilde{\boldsymbol{\chi}} := \boldsymbol{\chi} - \hat{\boldsymbol{\chi}}$$

represent the predictor state error. If $\tilde{\boldsymbol{\chi}} = \mathbf{0}$, then the information states are coherent, i.e, the predictor states equal the actual states. Considering (15), the error dynamics for $\boldsymbol{\theta}$ and $\tilde{\boldsymbol{\chi}}$ are given by

$$\dot{\tilde{\boldsymbol{\chi}}} = -CK_cF\boldsymbol{\theta} - CK_cA\tilde{\boldsymbol{\chi}} + CW\boldsymbol{\zeta}\,,$$
$$\dot{\boldsymbol{\theta}} = -GCK_cF\boldsymbol{\theta} - GCK_cA\tilde{\boldsymbol{\chi}} + GCW\boldsymbol{\zeta}\,,$$

where we used the fact that $L\boldsymbol{\chi} = FG\boldsymbol{\chi} = F\boldsymbol{\theta}$. Defining the aggregated state variable $\boldsymbol{x}_C := (\boldsymbol{\theta}, \tilde{\boldsymbol{\chi}})$, the above error dynamics can be written as

$$\begin{cases} \dot{\boldsymbol{x}}_C = \Lambda \boldsymbol{x}_C + B\boldsymbol{\zeta}, & t \in [t_k, t_{k+1}) \\ \boldsymbol{x}_C(t) = (\boldsymbol{\theta}(t^-), \mathbf{0}), & t = t_k \end{cases} \tag{16}$$

where

$$\Lambda := \begin{bmatrix} -GCK_cF & -GCK_cA \\ -CK_cF & -CK_cA \end{bmatrix}, \tag{17}$$

$$B := \begin{bmatrix} GCW \\ CW \end{bmatrix}. \tag{18}$$

The dynamic system (16) is a linear impulsive system which is ISS if a certain condition is verified.

Lemma 2. The system described by (16) is ISS with respect to state $\boldsymbol{x}_C$ and input $\boldsymbol{\zeta}$, if

$$\Phi := \begin{bmatrix} I_{n-1} & 0 \\ 0 & 0 \end{bmatrix} e^{\Lambda h} \tag{19}$$

is a *convergent matrix* (all its eigenvalues are strictly inside the unit circle).

The following lemma gives conditions under which matrix Φ is guaranteed to be a convergent matrix.

Lemma 3. For any communication graph with at least one globally reachable vertex, matrix Φ defined in (19) is a convergent matrix.

The next theorem states that the control structure proposed in Section 5, with the control laws developed in Sections 6 and 7, solves the CPF problem presented in Section 4. Its proof is a straightforward application of the small-gain theorem (see, e.g, Isidori [1999]).

Theorem 4. The overall system formed by the interconnection of the n combined PF/CC systems is globally asymptotically stable.

8. ILLUSTRATIVE EXAMPLE

We consider a group of three identical vehicles whose kinematic and dynamic equations of motion can be written as in (1)-(2), with

$$M = \mathrm{diag}(m_u, m_v, I_r)\,, \text{ and}$$

$$\boldsymbol{f}(\boldsymbol{\nu}, \boldsymbol{\eta}) = \begin{bmatrix} X_u + X_{|u|u}|u| & -m_v r & 0 \\ m_u r & Y_v + Y_{|v|v}|v| & 0 \\ 0 & 0 & N_r + N_{|r|r}|r| \end{bmatrix} \boldsymbol{\nu}\,.$$

In the simulations presented, the physical parameters are

$$\begin{array}{lll} X_u = -1\,\mathrm{kg/s}\,, & Y_{|v|v} = -200\,\mathrm{kg}\,, & m_u = 500\,\mathrm{kg}\,, \\ X_{|u|u} = -25\,\mathrm{kg/m}\,, & N_r = -0.5\,\mathrm{kgm^2/s}\,, & m_v = 1000\,\mathrm{kg}\,, \\ Y_v = -10\,\mathrm{kg/s}\,, & N_{|r|r}\,, = -1500\,\mathrm{kgm^2} & I_r = 700\,\mathrm{kgm^2}\,. \end{array}$$

The communication graph is illustrated in Fig. 2. The communication period is set to $h = 1\,\mathrm{s}$.

