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Tracking Control for Port-Hamiltonian Systems using Feedforward and Feedback Control and a State Observer ⋆

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Abstract:
This contribution is about the combination of a feedforward and a feedback controller and a reduced state observer in order to stabilize the trajectories of a nonlinear plant. Port-Hamiltonian systems provide some special mathematical properties and have turned out beneficial for the stability analysis of nonlinear control systems. The combination of a feedforward and feedback controller allows us to achieve good tracking and the rejection of disturbances and parameter variations. In addition the extension of the nonlinear control scheme with a state observer allows a reduction of the number of measured quantities. This approach will be shown for the example Ball on the Wheel.

Keywords: Nonlinear Control; Port-Hamiltonian Systems; Output Tracking; Mechatronics;

1. INTRODUCTION

This contribution is about the tracking controller design for nonlinear lumped-parameter systems. The goal of the controller design is that the considered system output y tracks a given, admissible trajectory. In order to achieve this behavior we use a control scheme as sketched in Fig. 1, which consists of a feedforward controller and a state feedback controller with an observer.

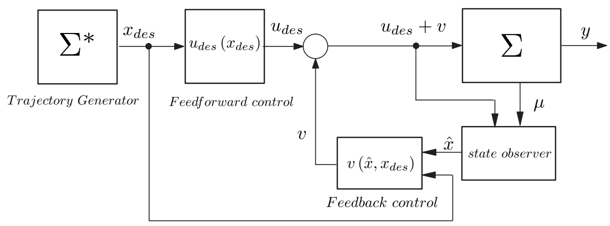

Fig. 1. Structure of the control scheme with observer

The design of a feedforward controller for a system like

$$\dot{x} = f(x, u) \qquad y = h(x) \qquad x(t_0) = x_0 \tag{1}$$

could be very tricky in the general case. During the late 80's Fliess and his co-worker, see Fliess et al. [1995] introduced the concept of flat systems. By means of a so-called flat output one is able to calculate an openloop control law $u_{des}(t)$ for (1) provided the reference curve is sufficiently smooth. Here we assume that a flat output exists for (1).

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In this contribution we combine the approach of passivity based control with flatness. In detail we extend the IDA-PBC-approach, see van der Schaft [2000], Ortega et al. [2002] in order to include the feedforward control law u_{des} in the Hamiltonian framework. One advantage of the IDA-PBC design is that it does not require the measurement of the whole system state in general.

This paper is organized as follows. After an introduction we give a short overview on Port-Hamiltonian systems (PH-systems) and we derive a Port-Hamiltonian representation for an Euler-Lagrange system. In section 3 we investigate coordinate transformations, which preserve the Hamiltonian structure. Section 4 is devoted to the IDA-PBC design and the stability analysis for nonlinear time variant PH-systems. The design of a reduced state observer is shown in section 5 and we investigate the closed loop stability for the state feedback controller and the state observer. Finally we apply the proposed approach to the nonlinear example Ball on the Wheel (BoW) and end with a short conclusion.

2. EULER-LAGRANGE SYSTEMS AND PORT-HAMILTONIAN REPRESENTATION

In this contribution we focus on nonlinear systems (1), which admit a representation as Port-Hamiltonian system. PH-systems are a generalization of the class of conservative Hamilton systems, which are usually given in the form

$$\dot{q}_i = (\partial_{p_i} H) \qquad \dot{p}_i = -(\partial_{q_i} H) \qquad i = 1, \ldots, \nu \quad . \tag{2}$$

For the rest of the paper we write I for the identity matrix and $(\partial_q H)$ for the Jacobian $(\partial_q H) = \left[\frac{\partial H}{\partial q_1}, \ldots, \frac{\partial H}{\partial q_n}\right]$ of the Hamiltonian function $H \in C^1(\mathbb{R}^{2\nu})$ with respect to the coordinates q respectively p. The generalized coordinates q and momenta p of (2) are ν-dimensional vectors $q(t), p(t) \in \mathbb{R}^\nu$. If the Hamiltonian $H > 0$ is positive definite (p.d.), then the system (2) is stable in the sense of Lyapunov, because of a negative semidefinite (nsd.) derivative $\dot{H} = 0$. A generalization of (2) is given by

$$\begin{aligned} \dot{x} &= (J(x) - R(x))(\partial_x H)^T + G(x) u \\ y &= G^T(x)(\partial_x H)^T \quad , \end{aligned} \tag{3}$$

which is called a Port-Hamiltonian system, see van der Schaft [2000], where $J = -J^T$ and $R = R^T \geq 0$ is met. In (3) y is also called the collocated output and the assumption $x \in \mathbb{R}^{2\nu}$ can be dropped for (3). If $H \geq C_0$ is bounded from below, then a PH-system is a passive system, because the passivity inequality $\frac{\mathrm{d}}{\mathrm{d}t} H(q(t), p(t)) \leq \langle y, u \rangle$

$$\dot{H} = \underbrace{(\partial_x H) J (\partial_x H)^T}_{0} - \underbrace{(\partial_x H) R (\partial_x H)^T}_{\geq 0} + \underbrace{(\partial_x H) G}_{y^T} u \tag{4}$$

holds. The pairing $\langle \cdot \rangle$ is given by the canonical product $\langle w, v \rangle = \sum_{i=1}^n w_i v_i$, $w \in (\mathbb{R}^n)^*$, $v \in \mathbb{R}^n$.

Now we consider Euler-Lagrange Systems (EL-systems). We confine ourselves to EL-systems, where the Lagrangian function $\mathcal{L} \in C^1(\mathbb{R}^n)$ is of the form $\mathcal{L} = T(q, v) - V(q) = \frac{1}{2} v^T M(q) v + V(q)$. Together with a Rayleigh function $\mathcal{R} \in C^1(\mathbb{R}^n)$, $(\partial_v \mathcal{R}) v \geq 0$ the equation of motion (EoM) take the form

$$\frac{\mathrm{d}}{\mathrm{d}t} (\partial_v \mathcal{L})^T - (\partial_q \mathcal{L})^T + (\partial_v \mathcal{R})^T = \mathcal{M} Q_e \quad . \tag{5}$$

with the generalized external forces $Q_e \in \mathbb{R}^m$. Based on the EoM of the EL-system we derive a PH-representation for (5), which uses the state variables q and v and the Hamiltonian $H = T + V$

$$H = \frac{1}{2} \sum_{i,j} m_{ij}(q) v_i v_j + V(q) \qquad i, j = 1, \ldots, n \quad . \tag{6}$$

According to (3) we look for matrices $J_{ij} = -J_{ij}^T$, $R_{ij} = R_{ij}^T$ and G_{ij}, i,j = 1,2, such that the EL-equations (5) take the form

$$\begin{bmatrix} \dot{q} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} J_{11} - R_{11} & J_{12} - R_{12} \\ -J_{12}^T - R_{12} & J_{22} - R_{22} \end{bmatrix} \begin{bmatrix} (\partial_q H)^T \\ (\partial_v H)^T \end{bmatrix} + \begin{bmatrix} G_{11} \\ G_{22} \end{bmatrix} Q_e. \tag{7}$$

by means of $\dot{q} = v$ the set of second order ode's of (5) becomes a set of $2n$ first order ode's, which leads immediately to the relation

$$\begin{aligned} \dot{q} &= (J_{11} - R_{11})(\partial_q H)^T \\ &\quad + (J_{12} - R_{12})(\partial_v H)^T + G_{11} Q_e \stackrel{!}{=} v \quad , \end{aligned} \tag{8}$$

for equivalence. It turns out, that for $J_{11} = R_{11} = R_{12} = G_{11} = 0$ and $J_{12} = M^{-1}(q) = \bar{M}(q)$ the expression (8) holds. The calculations for the lower n-ode's of (7) are more involved. We consider the explicit form of the EL-system given by (5)

$$\dot{v} = \bar{M} \left(\left(\tfrac{\mathrm{d}}{\mathrm{d}t} M(q) \right) v + (\partial_q \mathcal{L})^T - (\partial_v \mathcal{R})^T + \mathcal{M} Q_e \right) \tag{9}$$

with $(\partial_q \mathcal{L})^T = (\partial_q T)^T - (\partial_q V)^T$. If we plug in the result of (8), then (9) should be equal to

$$\begin{aligned} \dot{v} &= \underbrace{-\bar{M}(\partial_q T)^T + 2\bar{M}(\partial_q T)^T}_{\bar{M}(\partial_q T)^T} - 2\bar{M}(\partial_q T)^T \\ &\quad -\bar{M}(\partial_q V)^T + (J_{22} - R_{22}) M v + G_{22} Q_e \quad . \end{aligned} \tag{10}$$

By comparison of coefficients for (10) we get 3 relations for the equivalence:

$$\begin{aligned} &i)\ \bar{M} \left(\frac{\mathrm{d}}{\mathrm{d}t} M(q) \right) v = -2\bar{M}(\partial_q T)^T + J_{22} M v \\ &ii)\ \bar{M}(\partial_v \mathcal{R})^T = R_{22} M v \quad iii)\ \bar{M} \mathcal{M} Q_e = G_{22} Q_e \ , \end{aligned} \tag{11}$$

where $iii)$ is easy to solve for $G_{22} = \bar{M}\mathcal{M}$. The inclusion of a general Rayleigh dissipation $\mathcal{R}$ in the PH-representation demands a solution for the expression $ii)$. It has to be checked separately, if a solution for $ii)$ exists. Otherwise this approach fails. If we consider a quadratic Rayleigh function $\mathcal{R} = \frac{1}{2} v^T \mathcal{F}(q) v$, then we get the simple result $R_{22} = \bar{M}\mathcal{F}\bar{M}$. For the analysis of the left hand side of $i)$ we use an indexed notation

$$\bar{M} \left(\frac{\mathrm{d}}{\mathrm{d}t} M(q) \right) v = -\bar{M} \left(\sum_{i,j} \frac{\partial m_{ij}}{\partial q_k} v_i - \sum_{i,j} \frac{\partial m_{ki}}{\partial q_j} v_i \right) v_j \ . \tag{12}$$

If we add $\sum_j \frac{\partial m_{kj}}{\partial q_i} v_i - \sum_j \frac{\partial m_{kj}}{\partial q_i} v_i = 0$ to (12) and rewrite the left hand side of $i)$ of (11), then we end up with

$$-\bar{M} \left(\sum_{i,j} \frac{\partial m_{kj}}{\partial q_i} - 2 c_{ijk}(q) \right) v_i v_j = \sum_{i,j} J_{22_{ki}} m_{ij} v_j \tag{13}$$