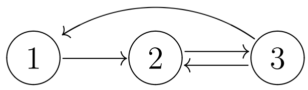

Fig. 2. Graph representing the communications network.

To illustrate the compliance of the control system, that is, to show why the existence of feedback from the PF subsystem to the CC subsystem is important, we consider a simulation where the vehicles must follow straight lines parallel to the x-axis, while the propulsion system of vehicle-2 has a force limitation of 20 N along the x-direction (that is, $|\tau_{u2}(t)| \leq 20\,\mathrm{N}$ for all $t \geq 0$). This keeps vehicle-2 from acquiring the necessary speed to hold the desired formation pattern. The initial conditions of each vehicle are $\boldsymbol{\eta}_1(t_0) = (-5\,\mathrm{m}, -5\,\mathrm{m}, \pi/3\,\mathrm{rad})$, $\boldsymbol{\eta}_2(t_0) = (-5\,\mathrm{m}, 15\,\mathrm{m}, -\pi/4\,\mathrm{rad})$, $\boldsymbol{\eta}_3(t_0) = (5\,\mathrm{m}, 17\,\mathrm{m}, 2\pi/3\,\mathrm{rad})$, $u_i(t_0) = v_i(t_0) = 0\,\mathrm{m/s}$ and $r_i(t_0) = 0\,\mathrm{rad/s}$ for $i = 1, 2, 3$. The initial condition for $\boldsymbol{\gamma}$ is chosen so that for each vehicle i, γ_i yields the closest point on the respective path, which gives $\boldsymbol{\gamma}(t_0) = (-5, -5, 5)$ [m]. The reference speed is set to $v_{\mathcal{L}} = 1\,\mathrm{s}^{-1}$. The vehicles are required to keep an in-line formation pattern, that is, $\boldsymbol{\phi}_0 = (0, 0, 0)$. Figure 3(a) illustrates the initial state of the vehicles.

Next, we consider two cases.

(1) The gains are $K_1 = 0.15I_3$, $K_2 = 200I_3$, $k_\zeta = 1$, $K_c = 0.5/hD^+$, and $W = 0$. In this case, there is no feedback from the PF subsystem to the CC subsystem (situation studied in Almeida et al. [2007]). Therefore, vehicle-2 is left behind and its position error increases continuously as shown in Fig. 4. Figure 3(c) illustrate the final state of the vehicles.

(2) The gains are $K_1 = 0.15I_3$, $K_2 = 200I_3$, $k_\zeta = 1$, $K_c = 0.5/hD^+$, and $W = I_3$. Due to the existence of feedback, the vehicles reach a compromise between converging to the paths and achieving coordination as can be seen in Fig. 5, where both the PF and CC errors do not tend to zero but are bounded. Figure 3(d) illustrate the final state of the vehicles.

For comparison purposes, Figure 3(b) illustrates the vehicles' final state for the ideal scenario where vehicle-2 has no force restrictions, using the gains of Case (2).

9. CONCLUSION

We addressed the problem of coordinated path-following for a group of autonomous vehicles using periodic communications. We proposed and analyzed a decentralized control structure formed by an interconnection of two subsystems named path-following and coordination control. Stability and convergence of the overall system are guaranteed for any fixed update period, as long as the graph induced by the communication network has at least one globally reachable vertex.

Future work includes the study of the applicability of the proposed control structure to address coordination problems involving time delays, communication failures, and asynchronous updates.