Obviously $c_{ijk}(q) = \frac{1}{2} \left(\frac{\partial m_{kj}}{\partial q_i} + \frac{\partial m_{ki}}{\partial q_j} - \frac{\partial m_{ij}}{\partial q_k} \right)$ are nothing else than the *Christoffel symbols* of the first kind. Eq. (13) takes the compact form $\dot{m}_{kj} - 2c_{kj}$, if we define $c_{kj} = \sum_i c_{ijk}(q) v_i$. In order to fulfill $i)$ of (11) J_{22} is equal to $J_{22} = -\bar{M}\left(\dot{M} - 2C\right)\bar{M}$, where c_{kj} are the components of the matrix $C(q, v)$. It is left to check if J_{22} is skew-symmetric, but this is done by means of a short calculation and the well-known result that $\dot{M} - 2C$ is skewsymmetric. To start with the definition of skewsymmetry $J_{22} + J_{22}^T = 0$ we get

$$\begin{aligned} &-\bar{M}\left(\dot{M} - 2C\right)\bar{M} - \bar{M}^T \left(\dot{M} - 2C\right)^T \bar{M}^T = \\ &-\underbrace{\bar{M}\left(\left(\dot{M} - 2C\right) + \left(\dot{M} - 2C\right)^T\right)\bar{M}}_{0} = 0 \end{aligned} \tag{14}$$

and $\left(\dot{M} - 2C\right) = -\left(\dot{M} - 2C\right)^T$ completes the proof.

Now one can write a Hamiltonian representation for the EL-equation (5)

$$\begin{aligned} &\begin{bmatrix} \dot{q} \\ \dot{v} \end{bmatrix} = (J_{EL} - R_{EL}) \begin{bmatrix} (\partial_q H)^T \\ (\partial_v H)^T \end{bmatrix} + \begin{bmatrix} 0 \\ \bar{M}\mathcal{M} \end{bmatrix} Q_e \\ &\text{with} \quad J_{EL} = \begin{bmatrix} 0 & \bar{M} \\ -\bar{M} & -\bar{M}\left(\dot{M} - 2C\right)\bar{M} \end{bmatrix} \quad , \end{aligned} \tag{15}$$

$R_{EL} = \mathrm{diag}\{0, R_{22}\}$. Although the coordinates $[q, p]$ have many theoretical advantages for EL-systems the choice of $[q, v]$ fits directly with the measured quantities. This is of interest for the observer design. Clearly the usage of the (inverse) Legendre-transform for (5) leads to a similar result as given in (15).

Example 1: As illustrative example we treat the Cart-Pole system, see Fantoni and Lozano [2001]. We skip the modelling at all and deal with the Lagrangian $\mathcal{L} = \frac{1}{2} v^T M(q) v - V(q)$, which reads in detail

$$\begin{aligned} q^T &= [\, q_1 \;\; q_2 \,] \quad v^T = [\, v_1 \;\; v_2 \,] \quad x^T = \left[\, q^T \;\; v^T \,\right] \\ M = M^T &= \begin{bmatrix} m_1 & m_{12}\cos(q_2) \\ m_{12}\cos(q_2) & m_2 \end{bmatrix} > 0 \qquad (16) \\ V(q) &= -m_{12} g \sin(q_2) \qquad m_1,\, m_{12},\, m_2,\, g \in \mathbb{R}^+ \end{aligned}$$

and the EoM are given by

$$M \begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} + \begin{bmatrix} -m_{12}\sin(q_2)\, v_2^2 \\ m_{12} g \sin(q_2) \end{bmatrix} - \mathcal{F} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} F_x \\ 0 \end{bmatrix} \quad , \qquad (17)$$

where a viscous friction is included by means of the Rayleigh function $\mathcal{R} = \frac{1}{2} v^T \mathcal{F} v$, $\mathcal{F} = \mathrm{diag}\{r_1, r_2\}$. In order to derive J_{22} we have to calculate $\dot{M}(q)$ and the Christoffel symbols $c_{ijk}(q)$. It turns out that all $c_{ijk}(q)$ vanish except $c_{221} = -m_{12}\sin(q_2)$. Only c_{21} of C is non-zero and J_{22} becomes

$$J_{22} = \bar{M} \begin{bmatrix} 0 & 0 \\ -m_{12}\sin(q_2) & 0 \end{bmatrix} \bar{M} = \begin{bmatrix} 0 & \bar{j}_{12} \\ -\bar{j}_{12} & 0 \end{bmatrix} \qquad (18)$$

for $\bar{j}_{12} = m_{12} \det(M)^{-1} \sin(q_2)\, v_2$. One can easy convince himself, that the PH-representation of (15) is equivalent to the EL-equations (5) by calculating $(\partial_x H)$ for the Hamiltonian $H = T + V$. For the sake of completeness the introduction of the generalized momentum $p = Mv$ via the Legendre transform leads of course to the well-known canonical PH-form.

3. TRANSFORMATIONS FOR PH-SYSTEMS

The concept of state and input transformations is well-established in control theory and plays a key role for many nonlinear control design methods. If one carries out a change of coordinates, then it is an important question which transformations are admissible for (3), such that the transformed system preserves the Port-Hamiltonian structure. The second question is how does the collocated output y transform ?

3.1 Affine time invariant Transformations

PH-systems like (3) have an affine structure and it is of interest to investigate affine transformations of the form

$$z = \varphi(x) \qquad \bar{u} = \Lambda(x)\, u + \delta(x) \qquad \Lambda\bar{\Lambda} = I \quad . \qquad (19)$$

A short calculation leads to the PH-system written in new coordinates

$$\begin{aligned} \dot{z} &= (\partial_x \varphi)(J - R)(\partial_x \varphi)^T \circ \varphi^{-1} \left(\partial_z \left(H \circ \varphi^{-1}\right)\right)^T \\ &\quad + \left((\partial_x \varphi) G \bar{\Lambda}\right) \circ \varphi^{-1} \bar{u} - \left((\partial_x \varphi) G \bar{\Lambda} \delta\right) \circ \varphi^{-1} \qquad (20) \\ \dot{z} &= \left(\bar{J} - \bar{R}\right)\left(\partial_z \bar{H}\right)^T + \bar{G}\bar{u} - \bar{G}\delta \circ \varphi^{-1} \quad . \end{aligned}$$

Eq. (20) includes an affine term $\bar{G}\delta$, which comes from the affine input δ and does not fit to the structure of a PH-system. In many cases the structure matrix J fulfills the *Jacobi*-identity and defines a Poisson structure. This property should be preserved throughout the following calculations. The system (20) can be rewritten in the favoured affine form, if the linear pde

$$\begin{aligned} &\left(\bar{J} - \bar{R}\right)\left(\partial_z \bar{H}_\varphi\right)^T + \bar{G}\delta \circ \varphi^{-1} = 0 \quad \text{resp.} \\ &(\partial_x \varphi) \underbrace{\left((J - R)(\partial_x H_\varphi)^T + G\bar{\Lambda}\delta\right)}_{0} \circ \varphi^{-1} = 0 \qquad (21) \end{aligned}$$

admits a solution. Due to the transformation rules (21) can be solved in the original coordinates x and the solution $H_\varphi(x)$ is mapped to the new coordinates $\bar{H}_\varphi(z) = H_\varphi(x) \circ \varphi^{-1}$. By means of a modified Hamiltonian function one is able to rewrite (20) as PH-system

$$\begin{aligned} \dot{z} &= \left(\bar{J} - \bar{R}\right)\left(\partial_z \bar{H}\right)^T + \bar{G}\bar{u} + \underbrace{\left(\bar{J} - \bar{R}\right)\left(\partial_z \bar{H}_\varphi\right)^T}_{\bar{G}\delta \circ \varphi^{-1}} \qquad (22) \\ \dot{z} &= \left(\bar{J} - \bar{R}\right)\left(\partial_z \left(\bar{H} + \bar{H}_\varphi\right)\right)^T + \bar{G}\bar{u} \quad . \end{aligned}$$

A modification of the Hamiltonian like $H_z = (H + H_\varphi) \circ \varphi^{-1}$ provides a possibility to preserve the PH-structure for the affine transformation (19). For instance it can be checked by formal methods, if the up coming pde (21) has a solution or not.

The collocated output y has to be transformed affine too, as the following calculation shows

$$\dot{H}_z = -(\partial_z H_z)\, \bar{R}\, (\partial_z H_z) + \underbrace{\left(\partial_z \left(\bar{H} + \bar{H}_\varphi\right)\right) \bar{G}}_{\bar{y}^T} \bar{u} \quad . \qquad (23)$$

A closer look to $\bar{y}$ leads to

$$\begin{aligned} \langle \bar{y}, \bar{u} \rangle &= \left(\left(\partial_z \bar{H}\right)\bar{G} + \left(\partial_z \bar{H}_\varphi\right)\bar{G}\right)\bar{u} \\ &= \left(y^T \circ \varphi^{-1} + (\partial_x H_\varphi)(\partial_x \varphi)\, G \circ \varphi^{-1}\right)\bar{\Lambda}\bar{u} \end{aligned} \qquad (24)$$

and one concludes that $\bar{y}$ has to be transformed in the way

$$\bar{y} = \left(\bar{\Lambda}\right)^T \left(y + G^T (\partial_x H_\varphi)^T\right) \quad . \qquad (25)$$

The result of (25) demands some additional discussion. In the case of an affine transformation the product $\langle \bar{y}, \bar{u} \rangle$ does not longer match with the original one $\langle y, u \rangle$ as we see from

$$\begin{aligned} \langle y, u \rangle \neq \langle \bar{y}, \bar{u} \rangle &= \langle y + \delta_y, u + \delta_u \rangle \quad \text{for } \delta_y,\, \delta_u \neq 0 \\ &\neq \langle y, u \rangle + \langle y, \delta_u \rangle + \langle \delta_y, u \rangle + \langle \delta_y, \delta_u \rangle \quad . \end{aligned} \qquad (26)$$

This becomes clear, because of to the modification of the Hamiltonian function the affine term $\langle \delta_y, \delta_u \rangle$ is already covered by the system representation.

3.2 Time variant Transformations

Another important class of transformations are time variant transformations, which arise, if one deals with time variant systems or one is interested in a description of a time invariant system in a moving reference frame. Especially in the second case the transformation demands an extension of the variables $(x) \to (x, t)$. Since t is also the curve parameter we have to add the constraint $\dot{t} = 1$ and we confine ourselves to transformations of the form

$$\bar{t} = t \quad (t, z) = \psi(x, t) \quad \bar{u} = \Lambda(x, t)\, u + \delta(x, t) \quad , \qquad (27)$$

where the time coordinate t is not changed. This is a reasonable assumption for physical models. A time variant change of coordinates like (27) for (3) leads to

$$\begin{aligned} \dot{z} &= \left((\partial_x \psi)\, \dot{x} + (\partial_t \psi)\right) \circ \psi^{-1} \\ \dot{z} &= (\partial_t \psi) \circ \psi^{-1} + \left(\bar{J} - \bar{R}\right)\left(\partial_z \left(H \circ \psi^{-1}\right)\right)^T \qquad (28) \\ &\quad + \bar{G}\bar{u} - \bar{G}\delta \circ \psi^{-1} \end{aligned}$$