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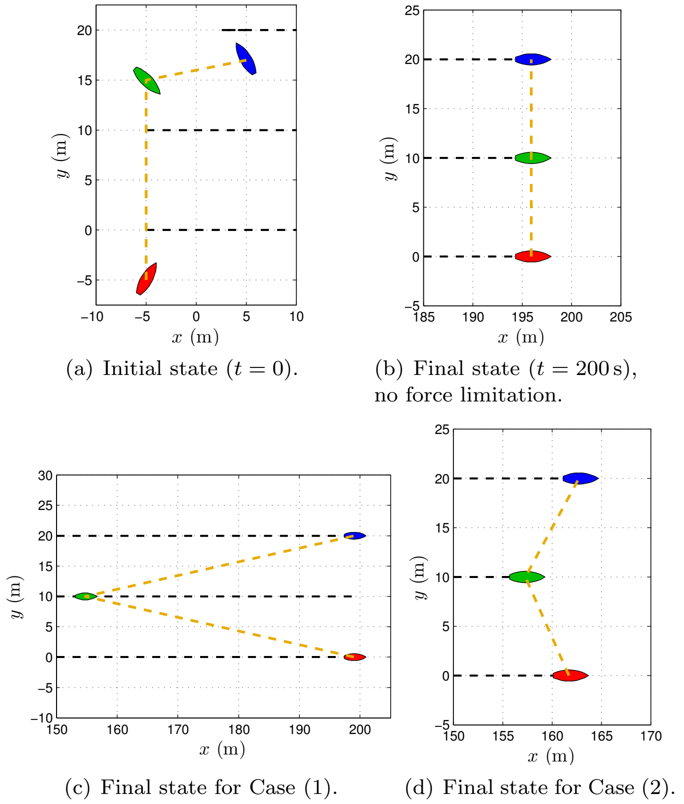

(a) Initial state ($t = 0$).

(b) Final state ($t = 200\,\mathrm{s}$), no force limitation.

(c) Final state for Case (1).

(d) Final state for Case (2).

Fig. 3. Comparison of the vehicles' final state for different scenarios.

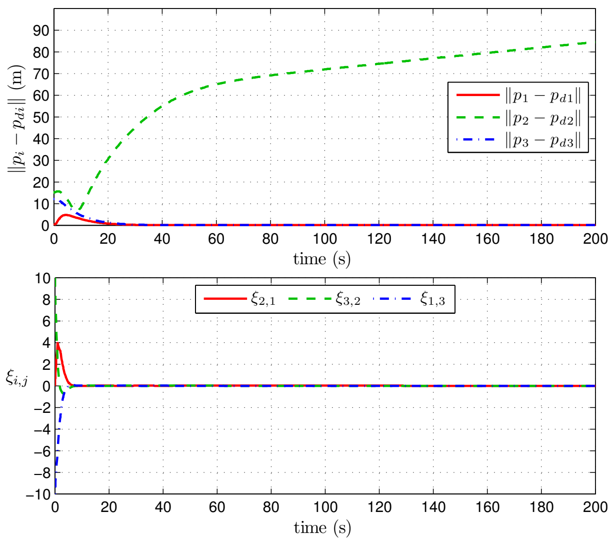

Fig. 4. Time evolution of the position errors (top plot) and "along-path" distances (bottom plot) for Case (1).

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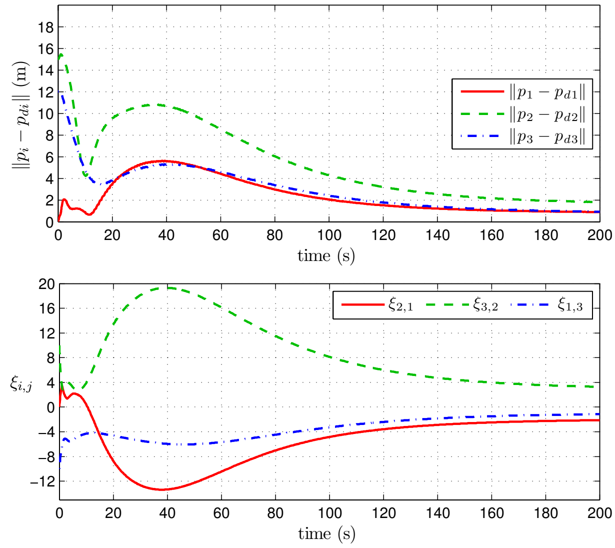

Fig. 5. Time evolution of the position errors (top plot) and "along-path" distances (bottom plot) for Case (2).

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