with a similar result to (20) $\bar{J} = (\partial_x \psi)\, J\, (\partial_x \psi)^T \circ \psi^{-1}$, $\bar{R} = (\partial_x \psi)\, R\, (\partial_x \psi)^T \circ \psi^{-1}$ and $\bar{G} = (\partial_x \psi)\, G\bar{\Lambda} \circ \psi^{-1}$. In addition one can easy convince himself that $\dot{\bar{t}} = \dot{t} = 1$

still holds. By means of the upper result we write the time variant Hamiltonian system in the form

$$\underbrace{(\dot{z} - \partial_t \psi) \circ \psi^{-1}}_{\dot{x}\circ\psi^{-1}} = \left(\bar{J} - \bar{R}\right)\left(\partial_z \bar{H}\right)^T + \bar{G}\bar{u} + \bar{G}\delta\circ\psi^{-1} \ . \quad (29)$$

According to the previous subsection the inclusion of the affine part $\bar{G}\delta$ is almost the same. Even if we consider an input transformation with $\delta = 0$ the resulting system (28) includes an affine term $\partial_t \psi$, which comes from the time dependency of the transformation. Roughly speaking this terms coincides with the motion of the reference frame and we see, that $\bar{J}$, $\bar{R}$, $\bar{G}$ as well as the Hamilton $\bar{H}$ become time dependent in the general case.

Remark: The resulting Hamiltonian system (29) allows two different interpretation. On the one hand we are interested in the transformed value of $\dot{x}\circ\psi^{-1}$ from (3), then the term $\partial_t\psi$ has to appear on the left hand side. A more intuitive interpretation makes this point clear. If we want to describe the velocity $\dot{x}$ in a moving reference frame, then we have to subtract the velocity of the moving frame in order to get the correct velocity. The second interpretation is more useful for the stabilization of the trajectories. The system (28) can be written as explicit time variant system of the form

$$\dot{z} = \left(\bar{J} - \bar{R}\right)\left(\partial_z \bar{H}_z\right)^T + \bar{G}\bar{u} \quad . \quad (30)$$

if the affine terms can be included in the Hamiltonian framework via a modified Hamiltonian $\bar{H}_z = H\circ\psi^{-1} + \bar{H}_\psi$ and a solution of the pde

$$\left(\bar{J} - \bar{R}\right)\left(\partial_z \bar{H}_\psi\right)^T - \left(\partial_t\psi - \bar{G}\delta\right)\circ\psi^{-1} = 0 \quad (31)$$

exists.

4. TRACKING CONTROL FOR PH-SYSTEMS

For the design of a tracking controller for (30) we introduce the so-called tracking error e via a special case of time variant transformation like (27)

$$e = \psi(x,t) = x - x_{des}(t) \qquad u = u_{des} + \bar{u} \quad . \quad (32)$$

The resulting PH-system becomes

$$\dot{e} = (J_e - R_e)\left(\partial_e\left(\bar{H}_z\circ\psi^{-1} + \bar{H}_\psi\right)\right)^T + G_e\bar{u} \quad (33)$$

because of $(\partial_x\psi) = I$. According to section 3 we assume that the linear pde

$$(J - R)\left(\partial_x \bar{H}_\psi\right)^T - \bar{G}u_{des} + \dot{x}_{des}(t) = 0 \quad (34)$$

has a solution and in general (33) is a time variant system, where the matrices $J_e(e,t)$, $R_e(e,t)$ and $G_e(e,t)$ contain continuous differentiable functions of the tracking error e and the time t. Please note that transformation (32) and the tracking controller design presented here is quite different to the work of Fujimoto et al. [2001].

In order to follow the desired trajectory x_{des} a controller has to stabilize the origin of (33). According to the IDA-PBC approach the selection of the closed loop dynamics

$$\dot{e} = (J_d(e,t) - R_d(e,t))\left(\partial_e H_d(e,t)\right)^T \quad (35)$$

with a p.d. Hamiltonian $H_d > 0$ leads to a set of restrictions

$$(J_e - R_e)\left(\partial_e H_e\right)^T + G_e\bar{u} = (J_d - R_d)\left(\partial_e H_d\right)^T \quad (36)$$

see also Ortega et al. [2002]. By means of a left hand annihilator $G_e^\perp$, $G_e^\perp G_e = 0$, the restrictions of the IDA-PBC

$$G_e^\perp\left((J_d - R_d)\left(\partial_e H_d\right)^T - (J_e - R_e)\left(\partial_e H_e\right)^T\right) = 0 \quad (37)$$

reduce to a set of $n-m$ equations, which has to be fulfilled. The control law is calculated from the restrictions (36)

$$\bar{u} = \left(G_e {G_e}^T\right)^{-1} {G_e}^T \Big((J_d - R_d)\left(\partial_e H_d\right)^T - (J_e - R_e)\left(\partial_e H_e\right)^T\Big) \quad , \quad (38)$$

because $G_e G_e^T$ has always an inverse provided that G_e has a full column rank. An extension of the control law like

$$\tilde{u} = \bar{u} - R_{di}y = \bar{u} - R_{di}G_e^T\left(\partial_e H_d\right)^T \quad (39)$$

includes a feedback of the collocated output y and the psd. matrix $R_{di} \geq 0$ leads to the closed loop dynamics

$$\dot{e} = \left(J_d - \left(R_d + G_e R_{di} G_e^T\right)\right)\left(\partial_e H_d\right)^T \quad . \quad (40)$$

This approach is known as Damping Injection and allows a modification of the dissipative terms of the closed loop provided that the collocated output is measurable. It is left to check, if $\dot{H}_d$ is at least nsd.

$$\dot{H}_d = -\left(\partial_e H_d\right)\underbrace{\left(R_d + G_e R_{di} G_e^T\right)}_{\tilde{R}_d \geq 0}\left(\partial_e H_d\right)^T + \partial_t H_d \leq 0 \quad (41)$$

along the trajectories of (40). It is clear that the term $\partial_t H_d$ requires an accurate investigation in order to guarantee a nsd. derivative $\dot{H}_d$.

5. NONLINEAR OBSERVER DESIGN

It is of practical interest to reduce the number of measurements for the implementation of the control law, but this leads directly to the observer design problem, which is hard to solve in the nonlinear case. For instance a nonlinear observer design based on a Lagrangian structure can be found in Aghannan and Rouchon [2003]. In this work we are interested in the derivation of a nonlinear velocity observer for a special class of EL-systems. The system dynamics is given in the form of (15) and we assume that some preliminary assumptions are fulfilled:

Assumption A 1.1: The derivatives $(\partial_q T) = 0$ of the kinetic energy T with respect to the generalized coordinates q vanish. Roughly speaking the inertia matrix in sensor coordinates $x_s^T = \left[\, \eta^T \ \mu^T \,\right]$ should be independent of the state variables x_s. Here we denote μ as the measurable variables and the variables η have to be estimated for the control law.

Assumption A 1.2: The control law $u(\mu, \eta)$ leads to an asymptotically stable equilibrium point $x_s = 0$ for the closed loop system $\dot{x}_s = f(0,t)$.

Assumption A 1.3: All the components of μ can be used for the control law and the observer design.

The observer design can be carried out in the following way. By means of assumption **A 1.1** $\dot{M} - 2C = 0$ vanishes for (15) and an extension of the reduced Luenberger-observer is possible for the nonlinear case. According to Luenberger [1979] an estimator for the generalized velocities $\eta = v$ of the PH-representation (15) works in the following way. The introduction of the observer state $\zeta = M\eta + K_{\mathcal{O}}\mu$ leads to a dynamic system

$$\dot{\zeta} = -M\left(\bar{M}\left(\partial_\mu V\right)^T + \bar{M}\mathcal{F}\bar{M}\right)(M\eta) + \underbrace{J_{22}\left(Mv\right)}_{0} + \underbrace{M\bar{M}}_{I}\mathcal{M}Q_e + K_{\mathcal{O}}\bar{M}\left(M\eta\right) \quad . \tag{42}$$

for the observer states, which can be rewritten as

$$\dot{\zeta} = \underbrace{\left(-\mathcal{F} + K_{\mathcal{O}}\right)\bar{M}}_{A_{\mathcal{O}}}\zeta + \left(\mathcal{F} - K_{\mathcal{O}}\right)\bar{M}K_{\mathcal{O}}\mu - \left(\partial_\mu V\right)^T + \mathcal{M}Q_e \qquad \hat{\eta} = \bar{M}\left(\zeta - K_{\mathcal{O}}\mu\right) \tag{43}$$

with the estimated values $\hat{\eta}$ for the generalized velocities v. The observation error $e_{\mathcal{O}} = \eta - \hat{\eta}$ is exponentially decreasing, if all eigenvalues λ_i of $A_{\mathcal{O}}$ are placed in the left half plane by means of the design parameter $K_{\mathcal{O}}$, such that $\mathrm{Re}\{\lambda_i\} < 0$ holds. A short calculation shows, that the observer dynamics $\dot{e}_{\mathcal{O}} = A_{\mathcal{O}}e_{\mathcal{O}}$ gets independent of the state variables x_s. This has an interesting consequence for the combination of the control system (15) and the observer dynamics (43). Due to the assumption **A 1.2** a replacement of η by the estimated states $\hat{\eta} = \bar{M}\left(\zeta - K_{\mathcal{O}}\mu\right)$ in the control law $u(\mu, \hat{\eta})$ and the introduction of the observation error $e_{\mathcal{O}} = \eta - \bar{M}\zeta + \bar{M}K_{\mathcal{O}}\mu$ leads to a cascaded system for the closed loop

$$\begin{bmatrix} \dot{x} \\ \dot{e}_{\mathcal{O}} \end{bmatrix} = \begin{bmatrix} f\left(\mu, e_{\mathcal{O}}, t, u\left(\mu, e_{\mathcal{O}}, t\right)\right) \\ A_{\mathcal{O}}e_{\mathcal{O}} \end{bmatrix} \quad , \tag{44}$$

which has a locally asymptotically stable origin $\{x, e_{\mathcal{O}}\} = (0, 0)$ provided that $f\left(x, t, e_{\mathcal{O}}\right)$ is locally Lipschitz on $\mathcal{D} \in \mathbb{R}^n$. A prove can be found in Isidori [1999]. In this case the design of the observer is independent from the controller design and the assumptions **A 1.1** to **A 1.3** provide some kind of separation known from the linear case.

Example 2: In order to show the presented controller design we deal with the nonlinear, unstable, underactuated example Ball on the Wheel (BoW), which is sketched in Fig. 2.

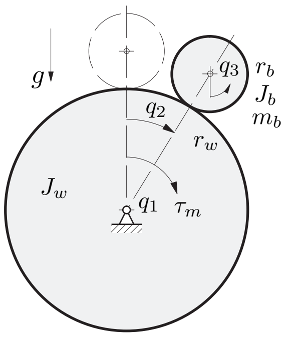

Fig. 2. Euler-Lagrange system Ball on the Wheel

We skip the modeling and refer the interested reader to the contribution of Fuchshumer et al. [2004]. The BoW is a mechanical 2-degree-of-freedom system and if we assume that the ball does not slip on the wheel. The roll condition provides an algebraic restriction

$$r_w q_1 - \left(r_w + r_b\right) q_2 - r_b q_3 = 0 \tag{45}$$

which allows an elimination of q_3 and to deal with the state vector $x^T = [\, q_1 \; q_2 \; v_1 \; v_2 \,]$ for the BoW-system. All the geometric parameters like r_w, r_b, the inertias J_w and J_b, and mass of the ball m_B are positive real values. As given in Fuchshumer et al. [2004] the Lagrangian takes the form

$$\mathcal{L} = \frac{1}{2}\left(m_1 v_1^2 - m_{12}v_1 v_2 + m_2 v_2^2\right) + V\left(q_2\right) \quad ,$$

where $m_1 = \left(J_w + J_b \frac{r_w^2}{r_b^2}\right)$, $m_{12} = m_{21} = J_b \frac{(r_w + r_b) r_w}{r_b^2}$, $m_2 = \left(r_w + r_b\right)^2 \left(\frac{J_b}{r_b^2} + m_b\right)$ are introduced as the components of the inertia matrix M for a short notation. Finally the potential energy of the ball $V\left(q_2\right)$ is given by $V\left(q\right) = m_r \cos\left(q_2\right)$, $m_r = m_B\left(r_w + r_b\right) g$. For the sake of simplicity we assume that the torque of the DC-drive serves directly as control input, such that $\mathcal{M}Q_e$ becomes $e_{01}\tau_m$. Here e_{01} denotes the unit vector in 1-direction $e_{01}^T = [\, 1 \; 0 \,]$. Additionally we include a viscous friction term $Q_d = d_w v_1 e_{01}$, $d_w > 0$ for the bearings of the wheel. According to (5) the EoM for the BoW become

$$M\dot{v} - \left(\partial_q V\right)^T + Q_d = e_{01}\tau_m \quad . \tag{46}$$

One derives the equivalent PH-representation for the EL-system as given in (15) as well as the canonical PH-form with the Hamiltonian $H_p = \frac{1}{2}p^T\bar{M}p + V$.

Here we are not interested in the control of the angle of the wheel q_1 and we deal with a reduced third order system

$$\dot{x}_r = \left(J_r - R_r\right)\left(\partial_{x_r} H_p\right)^T + G_r\tau_m$$
$$J_r = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} \qquad \begin{matrix} R_r = \mathrm{diag}\{0, dw, 0\} \\ G_r = e_{02}^T \end{matrix} \tag{47}$$

for the BoW with the system state $x_r^T = [\, q_2 \; p_1 \; p_2 \,]$. Any function $y_f = \phi\left(p_2\right)$ of the momentum of the ball p_2 is a flat output for the reduced BoW-system (47), which allows a parametrization of the variables x_r and τ_m. A 3-times continuous differentiable function for the evolution of the desired flat output $y_{f_{des}}\left(t\right)$ provides an useful feedforward control, which can to be included in a Hamiltonian description for the tracking error. The selection of $\dot{x}_{des}\left(t\right) = \dot{x}_r|_{x_r = x_{r_{des}}(t)}$ and $\tau_m = \tau_{m_{des}}\left(t\right) + \Delta\tau_m$ leads to a dynamic system, which describes the evolution tracking error $e_r = x_r - x_{r_{des}}\left(t\right)$

$$\dot{e}_r = \left(J_r - R_r\right)\left(\partial_e H_e\right)^T + G_r\Delta\tau_m \quad . \tag{48}$$

The Hamiltonian for (48)

$$H_e = \frac{1}{2}e_p^T\bar{M}e_p + V_e\left(e_1, t\right) \qquad e_p = [\, e_2 \; e_3 \,] \tag{49}$$

follows from the calculations given in section 3 and 4. In (49) $V_e = \kappa\left(t\right) m_r \sin\left(e_1\right) + m_2\dot{y}_f\left(1 - \cos\left(e_1\right)\right)$ becomes explicit time dependent and the control input τ_m consists of the FB-openloop part $\tau_{m_{des}}\left(t\right)$ and an arbitrary state feedback $\Delta\tau_m$. Here $\kappa\left(t\right) = \sqrt{1 - \left(\frac{m_{22}}{m_r}\dot{y}_{fd}\right)^2} > 0$ is positive and real for a bounded reference curve $|\dot{y}_{fd}| < \frac{m_r}{m_{22}}$. For the controller design we suggest a splitting of the time variant system (48) like $\dot{e} = \sigma\left(e, t, \Delta\tau_m\right) + \gamma\left(e, t\right)$, where $\gamma = m_{22}\dot{y}_f\left(1 - \cos\left(e_1\right)\right)e_{03}$ vanishes at the origin $\lim_{e\to 0}\gamma\left(e, t\right) = 0$ and meets the Lipschitz condition

$$\|\gamma(e', t) - \gamma(e'', t)\| \leq L_0\left\|e' - e''\right\| \quad . \tag{50}$$

By means of this assumption we derive a time variant controller for $\dot{e} = \sigma\left(e, t, \Delta\tau_m\right)$ and check the stability of the perturbed closed loop system $\dot{e} = \sigma + \gamma$ afterwards. A Hamiltonian H_σ for σ is given by $H_\sigma = H_e - H_\gamma$, if $\left(J_r - R_r\right)\left(\partial_e H_\gamma\right)^T = \gamma$ holds. The selection of the closed loop matrices J_d and R_d for the IDA-PBC

$$J_d = \begin{bmatrix} 0 & \frac{\kappa}{\alpha\beta} & \frac{\kappa}{\alpha} \\ -\frac{\kappa}{\alpha\beta} & 0 & 0 \\ -\frac{\kappa}{\alpha} & 0 & 0 \end{bmatrix} \qquad R_d = \mathrm{diag}\{0, r_{22}, 0\} \tag{51}$$

and the time depended closed loop Hamiltonian

$$H_d = \alpha m_r \left(1 - \cos\left(e_1\right)\right) + e_p^T P_d\left(t\right) e_p$$

$$P_d = \begin{bmatrix} \frac{\alpha\beta(\beta m_{11}+m_{12})}{\kappa\det(M)}+k_r\beta^2 & \frac{\alpha\beta m_{11}}{\kappa\det\left(M\right)}+k_r\beta \\ \frac{\alpha\beta m_{11}}{\kappa\det\left(M\right)}+k_r\beta & k_r \end{bmatrix} \tag{52}$$

fulfill the IDB-PBC restrictions (37) and the control law $\Delta\tau_m\left(e,t\right)$ follows from (38). The Hamiltonian (52) is a Lyapunov function, if the parameters α, β, $k_r > 0$ are chosen such that P_d gets p.d. One finds a constant $\bar{\kappa} \leq \kappa\left(t\right)$, $\bar{\kappa} \in \mathbb{R}^+$ and time invariant bounds for H_d

$$H_d|_{\kappa^{-1}=1} \leq H_d\left(e,t\right) \leq H_d|_{\kappa^{-1}=\bar{\kappa}} \tag{53}$$

such that $1 \leq \frac{1}{\kappa} \leq \frac{1}{\bar{\kappa}}$ holds. A short calculation leads to a nsd. result for $\dot{H}_d = -\left(\partial_e H_d\right) R_d \left(\partial_e H_d\right)^T + \left(\partial_t H_d\right)$

$$\dot{H}_d = \leq -\frac{1}{2} e_p^T Q_d\left(t\right) e_p \leq 0 \tag{54}$$

and it is possible to find parameters α, β, k_r, $r_{22} > 0$, such that $Q_d\left(t\right) > 0$ gets p.d. for a bounded curve $|\ddot{y}_{fd}| \leq \ddot{y}_{fd\,\max}$. Due to (53) the closed loop system is uniformly stable . It is left to check that the origin is asymptotically stable and the perturbation γ is dominated. By means of a Lyapunov function $H_{Lin} = \frac{1}{2} e^T P_L e$ for the linearized closed loop system one shows that the equilibrium $e = 0$ of is locally exponentially stable (les.) and we conclude that a les. system dominates any perturbation γ, which satisfies the Lipschitz condition (50) within a small ball $\mathcal{D} \in \mathbb{R}^n$ around the origin. The stability theorem as well as the prove can be found in Khalil [2002].

The BoW-model fulfills the assumptions **A 1.1** to **A 1.3** for the observer design and one derives an estimator for the velocities as presented in section 5. The calculations are straight forward and will not be given here. The result presented in Fig. 3 shows the behavior of the closed loop system, where the controller includes a flatness-based feedforward control $\tau_{m_{des}}$, a passivity-based state feedback $\Delta\tau_m$ and a velocity observer for $\hat{v}$. The result given in Fig 3 treats the following experiment. The desired trajectory starts at a mechanical limiter $x_0^T = [\varphi_{20}\ 0\ 0]$, $\varphi_{20} \neq 0$ and tracks to the upper equilibrium. At $t = 2s$ the ball starts following a smooth curve and the motion ends at the upper equilibrium point at $t = 2 + 2\pi$. A rectangular disturbance $F_\delta = \frac{1}{5}\left(\sigma\left(t - 4.5\right) - \sigma\left(t - 4.6\right)\right)$ acts on the ball at $t = 4.5$. As we can see in Fig. (3) the initial error is canceled by the controller.

6. CONCLUSION

The presented tracking controller takes care for flatness-based feedforward part, which is derived without an integration of ode's. We have introduced the tracking error $e = x - x_{des}\left(t\right)$ by means of a time variant transformation and we end up with a time variant control system. The IDA-PBC is used for the design of the stabilizing controller, but due to the time dependency of the Hamiltonian, the stability analysis is more involved. The approach based on PH-systems provides an systematic way to derive a tracking controller, but one has to overcome the lack of the time dependency in the general case.

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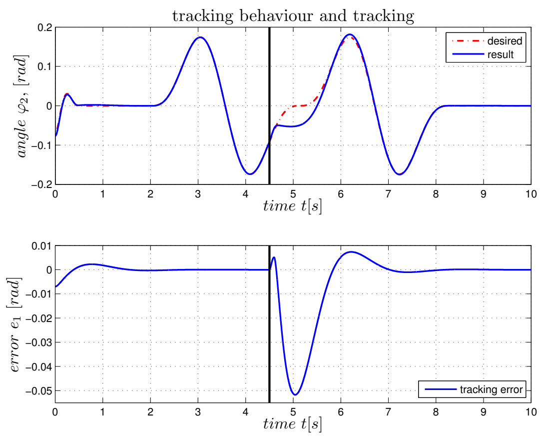

Fig. 3. Output tracking using feedforward and feedback control and a velocity observer